DATE: 22/07/2025 Exercise 1.2 DAY: 01

Equality of two complex number.

Let $z_1 = a+bi$ and $z_2 = c+di$

if $z_1 = z_2$

Then $a+bi = c+di \Leftrightarrow a=c$ and $b=d$

1) Find the value of x and y in each of following.

(i) $x+iy+2-3i = i(5-i)(3+4i)$

$(x+2)+i(y-3) = i(15+20i-3i-4i^2)$

$(x+2)+i(y-3) = i(19+17i)$

$(x+2)+i(y-3) = -17+19i$

$x+2=-17$ and $y-3=19$

$x=-17-2$ $\quad$ $y=19+3$

$x=-19$ $\quad$ $y=22$

(ii) $(x+iy)(1-i) = (2-3i)(-5+5i)\left(-i\frac{3}{5}\right)$

$x-xi+yi-yi^2 = (-10+10i+15i-15i^2)\left(-\frac{3i}{5}\right)$

$x+y+(y-x)i = (5+25i)\left(-\frac{3i}{5}\right)$

$x+y+(y-x)i = -3i-15i^2$

$x+y+(y-x)i = 15-3i$

$x+y=15$ and $y-x=-3$

Adding $\quad x+y=15$

$\quad -x+y=-3$

$2y=12$

$\boxed{y=6}$

$x+6=15 \Rightarrow x=15-6$

$\boxed{x=9}$

(iii) $\frac{x}{2+i} + \frac{y}{3-i} = 4+5i$

$\frac{x(3-i) + y(2+i)}{(2+i)(3-i)} = 4+5i$

$\frac{3x - xi + 2y + yi}{6-2i+3i-i^2} = 4+5i$

$\frac{(y-x)i + 3x + 2y}{7+i} = 4+5i$

$3x+2y + (y-x)i = (4+5i)(7+i)$

$3x+2y + (y-x)i = 28+4i+35i+5i^2$

$3x+2y + (y-x)i = 23+39i$

$3x+2y = 23$ and $y-x = 39$ — (i)

$3x+2y = 23$ — ① $\quad -2x+2y = 78$ — ②

$-2x+2y = 78$
$\pm 3x \pm 2y \quad \pm 23$

$-5x = 55$

$\boxed{x=-11}$

putting (i)

$y-(-11) = 39$

$y = 39-11 = 28$

So $\boxed{x=-11}$ $\boxed{y=28}$

Q2) If $z_1 = -13+24i$ and $z_2 = x+iy$ find The real value of x and y Such that $z_1 - z_2 = -27+15i$

Sol As $z_1 - z_2 = -27+15i$

$-13+24i - x - iy = -27+15i$

$-x-13+(24-y)i = -27+15i$

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So $-x-13=-27$ $\quad$ $24-y=15$

$27-13=x$ $\quad$ $24-15=y$

$\boxed{14=x}$ $\quad$ $\boxed{9=y}$

Square root of complex Number

if $z = x+iy$

then

$$\sqrt{z} = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

where $|z| = \sqrt{x^2+y^2}$

QNo3:- Find the real value of x and y if

(i) $(x+iy)^2 = 25+60i$

$$x+iy = \pm\sqrt{25+60i} \quad \text{--- (1)}$$

Let $z = 25+60i$

Then $\sqrt{z} = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$

here $|z| = |5(5+12i)|$

$= 5\sqrt{25+144}$

$= 5\times 13 = 65$

$$\sqrt{25+60i} = \pm\left(\sqrt{\frac{65+25}{2}} + i\frac{60}{60}\sqrt{\frac{65-25}{2}}\right)$$

$= \pm(\sqrt{45} \pm i\sqrt{20})$

$= \pm(3\sqrt{5} \pm 2\sqrt{5}i)$

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~~Square~~ Putting in ①

$$x+iy = \pm(3\sqrt{5} \pm 2\sqrt{5}i)$$

So $x = \pm 3\sqrt{5}$ $y = \pm 2\sqrt{5}$

(ii) $(x+iy)^2 = 64+48i$

then,

$$x+iy = \sqrt{64+48i} \longrightarrow ①$$

$$z = 64+48i$$

$$z = 16(4+3i)$$

$$|z| = 16|4+3i| = 16\sqrt{16+9} = 16\times5 = 80$$

$$\sqrt{z} = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{iy}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

$$\sqrt{64+48i} = \pm\left(\sqrt{\frac{80+64}{2}} + \frac{i48}{48}\sqrt{\frac{80-64}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{144}{2}} + i\sqrt{\frac{16}{2}}\right)$$

$$= \pm(\sqrt{72} + i\sqrt{8}$$

$$\sqrt{64+48i} = \pm(6\sqrt{2} + 2\sqrt{2}i)$$

Putting in ①

$$x+iy = \pm(6\sqrt{2} + 2\sqrt{2}i)$$

$$x = \pm 6\sqrt{2} \qquad y = 2\sqrt{2}$$

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(iii) $(x+iy)^2 = \frac{2i-3}{3+i}$

$(x+iy)^2 = \frac{-3+2i}{3-i} \times \frac{3+i}{3+i}$

$= \frac{-9-3i+6i+2i^2}{9+1}$

$(x+iy)^2 = \frac{-11+3i}{10}$

$x+iy = \sqrt{\frac{-11}{10}+\frac{3}{10}i}$

$z = \frac{-11}{10} + \frac{3}{10}i$

$|z| = \sqrt{\frac{121}{100}+\frac{9}{100}} = \sqrt{\frac{130}{100}} = \sqrt{\frac{13}{10}}$

$\sqrt{z} = \pm\left(\sqrt{\frac{|z|+x}{2}} + i\frac{y}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$

$= \pm\left(\sqrt{\frac{\sqrt{13/10}-\frac{11}{10}}{2}} \pm i\frac{3/10}{|3/10|}\sqrt{\frac{\sqrt{13/10}+\frac{11}{10}}{2}}\right)$

$= \pm\left(\sqrt{\frac{\sqrt{1.3}-1.1}{2}} \pm i\sqrt{\frac{\sqrt{1.3}+1.1}{2}}\right)$

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Q4: If $z_1 = 2+3i$ and $z_2 = 1-d$, find the real value of d, such that $\text{Im}(z_1 z_2) = 7$

Soln As $\text{Im}(z_1 z_2) = 7 \quad -①$

$z_1 z_2 = (2+3i)(1-d)$

$z_1 z_2 = 2 - 2d + 3i - 3di$

$z_1 z_2 = 2 - 2a + 3(1-d)i$

So $\text{Im}(z_1 z_2) = 3(1-d)$

$7 = 3 - 3a$ using ①

$3d = 3 - 7$

$3d = -4 \quad \boxed{d = -4/3}$

QNo5:- If $z_1 = x+iy$ and $z_2 = a+bi$ find x, y, a and b such that $z_1 + z_2 = 10+4i$, $z_1 - z_2 = 6+2i$.

Soln As

$$\begin{array}{rl} & z_1 + z_2 = 10+4i \\ + & z_1 - z_2 = 6+2i \\ \hline & 2z_1 = 16+6i \end{array}$$

$\boxed{z_1 = 8+3i}$

$$\begin{array}{rl} & z_1 + z_2 = 10+4i \\ & z_1 - z_2 = 6+2i \\ & - \quad + \quad - \quad - \\ \hline & 2z_2 = 4+2i \end{array}$$

$z_2 = 2+i$

As $z_1 = 8+3i$

$x + iy = 8+3i$

$x = 8 \quad y = 3$

$z_2 = 2+i$

$a+bi = 2+i$

$a = 2 \quad b = 1$

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Q6:- Show that $\forall\ z_1, z_2 \in \mathbb{C},\ \overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

Soln: Let $z_1 = a+bi \quad z_2 = c+di$

Then $z_1 z_2 = (a+bi)(c+di)$

$z_1 z_2 = ac - bd + i(bc + ad)$

$\overline{z_1 z_2} = \overline{ac - bd + i(bc+ad)}$

$\overline{z_1 z_2} = ac - bd - i(bc + ad)$

$\overline{z_1 z_2} = ac - bd - ibc - iad.$

$\overline{z_1 z_2} = ac - ibc - bd - adi$

$\overline{z_1 z_2} = c(a-bi) + bd(-1) - adi$

$\overline{z_1 z_2} = c(a-bi) - adi + bd\,i^2 \quad \therefore -1 = i^2$

$\overline{z_1 z_2} = c(a-bi) - di(a-bi)$

$\overline{z_1 z_2} = (a-bi)(c-di)$ — ①

As $z_1 = a+bi$ and $z_2 = c+di$

Then $\overline{z_1} = a-bi$ and $\overline{z_2} = c-di$

Putting value in ①

$\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$ (Hence the result)

QNo7:- Find the square root of the following complex numbers:-

(i) $-7-24i$

Let $z = -7 - 24i$

$|z| = \sqrt{(-7)^2 + (-24)^2} = 25$

$\sqrt{z} = \pm\left(\sqrt{\dfrac{|z| + x}{2}} + \dfrac{yi}{|y|}\sqrt{\dfrac{|z| - x}{2}}\right)$

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So $\sqrt{-7-24i} = \pm\left(\sqrt{\frac{25+(-7)}{2}} + \frac{(-24)}{|-24|}i\sqrt{\frac{25-(-7)}{2}}\right)$

$\sqrt{-7-24i} = \pm\left(\sqrt{\frac{18}{2}} + -i\sqrt{\frac{32}{2}}\right)$

$= \pm(3-4i)$

So $3-4i$ and $-3+4i$ are squareroot of $-7-24i$.

(ii) $8-6i$

Let $z = 8-6i$

$|z| = \sqrt{(8)^2+(-6)^2} = \sqrt{64+36}$

$= \sqrt{100} = 10$

As $\sqrt{z} = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{y}{|y|}i\sqrt{\frac{|z|-x}{2}}\right)$

$\sqrt{8-6i} = \pm\left(\sqrt{\frac{10+8}{2}} + \frac{-6}{|-6|}i\sqrt{\frac{10-8}{2}}\right)$

$= \pm(3-i\sqrt{1})$

$= \pm(3-i)$

So root are $3-i$ and $-3+i$

(iii) $-15-36i$

$z = -15-36i$

$|z| = \sqrt{(-15)^2+(-36)^2}$

$= \sqrt{225+1296} = 39$

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$$\sqrt{z} = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{yi}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

$$\sqrt{-15-36i} = \pm\left(\sqrt{\frac{39+(-15)}{2}} + \frac{-36}{|-36|}i\sqrt{\frac{39+15}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{24}{2}} - i\sqrt{\frac{54}{2}}\right)$$

$$= \pm\left(\sqrt{12} - i\sqrt{27}\right)$$

$$\sqrt{-15-36i} = \pm\left(2\sqrt{3} - i3\sqrt{3}\right)$$

So the roots are $2\sqrt{3} - i3\sqrt{3}$ and $-2\sqrt{3} + i3\sqrt{3}$

(iv) $119+120i$

Let $z = 119+120i$

$$|z| = \sqrt{119^2+120^2} = 169$$

$$\sqrt{z} = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{yi}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

$$\sqrt{z} = \pm\left(\sqrt{\frac{169+119}{2}} + \frac{120}{|120|}i\sqrt{\frac{169-119}{2}}\right)$$

$$\sqrt{119+120i} = \pm\left(\sqrt{\frac{288}{2}} + i\sqrt{\frac{50}{2}}\right)$$

$$= \pm\left(\sqrt{144} + i\sqrt{25}\right)$$

$$= \pm\left(12 + 5i\right)$$

So the square root are $12+5i$ and $-12-5i$

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Q8 Find the square root of $13-20\sqrt{3}i$ and represent it on an Argand diagram.

Soln Let $z = 13-20\sqrt{3}i$

then $|z| = \sqrt{(13)^2+(-20\sqrt{3})^2}$

$= \sqrt{169+1200} = \sqrt{1369} = 37.$

$$\sqrt{z} = \pm\left(\sqrt{\frac{|z|+x}{2}} + \frac{yi}{|y|}\sqrt{\frac{|z|-x}{2}}\right)$$

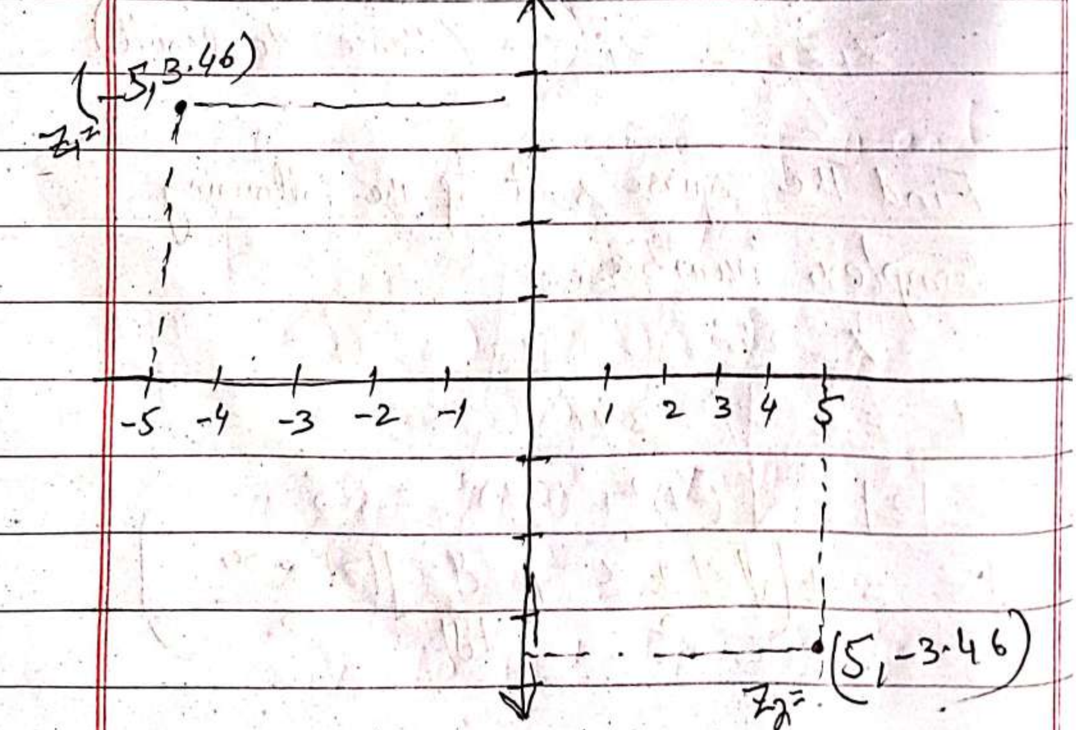
$$\sqrt{13-20\sqrt{3}i} = \pm\left(\sqrt{\frac{37+13}{2}} + \frac{-20\sqrt{3}i}{|-20\sqrt{3}|}\sqrt{\frac{37-13}{2}}\right)$$

$$= \pm\left(\sqrt{\frac{50}{2}} - i\sqrt{\frac{24}{2}}\right)$$

$$= \pm(5 - 2\sqrt{3}i)$$

So root $5-2\sqrt{3}i$ and $-5+2\sqrt{3}i$

$5-3.46i$ and $-5+3.46i$



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QNo9: Find the real value of x and y.

if $(-7+i)(x+iy)+(-1-5i)=i(11-i)$

$-7x-7iy+ix+i^2y-1-5i=11i-i^2$

$-7x-y-1+(x-7y-5)i=11i+1$

So $-7x-y-1=1$ & $x-7y-5=11$

$-7x-y=2$ — (1) $\quad x-7y=16$ — (2)

x with 7

$-49x-7y=14$ — (3) $\quad$ Eq (3) − (2)

$\underline{\overset{-}{+}x \overset{+}{-}7y=\overset{-}{16}}$ — (2)

$-50x=-2$

$x=1/25$

put $x=1/25$ in (2)

$\frac{1}{25}-7y=16$

$\frac{1}{25}-16=7y$

$\frac{1-400}{25}=7y$

$-\frac{399}{25\times 7}=y$

$y=-\frac{57}{25}$

QNo10 Find the real value of x and y

if $(5-2i)(x+iy)+3=i(11-i)-4i$

$5x+5yi-2xi-2i^2y+3=11i-i^2-4i$

$5x+2y+3+(5y-2x)i=1+7i$

$5x+2y+3=1$ — (1) $\quad 5y-2x=7$ — (2)

$5x+2y=-2$ $\quad -2x+5y=7$

x with 5

$5x+2y=-2$ | $-10x+25y=35$

$\times$ with 2.

$10x+4y=-4$ | $-10x+25y=35$

$$10x+4y=-4$$
$$+\;-10x+25y=35$$
$$\overline{\quad 29y=31\quad}$$

$$y=\frac{31}{29}$$

Put $y=\frac{31}{29}$ in $5x+2y=-2$

$$5x+2\left(\frac{31}{29}\right)=-2$$

$$5x=-2-\frac{62}{29}$$

$$5x=-\frac{120}{29}$$

$$x=-\frac{120}{5\times29}=\frac{-24}{29}$$

$$x=-\frac{24}{29}$$

Q.No 11 Find the real value of u and v

if $\frac{u-2}{2+i}+\frac{v-3}{2-i}=4i$

Soln

$$\frac{(u-2)(2-i)+(v-3)(2+i)}{(2+i)(2-i)}=4i$$

$$\frac{2u-ui-4+2i+2v+vi-6-3i}{2^2-i^2}=4i$$

$$\frac{2u+2v-10+(v-u+2-3)}{5}=4i$$

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$2u+2v-10+(v-u-5)i=20i$

$2u+2v-10=0 \quad | \quad v-u-5=20$

$u+v=5 \quad — (1) \quad | \quad -u+v=25 \quad — (2)$

Adding (1) + (2)

$u+v=5$

$-u+v=25$

$2v=30$

$\boxed{v=15}$

put $v=15$ in (1)

$u+15=5$

$u=5-15$

$\boxed{u=-10}$

Q12/ If $z_1=4+5i$ and $z_2=a-2i$ find the real values of a such that $Re(z_1z_2)=20$

Soln. $z_1z_2=(4+5i)(a-2i)$

$=4a-8i+5ai-10i^2$

$=4a-8i+5ai+10$

$z_1z_2=4a+10+(5a-8)i$

$Re(z_1z_2)=4a+10$

But given $Re(z_1z_2)=20$

$20=4a+10$

$10=4a$

$\frac{10}{4}=a$

$\boxed{5/2=a}$