10.1

Q1 Without using the tables, find the values of:

(i) $\cos(-1230^\circ)$

- $\cos(1230^\circ)$
- $\cos(1080^\circ + 150^\circ)$
- $\cos(3(360) + 150^\circ)$
- $\cos 150$
- $-\dfrac{\sqrt{3}}{2}$

(ii) $\tan(-1035^\circ)$

- $-(\tan(1035^\circ))$
- $-(\tan(1080^\circ - 45^\circ))$
- $-(\tan(3(360) - 45^\circ)$
- $-(\tan(-45^\circ)$
- $-(-\tan 45)$

1

(iii) $\sec(1140^\circ)$

- $\sec(1080 + 60)$
- $\sec(3(360) + 60)$
- $\sec 60$
- 2

(iv) $\text{Cosec}(-690^\circ)$

$-(\text{cosec}(690))$

$-(\text{cosec}(720-30))$

$-(\text{cosec}(2(360)-30)$

$-(\text{cosec}(-30))$

$-(-\text{cosec}(30))$

2

(v) $\cot(1320^\circ)$

$\cot(1440^\circ-120^\circ)$

$\cot(4(360)-120^\circ)$

$\cot(-120)$

$\frac{1}{\sqrt{3}}$

(vi) $\cos(-240^\circ)$

$\cos(240^\circ)$

$\cos(360-120)$

$\cos(1(360)-120)$

$\cos(-120^\circ)$

$\cos 120$

$-\frac{1}{2}$

2. Express each of the following as a trignometric function of an angle of positive measure of less than 45°.

(i) $\cos 168^\circ$

$\cos(180^\circ - 12^\circ)$

$\cos(2(90^\circ) - 12^\circ)$

$-\cos 12$

(ii) $\sin 192^\circ$

$\sin(180^\circ + 12^\circ)$

$\sin(2(90) + 12^\circ)$

$-\sin 12$

(iii) $\cos 333^\circ$

$\cos(360^\circ - 27^\circ)$

$\cos(4(90^\circ) - 27^\circ)$

$\cos 27^\circ$

(iv) $\tan 213^\circ$

$\tan(180^\circ + 33^\circ)$

$\tan(2(90^\circ) + 33^\circ)$

$\tan 33^\circ$

(v) $\cos(-435^\circ)$

$\cos(435^\circ)$

$\cos(450^\circ - 15^\circ)$

$\cos(5(90^\circ) - 15^\circ)$

$\sin 15^\circ$

(vi) $\sin 219^\circ$

$\sin(180^\circ + 39^\circ)$

$\sin(2(90) + 39^\circ)$

$-\sin 39^\circ$

(vii) $\tan(-597^\circ)$

$\tan(-630^\circ + 33^\circ)$

$\tan(-7(90) + 33)$

$\cancel{\tan 33}$ $-\cot 33^\circ$

(viii) $\cos(-111^\circ)$

$\cos(111)$

$\cos(90^\circ + 21^\circ)$

$\cos(1(90^\circ) + 21^\circ)$

$-\sin 21^\circ$

(ix) $\sin(-390^\circ)$

$\sin(-360^\circ - 30^\circ)$

$\sin(-4(90) - 30^\circ)$

$-\sin 30^\circ$

3. Prove the following:

(i) $\sin(180^\circ + \alpha)\sin(90^\circ - \alpha) = -\sin\alpha\cos\alpha$

$\sin(2(90) + \alpha)\sin(1(90) - \alpha)$

$-\sin\alpha\cos\alpha$

It is proved

(ii) $\sin 810^\circ \sin 630^\circ + \cos 135^\circ \sin 225^\circ = -\frac{1}{2}$

L.H.S

$\sin(8(90^\circ) + 90^\circ)\sin(6(90^\circ) + 90) + \cos(1(90^\circ) + 45^\circ)\sin(2(90) + 45^\circ)$

$(\sin 90^\circ)(-\sin 90^\circ) + (-\sin 45^\circ)(-\sin 45^\circ)$

$= (1)(-1) + \left(\frac{-1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right)$

$= -1 + \frac{1}{2}$

$= \frac{-2+1}{2}$

$= -\frac{1}{2}$

$R.H.S = -\frac{1}{2}$

It is proved.

(iii) $\tan 150^\circ \cot 330^\circ - 2 \sec 135^\circ \operatorname{cosec} = -3$

$\tan(90^\circ + 60^\circ) \cot(4(90) - 30) - 2 \sec(90 + 45)$
$\operatorname{cosec}(3(90) - 45^\circ)$

$(-\cot 60^\circ)(-\cot 30^\circ) - 2(-\operatorname{cosec} 45^\circ)$
$(-\sec 45^\circ)$

$= \frac{1}{\sqrt{3}} \times \sqrt{3} - 2(\sqrt{2})(\sqrt{2})$

$= 1 - 4$

$= -3$

$R.H.S = -3$

It is proved.

(iv) $\sin 210^\circ + \cos 240^\circ + \tan 225^\circ + \cot 225^\circ = 1$

$\sin(2(90) + 30^\circ) + \cos(2(90) + 60) + \tan(2(90) + 45)$
$+ \cot(2(90) + 45^\circ)$

$(-\sin 30) + (-\cos 60^\circ) + \tan 45^\circ + \cot 45^\circ$

$-\sin 30^\circ - \cos 60^\circ + \tan 45^\circ + \cot 45^\circ$

$-\frac{1}{2} - \frac{1}{2} + 1 + 1$

$$= \frac{-1-1+4}{2}$$

$$= \frac{\cancel{2}}{\cancel{2}}$$

$$= 1$$

R.H.S $= 1$

It is proved.

4. Prove that.

(i) $$\frac{\tan(180^\circ+\alpha)\cot(90^\circ-\alpha)}{\sin(360^\circ-\alpha)\cos(270^\circ+\alpha)} = -\sec^2\alpha$$

L.H.S

$$= \frac{\tan(2(90^\circ)+\alpha)\cot(1(90^\circ-\alpha)}{\sin(4(90^\circ)-\alpha)\cos(3(90^\circ)+\alpha)}$$

$$= \frac{(\tan\alpha)(\tan\alpha)}{(-\sin\alpha)(\sin\alpha)}$$

$$= -\frac{\tan^2\alpha}{\sin^2\alpha}$$

$$= -\frac{\frac{\sin^2\alpha}{\cos^2\alpha}}{\sin^2\alpha}$$

$$= \frac{-\cancel{\sin^2}\alpha}{\cos^2\alpha\ \cancel{\sin^2}\alpha}$$

$$= -\left(\frac{1}{\cos^2\alpha}\right)^2$$

$$= -\sec^2\alpha = R.H.S$$

It is proved.

(ii) $\dfrac{\sin^2(\pi+\theta)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\operatorname{cosec}(2\pi-\theta)} = \cos\theta$

L.H.S

$$= \frac{\left[\sin\left(2\left(\frac{\pi}{2}+\theta\right)\right)\right]^2(-\cot\theta)}{\left[\cot\left(3\left(\pi/2-\theta\right)\right)\right]^2\left[\cos\left(2\left(\pi/2-\theta\right)\right)\right]^2\left[\operatorname{cosec}\left(4\left(\pi/2-\theta\right)\right)\right]}$$

$$\frac{(-\sin\theta)^2(-\cot\theta)}{(\tan\theta)^2(-\cos\theta)^2(-\operatorname{cosec}\theta)}$$

$$\frac{\sin^2\theta\cot\theta}{\tan^2\theta\,.\cos\theta^2\,.\operatorname{cosec}\theta}$$

$$\frac{\sin^2\theta \times \frac{\cos\theta}{\cancel{\sin\theta}}}{\frac{\sin^2\theta}{\cancel{\cos^2\theta}}\,.\cancel{\cos^2\theta} \times \frac{1}{\cancel{\sin\theta}}}$$

$$\frac{\cancel{\sin\theta}\cos\theta}{\cancel{\sin\theta}}$$

$$= \cos\theta = R.H.S$$

It is proved

(iii) $\dfrac{\cos(90^\circ+\theta)\sec(-\theta)\tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\sin(180^\circ+\theta)\cot(90^\circ-\theta)} = -1$

L.H.S

$\dfrac{\cos(1(90^\circ)+\theta)\ \sec\theta\ \tan(2(90^\circ)-\theta)}{\sec(4(90^\circ)-\theta)\sin(2(90^\circ)+\theta)\cot(1(90^\circ)-\theta)}$

$\dfrac{(-\sin\theta)(\sec\theta)(-\tan\theta)}{(\sec\theta)(-\sin\theta)(\tan\theta)}$

$\dfrac{\cancel{\sin\theta}\ \cancel{\sec\theta}\ \cancel{\tan\theta}}{-\cancel{\sec\theta}\ \cancel{\sin\theta}\ \cancel{\tan\theta}}$

$= -1 = R.H.S$

It is proved.

5. Show that: $\sec\left(\frac{3\pi}{2}-\theta\right)\sec\left(\frac{5\pi}{2}-\theta\right) - \tan\left(\frac{3\pi}{2}-\theta\right)\tan\left(\frac{5\pi}{2}+\theta\right) = -1$

$(-\operatorname{cosec}\theta)(\operatorname{cosec}\theta) - (\cot\theta)(-\cot\theta)$

$-\operatorname{cosec}^2\theta + \cot^2\theta$

$\dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{1}{\sin^2\theta}$

$\dfrac{\cos^2\theta - 1}{\sin^2\theta}$

$\frac{\cancel{1} - \sin^2\theta - \cancel{1}}{\sin^2\theta}$

$-\frac{\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}} = -1 = R.H.S$

It is proved.

6. If α, β, γ are the angles of a triangle ABC, then prove that

(i) $\sin(\alpha+\beta) = \sin\gamma$

$\alpha+\beta+\gamma = 180°$

$\alpha+\beta = 180° - \gamma$

$\sin(180° - \gamma)$

$\sin(2(90°) - \gamma)$

$\sin\gamma \Rightarrow R.H.S$

Proved.

(ii) $\sec\left(\frac{\alpha+\beta}{2}\right) = \csc\frac{\gamma}{2}$

$\alpha+\beta+\gamma = 180°$

$\alpha+\beta = 180° - \gamma$

L.H.S =

$\sec\left(\frac{180-\gamma}{2}\right)$

$\sec(90° - \frac{\gamma}{2})$

$\cos\text{ec} \frac{\gamma}{2} \Rightarrow$ R.H.S

Proved

(iii) $\text{Cosec}\,\alpha = \frac{1}{\sin(\beta+\gamma)}$

$\alpha + \beta + \gamma = 180°$

$\beta + \gamma = 180° - \alpha$

$\sin(\beta+\gamma) = \sin(180° - \alpha)$

$= \sin(2(90) - \alpha)$

$\frac{\sin(\beta+\gamma)}{1} = \frac{\sin\alpha}{1}$

$\frac{1}{\sin(\beta+\gamma)} = \frac{1}{\sin\alpha}$

$\frac{1}{\sin(\beta+\gamma)} = \text{Cosec}\,\alpha$

or

$\text{Cosec}\,\alpha = \frac{1}{\sin(\beta+\gamma)}$

(iv) $\tan(\alpha+\beta) + \tan\gamma = 0$

$\alpha + \beta + \gamma = 180°$

$\alpha + \beta = 180 - \gamma$

$\tan(180 - \gamma) + \tan\gamma = 0$

$\tan(2(90) - \gamma) + \tan\gamma = 0$

$-\tan\gamma + \tan\gamma = 0$

$0 = 0$

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