DATE: (1) Ex. 1.4 DAY:

Find the three cube roots of.

(i) 8

Let x be cube roots of 8

$$x = 8^{1/3}$$

Taking cube

$$x^3 = 8$$

$$x^3 - 8 = 0$$

$$x^3 - 2^3 = 0$$

$$(x-2)(x^2+2x+4) = 0$$

$x - 2 = 0$ $\quad$ $x^2 + 2x + 4 = 0$

$x = 2$ $\quad$ $x = \dfrac{-2 \pm \sqrt{4 - 4(1)(4)}}{2(1)}$

$$x = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$x = \frac{-2 \pm \sqrt{-12}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{3}i}{2}$$

$$x = 2\left(\frac{-1 \pm \sqrt{3}i}{2}\right)$$

$$x = 2\left(\frac{-1+\sqrt{3}i}{2}\right), x = 2\left(\frac{-1-\sqrt{3}i}{2}\right)$$

$$x = 2\omega, x = 2\omega^2$$

So roots are $2, 2\omega, 2\omega^2$

$$S.S = \{2, 2\omega, 2\omega^2\}$$

(ii) -8

Let x be cube root of -8

$$x = (-8)^{1/3}$$

$$x^3 = -8$$

$$x^3 + 8 = 0$$

$$x^3 + 2^3 = 0$$

$$(x+2)(x^2 - 2x + 4) = 0$$

$x + 2 = 0$ $\quad$ $x^2 - 2x + 4 = 0$

$x = -2$ $\quad$ $x = \dfrac{-(-2) \pm \sqrt{4-16}}{2}$

$x = 2$, $x = -\left[\dfrac{-2 \mp 2\sqrt{3}i}{2}\right]$

$$x = -2\left[\frac{-1 \mp \sqrt{3}i}{2}\right]$$

$$x = -2\left[\frac{-1+\sqrt{3}}{2}\right],$$

$$x = -2\left[\frac{-1-\sqrt{3}i}{2}\right]$$

$$x = -2\omega, -2\omega^2$$

hence the roots

$$-2, -2\omega, -2\omega^2$$

$$S.S = \{-2, -2\omega, -2\omega^2\}$$

(iii) -27

Let x be cube root of -27

$x = (-27)^{1/3}$

$x^3 = -27$

$x^3 + 27 = 0$

$x^3 + 3^3 = 0$

$(x+3)(x^2-3x+9) = 0$

$x+3=0 \quad x^2-3x+9=0$

$x=-3 \quad x = \dfrac{-(-3) \pm \sqrt{9-36}}{2(1)}$

$x = -\left[\dfrac{-3 \mp \sqrt{-27}}{2}\right]$

$x = -\left[\dfrac{-3 \pm 3\sqrt{3}i}{2}\right]$

$x = -3\left[\dfrac{-1 \pm \sqrt{3}i}{2}\right]$

$x = -3\left[\dfrac{-1+\sqrt{3}i}{2}\right]$

& $x = -3\left[\dfrac{-1-\sqrt{3}i}{2}\right]$

$x = -3\omega, -3\omega^2$

So roots are

$-3, -3\omega, -3\omega^2$

$S.S = \{-3, -3\omega, -3\omega^2\}$

iv) 64

Let x be the cube root 64

$x = (64)^{1/3}$

$x^3 = 64$

$x^3 - 4^3 = 0$

$(x-4)(x^2+4x+16) = 0$

$x-4=0 \quad x^2+4x+16=0$

$x=4 \quad x = \dfrac{-4 \pm \sqrt{16-64}}{2(1)}$

$x = \dfrac{-4 \pm \sqrt{-48}}{2}$

$x = \dfrac{-4 \pm 4\sqrt{3}i}{2}$

$x = 4\left(\dfrac{-1 \pm \sqrt{3}i}{2}\right)$

$x = 4\left(\dfrac{-1+\sqrt{3}i}{2}\right), x = 4\left(\dfrac{-1-\sqrt{3}}{2}\right.$

$x = 4\omega, 4\omega^2$

hence the roots are

$4, 4\omega, 4\omega^2$

$S.S = \{4, 4\omega, 4\omega^2\}$

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iv) -125

let x be the cube root of -125 i.e $x=(-125)^{1/3}$

$$x^3=-125 \Rightarrow x^3+125=0$$

$$(x)^3+(5)^3=0$$

$$(x+5)(x^2-5x+25)=0$$

$$x+5=0 \qquad x^2-5x+25=0$$

$$x=-5, \quad x=\frac{-(-5)\pm\sqrt{25-4(25)}}{2}$$

$$x=-\left[\frac{-5\pm\sqrt{-75}}{2}\right]$$

$$=-5\left[\frac{-5\pm 5\sqrt{3}i}{2}\right]$$

$$x=-5, \; x=-5\left[\frac{-1+\sqrt{3}i}{2}\right], \; x=-5\left[\frac{-1-\sqrt{3}i}{2}\right]$$

$$x=-5 \quad x=-5\omega, \quad x=-5\omega^2$$

$$S.S=\{-5,-5\omega,-5\omega^2\}$$

Q2: Find the four fourth roots of 16, 81, 625. Also show that their sum is zero in each case.

Sol: Let 'x' be the fourth root of 16 so.

$$x=16^{1/4} \Rightarrow x^4=16$$

$$x^4-16=0 \Rightarrow (x^2)^2-(4)^2=0$$

$$(x^2-4)(x^2+4)=0$$

$$(x^2-2^2)(x^2-(2i)^2)=0$$

$$(x-2)(x+2)(x-2i)(x+2i)=0$$

$$x=2, \; x=-2, \; x=2i, \; x=-2i$$

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So the fourth roots are $2, -2, 2i, -2i$

$Sum = 2 + (-2) + 2i + (-2i)$

$Sum = 0$

(ii) Let the x be the fourth root of 81.

$x = 81^{1/4} \Rightarrow x^4 = 81$

$x^4 - 81 = 0 \Rightarrow (x^2)^2 - (9)^2 = 0$

$(x^2 - 9)(x^2 + 9) = 0$

$(x^2 - 3^2)(x^2 - (3i)^2) = 0$

$(x-3)(x+3)(x+3i)(x-3i) = 0$

So the roots are $3, -3, -3i, 3i$

and $Sum = 3 + (-3) + 3i + (-3i)$

$= 0$

(iii) Let 'x' be the fourth root of 625

$x = (625)^{1/4} \Rightarrow x^4 = 625$

$x^4 - 625 = 0 \Rightarrow (x^2)^2 - (25)^2 = 0$

$(x^2 - 25)(x^2 + 25) = 0$

$(x^2 - 5^2)(x^2 - (5i)^2) = 0$

$(x-5)(x+5)(x-5i)(x+5i) = 0$

So roots $5, -5, 5i, -5i$

$Sum = 5 + (-5) + 5i + (-5i)$

$= 0$

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Q3: If $1, \omega, \omega^2$ are cube root of unity show that $1+\omega^n+\omega^{2n}=3$ where n is a multiple of 3 respectively.

Soln $1+\omega^n+\omega^{2n}$

As n is multiple of 3 so $n=3n'$ where $n' \in Z$

$1+\omega^{3n'}+\omega^{2(3n')}$

$1+(\omega^3)^{n'}+(\omega^3)^{2n'}$

$\because \omega^3=1$

$1+(1)^{n'}+(1)^{2n'}$

$1+1+1=3$ hence $1+\omega^n+\omega^{2n}=3$

Q4:- Evaluate

(i) $\left(\frac{-1+\sqrt{-3}}{2}\right)^7+\left(\frac{-1-\sqrt{-3}}{2}\right)^7$

$\left(\frac{-1+\sqrt{3}i}{2}\right)^7+\left(\frac{-1-\sqrt{3}i}{2}\right)^7$

$\omega^7+(\omega^2)^7$ $\quad \because \omega=\frac{-1+\sqrt{3}i}{2}$

$\omega^2=\frac{-1-\sqrt{3}i}{2}$

$\omega^7+\omega^{14}$

$\omega^6 \cdot \omega+\omega^{12} \cdot \omega^2$

$(\omega^3)^2 \cdot \omega+(\omega^3)^4 \cdot \omega^2$ $\quad \because \omega^3=1$

$(1)^2 \cdot \omega+(1)^4 \cdot \omega^2$

$\omega+\omega^2$ $\quad \because 1+\omega+\omega^2=0$

-1 $\quad \omega+\omega^2=-1$

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(ii) $(-1+\sqrt{-3})^5 + (-1-\sqrt{-3})^5$

$(-1+\sqrt{3}i)^5 + (-1-\sqrt{3}i)^5$

$(2\omega)^5 + (2\omega^2)^5$ As $2\omega = -1+\sqrt{3}i$

$2\omega^2 = -1-\sqrt{3}i$

$32\omega^5 + 32\omega^{10}$

$32 \cdot \omega^3 \cdot \omega^2 + 32\omega^9 \cdot \omega$

$32(1) \cdot \omega^2 + 32(\omega^3)^3 \cdot \omega$

$32(\omega^2+\omega)$ As $1+\omega+\omega^2=0$

$32(-1)$ $\omega+\omega^2=-1$

$\boxed{-32}$

QNo5: Show that $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)$
$(1-\omega^8+\omega^{16})$ --- to $2n$ factor $= 2^{2n}$

$(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)(1-\omega^8+\omega^{16})\ldots$

$(1+\omega^2-\omega)(1+\omega^3\cdot\omega-\omega^2)(1+\omega^6\cdot\omega^2-\omega^3\cdot\omega)(1+\omega^{15}\cdot\omega-\omega^4)$

$(\underline{1+\omega^2}-\omega)(\underline{1+\omega}-\omega^2)(\underline{1+\omega^2}-\omega)(\underline{1+\omega}-\omega^2)$

$(-\omega-\omega)(-\omega^2-\omega^2)(-\omega-\omega)(-\omega^2-\omega^2)$

$(-2\omega)(-2\omega^2)(-2\omega)(-2\omega^2)$ --- $2n$

$(4\omega^3)(4\omega^3)$ --- n factor

$(4)(4)(4)$ --- n factor

4^n

$(2^2)^n$

$\boxed{2^{2n}}$

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Q6 Prove that $\left(\frac{i+\sqrt{3}}{2}\right)^8 + \left(\frac{i-\sqrt{3}}{2}\right)^8 = -1$

$$\left(\frac{i+\sqrt{3}}{2}\right)^8 + \left(\frac{i-\sqrt{3}}{2}\right)^8$$

$$1\cdot\left(\frac{i+\sqrt{3}}{2}\right)^8 + 1\cdot\left(\frac{i-\sqrt{3}}{2}\right)^8$$

As $i^8 \Rightarrow (i^2)^4 \Rightarrow (-1)^4 \Rightarrow 1$.

So $i^8 = 1$

$$i^8\cdot\left(\frac{i+\sqrt{3}}{2}\right)^8 + i^8\left(\frac{i-\sqrt{3}}{2}\right)^8$$

$$\left(\frac{i(i+\sqrt{3})}{2}\right)^8 + \left(\frac{i(i-\sqrt{3})}{2}\right)^8$$

$$\left(\frac{i^2+\sqrt{3}i}{2}\right)^8 + \left(\frac{i^2-\sqrt{3}i}{2}\right)^8$$

$$\left(\frac{-1+\sqrt{3}i}{2}\right)^8 + \left(\frac{-1-\sqrt{3}i}{2}\right)^8$$

$$\omega^8 + (\omega^2)^8$$

$$\omega^6\cdot\omega^2 + \omega^{16}.$$

$$(\omega^3)^2\cdot\omega^2 + \omega^{15}\cdot\omega \qquad \because \omega^3 = 1$$

$$\omega^2 + \omega \qquad \because 1+\omega+\omega^2 = 0$$

$$-1$$

Q.No7 Evaluate $\sum_{k=0}^{5} \omega^{2k}$, where ω is cube root of unity

DATE: 08. DAY:

$$\sum_{k=0}^{5} \omega^{2k}$$

$= \omega^{2(0)} + \omega^{2(1)} + \omega^{2(2)} + \omega^{2(3)} + \omega^{2(4)} + \omega^{2(5)}$

$= \omega^{0} + \omega^{2} + \omega^{4} + \omega^{6} + \omega^{8} + \omega^{10}$

$= 1 + \omega^{2} + \omega^{3}.\omega + (\omega^{3})^{2} + \omega^{6}.\omega^{2} + \omega^{9}.\omega$

$= 1 + \omega^{2} + \omega + 1 + \omega^{2} + \omega$

$= (1+\omega+\omega^{2}) + (1+\omega+\omega^{2})$

$= 0 + 0$

$= 0$

Q8:- If ω is an imaginary cube roots of unity, prove that $\dfrac{a+b\omega^{2}+c\omega}{a\omega^{2}+b\omega+c} = \omega$

$$\frac{a+b\omega^{2}+c\omega}{a\omega^{2}+b\omega+c}$$

$$\frac{a.1+b\omega^{2}+c\omega}{a\omega^{2}+b\omega+c}$$

$$\frac{a.\omega^{3}+b\omega^{2}+c\omega}{a\omega^{2}+b\omega+c} \quad \text{As } \omega^{3}=1$$

$$\frac{\omega\cancel{(a\omega^{2}+b\omega+c)}}{\cancel{(a\omega^{2}+b\omega+c)}}$$

ω

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If ω is a cube root of unity Prove that

$$\frac{a\omega^{12}+b\omega^{17}+c\omega^{19}}{a\omega^{14}+b\omega^{22}+c\omega^{30}}=\omega$$

$$\frac{a\cdot w^{9}\cdot w^{3}+bw^{15}\cdot w^{2}+cw^{18}\cdot w}{a\cdot w^{12}\cdot w^{2}+bw^{21}\cdot w+c(w^{3})^{10}}$$

$$\frac{a\cdot(w^{3})^{4}+b(w^{3})^{15}\cdot w^{2}+c(w^{3})^{6}\cdot w}{a(w^{3})^{4}w^{2}+b(w^{3})^{7}\cdot w+c(w^{3})^{10}}$$

$$\frac{a+bw^{2}+cw}{aw^{2}+bw+c}$$

$$\frac{a(1)+bw^{2}+cw}{aw^{2}+bw+c}$$

$\because 1=w^{3}$

$$\frac{aw^{3}+bw^{2}+cw}{aw^{2}+bw+c}$$

$$\frac{w\cancel{(aw^{2}+bw+c)}}{\cancel{aw^{2}+bw+c}}$$

w.