# Exercise # 10.1

**Q:1** Prove that

(i) $\sin(\pi+\theta) = -\sin\theta$

L.H.S $\sin(\pi+\theta)$ Apply formula

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \sin\pi\cos\theta + \cos\pi\sin\theta$

$= 0\cos\theta + (-1)\sin\theta$

$= 0 - \sin\theta = -\sin\theta = R.H.S$

(ii) $\cos(\pi+\theta) = -\cos\theta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

L.H.S $\cos(\pi+\theta)$

$= \cos\pi\cos\theta - \sin\pi\sin\theta$

$\cos\pi = -1$
$\sin\pi = 0$

$= (-1)\cos\theta - 0\sin\theta$

$= -\cos\theta = R.H.S$

(iii) $\tan(\frac{\pi}{2}+\theta) = -\cot\theta$

$\tan\frac{\pi}{2} = \infty$

So we can't used the formula

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

L.H.S $\tan(\frac{\pi}{2}+\theta)$

$= \dfrac{\sin(\frac{\pi}{2}+\theta)}{\cos(\frac{\pi}{2}+\theta)}$

By formulae

$= \dfrac{\sin\frac{\pi}{2}\cos\theta + \cos\frac{\pi}{2}\sin\theta}{\cos\frac{\pi}{2}\cos\theta - \sin\frac{\pi}{2}\sin\theta}$

$\sin\frac{\pi}{2} = 1$
$\cos\frac{\pi}{2} = 0$

$= \dfrac{1\cos\theta + 0\sin\theta}{0\cos\theta - 1\sin\theta}$

$= \dfrac{\cos\theta}{-\sin\theta} = -\cot\theta = R.H.S$

(iv) $\cos(270°+\theta) = \sin\theta$

$\cos 270° = 0$
$\sin 270° = -1$

L.H.S $\cos(270°+\theta)$

$= \cos 270°\cos\theta - \sin 270°\sin\theta$

$= 0\cos\theta - (-1)\sin\theta$

$= 0+\sin\theta = \sin\theta = R.H.S$

(v) $\tan(270°+\theta) = -\cot\theta$

L.H.S $\tan(270°+\theta)$

$= \dfrac{\sin(270°+\theta)}{\cos(270°+\theta)}$

$\cos 270° = 0$
$\sin 270° = -1$

$= \dfrac{\sin 270°\cos\theta + \cos 270°\sin\theta}{\cos 270°\cos\theta - \sin 270°\sin\theta}$

$= \dfrac{(-1)\cos\theta + 0\sin\theta}{0\cos\theta - (-1)\sin\theta}$

$= \dfrac{-\cos\theta + 0}{0+\sin\theta}$

$= -\dfrac{\cos\theta}{\sin\theta}$

$= -\cot\theta = R.H.S$

(vi) $\sin(360^\circ + \theta) = +\sin\theta$

L.H.S $\sin(360^\circ + \theta)$

$= \sin 360^\circ \cos\theta + \cos 360^\circ \sin\theta$

$= 0 \cdot \cos\theta + 1 \sin\theta$

$= \sin\theta = R.H.S$

(vi) $\sin(360^\circ - \theta) = -\sin\theta$

L.H.S $\sin(360^\circ - \theta)$

$= \sin 360^\circ \cos\theta - \cos 360^\circ \sin\theta$

$= 0 \cos\theta - 1 \sin\theta$

$= -\sin\theta = R.H.S$

(vii) $\cot(360^\circ + \theta) = \cot\theta$

L.H.S $\cot(360^\circ + \theta)$

$$= \frac{\cos(360^\circ+\theta)}{\sin(360^\circ+\theta)}$$

$$= \frac{\cos 360^\circ \cos\theta - \sin 360^\circ \sin\theta}{\sin 360^\circ \cos\theta + \cos 360^\circ \sin\theta}$$

$$= \frac{1 \cos\theta - 0 \sin\theta}{0 \cos\theta + 1 \sin\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta = R.H.S$$

Q.2: Show that

(i) $\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$

L.H.S $\sin(\alpha+\beta) + \sin(\alpha-\beta)$

$= (\sin\alpha\cos\beta + \cancel{\cos\alpha\sin\beta}) + (\sin\alpha\cos\beta - \cancel{\cos\alpha\sin\beta})$

$= \sin\alpha\cos\beta + \sin\alpha\cos\beta$

$= 2\sin\alpha\cos\beta$

$= R.H.S$

(ii) $\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\sin\alpha\sin\beta$

L.H.S $\cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= (\cos\alpha\cos\beta - \sin\alpha\sin\beta) - (\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$= \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta - \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta$

$= -2\sin\alpha\sin\beta = R.H.S$

Q.3 Show that

(i) $\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1 = 1 - 2\sin^2\frac{\alpha}{2}$

Sol

$\cos\alpha = \cos 2\left(\frac{\alpha}{2}\right)$

$= \cos\left(\frac{\alpha+\alpha}{2}\right)$

$= \cos\left(\frac{\alpha}{2} + \frac{\alpha}{2}\right)$

$= \cos\frac{\alpha}{2}\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\sin\frac{\alpha}{2}$

$= \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}$ But $\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$= \cos^2\frac{\alpha}{2} - \left(1 - \cos^2\frac{\alpha}{2}\right)$

$= \cos^2\frac{\alpha}{2} - 1 + \cos^2\frac{\alpha}{2}$

$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1 \longrightarrow (1)$

$= 2\left(1 - \sin^2\frac{\alpha}{2}\right) - 1$

$= 2 - 2\sin^2\frac{\alpha}{2} - 1$

$= 1 - 2\sin^2\frac{\alpha}{2} \longrightarrow (2)$

From (1) and (2), it is proved

$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1 = 1 - 2\sin^2\frac{\alpha}{2}$.

(ii) $\sin(\alpha+\beta)\ \sin(\alpha-\beta) = \cos^2\beta - \cos^2\alpha$

L.H.S $\quad \sin(\alpha+\beta)\cdot\sin(\alpha-\beta)$

$= (\sin\alpha\cos\beta + \cos\alpha\sin\beta)\cdot(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$

$(a+b)(a-b) = a^2 - b^2$

$= (\sin\alpha\cos\beta)^2 - (\cos\alpha\sin\beta)^2$

$= \sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta$

$= (1-\cos^2\alpha)\cos^2\beta - \cos^2\alpha(1-\cos^2\beta)$

$= \cos^2\beta - \cos^2\alpha\cos^2\beta - \cos^2\alpha + \cos^2\alpha\cos^2\beta$

$= \cos^2\beta - \cos^2\alpha = $ R.H.S.



Q.4 If $\sin\alpha = 4/5$ and $\sin\beta = 12/13$ and both $\alpha$ & $\beta$ are measures of 1st quadrant angles, then find

(i) $\sin(\alpha+\beta)$ (ii) $\cos(\alpha+\beta)$ (iii) $\tan(\alpha+\beta)$

Sol First find $\cos\alpha$ and $\cos\beta$ from formula $\sin^2\theta+\cos^2\theta = 1$

$\sin^2\alpha + \cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha = 1-\sin^2\alpha$

$\Rightarrow \cos\alpha = \pm\sqrt{1-\sin^2\alpha}$

$\Rightarrow \cos\alpha = \pm\sqrt{1-\left(\frac{4}{5}\right)^2}$

$\Rightarrow \cos\alpha = \pm\sqrt{1-\frac{16}{25}}$

$\Rightarrow \cos\alpha = \pm\sqrt{\frac{25-16}{25}}$

$= \pm\sqrt{\frac{9}{25}}$

$\cos\alpha = \pm 3/5$

$\Rightarrow \boxed{\cos\alpha = 3/5}$ $\because$ 1st Q angle

& $\sin^2\beta + \cos^2\beta = 1$

$\Rightarrow \cos^2\beta = 1-\sin^2\beta$

$\Rightarrow \cos\beta = \pm\sqrt{1-\sin^2\beta}$

$\Rightarrow \cos\beta = \pm\sqrt{1-\left(\frac{12}{13}\right)^2}$

$\Rightarrow \cos\beta = \pm\sqrt{1-\frac{144}{169}}$

$\cos\beta = \pm\sqrt{\frac{169-144}{169}}$

$\cos\beta = \pm\sqrt{\frac{25}{169}}$

$\cos\beta = \pm 5/13$

$\Rightarrow \boxed{\cos\beta = 5/13}$ $\because$ 1st Q

Now (i) $\sin(\alpha+\beta)$

$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{20}{65} + \frac{36}{65}$

$= \frac{20+36}{65}$

$= 56/65$ Ans

(ii) $\cos(\alpha+\beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{15}{65} - \frac{48}{65}$

$= \frac{15-48}{65}$

$= -33/65$ Ans

(iii) $\tan(\alpha+\beta)$

$= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$

$= \frac{56/65}{-33/65}$

$= -\frac{56}{33}$ Ans

Q.5 If $\tan\alpha = 3/4$ and $\sec\beta = 13/5$ and neither the terminal side of $\alpha$ nor $\beta$ is in 1st quadrant then find

(i) $\sin(\alpha+\beta)$ (ii) $\cos(\alpha+\beta)$ (iii) $\tan(\alpha+\beta)$

Sol $\tan\alpha = 3/4 \Rightarrow \alpha$ is in 1st or 3rd quadrant but given that $\alpha$ is not in 1st Q $\Rightarrow \alpha$ must be in 3rd Q

$\sec\beta = 13/5 \Rightarrow \beta$ is in 1st or 4th Q but given that $\beta$ is not in 1st Q $\Rightarrow \beta$ is in 4th Q

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$\because$ Now $\tan\alpha = 3/4$

Now by formula

$$1+\tan^2\alpha = \sec^2\alpha$$

$\Rightarrow \sec^2\alpha = 1+(\tan\alpha)^2$

$\Rightarrow (\sec\alpha)^2 = 1+(3/4)^2$

$\Rightarrow (\sec\alpha)^2 = 1+\frac{9}{16} = \frac{16+9}{16}$

$\Rightarrow (\sec\alpha)^2 = \frac{25}{16}$ take sq. root

$\Rightarrow \sec\alpha = \pm\sqrt{25/16}$

$\Rightarrow \sec\alpha = \pm 5/4$

$\Rightarrow \sec\alpha = -5/4 \quad \because$ 3rd Q

$\Rightarrow \boxed{\cos\alpha = -\frac{4}{5}}$

Now $\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha = 1-\cos^2\alpha$

$\sin^2\alpha = 1-\left(\frac{-4}{5}\right)^2$

$\sin^2\alpha = 1-\frac{16}{25}$

$\sin^2\alpha = \frac{9}{25}$

$\Rightarrow \sin\alpha = \pm 3/5$

$\Rightarrow \boxed{\sin\alpha = -3/5} \quad \because$ 3rd Q

& $\sec\beta = 13/5$

$\Rightarrow \boxed{\cos\beta = 5/13}$

Now $\sin^2\beta + \cos^2\beta = 1$

$\sin^2\beta = 1-\cos^2\beta$

$\sin^2\beta = 1-\left(\frac{5}{13}\right)^2$

$\sin^2\beta = 1-\frac{25}{169}$

$\sin^2\beta = \frac{169-25}{169}$

$\sin^2\beta = \frac{144}{169}$

$\Rightarrow \sin\beta = \pm 12/13$

$\Rightarrow \boxed{\sin\beta = \frac{-12}{13}} \quad \because$ 4th Q

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Now

(i) $\sin(\alpha+\beta)$

$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{-3}{5}\right)\left(\frac{5}{13}\right)+\left(\frac{-4}{5}\right)\left(\frac{-12}{13}\right)$

$= \frac{-15}{65}+\frac{48}{65}$

$= \frac{-15+48}{65}$

$= 33/65$ Ans.

(ii) $\cos(\alpha+\beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{-4}{5}\right)\left(\frac{5}{13}\right)-\left(\frac{-3}{5}\right)\left(\frac{-12}{13}\right)$

$= \frac{-20}{65}-\frac{36}{65}$

$= \frac{-20-36}{65}$

$= \frac{-56}{65}$ Ans

(iii) $\tan(\alpha+\beta)$

$= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$

$= \frac{33/65}{-56/65}$

$= -33/56$ Ans

Q:6 Show that

(i) $\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$

R.H.S

$$\frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$$

$$= \frac{\frac{\cos\alpha}{\sin\alpha}\cdot\frac{\cos\beta}{\sin\beta}-1}{\frac{\cos\alpha}{\sin\alpha}+\frac{\cos\beta}{\sin\beta}}$$

$$= \frac{\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\frac{\cos\alpha\sin\beta+\cos\beta\sin\alpha}{\sin\alpha\sin\beta}} = \frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\sin\beta+\cos\beta\sin\alpha}$$

$$= \frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)} = \cot(\alpha+\beta)$$

$=$ L.H.S

(ii) $\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

L.H.S $\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$

$= \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$

$= \dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$

$= \dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\sin\beta}{\cos\beta}$

$= \tan\alpha + \tan\beta =$ R.H.S



Q.7: Prove that

(i) $\tan(\frac{\pi}{4}+\theta) = \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

Sol L.H.S $\tan(\frac{\pi}{4}+\theta)$

Formula $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$= \dfrac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}$

$= \dfrac{1+\tan\theta}{1-(1)\tan\theta} = \dfrac{1+\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin\theta}{\cos\theta}} = \dfrac{\frac{\cos\theta+\sin\theta}{\cancel{\cos\theta}}}{\frac{\cos\theta-\sin\theta}{\cancel{\cos\theta}}}$

$= \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} =$ R.H.S

(ii) $\tan(\frac{\pi}{4}-\theta) = \dfrac{1-\tan\theta}{1+\tan\theta}$

L.H.S $\tan(\frac{\pi}{4}-\theta)$

$\tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$= \dfrac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$

& $\tan\frac{\pi}{4} = 1$

$= \dfrac{1-\tan\theta}{1+\tan\theta} =$ R.H.S

(iii) $\dfrac{\tan(\alpha+\beta)}{\cot(\alpha-\beta)} = \dfrac{\tan^2\alpha-\tan^2\beta}{1-\tan^2\alpha\tan^2\beta}$

L.H.S $\dfrac{\tan(\alpha+\beta)}{\cot(\alpha-\beta)} = \tan(\alpha+\beta)\cdot\dfrac{1}{\cot(\alpha-\beta)}$

$= \tan(\alpha+\beta)\cdot\tan(\alpha-\beta)$

$= \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\cdot\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$= \dfrac{(\tan\alpha)^2-(\tan\beta)^2}{(1)^2-(\tan\alpha\tan\beta)^2}$

$= \dfrac{\tan^2\alpha-\tan^2\beta}{1-\tan^2\alpha\tan^2\beta}$

$=$ R.H.S

**Q:8** Prove that

$$\frac{\sin\theta}{\sec 4\theta} + \frac{\cos\theta}{\operatorname{cosec} 4\theta} = \sin 5\theta$$

**L.H.S** $\quad \dfrac{\sin\theta}{\sec 4\theta} + \dfrac{\cos\theta}{\operatorname{cosec} 4\theta}$

$$= \sin\theta\left(\frac{1}{\sec 4\theta}\right) + \cos\theta\left(\frac{1}{\operatorname{cosec} 4\theta}\right)$$

$$= \sin\theta\cos 4\theta + \cos\theta\sin 4\theta$$

$$= \sin(\theta + 4\theta)$$

$$= \sin 5\theta = R.H.S$$

**Q.9** Show that $\dfrac{\sin(180^\circ-\alpha)\cos(270^\circ-\alpha)}{\sin(180^\circ+\alpha)\cos(270^\circ+\alpha)} = 1$

**L.H.S** $\quad \dfrac{\sin(180^\circ-\alpha)\cos(270^\circ-\alpha)}{\sin(180^\circ+\alpha)\cos(270^\circ+\alpha)}$ by formulae

$$= \frac{(\sin 180^\circ\cos\alpha - \cos 180^\circ\sin\alpha)(\cos 270^\circ\cos\alpha + \sin 270^\circ\sin\alpha)}{(\sin 180^\circ\cos\alpha + \cos 180^\circ\sin\alpha)(\cos 270^\circ\cos\alpha - \sin 270^\circ\sin\alpha)}$$

$\sin 180^\circ = 0,\ \cos 180^\circ = -1,\ \cos 270^\circ = 0,\ \sin 270^\circ = -1$

$$= \frac{(0\cdot\cos\alpha - (-1)\sin\alpha)(0\cos\alpha + (-1)\sin\alpha)}{(0\cos\alpha + (-1)\sin\alpha)(0\cos\alpha - (-1)\sin\alpha)}$$

$$= \frac{(0+\sin\alpha)(0-\sin\alpha)}{(0-\sin\alpha)(0+\sin\alpha)}$$

$$= \frac{(\sin\alpha)(-\sin\alpha)}{(-\sin\alpha)(\sin\alpha)}$$

$$= 1 = R.H.S$$

**Q:10** Express each of the following in the form $r\sin(\theta+\phi)$ where the terminal ray of $\theta$ and $\phi$ are in the 1st quadrants.

(i) $4\sin\theta + 3\cos\theta$

**Sol** $\times$ and $\div$ by $\sqrt{4^2+3^2}$

$$= \frac{\sqrt{4^2+3^2}}{\sqrt{4^2+3^2}}(4\sin\theta + 3\cos\theta)$$

$$= \frac{5}{5}(4\sin\theta + 3\cos\theta)$$

$$= 5\left(\sin\theta\,\frac{4}{5} + \cos\theta\,\frac{3}{5}\right)$$

Let $5 = r$, $\cos\phi = 4/5$ and $\sin\phi = 3/5$, we get

$$= r(\sin\theta\cos\phi + \cos\theta\sin\phi)$$

$$= r\sin(\theta+\phi) = R.H.S$$

(ii) $15\sin\theta + 8\cos\theta$.

**Sol** $\times$ and $\div$ by $\sqrt{15^2+8^2}$

$$= \frac{\sqrt{15^2+8^2}}{\sqrt{15^2+8^2}}(15\sin\theta + 8\cos\theta)$$

$$= \frac{\sqrt{289}}{\sqrt{289}}\left(15\sin\theta + 8\cos\theta\right)$$

$$= \frac{17}{17}\left(\sin\theta \cdot 15 + \cos\theta \cdot 8\right)$$

$$= 17\left(\sin\theta \cdot \frac{15}{17} + \cos\theta \cdot \frac{8}{17}\right)$$

$$= r\left(\sin\theta\cos\phi + \cos\theta\sin\phi\right) = r\sin(\theta+\phi)$$

$\because$ $r = 17$, $\cos\phi = 15/17$ and $\sin\phi = 8/17$

(iii) $2\sin\theta - 5\cos\theta$

$$= 2\sin\theta + (-5)\cos\theta$$

$\times$ and $\div$ by $\sqrt{(2)^2+(-5)^2}$

$$= \frac{\sqrt{(2)^2+(-5)^2}}{\sqrt{(2)^2+(-5)^2}}\left\{2\sin\theta + (-5)\cos\theta\right\}$$

$$= \frac{\sqrt{29}}{\sqrt{29}}\left\{2\sin\theta + (-5)\cos\theta\right\}$$

$$= \sqrt{29}\left\{\sin\theta \cdot \frac{2}{\sqrt{29}} + \cos\theta\left(\frac{-5}{\sqrt{29}}\right)\right\}$$

Let $\sqrt{29} = r$, $\frac{2}{\sqrt{29}} = \cos\phi$, $\frac{-5}{\sqrt{29}} = \sin\phi$

$$= r\left\{\sin\theta\cos\phi + \cos\theta\sin\phi\right\}$$

$$= r\sin(\theta+\phi)$$



(iv) $\sin\theta + \cos\theta$

Sol $1\sin\theta + 1\cos\theta$

$\times$ and $\div$ by $\sqrt{1^2+1^2}$

$$= \frac{\sqrt{1^2+1^2}}{\sqrt{1^2+1^2}}\left(1\sin\theta + 1\cos\theta\right)$$

$$= \frac{\sqrt{2}}{\sqrt{2}}\left(1\sin\theta + 1\cos\theta\right)$$

$$= \sqrt{2}\left(\sin\theta\,\frac{1}{\sqrt{2}} + \cos\theta\,\frac{1}{\sqrt{2}}\right)$$

Let $\sqrt{2} = r$, $\frac{1}{\sqrt{2}} = \cos\phi$ and $\frac{1}{\sqrt{2}} = \sin\phi$

$$= \sqrt{2}\left(\sin\theta\cos\phi + \cos\theta\sin\phi\right)$$

$$= \sqrt{2}\,\sin(\theta+\phi)$$

$$= r\,\sin(\theta+\phi)$$

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Quote: I do not know the key to success, but the key to failure is trying to please every body (Bill Cosby 1937)

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# Exercise # 10.2

**Q:1** If $\sin\theta = 5/13$ and terminal ray of $\theta$ is in second quadrant, then find

(i) $\sin 2\theta$ (ii) $\cos 2\theta$ (iii) $\tan 2\theta$

**Sol** 1st find $\cos\theta$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \cos^2\theta = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{169-25}{169} = \frac{144}{169}$$

$$\Rightarrow \cos^2\theta = \frac{144}{169}$$

$$\Rightarrow \sqrt{\cos^2\theta} = \pm\sqrt{\frac{144}{169}}$$

$$\Rightarrow \cos\theta = \pm 12/13$$

$$\Rightarrow \cos\theta = \frac{-12}{13} \quad (\because \text{2nd quadrant})$$

(i) $\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\frac{5}{13}\right)\left(\frac{-12}{13}\right)$

$= \frac{-120}{169}$

(ii) $\cos 2\theta$

$= \cos^2\theta - \sin^2\theta$

$= \left(\frac{-12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{144}{169} - \frac{25}{169}$

$= \frac{144-25}{169}$

$= \frac{119}{169}$

(iii) $\tan 2\theta$

$= \frac{\sin 2\theta}{\cos 2\theta}$

$= \frac{-120/169}{119/169}$

$= -120/119$ Ans

**Q:2** If $\sin\theta = 4/5$ and $\theta$ in 2nd, then find

(i) $\sin 2\theta$ (ii) $\cos\frac{\theta}{2}$

**Sol** First find $\cos\theta$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$$

$$\cos^2\theta = 1 - \left(\frac{4}{5}\right)^2$$

$$\Rightarrow \cos^2\theta = 1 - \frac{16}{25} = \frac{25-16}{25} = 9/25$$

$$\Rightarrow \cos^2\theta = 9/25, \text{ take sq. root}$$

$$\Rightarrow \cos\theta = \pm 3/5$$

$$\Rightarrow \boxed{\cos\theta = -3/5} \quad \because \text{2nd quadrant}$$

(i) $\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right)$

$= \frac{-24}{25}$ Ans

(ii) $\cos\frac{\theta}{2}$

$= \sqrt{\frac{1+\cos\theta}{2}}$

$= \sqrt{\frac{1+(-3/5)}{2}}$

$= \sqrt{\frac{\frac{5-3}{5}}{2}} = \sqrt{\frac{2}{5}\times\frac{1}{2}}$

$= \frac{1}{\sqrt{5}}$ Ans

**Q:3** If $\cos\theta = -3/7$ and $\theta$ in 3rd quadrant. Find $\sin\frac{\theta}{2}$.

**Sol** By formula

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-(-3/7)}{2}} = \sqrt{\frac{\frac{7+3}{7}}{2}}$$

$$= \sqrt{\frac{10}{7}\times\frac{1}{2}} = \sqrt{5/7} \text{ Ans}$$

**Q:4** Use double angle identities, find the values of

(i) $\sin\frac{2\pi}{3}$

**Sol** $\sin\frac{2\pi}{3}$

$$= \sin 2\left(\frac{\pi}{3}\right)$$

$$= 2\sin\frac{\pi}{3}\cos\frac{\pi}{3}$$

$$= 2\sin 60^\circ \cos 60^\circ$$

$$= 2\left(\sqrt{3}/2\right)\left(\frac{1}{2}\right)$$

$$= \sqrt{3}/2$$

(ii) $\cos\frac{2\pi}{3}$

**Sol** $\cos 2\left(\frac{\pi}{3}\right)$

$$= \cos^2\frac{\pi}{3} - \sin^2\frac{\pi}{3}$$

$$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} - \frac{3}{4}$$

$$= \frac{1-3}{4} = \frac{-2}{4} = -\frac{1}{2} \text{ Ans}$$

Prove the following identities

(5) $(\sin\theta - \cos\theta)^2 = 1 - \sin 2\theta$

**L.H.S** $(\sin\theta - \cos\theta)^2$

$$= (\sin\theta)^2 + (\cos\theta)^2 - 2\sin\theta\cos\theta$$

$$= \sin^2\theta + \cos^2\theta - \sin 2\theta = 1 - \sin 2\theta = R.H.S$$



(6) $\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$

**L.H.S** $\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\tan\theta}{\sec^2\theta}$

$$= 2\tan\theta \cdot \cos^2\theta$$

$$= 2\frac{\sin\theta}{\cos\theta}\cdot\cos^2\theta$$

$$= 2\sin\theta\cos\theta = \sin 2\theta = R.H.S$$

(7) $\frac{1}{\sec 2\alpha} = \cos^4\alpha - \sin^4\alpha$

**R.H.S** $\cos^4\alpha - \sin^4\alpha$

$$= (\cos^2\alpha)^2 - (\sin^2\alpha)^2 \qquad a^2-b^2 \text{ formula}$$

$$= (\cos^2\alpha + \sin^2\alpha)(\cos^2\alpha - \sin^2\alpha)$$

$$= (1)\cdot(\cos 2\alpha)$$

$$= \cos 2\alpha = \frac{1}{\sec 2\alpha} = L.H.S$$

(8) $\frac{1+\cos 2\theta}{\sin 2\theta} = \cot\theta$

**L.H.S** $\frac{1+\cos 2\theta}{\sin 2\theta}$ Apply double angle formulae

$$= \frac{1+\cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{1-\sin^2\theta+\cos^2\theta}{2\sin\theta\cos\theta} = \frac{\cos^2\theta+\cos^2\theta}{2\sin\theta\cos\theta} = \frac{2\cos^2\theta}{2\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta = R.H.S$$

(9) $\operatorname{cosec} 2\alpha - \cot 2\alpha = \tan\alpha$

L.H.S $\quad \operatorname{cosec} 2\alpha - \cot 2\alpha$

$$= \frac{1}{\sin 2\alpha} - \frac{\cos 2\alpha}{\sin 2\alpha}$$

$$= \frac{1 - \cos 2\alpha}{\sin 2\alpha}$$

$$= \frac{1-(\cos^2\alpha - \sin^2\alpha)}{2\sin\alpha\cos\alpha}$$

$$= \frac{1-\cos^2\alpha + \sin^2\alpha}{2\sin\alpha\cos\alpha}$$

$$= \frac{\sin^2\alpha + \sin^2\alpha}{2\sin\alpha\cos\alpha}$$

$$= \frac{2\sin^2\alpha}{2\sin\alpha\cos\alpha}$$

$$= \frac{\sin\alpha}{\cos\alpha}$$

$$= \tan\alpha = R.H.S$$

(10) $\dfrac{\sin 3\beta}{\sin\beta} - \dfrac{\cos 3\beta}{\cos\beta} = 2$

L.H.S $\quad \dfrac{\sin 3\beta}{\sin\beta} - \dfrac{\cos 3\beta}{\cos\beta}$

$$= \frac{\sin 3\beta\cos\beta - \cos 3\beta\sin\beta}{\sin\beta\cos\beta}$$

$$= \frac{\sin(3\beta - \beta)}{\sin\beta\cos\beta}$$

$$= \frac{\sin 2\beta}{\sin\beta\cos\beta}$$

$$= \frac{2\sin\beta\cos\beta}{\sin\beta\cos\beta}$$

$$= 2 = R.H.S$$

(11) $\dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = \dfrac{2+\sin 2\theta}{2}$

L.H.S $\quad \dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} \qquad a^3 - b^3 = (a-b)(a^2+b^2+ab)$

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \cos\theta\sin\theta)}{(\cos\theta - \sin\theta)}$$

$$= \cos^2\theta + \sin^2\theta + \cos\theta\sin\theta$$

$$= 1 + \cos\theta\sin\theta$$

$\times$ and $\div$ by 2

$$= \frac{2}{2}(1 + \cos\theta\sin\theta)$$

$$= \frac{2 + 2\cos\theta\sin\theta}{2}$$

$$= \frac{2 + \sin 2\theta}{2}$$

$$= R.H.S$$

(12) $\dfrac{\sin 3\theta}{\cos\theta} + \dfrac{\cos 3\theta}{\sin\theta} = 2\cot 2\theta$

L.H.S $\quad \dfrac{\sin 3\theta}{\cos\theta} + \dfrac{\cos 3\theta}{\sin\theta}$

$$= \frac{\sin 3\theta\sin\theta + \cos 3\theta\cos\theta}{\cos\theta\sin\theta}$$

$$= \frac{\cos(3\theta - \theta)}{\cos\theta\sin\theta}$$

$$= \frac{\cos 2\theta}{\cos\theta\sin\theta}$$

$\times$ & $\div$ by 2

$$= \frac{2\cos 2\theta}{2\cos\theta\sin\theta}$$

$$= \frac{2\cos 2\theta}{\sin 2\theta} = 2\cot 2\theta = R.H.S$$

(13) $\tan\theta \cdot \tan\frac{\theta}{2} = \sec\theta - 1$

L.H.S $\quad \tan\theta \cdot \tan\frac{\theta}{2}$

$$= \frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta/2}{\cos\theta/2}$$

$$= \frac{\sin 2\left(\frac{\theta}{2}\right)}{\cos\theta} \cdot \frac{\sin\theta/2}{\cos\theta/2}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\theta} \cdot \frac{\sin\theta/2}{\cos\theta/2}$$

$$= \frac{2\sin\frac{\theta}{2} \cdot \sin\frac{\theta}{2}}{\cos\theta}$$

$$= \frac{2\sin^2\theta/2}{\cos\theta}$$

$$= {}^{*}\ \frac{1-\cos\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta}$$

$$= \sec\theta - 1$$

$$= R.H.S$$

$\star$ Half angle formula $\sin\frac{\theta}{2} = \sqrt{\dfrac{1-\cos\theta}{2}}$

(14) $$\frac{\sin\alpha+\cos\alpha}{\cos\alpha-\sin\alpha}+\frac{\sin\alpha-\cos\alpha}{\cos\alpha+\sin\alpha}=2\tan2\alpha$$

L.H.S $$\frac{\sin\alpha+\cos\alpha}{\cos\alpha-\sin\alpha}+\frac{\sin\alpha-\cos\alpha}{\cos\alpha+\sin\alpha}$$

$$=\frac{(\sin\alpha+\cos\alpha)(\cos\alpha+\sin\alpha)+(\sin\alpha-\cos\alpha)(\cos\alpha-\sin\alpha)}{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}$$

$$=\frac{(\sin\alpha+\cos\alpha)^2+(\sin\alpha\cos\alpha-\sin^2\alpha-\cos^2\alpha+\cos\alpha\sin\alpha)}{(\cos\alpha)^2-(\sin\alpha)^2}$$

$$=\frac{(\cancel{\sin^2\alpha}+\cancel{\cos^2\alpha}+2\sin\alpha\cos\alpha)+(2\sin\alpha\cos\alpha-\cancel{\sin^2\alpha}-\cancel{\cos^2\alpha})}{\cos2\alpha}$$

$$=\frac{4\sin\alpha\cos\alpha}{\cos2\alpha}=\frac{2(2\sin\alpha\cos\alpha)}{\cos2\alpha}=\frac{2\sin2\alpha}{\cos2\alpha}=2\tan2\alpha$$
$$=R.H.S$$

(15) $$\frac{\cot^2\beta-1}{\operatorname{cosec}^2\beta}=\cos2\beta$$

L.H.S $$\frac{\cot^2\beta-1}{\operatorname{cosec}^2\beta}$$

$$=(\cot^2\beta-1)\times\frac{1}{\operatorname{cosec}^2\beta}$$

$$=\left(\frac{\cos^2\beta}{\sin^2\beta}-1\right)\times\sin^2\beta$$

$$=\left(\frac{\cos^2\beta-\sin^2\beta}{\sin^2\beta}\right)\times\sin^2\beta$$

$$=\cos^2\beta-\sin^2\beta$$

$$=\cos2\beta$$

$$=R.H.S$$

(16) $$\sin\theta=\frac{2}{\tan\frac{\theta}{2}+\cot\frac{\theta}{2}}$$

R.H.S $$\frac{2}{\tan\frac{\theta}{2}+\cot\frac{\theta}{2}}=\frac{2}{\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}+\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}}=$$

$$=\frac{2}{\frac{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}}{\cos\frac{\theta}{2}\cdot\sin\frac{\theta}{2}}}=\frac{2}{\frac{1}{\cos\frac{\theta}{2}\sin\frac{\theta}{2}}}=2\cos\frac{\theta}{2}\sin\frac{\theta}{2}$$
$$=\sin2\left(\frac{\theta}{2}\right)$$
$$=\sin\theta=L.H.S$$

(17) $$\sin^2\frac{\theta}{2}=\frac{\sin\theta\tan\frac{\theta}{2}}{2}$$

R.H.S $$\frac{\sin\theta\tan\frac{\theta}{2}}{2}$$

$$=\frac{1}{2}\sin2\frac{\theta}{2}\tan\frac{\theta}{2}$$

$$=\frac{1}{2}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\cdot\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)$$

$$=\sin\frac{\theta}{2}\cdot\sin\frac{\theta}{2}=\sin^2\frac{\theta}{2}=L.H.S$$

(18) $$\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\cos\alpha$$

L.H.S $$\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\frac{1-\frac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{1+\frac{\sin^2\frac{\alpha}{2}}{\cos^2\alpha/2}}=\frac{\frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{\frac{\cos^2\alpha/2+\sin^2\alpha/2}{\cos^2\alpha/2}}$$

$$= \frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}} = \frac{\cos 2\frac{\alpha}{2}}{1} = \cos\alpha = R.H.S$$

(19) $$\tan 2\beta + \sec 2\beta = \frac{\cos\beta + \sin\beta}{\cos\beta - \sin\beta}$$

<u>L.H.S</u> $\tan 2\beta + \sec 2\beta$

$$= \frac{\sin 2\beta}{\cos 2\beta} + \frac{1}{\cos 2\beta}$$

$$= \frac{\sin 2\beta + 1}{\cos 2\beta} = \frac{2\sin\beta\cos\beta + 1}{\cos 2\beta}$$

$$= \frac{(2\sin\beta\cos\beta) + (\cos^2\beta + \sin^2\beta)}{\cos^2\beta - \sin^2\beta}$$

$$= \frac{(\cos\beta + \sin\beta)^2}{(\cos\beta + \sin\beta)(\cos\beta - \sin\beta)}$$

$$= \frac{\cos\beta + \sin\beta}{\cos\beta - \sin\beta}$$

$= R.H.S$

(20) $$\cos^4\theta = \frac{3}{8} + \frac{\cos 2\theta}{8} + \frac{\cos 4\theta}{8}$$

<u>L.H.S</u> $\cos^4\theta$

$$= (\cos^2\theta)^2$$

$$= \left(\frac{1 + \cos 2\theta}{2}\right)^2$$

$$= \frac{1^2 + \cos^2 2\theta + 2\cos 2\theta}{4}$$

$$= \frac{1}{4}\left\{1 + \cos^2 2\theta + 2\cos 2\theta\right\}$$

$$= \frac{1}{4}\left\{1 + \frac{1 + \cos 4\theta}{2} + 2\cos 2\theta\right\}$$

$$= \frac{1}{4}\left\{1 + \frac{1}{2} + \frac{\cos 4\theta}{2} + 2\cos 2\theta\right\}$$

$$= \frac{1}{4}\left\{\frac{3}{2} + \frac{\cos 4\theta}{2} + 2\cos 2\theta\right\}$$

$$= \frac{3}{8} + \frac{\cos 4\theta}{8} + \frac{2\cos 2\theta}{4}$$

$$= \frac{3}{8} + \frac{\cos 4\theta}{8} + \frac{\cos 2\theta}{2} = R.H.S$$

<u>Half angle identity</u>

$$\cos\frac{\theta}{2} = \sqrt{\frac{1 + \cos\theta}{2}}$$

$$\cos^2\frac{\theta}{2} = \frac{1 + \cos\theta}{2}$$

for $2\theta$

$$\cos^2\frac{2\theta}{2} = \frac{1 + \cos 2\theta}{2}$$

$$\boxed{\cos^2\theta = \frac{1 + \cos 2\theta}{2}}$$

for $4\theta$

$$\cos^2\frac{4\theta}{2} = \frac{1 + \cos 4\theta}{2}$$

$$\cos^2 2\theta = \frac{1 + \cos 4\theta}{2}$$

<u>Golden words</u>

The foundation of every state is the education of its youth. (Diogenes Laertius)

# Exercise # 10.3

**Q.1** Express the following products as sums/difference.

(i) $2\sin 60^\circ \sin 20^\circ$ $\quad 2SC = S + S$

$= \sin(60^\circ + 20^\circ) + \sin(60^\circ - 20^\circ)$

$= \sin 80^\circ + \sin 40^\circ$

(ii) $2\cos 8\theta \sin 4\theta$ $\quad 2CS = S - S$

$= \sin(8\theta + 4\theta) - \sin(8\theta - 4\theta)$

$= \sin 12\theta - \sin 4\theta$

(iii) $2\cos 75\alpha \sin 25\alpha$

$= \sin(75\alpha + 25\alpha) - \sin(75\alpha - 25\alpha)$

$= \sin 100\alpha - \sin 50\alpha$

(iv) $\sin 32^\circ \cos 24^\circ$

$\times$ and $\div$ by 2

$= \frac{2}{2}(\sin 32^\circ \cos 24^\circ)$

$= \frac{1}{2}(2\sin 32^\circ \cos 24^\circ)$

$= \frac{1}{2}\{\sin(32^\circ + 24^\circ) + \sin(32^\circ - 24^\circ)\}$

$= \frac{1}{2}\{\sin 56^\circ + \sin 8^\circ\}$

(v) $\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$= \frac{1}{2}\left\{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right\}$

$= \frac{1}{2}\left\{\sin\left(\frac{A+B}{2} + \frac{A-B}{2}\right) + \sin\left(\frac{A+B}{2} - \frac{A-B}{2}\right)\right\}$

$= \frac{1}{2}\left\{\sin\left(\frac{A+B+A-B}{2}\right) + \sin\left(\frac{A+B-A+B}{2}\right)\right\}$

$= \frac{1}{2}\left\{\sin\frac{2A}{2} + \sin\frac{2B}{2}\right\}$

$= \frac{1}{2}\{\sin A + \sin B\}$

(vi) $\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$ $\quad 2CC = C + C$

$= \frac{1}{2}\left\{2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)\right\}$

$= \frac{1}{2}\left\{\cos\left(\frac{P+Q}{2} + \frac{P-Q}{2}\right) + \cos\left(\frac{P+Q}{2} - \frac{P-Q}{2}\right)\right\}$

$= \frac{1}{2}\left\{\cos\left(\frac{P+Q+P-Q}{2}\right) + \cos\left(\frac{P+Q-P+Q}{2}\right)\right\}$

$= \frac{1}{2}\left\{\cos\left(\frac{2P}{2}\right) + \cos\left(\frac{2Q}{2}\right)\right\}$

$= \frac{1}{2}\{\cos P + \cos Q\}$

**Q.2** Convert the following sums/difference into products

(i) $\sin 94^\circ - \sin 86^\circ$ $\quad S - S = 2CS$

$= 2\cos\left(\frac{94^\circ + 86^\circ}{2}\right)\sin\left(\frac{94^\circ - 86^\circ}{2}\right)$

$= 2\cos\left(\frac{180^\circ}{2}\right)\sin\left(\frac{8^\circ}{2}\right)$

$= 2\cos 90^\circ \sin 4^\circ$

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(ii) $\cos 86^\circ + \cos 22^\circ$ $\qquad C+C=2CC$

$= 2\cos\left(\frac{86^\circ+22^\circ}{2}\right)\cos\left(\frac{86^\circ-22^\circ}{2}\right)$

$= 2\cos\left(\frac{108^\circ}{2}\right)\cos\left(\frac{64^\circ}{2}\right)$

$= 2\cos 54^\circ \cos 32^\circ$ Ans

(iii) $\cos 95^\circ - \cos 41^\circ$ $\qquad C-C=-2SS$

$= -2\sin\left(\frac{95^\circ+41^\circ}{2}\right)\sin\left(\frac{95^\circ-41^\circ}{2}\right)$

$= -2\sin\left(\frac{136^\circ}{2}\right)\sin\left(\frac{54^\circ}{2}\right)$

$= -2\sin 68^\circ \sin 27^\circ$ Ans

(iv) $\sin\left(\frac{P+Q}{2}\right) - \sin\left(\frac{P-Q}{2}\right)$ $\qquad S-S=2CS$

$= 2\cos\left(\frac{\frac{P+Q}{2}+\frac{P-Q}{2}}{2}\right)\sin\left(\frac{\frac{P+Q}{2}-\frac{P-Q}{2}}{2}\right)$

$= 2\cos\left(\frac{\frac{P+Q+P-Q}{2}}{2}\right)\sin\left(\frac{\frac{P+Q-P+Q}{2}}{2}\right)$

$= 2\cos\left(\frac{2P}{2}\times\frac{1}{2}\right)\sin\left(\frac{2Q}{2}\times\frac{1}{2}\right)$

$= 2\cos\left(\frac{P}{2}\right)\sin\left(\frac{Q}{2}\right)$ Ans

(v) $\cos 84^\circ + \cos 76^\circ$ $\qquad C+C=2CC$

$= 2\cos\left(\frac{84^\circ+76^\circ}{2}\right)\cos\left(\frac{84^\circ-76^\circ}{2}\right)$

$= 2\cos\left(\frac{160^\circ}{2}\right)\cos\left(\frac{8^\circ}{2}\right)$

$= 2\cos 80^\circ \cos 4^\circ$

(vi) $\cos\left(\frac{A+B}{2}\right) + \cos\left(\frac{A-B}{2}\right)$

$= 2\cos\left(\frac{\frac{A+B}{2}+\frac{A-B}{2}}{2}\right)\cos\left(\frac{\frac{A+B}{2}-\frac{A-B}{2}}{2}\right)$

$= 2\cos\left(\frac{\frac{A+B+A-B}{2}}{2}\right)\cos\left(\frac{\frac{A+B-A+B}{2}}{2}\right)$

$= 2\cos\left(\frac{2A}{2}\times\frac{1}{2}\right)\cos\left(\frac{2B}{2}\times\frac{1}{2}\right)$

$= 2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)$ Ans

Prove the following identities

Q:3 $\dfrac{\sin\alpha - \sin\beta}{\cos\alpha + \cos\beta} = \tan\left(\frac{\alpha-\beta}{2}\right)$

L.H.S $\dfrac{\sin\alpha - \sin\beta}{\cos\alpha + \cos\beta}$ $\qquad S-S=2CS$, $C+C=2CC$

$= \dfrac{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$

$= \dfrac{\sin\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} = \tan\left(\frac{\alpha-\beta}{2}\right) = R.H.S$

Q:4 $\dfrac{\cos 5\theta + \cos 3\theta}{\sin 5\theta - \sin 3\theta} = \cot\theta$

L.H.S $\dfrac{\cos 5\theta + \cos 3\theta}{\sin 5\theta - \sin 3\theta}$ $\qquad C+C=2CC$, $S-S=2CS$

$$= \frac{2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)}{2\cos\left(\frac{5\theta+3\theta}{2}\right)\sin\left(\frac{5\theta-3\theta}{2}\right)} = \frac{2\cos\frac{8\theta}{2}\cos\frac{2\theta}{2}}{2\cos\frac{8\theta}{2}\sin\frac{2\theta}{2}}$$

$$= \frac{2\cos 4\theta\cos\theta}{2\cos 4\theta\sin\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta = R.H.S$$

**Q:5** $\dfrac{\sin\alpha + \sin 9\alpha}{\cos\alpha + \cos 9\alpha} = \tan 5\alpha$

L.H.S $\dfrac{\sin\alpha + \sin 9\alpha}{\cos\alpha + \cos 9\alpha}$

$$= \frac{\sin 9\alpha + \sin\alpha}{\cos 9\alpha + \cos\alpha}$$

$$= \frac{2\sin\left(\frac{9\alpha+\alpha}{2}\right)\cos\left(\frac{9\alpha-\alpha}{2}\right)}{2\cos\left(\frac{9\alpha+\alpha}{2}\right)\cos\left(\frac{9\alpha-\alpha}{2}\right)}$$

$$= \frac{2\sin 5\alpha\cos 4\alpha}{2\cos 5\alpha\cos 4\alpha} = \frac{\sin 5\alpha}{\cos 5\alpha} = \tan 5\alpha = R.H.S$$

**Q:6** $\dfrac{\cos\beta + \cos 3\beta + \cos 5\beta}{\sin\beta + \sin 3\beta + \sin 5\beta} = \cot 3\beta$

L.H.S $\dfrac{\cos\beta + \cos 3\beta + \cos 5\beta}{\sin\beta + \sin 3\beta + \sin 5\beta}$

Rearranging the terms, we get

$$= \frac{\cos 5\beta + \cos\beta + \cos 3\beta}{\sin 5\beta + \sin\beta + \sin 3\beta}$$



$$= \frac{2\cos\left(\frac{5\beta+\beta}{2}\right)\cos\left(\frac{5\beta-\beta}{2}\right) + \cos 3\beta}{2\sin\left(\frac{5\beta+\beta}{2}\right)\cos\left(\frac{5\beta-\beta}{2}\right) + \sin 3\beta}$$

$$= \frac{2\cos 3\beta\cos 2\beta + \cos 3\beta}{2\sin 3\beta\cos 2\beta + \sin 3\beta}$$

take $\cos 3\beta$ common from nominator and $\sin 3\beta$ from denominator

$$= \frac{\cos 3\beta\{2\cos 2\beta + 1\}}{\sin 3\beta\{2\cos 2\beta+1\}} = \frac{\cos 3\beta}{\sin 3\beta} = \cot 3\beta = R.H.S$$

**Q:7** $\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta = 4\cos\theta\sin 6\theta\cos 2\theta$

L.H.S $(\sin 3\theta + \sin 5\theta) + (\sin 7\theta + \sin 9\theta)$

$$= \left\{2\sin\left(\frac{3\theta+5\theta}{2}\right)\cos\left(\frac{3\theta-5\theta}{2}\right)\right\} + \left\{2\sin\left(\frac{7\theta+9\theta}{2}\right)\cos\left(\frac{7\theta-9\theta}{2}\right)\right\}$$

$$= 2\sin 4\theta\cos(-\theta) + 2\sin 8\theta\cos(-\theta)$$

since $\cos(-\theta) = \cos\theta$

$$= 2\sin 4\theta\cos\theta + 2\sin 8\theta\cos\theta$$

take common

$$= 2\cos\theta\{\sin 4\theta + \sin 8\theta\}$$

$$= 2\cos\theta\left\{2\sin\left(\frac{4\theta+8\theta}{2}\right)\cos\left(\frac{4\theta-8\theta}{2}\right)\right\}$$

$$= 2\cos\theta\{2\sin 6\theta\cos(-2\theta)\}$$

$$= 4\cos\theta\sin 6\theta\cos 2\theta = R.H.S$$

**Q:8** $\dfrac{\cos 75^\circ + \cos 15^\circ}{\sin 75^\circ - \sin 15^\circ} = \sqrt{3}$

**L.H.S** $\dfrac{\cos 75^\circ + \cos 15^\circ}{\sin 75^\circ - \sin 15^\circ}$

$$= \frac{2\cos\left(\frac{75^\circ+15^\circ}{2}\right)\cos\left(\frac{75^\circ-15^\circ}{2}\right)}{2\cos\left(\frac{75^\circ+15^\circ}{2}\right)\sin\left(\frac{75^\circ-15^\circ}{2}\right)} = \frac{2\cos 45^\circ \cos 30^\circ}{2\cos 45^\circ \sin 30^\circ}$$

$$= \frac{\cos 30^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} = R.H.S$$

**Q:9** $\cos 3\alpha(1-2\sin\alpha) = \cos 3\alpha - (\sin 4\alpha - \sin 2\alpha)$

**L.H.S** $\cos 3\alpha\,(1-2\sin\alpha)$

$= \cos 3\alpha - 2\cos 3\alpha \sin\alpha$ $\qquad 2CS = S - S$

$= \cos 3\alpha - \{\sin(3\alpha+\alpha) - \sin(3\alpha-\alpha)\}$

$= \cos 3\alpha - \{\sin 4\alpha - \sin 2\alpha\} = R.H.S$

**Q:10** $\cos\beta + \cos 2\beta + \cos 5\beta = \cos 2\beta\,(1+2\cos 3\beta)$

**L.H.S** $\cos\beta + \cos 2\beta + \cos 5\beta$

$= \cos 5\beta + \cos\beta + \cos 2\beta$

$= 2\cos\left(\frac{5\beta+\beta}{2}\right)\cos\left(\frac{5\beta-\beta}{2}\right) + \cos 2\beta$

$= 2\cos 3\beta \cos 2\beta + \cos 2\beta$

take $\cos 2\beta$ as common

$= \cos 2\beta\,(2\cos 3\beta + 1) = R.H.S$

**Q:11** $\sin 5\theta + \sin\theta + 2\sin 3\theta = 4\sin 3\theta\cos^2\theta$

**L.H.S** $\sin 5\theta + \sin\theta + 2\sin 3\theta$

$= (\sin 5\theta + \sin\theta) + 2\sin 3\theta$

$= 2\sin\left(\frac{5\theta+\theta}{2}\right)\cos\left(\frac{5\theta-\theta}{2}\right) + 2\sin 3\theta$

$= 2\sin 3\theta\cos 2\theta + 2\sin 3\theta$

$= 2\sin 3\theta\,\{\cos 2\theta + 1\}$

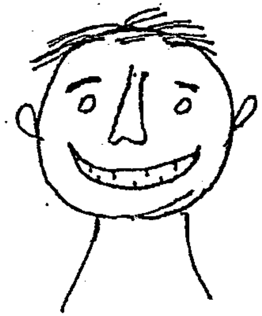
$= 2\sin 3\theta\,\{\cos^2\theta - \sin^2\theta + 1\}$

$= 2\sin 3\theta\,\{\cos^2\theta + 1 - \sin^2\theta\}$

$= 2\sin 3\theta\,\{\cancel{\sin^2\theta}\cos^2\theta + \cos^2\theta\}$

$= 2\sin 3\theta\,(2\cos^2\theta)$

$= 4\sin 3\theta\cos^2\theta = R.H.S$



End of chapter 10.
Good Luck.