Exercise 1.3 (Solutions)
Mathematics 11 (PECTAA)
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Available at MathCity.org

Exercise 1.3 DAY:________

Q1 Factorize the following.

(i) a^2+4b^2

$(a)^2-(-4b^2)$

$a^2-(i^2 4b^2)$

$a^2-(2bi)^2$

$(a-2bi)(a+2bi)$

(ii) $9a^2+16b^2$

$(3a)^2-(-16b^2)$

$(3a)^2-(4^2b^2i^2)$

$(3a)^2-(4bi)^2$

$(3a-4bi)(3a+4bi)$

(iii) $3x^2+3y^2$

$3(x^2+y^2)$

$3(x^2-(iy)^2)$

$3(x-iy)(x+iy)$

(iv) $144x^2+225y^2$

$(12x)^2-(15^2y^2i^2)$

$(12x)^2-(15yi)^2$

$(12x-15yi)(12x+15yi)$

$3(4x-5yi)3(4x+5yi)$

$9(4x-5yi)(4x+5yi)$

v) $z^2-2iz-1$

$z^2-2iz+(i)^2$

$(z-i)^2$

$(z-i)(z-i)$

vi) $z^2+6z+13$

$z^2+6z+9+4$

$z^2+2(3)z+(3)^2-(4i^2)$

$(z+3)^2-(2i)^2$

$(z+3-2i)(z+3+2i)$

vii) z^2+4z+5

$z^2+4z+4+1$

$z^2+2(2)z+(2)^2-(i^2)$

$(z+2)^2-i^2$

$(z+2-i)(z+2+i)$

viii) $2z^2-22z+65$

$2z^2-22z+64+1$

$2\left(z^2-11z+32+\frac{1}{2}\right)$

$2\left(z^2-11z+\frac{121}{4}+\frac{65}{2}-\frac{121}{4}\right)$

$2\left(\left(z-\frac{11}{2}\right)^2+\frac{9}{4}\right)$

$2\left(\left(z-\frac{11}{2}\right)^2-\left(\frac{3i}{2}\right)^2\right)$

$2\left(z-\frac{11}{2}-\frac{3i}{2}\right)\left(z-\frac{11}{2}+\frac{3i}{2}\right)$

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Q2 Factorize the following polynomial into its linear factors.

(i) z^3+8

z^3+2^3

$(z+2)(z^2-2z+4)$

$(z+2)(z^2-2z+1+3)$

$(z+2)\left((z-1)^2-(\sqrt{3}i)^2\right)$

$(z+2)(z-1-\sqrt{3}i)(z-1+\sqrt{3}i)$

(ii) z^3+27.

z^3+3^3

$(z+3)(z^2-3z+9)$

$(z+3)\left(z^2-3z+(3/2)^2+9-\left(\frac{3}{2}\right)^2\right)$

$(z+3)\left((z-3/2)^2+\frac{36-9}{4}\right)$

$(z+3)\left((z-3/2)^2+27/4\right)$

$(z+3)\left((z-3/2)^2-\left(\frac{3\sqrt{3}}{2}i\right)^2\right)$

$(z+3)\left(z-3/2-\frac{3\sqrt{3}}{2}i\right)\left(z-\frac{3}{2}+\frac{3\sqrt{3}}{2}i\right)$

(iii) $z^3-2z^2+16z-32$

$z^2(z-2)+16(z-2)$

$(z-2)(z^2+16)$

$(z-2)(z^2-(4i)^2)$

$(z-2)(z-4i)(z+4i)$

(iv) z^4+21z^2-100

$z^4+25z^2-4z^2-100$

$z^2(z^2+25)-4(z^2+25)$

$(z^2+25)(z^2-4)$

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$(z^2 - 25i^2)(z^2 - 4)$

$(z^2 - (5i)^2)(z^2 - (2)^2)$

$(z - 5i)(z + 5i)(z - 2)(z + 2)$

v) $z^4 - 16$.

$(z^2)^2 - (4)^2$

$(z^2 - 4)(z^2 + 4)$

$(z^2 - 2^2)(z^2 - 4i^2)$

$(z - 2)(z + 2)(z - 2i)(z + 2i)$

vi) $z^4 + 3z^2 - 4$

$z^4 + 4z^2 - z^2 - 4$

$z^2(z^2 + 4) - 1(z^2 + 4)$

$(z^2 + 4)(z^2 - 1)$

$(z^2 - 4i^2)(z^2 - (1)^2)$

$(z - 2i)(z + 2i)(z - 1)(z + 1)$

vii) $z^4 + 5z^2 + 6$.

$z^4 + 3z^2 + 2z^2 + 6$

$z^2(z^2 + 3) + 2(z^2 + 3)$

$(z^2 + 3)(z^2 + 2)$

$(z^2 - 3i^2)(z^2 - 2i^2)$

$((z)^2 - (\sqrt{3}i)^2)(z^2 - (\sqrt{2}i)^2)$

$(z - \sqrt{3}i)(z + \sqrt{3}i)(z - \sqrt{2}i)(z + \sqrt{2}i)$

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viii) $z^4 - 32z^2 - 3969$

$z^4 - 81z^2 + 49z^2 - 3969$

$z^2(z^2 - 81) + 49(z^2 - 81)$

$(z^2 + 49)(z^2 - 81)$

$(z^2 - (7i)^2)(z^2 - (9)^2)$

$(z - 7i)(z + 7i)(z - 9)(z + 9)$

(Book answer is wrong).

(03) Find the roots of $z^4 + 7z^2 - 144 = 0$ and hence express it as product of linear factors:-

Sol:

$z^4 + 7z^2 - 144 = 0$

$z^4 + 16z^2 - 9z^2 - 144 = 0$

$z^2(z^2 + 16) - 9(z^2 + 16) = 0$

$(z^2 - 9)(z^2 + 16) = 0$

$((z)^2 - (3)^2)(z^2 - (4i)^2) = 0$

$(z - 3)(z + 3)(z - 4i)(z + 4i) = 0.$

(04) Solve the following complex quadratic equation by completing square method.

(i) $2z^2 - 3z + 4 = 0$

$2z^2 - 3z = -4$

$\frac{2z^2}{2} - \frac{3z}{2} = \frac{-4}{2}$

$z^2 - \frac{3}{2}z = -2$

$z^2 - \frac{3}{2}z + \left(\frac{3}{4}\right)^2 = -2 + \left(\frac{3}{4}\right)^2$

$\left(z - \frac{3}{4}\right)^2 = 2 + \frac{9}{16}.$

$\left(z - \frac{3}{4}\right)^2 = -\frac{23}{16}.$

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$$\left(z-\frac{3}{4}\right)=\pm\frac{\sqrt{23}\,i}{4}$$

$$z=\frac{3}{4}\pm\frac{\sqrt{23}\,i}{4}$$

$$z=\frac{3+\sqrt{23}\,i}{4} \qquad z=\frac{3-\sqrt{23}\,i}{4}$$

So solution are $\left\{\frac{3+\sqrt{23}\,i}{4}, \frac{3-\sqrt{23}}{4}\right\}$

(ii) $z^2-6z+30=0$

$$z^2-6z=-30$$

$$z^2-2(3)z=-30$$

$$z^2-2(3)z+(3)^2=-30+(3)^2$$

$$(z-3)^2=-30+9$$

$$(z-3)^2=-21$$

$$z-3=\pm\sqrt{21}\,i$$

$$z=3\pm\sqrt{21}\,i$$

$$z_1=3+\sqrt{21}\,i \quad \& \quad z=3-\sqrt{21}\,i$$

$$S.S=\{3+\sqrt{21}\,i,\ 3-\sqrt{21}\,i\}$$

(iii) $3z^2-18z+50=0$.

$$\frac{3z^2}{3}-\frac{18z}{3}+\frac{50}{3}=0$$

$$z^2-6z=-\frac{50}{3}$$

$$z^2-2(3)z+(3)^2=-\frac{50}{3}+(3)^2$$

$$(z-3)^2=-\frac{50}{3}+9$$

$$(z-3)^2=\frac{-50+27}{3}$$

$$(z-3)^2=-\frac{23}{3}$$

$$z-3=\pm\sqrt{\frac{23}{3}}\,i$$

$$z-3=\pm\frac{\sqrt{69}}{3}\,i$$

$$z = 3 \pm \frac{\sqrt{69}\,i}{3}$$

$$z = \frac{9 \pm \sqrt{69}\,i}{3}$$

$$z = \frac{9+\sqrt{69}\,i}{3} \quad \text{and} \quad z = \frac{9 - \sqrt{69}\,i}{3}.$$

(iv) $z^2+4z+13=0$

$$z^2+4z = -13$$

$$z^2+2(2)z = -13$$

$$z^2+2(2)z+(2)^2 = -13+(2)^2$$

$$(z+2)^2 = -13+4$$

$$(z+2)^2 = -9$$

$$z+2 = \pm\sqrt{9}\,i$$

$$z+2 = \pm 3i$$

$$z = -2 \pm 3i$$

$$z = -2+3i, \quad z = -2-3i$$

$$S.S = \{-2+3i, -2-3i\}$$

v) $2z^2+6z+9=0$

$$2z^2+6z = -9$$

$$\frac{\cancel{2}z^2}{\cancel{2}} + \frac{\overset{3}{\cancel{6}}z}{\cancel{2}} = -9/2$$

$$z^2+3z = -9/2$$

$$z^2+2(3/2)z = -9/2$$

$$z^2+2(3/2)z+(3/2)^2 = -\frac{9}{2}+(3/2)^2$$

$$(z+3/2)^2 = -\frac{9}{2}+\frac{9}{4}$$

$$(z+3/2)^2 = -\frac{9}{4}$$

$$(z+3/2) = \pm\frac{3}{2}i$$

$$z = -\frac{3}{2} \pm \frac{3i}{2}$$

$$z = \frac{-3+3i}{2} \quad z = \frac{-3-3i}{2}$$

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vi) $3z^2-5z+7=0$

$3z^2-5z=-7$

$\frac{3z^2}{3}-\frac{5z}{3}=-\frac{7}{3}$

$z^2-\frac{5}{3}z=-\frac{7}{3}$

$z^2-2\left(\frac{5}{6}\right)z+\left(\frac{5}{6}\right)^2=-\frac{7}{3}+\left(\frac{5}{6}\right)^2$

$\left(z-\frac{5}{6}\right)^2=-\frac{7}{3}+\frac{25}{36}$

$\left(z-\frac{5}{6}\right)^2=\frac{-84+25}{36}$

$\left(z-\frac{5}{6}\right)^2=\frac{-59}{36}$

$z-\frac{5}{6}=\pm\frac{\sqrt{59}\,i}{6}$

$z=\frac{5}{6}\pm\frac{\sqrt{59}\,i}{6}$

$z=\frac{5\pm\sqrt{59}\,i}{6}$

$z=\frac{5+\sqrt{59}\,i}{6}$ $\quad z=\frac{5-\sqrt{59}\,i}{6}$

QNO5 Solve the following equation.

(i) $2z^4-32=0$

$2(z^2-16)=0$

$z^2-16=0$

$z^2-(4)^2=0$

$(z^2-4)(z^2+4)=0$

$(z^2-(2)^2)(z^2-(2i)^2)=0$

$(z-2)(z+2)(z-2i)(z+2i)=0$

ii) $3z^5-243z=0$

$3z(z^4-81)=0$

(ii) $z((z^2)^2-(9)^2) = 0$

$z(z^2-9)(z^2+9) = 0$

$z(z^2-3^2)(z^2-9i^2) = 0$

$z(z-3)(z+3)(z-3i)(z+3i) = 0$

So $z=0$, $z-3=0$ $z+3=0$ $z-3i=0$ $z+3i=0$

$z=0$, $z=3$, $z=-3$, $z=3i$, $z=-3i$

So roots are $0, 3, -3, 3i, -3i$

(iii) $5z^5 - 5z = 0$

$5z(z^4-1) = 0$

$z((z^2)^2-1) = 0$

$z(z^2-1)(z^2+1) = 0$

$z(z-1)(z+1)(z^2-i^2) = 0$

$z(z-1)(z+1)(z-i)(z+i) = 0$

$z=0$, $z=1$, $z=-1$, $z=i$, $z=-i$

So roots are $0, 1, -1, i, -i$,

(iv) $z^3-5z^2+z-5 = 0$

$z^2(z-5)+1(z-5) = 0$

$(z-5)(z^2+1) = 0$

$(z-5)(z^2-i^2) = 0$

$(z-5)(z-i)(z+i) = 0$

$z=5$ $z=i$ $z=-i$

So roots are $5, i, -i$.

v) $4z^4 - 25z^2 - 21 = 0$

$4z^4 - 28z^2 + 3z^2 - 21 = 0$

$4z^2(z^2 - 7) + 3(z^2 - 7) = 0$

$(z^2 - 7)(4z^2 + 3) = 0$

$(z^2 - (\sqrt{7})^2)((2z)^2 - (\sqrt{3}i)^2) = 0$

$(z - \sqrt{7})(z + \sqrt{7})(2z + \sqrt{3}i)(2z - \sqrt{3}i) = 0$

So roots $\sqrt{7}, -\sqrt{7}, -\frac{\sqrt{3}i}{2}, \frac{\sqrt{3}i}{2}$

vi) $z^3 + z^2 + z + 1 = 0$

$z^2(z+1) + 1(z+1) = 0$

$(z+1)(z^2+1) = 0$

$z + 1 = 0 \qquad z^2 + 1 = 0$

$z = -1 \qquad z^2 - i^2 = 0$

$(z - i)(z + i) = 0$

$z = i \quad z = -i$

$-1, i, i$ are the roots.

Q6. Find the ~~a~~ polynomial $P(z)$ of degree 3 with zero $3, -2i, 2i$ and statifying $P(1) = 20$.

Soln:- let the polynomial be.

$P(z) = a(z - \alpha_1)(z - \alpha_2)(z - \alpha_3)$

where a is constant $\alpha_1, \alpha_2, \alpha_3$ are roots of polynomial.

~~In this~~ Here $\alpha_1 = 3 \quad \alpha_2 = -2i \quad \alpha_3 = 2i$

$P(z) = a(z-3)(z+2i)(z-2i)$

Put $z = 1$

$P(1) = a(1-3)(1+2i)(1-2i)$

$P(1) = -2a(1+4)$

$P(1) = -10a$

As $P(1) = 20$ So

$20 = -10a$

$\boxed{a = -2}$

$P(z) = -2(z-3)(z+2i)(z-2i)$

$= -2(z-3)(z^2+4)$

$= -2(z^3-3z^2+4z-12)$

$P(z) = -2z^2+6z^2-8z+24$

Q7:- Find a polynomial $P(z)$ of degree 4 with zeros $2i, -2i, 1, -1$ and statisfying $P(2) = 240$.

Sol/ns. Let $P(z) = a(z-a_1)(z-a_2)(z-a_3)(z-a_4)$

$P(z) = a(z-2i)(z+2i)(z-1)(z+1)$

$P(z) = a(z^2+4)(z^2-1)$ — 1

~~As~~ Put $z=2$

$P(2) = a(4+4)(4-1)$

$240 = a(24)$

$\frac{240}{24} = a$

$10 = a$

Put in 1

$P(z) = 10(z^2+4)(z^2-1)$

$P(z) = 10(z^4+3z^2-4)$

$= 10z^4+30z^2-40$.

Q8 Find the polynomial of degree 4 with zeros $4, -4, 1+i, 1-i$ and statisfying $P(2)=72$.

Solns- Let polynomial of degree '4' be

$$P(z) = a(z-\alpha_1)(z-\alpha_2)(z-\alpha_3)(z-\alpha_4)$$
$$= a(z-4)(z+4)(z-1-i)(z-1+i)$$
$$P(z) = a(z^2-16)((z-1)^2-i^2)$$
$$P(z) = a(z^2-16)(z^2-2z+1+1)$$
$$P(z) = a(z^2-16)(z^2-2z+2)$$
$$P(2) = a(2^2-16)(2^2-2\times 2+2) \quad \text{--- (1)}$$

As $P(2)=72$

$$72 = a(-12)(2)$$
$$72 = -24a$$
$$\boxed{-3 = a}$$

Put $a=-3$ in (1)

$$P(z) = -3(z^2-16)(z^2-2z+2)$$
$$= -3(z^4-2z^3+2z^2-16z^2+32z-32)$$
$$= -3(z^4-2z^3-14z^2+32z-32)$$
$$\boxed{P(z) = -3z^4+6z^3+42z^2-96z+96}$$

The End

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