Date: 01-07-2015 Day: Tuesday 01

Unit # 1 Complex Numbers:-

Complex Number: A number of the form $z = a + bi$, where $a, b \in R$ and $i = \sqrt{-1}$ is called complex number.

For example $3+4i, 2-5/7 i, -7-2i \in C$ (set of complex number).

In $z = a+bi$ 'a' is real part and 'b' is called imaginary part

e.g, $2i, -3i, \sqrt{5}i, -1/2 i$ are imaginary numbers. $3+4i$, Real part $=3$ imaginary part $=4$

Conjugate of complex Number: Let $z = a+bi$ be a complex number, then $a-ib$ is complex conjugate and is denoted by $\bar{z}$.

e.g $z_1 = 4-5i$ $\bar{z}_1 = 4+5i$ $z_2 = 1/2 + \frac{4}{5}i$

$\bar{z}_2 = 1/2 - 4/5 i$,

Operation on complex numbers:-

(i) Addition $\Rightarrow (a+bi)+(c+di) = a+c+(b+d)i$

OR $(a,b)+(c,d) = (a+b, c+d)$ $\because (a,b) = a+bi$

ii Scalar Multiplication $=$ $K(a+ib) = Ka + iKa$.

OR $K(a,b) = (Ka, Kb)$

(iii) Subtraction:- $(a+bi)-(c+di) = a-c+(b-d)i$

OR $(a,b)-(c,d) = (a-c, b-d)$

(iv) Multiplication:- $(a+bi)\times(c+di) = ac-bd+i(ad+bc)$

OR $(a,b)\times(c,d) = (ac-bd, ad+bc)$

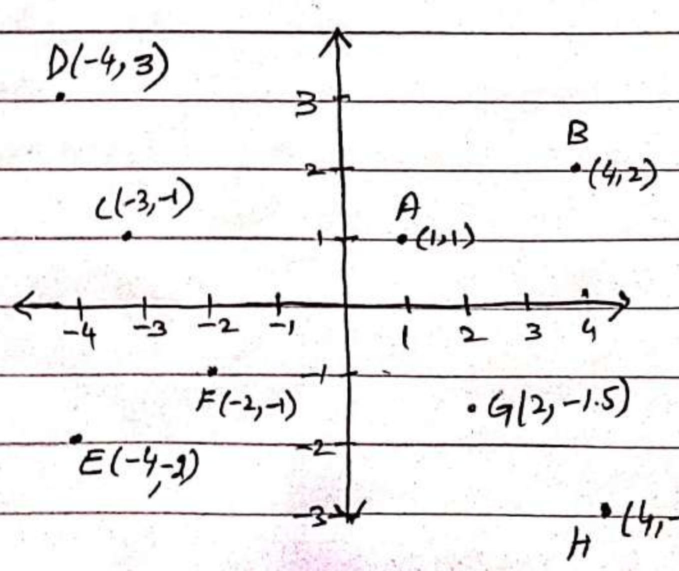
Properties of complex number.

w.r.t '+'	w.r.t '×'
(i) Closure: $\forall z_1, z_2 \in C$	$\forall z_1, z_2 \in C$
$z_1 + z_2 \in C$	$z_1 \times z_2 \in C$.

ii	Associative: $\forall\ z_1, z_2, z_3 \in C$ $\Rightarrow z_1+(z_2+z_3) = (z_1+z_2)+z_3$	$\forall\ z_1, z_2, z_3 \in C$ $z_1(z_2 z_3) = (z_1 z_2) z_3$
(iii)	Additive Identity $0 = 0+0i \in C$ $\Rightarrow z+0 = 0+z = z \quad \forall z \in C$	Multiplicative Identity $1+0i \in C$ $z+(1+0i) = 1+0i+z = z$
(iv)	Additive Inverse: $\forall\ a+bi \in C \ \exists\ -a-bi \in C$ Such that $a+bi+(-a-bi) = 0 \in C$	Multiplicative Inverse $\forall z = a+bi \in C \ \exists$ $z' = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2} i \in C$ $z \times z' = z' \times z = 1$

(v) Multiplication is distributive over addition.
$(a,b)[(c,d) \pm (e,f)] = (a,b)(c,d) \pm (a,b)(e,f)$
Hence complex number form field (∵ it statisfies above mention (xi) property.

Argand Diagram:
There is one-to-one corresponding between the complex number and point on the XY-plane: (i-e) For every complex number there is one and only one point in plane and for every point in the plane there is one and only one ~~point~~ complex Number.

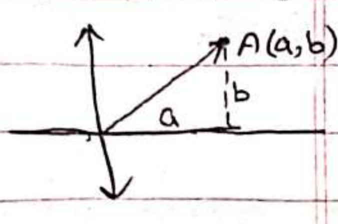


$\overrightarrow{OA} = 1+i$
$\overrightarrow{OB} = 4+2i$
$\overrightarrow{OC} = -3-i$
$\overrightarrow{OD} = -4+3i$

Date: ______ Day: ______

Modulus of a complex Number:

Modulus of complex number $z = a + bi$ is $\sqrt{a^2 + b^2}$.



$$|OA| = \sqrt{a^2 + b^2}.$$

Exercise: 1.1

Q1 Find the multiplicative inverse of the each of the following complex numbers.

(i) $(-4, 7)$

Multiplicative inverse of (a,b) is $\left(\dfrac{a}{a^2+b^2}, \dfrac{-b}{a^2+b^2}\right)$

So " " " $(-4,7)$ is $\left(\dfrac{-4}{(-4)^2+7^2}, \dfrac{-7}{(-4)^2+(7)^2}\right)$

$$\left(\frac{-4}{65}, \frac{-7}{65}\right).$$

Second method:

$= -4 + 7i = (-4, 7)$

M.I of $-4 + 7i$ is $\dfrac{1}{-4+7i}$

$$\frac{1}{-4+7i} \times \frac{-4-7i}{-4-7i}$$

$$\frac{-4-7i}{(-4)^2-(7i)^2} = \frac{-4-7i}{65} = \left(\frac{-4}{65}, \frac{-7}{65}\right).$$

(ii) $(\sqrt{2}, -\sqrt{5})$

M.I of (a,b) is $\left(\dfrac{a}{a^2+b^2}, \dfrac{-b}{a^2+b^2}\right)$

M.I " $(\sqrt{2}, -\sqrt{5})$ is $\left(\dfrac{\sqrt{2}}{(\sqrt{2})^2+(-\sqrt{5})^2}, \dfrac{-(-\sqrt{5})}{(\sqrt{2})^2+(-\sqrt{5})^2}\right)$

$$= \left(\frac{\sqrt{2}}{2+5}, \frac{\sqrt{5}}{2+5}\right)$$

$$= \left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right).$$

(iii) (1,0)

M.I of (1,0) is $\left(\frac{1}{1^2+0^2}, \frac{0}{1^2+0^2}\right)$

$= (1,0)$

QNo2) Separate into real and imaginary part:

(i) $\frac{2-7i}{4+5i} \Rightarrow \frac{2-7i}{4+5i} \times \frac{4-5i}{4-5i}$

$\Rightarrow \frac{8-10i-28i+35i^2}{(4)^2-(5i)^2} \Rightarrow \frac{-27-38i}{16+25} = \frac{-27}{41} - \frac{38}{41}i$

(ii) $\frac{(-2+3i)^2}{1+i} \Rightarrow \frac{4+9i^2-12i}{1+i} \Rightarrow \frac{4-9-12i}{1+i}$

$\Rightarrow \frac{-5-12i}{1+i} \times \frac{1-i}{1-i} \Rightarrow \frac{-5+5i-12i+12i}{1^2-i^2} \Rightarrow \frac{-17}{2} - \frac{7}{2}i$

(iii) $\frac{i}{1+i} \Rightarrow \frac{i}{1+i} \times \frac{1-i}{1-i} = \frac{i-i^2}{1^2-i^2} \Rightarrow \frac{1-i}{1+1} = \frac{1}{2} - \frac{i}{2}$

(iv) $\frac{(4+3i)^2}{4-3i} \Rightarrow \frac{16+9i^2+24i}{4-3i} \Rightarrow \frac{7+24i}{4-3i} \times \frac{4+3i}{4+3i}$

$\Rightarrow \frac{28+21i+96i+72i^2}{4^2-9i^2} = \frac{-44}{25} + \frac{117i}{25}$

QNo3 Prove that $\bar{z} = z$ iff z is real.

Proof: Let $z = a+bi$

Suppose $\bar{z} = z$ to prove z is real.

As $\bar{z} = a-bi$

So $a-bi = a+bi$

$0 = \cancel{a}+bi-\cancel{a}+bi$

$0 = 2bi$

$b = 0$

So $z = a+0i \Rightarrow z = a$ which is real.

Conversely:

Suppose z is real to prove $\bar{z} = z$

As z is real so $z = a$ (having 0 imaginary part)

$\bar{z} = a.$

hence $z = \bar{z}$

Q no 4: For $z \in C$ show that

(i) $\frac{z+\bar{z}}{2} = Re(z)$

Let $z = a+bi \implies Re(z) = a,\ Im(z) = b$

Now $\bar{z} = a-bi$

$\frac{z+\bar{z}}{2} \Rightarrow \frac{a+bi+a-bi}{2} \Rightarrow \frac{2a}{2} \Rightarrow a$

$a = Re(z)$ hence proved.

(ii) $\frac{z-\bar{z}}{2i} = Im(z)$

L.H.S

$\frac{a+bi-(a-bi)}{2i}$

$\frac{a+bi-a+bi}{2i}$

$\frac{2bi}{2i}$

b which is $Im(z)$.

(iii) $|z|^2 = z.\bar{z}$

As $z = a+bi$ since $a^2-(-b^2)$

$|z| = \sqrt{a^2+b^2}$ $a^2-(i^2b^2)$ $i^2=-1$

$|z|^2 = a^2+b^2$ $(a+bi)(a-bi)$

$|z|^2 = (a+bi)(a-bi)$

$|z|^2 = z.\bar{z}$

Second method: $z = a+bi,\ \bar{z} = a-bi$

$z.\bar{z}$

$(a+bi).(a-bi)$

$a^2+b^2 \Rightarrow (\sqrt{a^2+b^2})^2 = |z|^2.$

Date: Day:

Q5: If $z_1 = 2+i$, $z_2 = 3-2i$, $z_3 = 1+3i$ Then express $\frac{\bar{z}_1\bar{z}_3}{z_2}$ in the form of

Soln $\bar{z}_1 = 2-i$ $\bar{z}_2 = 3+2i$ $\bar{z}_3 = 1-3i$

Now $\frac{\bar{z}_1\bar{z}_3}{z_2} = \frac{(2-i)(1-3i)}{3-2i} = \frac{2-7i-3}{3-2i}$

$= \frac{-1-7i}{3-2i} \times \frac{3+2i}{3+2i} = \frac{-3+14-21i-2i}{9+4}$

$= \frac{11-23i}{13} = \frac{11}{13} - \frac{23}{13}i$

QNo6 If $z_1 = 2+7i$ $z_2 = -5+3i$ then evaluate the following

(i) $|2z_1 - 4z_2| = |4+14i+20-12i| = |24+2i|$

$= \sqrt{(24)^2+(2)^2} = \sqrt{576+4} = \sqrt{580} = \sqrt{4\times145} = 2\sqrt{145}$

(ii) $|3z_1 + 2\bar{z}_1| = |6+21i+4-14i| = |10+7i| = \sqrt{100+49}$

$= \sqrt{149}$

(iii) $|-7z_2 + 2\bar{z}_2| = |+35-21i-10-6i| = |25-27i|$

$= \sqrt{(25)^2+(27)^2} =$

(iv) $|(z_1+z_2)^3| = |z_1+z_2|^3 \Rightarrow |2+7i-5+3i|^3$

$= |-3+10i|^3 = \cancel{\left(\sqrt{(5)^2+(-1)^2}\right)^3 = (26^{1/2})^3 = 26^{3/2}}$

$= (\sqrt{9+100})^{3/2} = (\sqrt{109})^2\sqrt{109} = 109\sqrt{109}$

QNo7 Show that $i^{n+1} + i^{n+2} + i^{n+3} + i^{n+4} = 0$ for all $n \in N$

L.H.S $i^{n+1} + i^{n+2} + i^{n+3} + i^{n+4}$

$i^n(i) + i^n \cdot i^2 + i^n \cdot i^3 + i^n \cdot i^4$

$i^n[i + i^2 + i^3 + i^4]$

$i[i - 1 - i + 1]$

$i \times 0$

0