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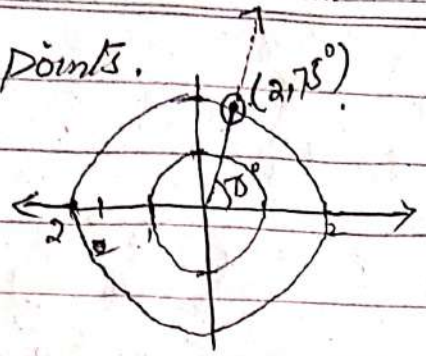
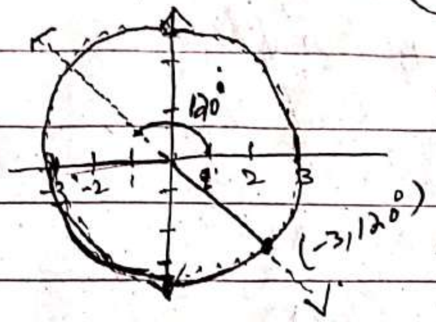
Ex: 1.5

Q.No1) Plot the following points.

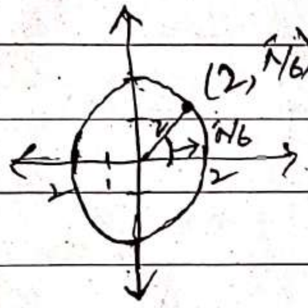
(i) $(2, 75^\circ)$



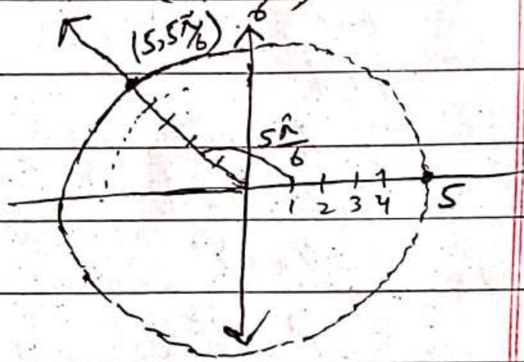
(ii) $(-3, 120^\circ)$



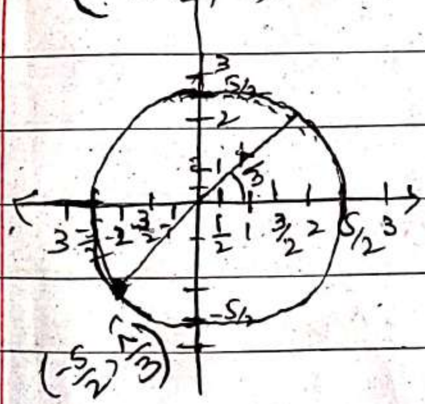
(iii) $(2, \pi/6)$



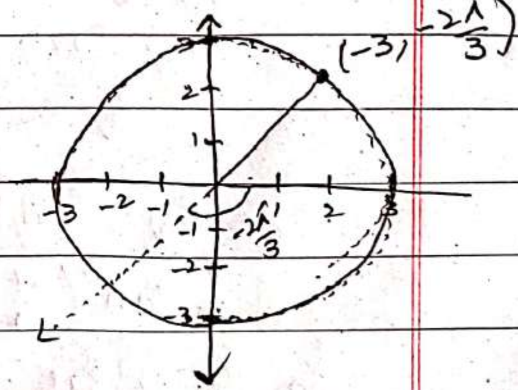
iv $(5, \frac{5\pi}{6})$



v) $(-5/2, \pi/3)$



vi) $(-3, -2\pi/3)$



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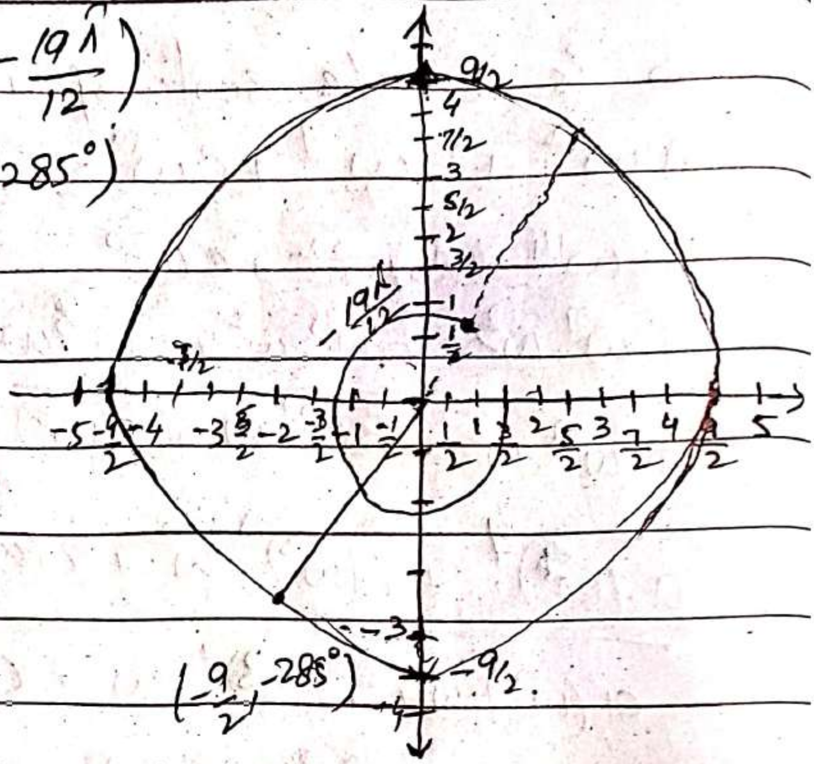
vii) $\left(-9/2, -\frac{19\pi}{12}\right)$

$(-4.5, -285°)$



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Q2 Express the following complex number in polar form.

i) $4+3i$ For $z = x+iy$

$r = \sqrt{4^2+3^2} = 5$ $r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}(3/4) = 36.87^\circ$ $\theta = \tan^{-1}(y/x)$

so polar form

$r(\cos\theta + i\sin\theta)$

$5(\cos 36.87^\circ + i\sin 36.87^\circ)$

ii) $1+i$

$r = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = 45^\circ$

so ~~pla~~ polar form is

$r(\cos\theta + i\sin\theta)$

$5(\cos 45^\circ + i\sin 45^\circ)$

(iii) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$ $\theta = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{1/2}\right)$

$= \sqrt{\frac{1}{4} + 3/4}$ $\theta = \tan^{-1}(\sqrt{3})$

$r = \sqrt{1} = 1$ $\theta = 60^\circ$

so polar form.

$r(\cos\theta + i\sin\theta)$

$1(\cos 60^\circ + i\sin 60^\circ)$

(iv) $-5/2 - \frac{5\sqrt{3}}{2}$

$r = \sqrt{(-5/2)^2 + \left(\frac{-5\sqrt{3}}{2}\right)^2}$

$= \sqrt{25/4 + \frac{25\times 3}{4}}$

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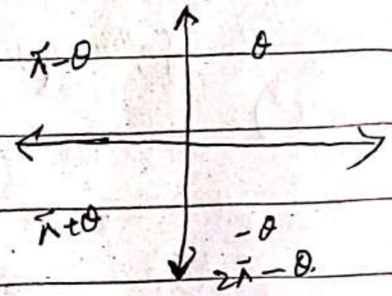
$r = \sqrt{\frac{100}{4}}$

$r = 10/2 = 5$



$\theta = \tan^{-1}\left(\frac{-5/2\sqrt{3}}{-5/2}\right)$

$\theta = \pi + \tan^{-1}\sqrt{3}$

$= \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$r(\cos\theta + i\sin\theta)$

$5\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

(v) $\frac{1-i}{1+i}$

$\frac{1-i}{1+i} \times \frac{1-i}{1-i}$

$\frac{(1-i)^2}{1^2 - i^2}$

$\frac{1+i^2-2i}{1+1}$

$= \frac{1-1-2i}{2}$

$-i$

$r = \sqrt{(0)^2 + (-1)^2} = \sqrt{1} = 1$

$\theta = \tan^{-1}\left(\frac{-1}{0}\right) = -\pi/2$

So The polar form

$r(\cos\theta + i\sin\theta)$

$1(\cos(-\pi/2) + i\sin(-\pi/2))$

vi) $\dfrac{\sqrt{3}+i}{1+\sqrt{3}i}$

$\dfrac{(\sqrt{3}+i)(1-\sqrt{3}i)}{(1+\sqrt{3}i)(1-\sqrt{3}i)}$

$\dfrac{\sqrt{3}-3i+i-\sqrt{3}i^2}{1-3i^2}$

$\dfrac{2\sqrt{3}-2i}{1+3} \Rightarrow \dfrac{2\sqrt{3}}{4}-\dfrac{2i}{4}$

$= \dfrac{\sqrt{3}}{2} - i/2.$

$r = \sqrt{\left(\sqrt{3}/2\right)^2 + (-1/2)^2} = \sqrt{\dfrac{3}{4}+\dfrac{1}{4}} = \sqrt{\dfrac{4}{4}} = 1$

$\theta = \tan^{-1}\left(\dfrac{-1/2}{\sqrt{3}/2}\right) = \tan^{-1}\left(\dfrac{-1}{\sqrt{3}}\right) = -\tan^{-1}\dfrac{1}{\sqrt{3}}$

$= -\dfrac{\pi}{6}$

So polar form is

$r(\cos\theta + i\sin\theta)$

$1.(\cos(-\pi/6) + i\sin(-\pi/6))$

vii) $\dfrac{3+4i}{4+3i}$

$\dfrac{(3+4i)(4-3i)}{(4+3i)(4-3i)} \Rightarrow \dfrac{12-9i+16i-12i^2}{16+9}$

$\dfrac{24+7i}{25} \Rightarrow 24/25 + \dfrac{7}{25}i$

$r = \sqrt{\left(\dfrac{24}{25}\right)^2 + \left(\dfrac{7}{25}\right)^2}$

$= \sqrt{\dfrac{576+49}{25^2}}$

$= \sqrt{\dfrac{625}{25^2}} = 1$

$\theta = \tan^{-1}\dfrac{7/25}{24/25}$

$\theta = \tan^{-1}\dfrac{7}{24}$

polar form

$1.(\cos(\tfrac{7}{24}) + i\sin(\tfrac{7}{24}))$

24
× 24
96
48×
576

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Q.No3. Convert each of the complex number z in the rectangular form $x+iy$.

(i) $4\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$

$4\cos\frac{5\pi}{3}+i\left(\sin\frac{5\pi}{3}\right)4$

$4\left(\frac{1}{2}\right)+4i\left(-\sqrt{3}/2\right)$

$2-2\sqrt{3}i$

(ii) $\frac{3}{2}\left(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}\right)$

$\frac{3}{2}\left(-\frac{\sqrt{3}}{2}\right)+i\,\frac{3}{2}\times\left(\frac{-1}{2}\right)$

$-\frac{3\sqrt{3}}{4}-\frac{3}{4}i$

(iii) $|z|=7$, $\arg(z)=\frac{23\pi}{12}$

$x=r\cos\theta$ $\qquad$ $y=r\sin\theta$

$=7\cos\frac{23\pi}{12}$ $\qquad$ $y=7\sin\frac{23\pi}{12}$

$=6.76$ $\qquad$ $=-1.81$

$x+iy$

$6.76-1.81i$

(iv) $|z|=11$, $\arg(z)=-\frac{11\pi}{12}$

$x=r\cos\theta$ $\qquad$ $y=r\sin\theta$

$x=11\cos\left(-\frac{11\pi}{12}\right)$ $\qquad$ $y=11\sin\left(-\frac{11\pi}{12}\right)$

$=-10.62$ $\qquad$ $=-2.85$

$x+iy$

$-10.62+i(-2.85)$

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v) $|z| = \frac{10}{3}$, $\arg(z) = -\frac{17\pi}{12}$

$x = r\cos\theta$. $y = r\sin\theta$.

$x = \frac{10}{3}\cos\left(-\frac{17\pi}{12}\right)$ $y = \frac{10}{3}\sin\left(-\frac{17\pi}{12}\right)$

$x = -0.86$ $y = -1.09$

Now $x + iy$

$-0.86 - 1.09i$

vi) $2\cos(-33) + i\,2\sin(-33)$

$1.68 + (-1.09)i$

$1.68 - 1.09i$

Q4) If $z_1 = 9\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$ and $z_2 = 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Then find.

(i) $z_1 + z_2 = 9\cos\frac{5\pi}{4} + 9i\sin\frac{5\pi}{4} + 5\cos\frac{\pi}{3} + i5\sin\frac{\pi}{3}$

$= \left(9\cos\frac{5\pi}{4} + 5\cos\frac{\pi}{3}\right) + i\left(9\sin\frac{5\pi}{4} + 5\sin\frac{\pi}{3}\right)$

$= -3.86 - 2.03i$

ii) $z_1 - z_2 = 9\cos 5\pi/4 + 9i\sin\frac{5\pi}{4} - 5\cos\frac{\pi}{3} - 5i\sin\frac{\pi}{3}$

$= \left(9\cos\frac{5\pi}{4} - 5\cos\frac{\pi}{3}\right) + i\left(9\sin\frac{5\pi}{4} - 5\sin\frac{\pi}{3}\right)$

$= -8.86 - 10.69i$

(iii) $z_1 \cdot z_2 = 9\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) \cdot 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= 45\left(\cos\left(\frac{5\pi}{4} + \frac{\pi}{3}\right) + i\sin\left(\frac{5\pi}{4} + \frac{\pi}{3}\right)\right)$

$= 45\left(\cos\left(\frac{19\pi}{12}\right) + i\sin\left(\frac{19\pi}{12}\right)\right)$

$= 11.64 - 43.47i$

(iv) $\frac{z_1}{z_2} = \frac{9\cos\frac{5\pi}{4} + 9\sin\frac{5\pi}{4}i}{5\cos\frac{\pi}{3} + 5\sin\frac{\pi}{3}i}$

$= \frac{9}{5}\left(\cos\left(\frac{5\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{5\pi}{4} - \frac{\pi}{3}\right)\right)$

$= \frac{9}{5}\left(\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11\pi}{12}\right)\right)$

Q5. If $z_1 = 7\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)$ and,

$z_2 = 11\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$

find the following and express in $x+iy$ form.

(i) $z_1 + z_2 = 7\cos\frac{23\pi}{12} + 7i\sin\frac{23\pi}{12} + 11\cos\frac{11\pi}{12}$
$+ 11i\sin\frac{11\pi}{12}$

$= \left(7\cos\frac{23\pi}{12} + 11\cos\frac{11\pi}{12}\right) + i\left(7\sin\frac{23\pi}{12} + 11\sin\frac{11\pi}{12}\right)$

$= -3.86 + 1.03i$

(ii) $z_1 - z_2 = \left(7\cos\frac{23\pi}{12} - 11\cos\frac{11\pi}{12}\right) + i\left(7\sin\frac{23\pi}{12} - 11\sin\frac{11\pi}{12}\right)$

$= 17.38 - 4.65i$

(iii) $z_1 z_2$

$= 7\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right) \cdot 11\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$

$= 77\left(\cos\left(\frac{23\pi}{12} + \frac{11\pi}{12}\right) + i\sin\left(\frac{23\pi}{12} + \frac{11\pi}{12}\right)\right)$

$= 77\left(\cos\frac{34\pi}{12} + i\sin\left(\frac{34}{12}\pi\right)\right)$

$= 77\left(\cos\frac{17\pi}{6} + i\sin\left(\frac{17\pi}{6}\right)\right)$

$-66.68 + 38.5i$

v) $\frac{z_1}{z_2} = \frac{7\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)}{11\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)}$

$= \frac{7}{11}\left(\cos\left(\frac{23\pi}{12} - \frac{11\pi}{12}\right) + i\sin\left(\frac{23\pi}{12} - \frac{11\pi}{12}\right)\right)$

$= \frac{7}{11}\left(\cos\left(\frac{23\pi - 11\pi}{12}\right) + i\sin\left(\frac{23\pi - 11\pi}{12}\right)\right)$

$= \frac{7}{11}\left(\cos\left(\frac{12\pi}{12}\right) + i\sin\left(\frac{12\pi}{12}\right)\right)$

$= \frac{7}{11}\left(\cos(\pi) + i\sin(\pi)\right)$

$= \frac{7}{11}\left(-1 + i(0)\right)$

$= -\frac{7}{11}$

Q6: If z_1 and z_2 are two complex number show that

(i) $Arg(z_1 z_2) = Arg\, z_1 + Arg\, z_2$.

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1 z_2 = r_1(\cos\theta_1 + i\sin\theta_1)\, r_2(\cos\theta_2 + i\sin\theta_2)$

$= r_1 r_2 [\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$

$+ i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)]$

$z_1 z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$

so $|z_1 z_1| = r_1 r_2$

$Arg(z_1 z_2) = \theta_1 + \theta_2$

$= Arg\, z_1 + Arg\, z_2$

(ii) $\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)}$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left(\frac{(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 - i\sin\theta_2)}{\cos^2\theta_2 + \sin^2\theta_2}\right)$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 + i(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)\right)$$

$$= \frac{r_1}{r_2}\left(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right)$$

so $\left|\frac{z_1}{z_2}\right| = \frac{r_1}{r_2}$

and $Arg\left(\frac{z_1}{z_2}\right) = \theta_1 - \theta_2$

$$\boxed{Arg\left(\frac{z_1}{z_2}\right) = Arg\, z_1 - Arg\, z_2}$$

Q7: Divide $z_1 = 6(\cos 150^\circ + i\sin 150^\circ)$

$z_2 = 3(\cos 30^\circ + i\sin 30^\circ)$

Soln: $\frac{z_1}{z_2} = \frac{\cancel{6}^{2}(\cos 150^\circ + i\sin 150^\circ)}{\cancel{3}(\cos 30^\circ + i\sin 30^\circ)}$

$$\frac{z_1}{z_2} = 2(\cos(150-30) + i\sin(150-30^\circ))$$

$$\frac{z_1}{z_2} = 2(\cos(120^\circ) + i\sin(120^\circ))$$

$$= 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

$$= -1 + i\sqrt{3}$$

Q8 Multiple $z_1 = 2(\cos 60^\circ + i\sin 60^\circ)$

$z_2 = 5(\cos 90^\circ + i\sin 90^\circ)$

and express in the form $x + iy$

Soln: $z_1 z_2 = 10(\cos 90\cos 60 - \sin 90\sin 60 + i(\sin 90\cos 60 + \cos 90\sin 60))$

$$= 10(\cos(90+60) + i\sin(90+60))$$

$$= 10(\cos 150 + i\sin 150)$$

$$= 10\left(-\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

$$= -5\sqrt{3} + 5i$$

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Q9: Find the modulus and argument of $z = -2 - 2i$.

Soln:- $|z| = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{2(4)} = 2\sqrt{2}$

$\theta = \tan\left(\frac{-2}{-2}\right) = \pi + \tan^{-1} 1 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$\theta = \frac{5\pi}{4} + 2n\pi$.

Q10) Write the equation $\arg(\bar{z} - 2 + i) = \frac{2\pi}{3}$ in cartesian form if $z = x + iy$

$\arg(x - iy - 2 + i) = \frac{2\pi}{3}$

$\arg((1-y)i + (x-2)) = \frac{2\pi}{3}$

$\tan^{-1} \frac{1-y}{x-2} = \frac{2\pi}{3}$

$\frac{1-y}{x-2} = \tan\left(\frac{2\pi}{3}\right)$

$\frac{1-y}{x-2} = -\sqrt{3}$

$1 - y = -\sqrt{3}x + 2\sqrt{3}$

$\sqrt{3}x - y = 2\sqrt{3} - 1$

$\sqrt{3}x - 2\sqrt{3} + 1 = y$

$y = \sqrt{3}x - 2\sqrt{3} + 1$

11) If $z = x + iy$ $\arg\left(\frac{\bar{z} - 1 + 2i}{\bar{z} + 1 - 2i}\right) = \frac{9\pi}{4}$

show that $x^2 + y^2 - 4x + 2y - 5 = 0$

Soln As $z = x + iy$ $\bar{z} = x - iy$

$\arg\left(\frac{x - iy - 1 + 2i}{x - iy + 1 - 2i}\right) = \frac{9\pi}{4}$

$\arg\left(\frac{x - 1 + (2-y)i}{x+1 - (2+y)i}\right) = \frac{9\pi}{4}$

$$Arg\left(\frac{(x-1)+(2-y)i}{(x+1)-(2+y)i} \times \frac{x+1+(2+y)i}{x+1+(2+y)i}\right) = \frac{9\pi}{4}$$

$$Arg\left(\frac{x^2-1-4+y^2+i((x-1)(2+y)+(x+1)(2-y))}{(x+1)^2+(2+y)^2}\right) = \frac{9\pi}{4}$$

$$Arg\left(\frac{x^2+y^2-5+i(2x+\cancel{xy}-\cancel{2}-y+2x-\cancel{xy}+\cancel{2}-y)}{(x+1)^2+(2+y)^2}\right)$$

$$Arg\left(\frac{x^2+y^2-5}{(x+1)^2+(2+y)^2} + i\frac{(4x-2y)}{(x+1)^2+(2+y)^2}\right) = \frac{9\pi}{4}$$

$$\tan^{-1}\left(\frac{4x-2y/\cancel{(x+1)^2+(2+y)^2}}{x^2+y^2-5/\cancel{(x+1)^2+(2+y)^2}}\right) = \frac{9\pi}{4}$$

$$\frac{4x-2y}{x^2+y^2-5} = \tan\left(\frac{9\pi}{4}\right)$$

$$\frac{4x-2y}{x^2+y^2-5} = 1$$

$$4x-2y = x^2+y^2-5$$

$$0 = x^2+y^2-4x+2y-5$$

$$\boxed{x^2+y^2-4x+2y-5=0}$$

hence proved.

12) If $z = x+iy$ and $\arg(z-2-3i) - \arg(z+2+3i) = 2\pi$ show that $2y = 3x$

Sol/n $\arg(x+iy-2-3i) - \arg(x+iy+2+3i) = 2\pi$

$\arg(x-2+i(y-3)) - \arg(x+2+i(y+3)) = 2\pi$

$\tan^{-1}\left(\frac{y-3}{x-2}\right) - \tan^{-1}\left(\frac{y+3}{x+2}\right) = 2\pi$

Taking tan to both side.

$\tan^{-1}\left(\tan^{-1}\left(\frac{y-3}{x-2}\right) - \tan^{-1}\left(\frac{y+3}{x+2}\right)\right) = \tan 2\pi$

$$\frac{\tan\left(\tan^{-1}\left(\frac{y-3}{x-2}\right)\right) - \tan\left(\tan^{-1}\left(\frac{y+3}{x+2}\right)\right)}{1 + \tan\left(\tan^{-1}\left(\frac{y-3}{x-2}\right)\right)\tan\left(\tan^{-1}\frac{y+3}{x+2}\right)} = 0$$

$$\frac{\frac{y-3}{x-2} - \frac{y+3}{x+2}}{1 + \frac{y-3}{x-2} \times \frac{y+3}{x+2}} = 0$$

$$\frac{\frac{(y-3)(x+2) - (y+3)(x-2)}{\cancel{(x-2)(x+2)}}}{\frac{(x-2)(x+2) + (y-3)(y+3)}{\cancel{(x-2)}(x+2)}} = 0$$

$$\frac{xy + 2y - 3x - 6 - (xy - 2y + 3x - 6)}{x^2 - 4 + y^2 - 9} = 0$$

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$$\cancel{xy} + 2y - 3x - \cancel{6} - \cancel{xy} + 2y - 3x + \cancel{6} = 0$$
$$4y - 6x = 0$$
$$4y = 6x$$
$$2y = 3x.$$

(13) Solve the equation $|z-2i| = |\bar{z}+2|$ for $z = x+iy$.

Soln: $|z-2i| = |\bar{z}+2|$

$$|x+iy-2i| = |x-iy+2|$$
$$|x+(y-2)i| = |x+2-iy|$$
$$\sqrt{x^2+(y-2)^2} = \sqrt{(x+2)^2+y^2}$$
$$x^2+(y-2)^2 = (x+2)^2+y^2$$
$$\cancel{x^2}+\cancel{y^2}-4y+\cancel{4} = \cancel{x^2}+\cancel{4}+4x+\cancel{y^2}$$
$$-\cancel{4}y = \cancel{4}x$$
$$\boxed{0 = x+y}$$

(14) For $z = x+iy$ solve the equation $|5z+4+i| = |5\bar{z}-3+2i|$

soln $|5(x+iy)+4+i| = |5(x-iy)-3+2i|$

$$|5x+4+i(5y+1)| = |5x-3+i(2-5y)|$$
$$\sqrt{(5x+4)^2+(5y+1)^2} = \sqrt{(5x-3)^2+(2-5y)^2}$$
$$25x^2+16+40x+25y^2+10y+1 = 25x^2-30x+9+4-20y+25y^2$$
$$40x+10y+17 = -30x-20y+13$$
$$70x+30y+4 = 0$$
$$35x+15y+2 = 0$$
$$15y = -35x-2$$
$$y = \frac{-35x}{15} - 2/15$$
$$\boxed{y = \frac{-7x}{3} - 2/15}$$

15) Determine the set of point $z = x+iy$ that statisfy the equation.

$$|3\bar{z} - 2 + i| = |3z + i|$$

$$|3(x-iy) - 2 + i| = |3x + 3iy + i|$$

$$|3x - 2 + i(1-3y)| = |3x + i(3y+1)|$$

$$\sqrt{(3x-2)^2 + (1-3y)^2} = \sqrt{(3x)^2 + (3y+1)^2}$$

$$9x^2 + 4 - 12x + 1 + 9y^2 - 6y = 9x^2 + 9y^2 + 1 + 6y$$

$$-12x + 5 - 6y = 1 + 6y.$$

$$-12x + 5 - 1 = 6y + 6y$$

$$-12x + 4 = 12y$$

$$\frac{-12x}{12} + \frac{4}{12} = y$$

$$\boxed{y = -x + \frac{1}{3}}$$

16) If $z = x+iy$ and $w = \frac{1-iz}{z-i}$ Show that $|w| = 1 \Rightarrow z$ is real.

Sol:- $w = \frac{1-iz}{z-i}$

$$|w| = \left|\frac{1-iz}{z-i}\right|$$

As $|i| = 1$ & $|-i| = 1$

$$= \frac{|1|\,|1-iz|}{|1|\,|z-i|} \quad \text{As } \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

$$= \frac{|1|\,|1-iz|}{|-i|\,|z-i|} = \frac{|1(1-iz)|}{|-i(z-i)|}$$

$$= \frac{\cancel{-1+z}}{} = \frac{|1-iz|}{|-1-iz|}$$

$$|w| = \frac{|1+y-ix|}{|1+y-1-ix|}$$

$$|w| = \frac{\sqrt{(1+y)^2 + x^2}}{\sqrt{(y-1)^2 + x^2}}$$

if $|w| = 1$

$$\sqrt{(1+y)^2 + x^2} = \sqrt{(y-1)^2 + x^2}$$

$$(1+y)^2 + \cancel{x^2} = (y-1)^2 + \cancel{x^2}$$

$$1 + y^2 + 2y = y^2 + 1 - 2y$$

$$4y = 0 \qquad y = 0$$

$z = x$ so z is real.

(17) If z_1 and z_2 are different complex number with $|z_2| = 1$ find $\left|\frac{z_2 - z_1}{1 - \bar{z}_1 z_2}\right|$

Soln Let $z_1 = x_1 + iy_1 \quad z_2 = x_2 + iy_2$

$|z_2|^2 = z_2 \bar{z}_2$ As $|z_2| = 1$ So $x_2^2 + y_2^2 = 1$

$$\frac{1}{1} \cdot \left|\frac{z_2 - z_1}{1 - \bar{z}_1 z_2}\right| \Rightarrow \frac{1}{|z_2|} \left|\frac{z_2 - z_1}{1 - \bar{z}_1 z_2}\right|$$

$$= \frac{|z_2 - z_1|}{|\bar{z}_2 - \bar{z}_1 |z_2^2||} = \frac{|z_2 - z_1|}{|\bar{z}_2 - \bar{z}_1|}$$

$$= \frac{|x_2 - x_1 + i(y_2 - y_1)|}{|x_2 - x_1 - i(y_2 - y_1)|}$$

$$= \frac{(x_2 - x_1)^2 + \cancel{(y_2 - y_1)^2}}{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 1$$

Ohm's Law = $V = IR$

If (AC) circuits Resistance is generalized by impedance (Z)

Reactance = Resistance of inductor − Resistance of capacitor = 1

$$X = X_L - X_C.$$

and $Z = R + iX$.

$$V = I.Z.$$

18) An AC source supplies a voltage of $V = 120(\cos\pi/4 + i\sin\pi/4)$ volt to a circuit with $Z = \frac{1+i\sqrt{3}}{2}$ ohms. Calculate the current in polar form.

Soln: As $Z = \frac{1}{2} + \frac{i\sqrt{3}}{2}$

$$|Z| = \sqrt{\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

$$Z = 1.(\cos\pi/3 + i\sin\pi/3)$$

$$V = I.Z.$$

$$I = \frac{V}{Z} = \frac{120(\cos\pi/4 + i\sin\pi/4)}{1(\cos\pi/3 + i\sin\pi/3)}$$

$$= 120\left(\cos\left(\frac{\pi}{4} - \pi/3\right) + i\left(\sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)\right)\right)$$

$$= 120\left(\cos\left(\frac{3\pi - 4\pi}{12}\right) + i\left(\sin\left(\frac{3\pi - 4\pi}{12}\right)\right)\right)$$

$$= 120\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)$$

$$= 120\left(\cos\frac{\pi}{12} - i\sin\left(\frac{\pi}{12}\right)\right)$$

DATE: 18 DAY:

19) An AC circuit has an impedance of $Z = 3-6i$ ohm and is connected to a voltage source of $V = 90+30i$ volt. Find the current in both rectangular and polar form.

Soln

$$Z = 3-6i \qquad V = 90+30i$$

$$V = I \cdot Z \Rightarrow I = \frac{V}{Z}$$

$$I = \frac{90+30i}{3-6i} = \frac{\overset{10}{\cancel{30}}(3+i)}{\cancel{3}(1-2i)}$$

$$I = \frac{10(3+i)}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{10(3+7i+2i^2)}{1+4}$$

$$I = 2(1+7i)$$

$$\boxed{I = 2+14i}$$

$$|I| = \sqrt{4+(14)^2} = \sqrt{4+192} = \sqrt{200}$$

$$= \sqrt{100 \times 2} = 10\sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{14}{2}\right) = \tan^{-1}(7) = 81.87^\circ$$

$$I = 10\sqrt{2}(\cos 81.87^\circ + i\sin 81.87^\circ)$$

DATE: 29 DAY:

(20) Encrpt The word 'CODE' by multiplying complex encryption key $K = 2 - i$
Then decrypt it back to the original word.

Letter	Complex No.	$Z \times K$	$\frac{Z}{K}$	Letter
C	$3+0i$	$6-3i$	$6-3i \times \left(\frac{2+i}{5}\right) = 3+0i$	C.
O	$15+0i$	$30-15i$	$30-15i\left(\frac{2+i}{5}\right) = 15+0i$	O
D	$4+0i$	$8-4i$	$8-4i \times \frac{2+i}{5} = 4+0i$	D
E	$5+0i$	$10-5i$	$10-5i \times \frac{2+i}{5} = 5+0i$	E

Here Key $K = 2 - i$

Inverse of $K = \frac{1}{2-i} \times \frac{2+i}{2+i} = \frac{2+i}{4+1} = \frac{2+i}{5}$

(21) Consider the complex encryption key $k = 3 - 3i$
Encrypt the word "QUIZ" and the recover the original word using the inverse key.

Soln $K = 3 - 3i$

Inverse of $K = \frac{1}{3-3i} \times \frac{3+3i}{3+3i} = \frac{3(1+i)}{9+9} = \frac{1+i}{6}$

Letter	Complex No. Z	$Z \times K$	$\frac{Z}{K}$	Letter
Q	17	$51-51i$	$51-51i \times \left(\frac{1+i}{6}\right)$	$17+0i =$ Q
U	21	$63-63i$	$63-63i \times \left(\frac{1+i}{6}\right)$	$21+0i =$ U
I	9	$27-27i$	$27-27i \times \frac{(1+i)}{6}$	$9+0i =$ I
Z	26	$78-78i$	$78-78i \times \frac{1+i}{6}$	$26+0i =$ Z

22) Encrypt the word "CLASS" by adding the Complex encryption key $K = -3 + 4i$
Then decrypt it back to the original word.

DATE: 20 DAY:

$K = -3+4i$

Inverse of $K = \frac{1}{K} = \frac{1}{-3+4i} \times \frac{-3-4i}{-3-4i}$

$= \frac{-3-4i}{9+16} = \frac{-3-4i}{25}$

Letter	Complex	$Z \times K$	Z/K	Letter
C	3	$-9+12i$	$-9+12i \times \frac{-3-4i}{25}$	$3+0i = C$
L	12	$-36+48i$	$-36+48i \times \frac{-3-4i}{25}$	$12+0i = L$
A	1	$-3+4i$	$-3+4i \times \frac{-3-4i}{25}$	$1+0i = A$
S	19	$-57+76i$	$-57+76i \times \frac{-3-4i}{25}$	$19+0i = S$
S	19	$-57+76i$	$-57+76i \times \frac{3-4i}{25}$	$19+0i = S$

Remember me in your prayer
(HASSAN MEHBOOB S.S.S (Maths))