10.3

1. Find the values of $\sin 2\alpha$, $\cos 2\alpha$, and $\tan 2\alpha$, when:

(i) $\sin\alpha = \frac{3}{5}$

$$\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$$

$$\cos\alpha = \sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$= \sqrt{1-\frac{9}{25}}$$

$$= \sqrt{\frac{25-9}{25}}$$

$$= \sqrt{\frac{16}{25}}$$

$$\cos\alpha = \frac{4}{5}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$$

$$\sin 2\alpha = \frac{24}{25}$$

$$\cos 2\alpha = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \frac{16-9}{25}$$

$$\cos 2\alpha = \frac{7}{25}$$

$$\tan 2\alpha = \frac{\sin 2a}{\cos 2\alpha}$$

$$= \frac{\cancel{7/25}}{\cancel{4/25}}$$

$$\tan 2\alpha = \frac{7}{4}$$

(ii) $\cos\alpha = \frac{4}{5}$, where $0 < \alpha < \frac{\alpha}{2}$

$$\sin\alpha = \pm\sqrt{1-\cos^2\alpha}$$

As α in I quadrant

$$\sin a = \sqrt{1-(4/5)^2}$$

$$= \sqrt{1-\frac{16}{25}}$$

$$= \sqrt{\frac{25-16}{25}}$$

$$= \sqrt{\frac{9}{25}}$$

$$\sin\alpha = \frac{3}{5}$$

$$\sin 2\alpha = 2\sin\alpha\cos a$$

$$= 2(3/5)(4/5)$$

$$\sin 2\alpha = 24/25$$

CamScanner

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$= \left(\tfrac{4}{5}\right)^2 - \left(\tfrac{3}{5}\right)^2$$
$$= \tfrac{16}{25} - \tfrac{9}{25}$$
$$= \frac{16-9}{25}$$

$$\cos 2\alpha = \frac{7}{25}$$

$$\tan 2a = \frac{\sin 2a}{\cos 2\alpha}$$
$$= \frac{24/25}{7/25}$$
$$\tan 2\alpha = \frac{24}{7}$$

2. Prove the following identities:

(i) $\cot\alpha - \tan\alpha = 2\cot 2a$

L.H.S

$= \cot\alpha - \tan\alpha$

$= \dfrac{\cos\alpha}{\sin\alpha} - \dfrac{\sin a}{\cos a}$

$= \dfrac{\cos^2 a - \sin^2\alpha}{\cos\alpha \sin\alpha}$

$= \dfrac{\cos 2\alpha}{\cos\alpha \sin\alpha}$

$$= \frac{2\cos 2\alpha}{2\sin\alpha\cos\alpha}$$

$$= \frac{2\cos 2\alpha}{\sin 2\alpha}$$

$= 2\cot 2\alpha \Rightarrow$ R.H.S

Proved

(ii) $\frac{\sin 2\alpha}{1+\cos 2\alpha} = \tan\alpha$

L.H.S

$$= \frac{\sin 2\alpha}{1+\cos 2\alpha}$$

$$= \frac{2\sin\alpha\cos\alpha}{\cancel{1}+2\cos^2\alpha-\cancel{1}}$$

$$= \frac{\cancel{2}\sin\alpha\cos\alpha}{\cancel{2}\cos^2\alpha}$$

$$= \frac{\sin\alpha}{\cos\alpha}$$

$= \tan\alpha \Rightarrow$ R.H.S

Proved

(iii) $\frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$

L.H.S

$= \dfrac{1-\cos\alpha}{\sin\alpha}$

$= \dfrac{1-(\cos^2\alpha/2 - \sin^2\alpha/2)}{2\sin\alpha/2 \cos\alpha/2}$

$= \dfrac{1-\cos^2\alpha/2 + \sin^2\alpha/2}{2\sin\alpha/2 \cos\alpha/2}$

$= \dfrac{\sin^2\alpha/2 + \sin^2\alpha/2}{2\sin\alpha/2 \cos\alpha/2}$

$= \dfrac{\cancel{2}\sin^2\alpha/2}{\cancel{2}\sin\alpha/2 \cos\alpha/2}$

$= \dfrac{\sin\alpha/2}{\cos\alpha/2}$

$= \tan\alpha/2 \Rightarrow$ R.H.S

Proved

(v) $\dfrac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha} = \sec 2\alpha - \tan 2\alpha$

R.H.S

$= \sec 2\alpha - \tan 2\alpha$

$= \dfrac{1}{\cos 2\alpha} - \dfrac{\sin 2\alpha}{\cos 2\alpha}$

$= \dfrac{1-\sin 2\alpha}{\cos 2\alpha}$

$$= \frac{1 - 2\sin\alpha\cos\alpha}{\cos^2\alpha - \sin^2\alpha}$$

$$= \frac{\cos^2\alpha + \sin^2\alpha - 2\sin\alpha\cos\alpha}{\cos^2\alpha - \sin^2\alpha}$$

$$\cancel{= \frac{(\cos\alpha - \sin\alpha)^2}{(\cos\alpha - \sin\alpha)(\cos\alpha + \sin\alpha)}}$$

$$\cancel{= \frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha}}$$

$$= \frac{1 - \sin 2\alpha}{\cos 2\alpha}$$

$$= \frac{1}{\cos 2\alpha} - \frac{\sin 2\alpha}{\cos 2\alpha}$$

$$= \sec 2\alpha - \tan 2\alpha \Rightarrow \text{R.H.S}$$

Proved

(v) $\sqrt{\dfrac{1+\sin\alpha}{1-\sin\alpha}} = \dfrac{\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}}$

L.H.S

$$= \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}$$

$$= \sqrt{\frac{1 + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{1 - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}}$$

$$= \sqrt{\frac{\sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\sin^2\alpha/2 + \cos^2\alpha/2 - 2\sin\alpha/2\cos\alpha/2}}$$

$$= \sqrt{\frac{(\sin\alpha/2 + \cos\alpha/2)^2}{(\sin\alpha/2 - \cos\alpha/2)^2}}$$

$$= \frac{\sin\alpha/2 + \cos\alpha/2}{\sin\alpha/2 - \cos\alpha/2} \Rightarrow R.H.S$$

Proved

ii) $\frac{\operatorname{Cosec}\theta + 2\operatorname{Cosec}2\theta}{\operatorname{Sec}\theta} = \cot\frac{\theta}{2}$

L.H.S

$$= \frac{\operatorname{Cosec}\theta + 2\operatorname{cosec}2\theta}{\operatorname{Sec}\theta}$$

$$= \frac{\frac{1}{\sin\theta} + \frac{2}{\sin 2\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\frac{1}{\sin\theta} + \frac{\cancel{2}}{\cancel{2}\sin\theta\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\frac{1}{\sin\theta} + \frac{1}{\sin\theta\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\frac{\cos\theta + 1}{\sin\theta\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \cancel{\cos\theta}\ \frac{1+\cos\theta}{\sin\theta}$$

$$= \cancel{\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}}\ \frac{1+\cos^2\theta/2 - \sin^2\theta/2}{2\sin\theta/2\cos\theta/2}$$

$$= \frac{\cancel{1}+\cos^2\theta/2 - \cancel{1} + \cos^2\theta/2}{2\sin\theta/2\cos\theta/2}$$

$$= \frac{\cancel{2}\cos^{\cancel{2}}\theta/2}{\cancel{2}\sin\theta/2\cancel{\cos\theta/2}}$$

$$= \frac{\cos\theta/2}{\sin\theta/2}$$

$$= \cot\theta/2 \Rightarrow R.H.S$$

Proved

(vii) $1+\tan\alpha\ \tan 2\alpha = \sec 2\alpha$

L.H.S

$= 1+\tan\alpha\ \tan 2\alpha$

$$= 1+\frac{\sin\alpha}{\cos\alpha} \times \frac{\sin 2\alpha}{\cos 2\alpha}$$

$$= \frac{\cos\alpha\cos 2\alpha + \sin\alpha\sin 2\alpha}{\cos\alpha\cos 2\alpha}$$

$$= \frac{\cos(\alpha - 2\alpha)}{\cos\alpha \cos 2\alpha}$$

$$= \frac{\cos(-\alpha)}{\cos\alpha \cos 2\alpha}$$

$$= \frac{\cos\alpha}{\cancel{\cos\alpha} \cos 2\alpha}$$

$$= \frac{1}{\cos 2\alpha}$$

$$= \sec 2\alpha \Rightarrow R.H.S$$

Proved

(viii) $\frac{2\sin\theta \sin 2\theta}{\cos\theta + \cos 3\theta} = \tan 2\theta \tan\theta$

L.H.S

$$= \frac{2\sin\theta \sin 2\theta}{\cos\theta + \cos 3\theta}$$

$$= \frac{2\sin\theta . \cancel{2}\sin^2\theta \cancel{\cos\theta}}{\cos\theta + 4\cos^3\theta - 3\cos\theta}$$

$$= \frac{\cancel{4\sin^2\theta \cos\theta}\ 2\sin\theta \sin 2\theta}{4\cos^3\theta - 2\cos\theta}$$

$$= \frac{\cancel{4\sin^2\theta \cos\theta}\ 2\sin\theta \sin 2\theta}{2\cos\theta(2\cos^2\theta - 1)}$$

$= \frac{\cancel{4\sin\theta\cos\theta}\ \cancel{2}\sin\theta \cdot \sin 2\theta}{\cancel{2}\cos\theta\cos 2\theta}$

$= \tan\theta \cdot \tan 2\theta$

$= \tan 2\theta \tan\theta \Rightarrow R.H.S$

Proved

(ix) $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$

L.H.S

$= \frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta}$

$= \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$

$= \frac{\sin(3\theta - \theta)}{\sin\theta\cos\theta}$

$= \frac{\sin 2\theta}{\sin\theta\cos\theta}$

$= \frac{2(\sin 2\theta)}{2\sin\theta\cos\theta}$

$= \frac{2\ \cancel{\sin 2\theta}}{\cancel{\sin 2\theta}}$

$= 2 \Rightarrow R.H.S$

Proved

(x) $\dfrac{\cos 3\theta}{\cos\theta} + \dfrac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta$

L.H.S

$= \dfrac{\cos 3\theta}{\cos\theta} + \dfrac{\sin 3\theta}{\sin\theta}$

$= \dfrac{\sin\theta\cos 3\theta + \sin 3\theta\cos\theta}{\cos\theta\sin\theta}$

$= \dfrac{\sin(3\theta+\theta)}{\sin\theta\cos\theta}$

$= \dfrac{\sin 4\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin 2(2\theta)}{\sin\theta\cos\theta}$

$= \dfrac{2\sin 2\theta\cos 2\theta}{\sin\theta\cos\theta}$

$= \dfrac{2\times 2\cancel{\sin\theta}\cancel{\cos\theta}\cdot\cos 2\theta}{\cancel{\sin\theta}\cancel{\cos\theta}}$

$= 4\cos 2\theta \Rightarrow$ R.H.S

Proved.

(xi) $\dfrac{\tan\frac{\theta}{2} + \cot\frac{\theta}{2}}{\cot\frac{\theta}{2} - \tan\frac{\theta}{2}} = \sec\theta$

L.H.S

$$= \dfrac{\dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} + \dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}}{\dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} - \dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}$$

$$= \dfrac{\dfrac{\sin^2\theta/2 + \cos^2\theta/2}{\cancel{\sin\theta/2 \cos\theta/2}}}{\dfrac{\cos^2\theta/2 - \sin^2\theta/2}{\cancel{\sin\theta/2 \cos\theta/2}}}$$

$$= \dfrac{\sin^2\theta/2 + \cos^2\theta/2}{\cos^2\theta/2 - \sin^2\theta/2}$$

$$= \dfrac{1}{\cos\theta}$$

$= \sec\theta \Rightarrow$ R.H.S

Proved

(xii) $\dfrac{\sin 3\theta}{\cos\theta} + \dfrac{\cos 3\theta}{\sin\theta} = 2\cot 2\theta$

L.H.S

$$= \dfrac{\sin 3\theta}{\cos\theta} + \dfrac{\cos 3\theta}{\sin\theta}$$

$= \dfrac{\sin 3\theta \sin\theta + \cos 3\theta \cos\theta}{\sin\theta \cos\theta}$

$= \dfrac{\cos(3\theta - \theta)}{\sin\theta \cos\theta}$

$= \dfrac{\cos 2\theta}{\frac{\sin 2\theta}{2}}$

$= \dfrac{2\cos 2\theta}{\sin 2\theta}$

$= 2\cot 2\theta \Rightarrow$ R.H.S

Proved

(a) $\dfrac{3+\cos 4\theta}{1-\cos 4\theta} = \dfrac{1}{2}(\tan^2\theta + \cot^2\theta)$

R.H.S

$= \dfrac{1}{2}\left(\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta}\right)$

$= \dfrac{1}{2}\left(\dfrac{\sin^4\theta + \cos^4\theta}{\cos^2\theta \sin^2\theta}\right)$

$= \dfrac{1}{2}\left[\dfrac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{\left(\frac{1+\cos 2\theta}{2}\right)\left(\frac{1-\cos 2\theta}{2}\right)}\right]$

$= \dfrac{1}{2}\left[\dfrac{\left(\frac{1-\cos 2\theta}{2}\right)^2 \left(\frac{1+\cos 2\theta}{2}\right)^2}{\frac{1-\cos^2 2\theta}{4}}\right]$

$$= \frac{1}{\cancel{2}} \times \cancel{4}^{2} \left[\frac{\frac{1+\cos^2 2\theta - 2\cos 2\theta}{4} + \frac{1+\cos^2 2\theta + 2\cos 2\theta}{4}}{1-\cos^2 2\theta} \right]$$

$$= 2 \left[\frac{\frac{1+\cos^2 2\theta - \cancel{2\cos 2\theta} + 1 + \cos^2 2\theta + \cancel{2\cos 2\theta}}{4}}{1 - \frac{1+\cos 4\theta}{2}} \right]$$

$$= 2 \left[\frac{\frac{2+2\cos^2 2\theta}{4}}{\frac{2-1-\cos 4\theta}{2}} \right]$$

$$= 2 \left[\frac{\frac{\cancel{2}(1+\cos^2 2\theta)}{4\cancel{2}}}{\frac{1-\cos 4\theta}{\cancel{2}}} \right]$$

$$= 2 \left[\frac{1+\cos^2 2\theta}{1-\cos 4\theta} \right]$$

$$= 2 \left[\frac{1 + \frac{1+\cos 4\theta}{2}}{1-\cos 4\theta} \right]$$

$$= \cancel{2} \left[\frac{\frac{2+1+\cos 4\theta}{\cancel{2}}}{1-\cos 4\theta} \right]$$

$$= \frac{3+\cos 4\theta}{1-\cos 4\theta}$$

(xiv) $\dfrac{1+\sin 2\theta}{1-\sin 2\theta} = \tan^2\left(\dfrac{\pi}{4}+\theta\right)$

R.H.S

$= \tan^2\left(\frac{\pi}{4}+\theta\right)$

$= \dfrac{\sin^2\left(\frac{\pi}{4}+\theta\right)}{\cos^2\left(\frac{\pi}{4}+\theta\right)}$

$= \dfrac{\left(\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}\sin\theta\right)^2}{\left(\cos\frac{\pi}{4}\cos\theta-\sin\frac{\pi}{4}\sin\theta\right)^2}$

$= \dfrac{\left(\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right)^2}{\left(\frac{1}{\sqrt{2}}\cos\theta-\frac{1}{\sqrt{2}}\sin\theta\right)^2}$

$= \dfrac{\cancel{\left(\frac{1}{\sqrt{2}}\right)^2}(\cos\theta+\sin\theta)^2}{\cancel{\left(\frac{1}{\sqrt{2}}\right)^2}(\cos\theta-\sin\theta)^2}$

$= \dfrac{\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta}$

$= \dfrac{1+\sin 2\theta}{1-\sin 2\theta} \Rightarrow$ L.H.S

Proved

(xv) $\cos^2\dfrac{\pi}{8}+\cos^2\dfrac{3\pi}{8}+\cos^2\dfrac{5\pi}{8}+\cos^2\dfrac{7\pi}{8}=2$

L.H.S

$= \cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \left(\cos\left(\pi - \frac{3\pi}{8}\right)\right)^2 + \left(\cos\left(\pi - \frac{\pi}{8}\right)\right)^2$

$= \cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \left(\cos\left(2\left(\frac{\pi}{2}\right) - \frac{3\pi}{8}\right)\right)^2 + \left(\cos\left(2\left(\frac{\pi}{2}\right) - \frac{\pi}{8}\right)\right)^2$

$= \cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \left(-\cos\frac{3\pi}{8}\right)^2 + \left(-\cos\frac{\pi}{8}\right)^2$

$= \cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{\pi}{8}$

$= \cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + 1 - \sin^2\frac{3\pi}{8} + 1 - \sin^2\frac{\pi}{8}$

$= 2 + \left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right) + \left(\cos^2\frac{3\pi}{8} - \sin^2\frac{3\pi}{8}\right)$

$= 2 + \cos 2\left(\frac{\pi}{8}\right) + \cos\left(2\left(\frac{3\pi}{8}\right)\right)$

$= 2 + \cos\frac{\pi}{4} + \cos\frac{3\pi}{4}$

$= 2 + \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} - \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$= 2$

Proved L.H.S = R.H.S

3. Show that: $2\cos\theta = \sqrt{2+\sqrt{2+2\cos 4\theta}}$

R.H.S

$= \sqrt{2+\sqrt{2+2\cos 4\theta}}$

$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$

$= \sqrt{2+\sqrt{2(2\cos^2 2\theta)}}$

$= \sqrt{2+\sqrt{4\cos^2\theta}}$

$= \sqrt{2+2\cos 2\theta}$

$= \sqrt{2(1+\cos 2\theta)}$

$= \sqrt{2(2\cos^2\theta)}$

$= \sqrt{4\cos^2\theta}$

$= 2\cos\theta$

Proved L.H.S = R.H.S

4. Reduce $\sin^4\theta$ to an expression involving only functions of multiples of θ, raised to the first power.

$= \sin^4\theta$

$= (\sin^2\theta)^2$

$= \left(\dfrac{1-\cos 2\theta}{2}\right)^2$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$= \dfrac{1+\cos^2 2\theta - 2\cos 2\theta}{4}$

$= \dfrac{1}{4}\left[1 + \dfrac{1+\cos 4\theta}{2} - 2\cos 2\theta\right]$

$= \dfrac{1}{4}\left[\dfrac{2+1+\cos 4\theta - 4\cos 2\theta}{2}\right]$

CamScanner

$$= \frac{3 + \cos 4\theta - 4\cos 2\theta}{8}$$

5. Find the value of $\sin\theta$ and $\cos\theta$ without using table or calculator when θ is:

(i) 18° (ii) 36° (iii) 54° (iv) 72°

Hence prove that: $\cos 36^\circ \cos 72^\circ \cos 108^\circ \cos 144^\circ = \frac{1}{16}$

(i) 18°

Let $\theta = 18^\circ$

$5\theta = 5 \times 18$

$3\theta + 2\theta = 90^\circ$

$3\theta = 90 - 2\theta$

$\sin 3\theta = \sin(90 - 2\theta)$

$\sin 3\theta = \cos 2\theta$

$3\sin\theta - 4\sin^3\theta = \cos 2\theta$

$3\sin\theta - 4\sin^3\theta = 1 - 2\sin^2\theta$

$0 = 4\sin^3\theta - 2\sin^2\theta - 3\sin\theta + 1$

$\sin\theta = 1$	4	-2	-3	$\cancel{1}$
$\sin\theta - 1 = 0$		4	2	$\cancel{1}$
	4	2	-1	0

$(\sin\theta - 1)(4\sin^2\theta + 2\sin\theta - 1) = 0$

$\sin\theta - 1 = 0$

$\sin\theta = 1$

$\theta = 90^\circ$

no relevent

$4\sin^2\theta + 2\sin^2\theta - 1 = 0$

$$\sin\theta = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-2 \pm \sqrt{4+16}}{8}$$

$$= \frac{-2 \pm \sqrt{20}}{8}$$

$$= \frac{-2 \pm 2\sqrt{5}}{8}$$

$$= \frac{\cancel{2}(-1 \pm \sqrt{5})}{\cancel{8}\,4}$$

$$\sin\theta = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin\theta = \frac{-1 - \sqrt{5}}{4}$$

As θ is 18° so

$\sin\theta > 0$

$$\sin 18^\circ = \frac{-1+\sqrt{5}}{4}$$

$$\cos 18^\circ = \sqrt{1 - \sin^2 18}$$

$$= \sqrt{1 - \left(\frac{-1+\sqrt{5}}{4}\right)^2}$$

$$= \sqrt{1-\left(\frac{(-1)^2+(\sqrt{5})^2+2(-1)(\sqrt{5})}{16}\right)}$$

$$= \sqrt{1-\left(\frac{1+5-2\sqrt{5}}{16}\right)}$$

$$= \sqrt{1-\left(\frac{6-2\sqrt{5}}{16}\right)}$$

$$= \sqrt{\frac{16-6+2\sqrt{5}}{16}}$$

$$= \sqrt{\frac{10+2\sqrt{5}}{16}}$$

$$\cancel{= \sqrt{\frac{2(5-\sqrt{5})}{16}} = \sqrt{\frac{5-\sqrt{5}}{8}}}$$

$$= \frac{\sqrt{10+2\sqrt{5}}}{4}$$

(ii) $36°$

$$\sin 36 = 2 \sin 18 \cos 18$$

$$= 2\left(\frac{-1+\sqrt{5}}{4}\right)\left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)$$

$$= \frac{2}{16}\left(\sqrt{(-1+\sqrt{5})^2(2)(5+\sqrt{5})}\right.$$

$$= \frac{1}{8}\left(\sqrt{2(1+5-2\sqrt{5})(5+\sqrt{5})}\right)$$

$$= \frac{1}{8}\left(\sqrt{2(6-2\sqrt{5})(5+\sqrt{5})}\right)$$

$$= \frac{1}{8}\left(\sqrt{2(2(3-\sqrt{5})(5+\sqrt{5})}\right)$$

$$= \frac{1}{8}\left(\sqrt{4(3-\sqrt{5})(5+\sqrt{5})}\right)$$

$$= \frac{1}{\cancel{8}\,4} \times \cancel{2}\left(\sqrt{15+3\sqrt{5}-5\sqrt{5}-5}\right)$$

$$= \frac{1}{4}\left(\sqrt{10-2\sqrt{5}}\right)$$

$$\sin 36 = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\cos 36 = \sqrt{1-\sin^2 36}$$

$$= \sqrt{1-\left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)^2}$$

$$= \sqrt{1-\frac{(10-2\sqrt{5})}{16}}$$

$$= \sqrt{1-\frac{10+2\sqrt{5}}{16}}$$

$$= \sqrt{\frac{16-10+2\sqrt{5}}{16}}$$

$$= \sqrt{\frac{6+2\sqrt{5}}{16}}$$

$$= \frac{\sqrt{1+5+2\sqrt{5}}}{4}$$

$$= \frac{\sqrt{(1)^2+(\sqrt{5})^2+2\sqrt{5}}}{4}$$

$$= \frac{\sqrt{(1+\sqrt{5})^2}}{4}$$

$$\cos 36^\circ = \frac{\sqrt{5}+1}{4}$$

$$\sin 54 = \sin(90-36)$$

$$\sin 54 = \cos 36$$

$$\sin 54 = \frac{\sqrt{5}+1}{4}$$

$$\cos 54 = \cos(90-36)$$

$$\cos 54 = \sin 36$$

$$\cos 54 = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\sin 72 = \sin(90-18)$$

$$\sin 72 = \cos 18$$

$$\sin 72 = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\cos 72 = \cos(90-18)$$

$$\cos 72 = \sin 18$$

$$\cos 72 = \frac{-1+\sqrt{5}}{4} = \frac{\sqrt{5}-1}{4}$$

$$\cos 36 \cos 72 \cos 108 \cos 144 = \frac{1}{16}$$

$$\cos 36 \cos\overset{72}{(90-18)} \cos(90+18) \cos(180-36)$$

$$\cos 36 \cdot \cos 72 \cdot -\sin 18 \cdot -\cos 36$$

$$= \frac{\sqrt{5}+1}{4} \cdot \frac{\sqrt{5}-1}{4} \cdot -\left(\frac{\sqrt{5}-1}{4}\right) \cdot -\frac{\sqrt{5}+1}{4}$$

$$= \frac{(\sqrt{5})^2-(1)^2}{16} \cdot \frac{(\sqrt{5})^2-(1)^2}{16}$$

$$= \frac{5-1}{16} \cdot \frac{5-1}{16}$$

$$= \frac{4}{16} \cdot \frac{4}{16}$$

$$= \frac{\cancel{16}}{16 \times \cancel{16}} = \frac{1}{16} \quad \text{Proved}$$

HASSAN MEHBOOB

S.S.S(Math)