# Chapter 2

Average rate of change $\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$

## Exercise 2.1

Q1. Find the average rate of change of the following functions over the indicated intervals:

a). $y=x^2+4$ from x=2 to x=3

Sol: We have $y=f(x)=x^2+4$

$\therefore$ average rate of change $\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$

With $\Delta x=x_{\max}-x_{\min}$ then $\Delta x=3-2=1$

$$\frac{\Delta y}{\Delta x}=\frac{f(3)-f(2)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x}=\frac{[3^2+4]-[2^2+4]}{1}$$

$$\frac{\Delta y}{\Delta x}=(9+4)-(4+4)=13-8$$

$$\frac{\Delta y}{\Delta x}=5$$

b). $y=x^2+\frac{1}{3}x$ from x=-3 to x=3

Solution: We have $y=f(x)=x^2+\frac{1}{3}x$

With $\Delta x=x_{\max}-x_{\min}$ then $\Delta x=3-(-3)=6$

$\therefore$ average rate of change $\frac{\Delta y}{\Delta x}=\frac{f(3)-f(-3)}{6}$

$$\frac{\Delta y}{\Delta x}=\frac{\left[(3)^2+\frac{1}{3}(3)\right]-\left[(-3)^2+\frac{1}{3}(-3)\right]}{6}$$

$$\frac{\Delta y}{\Delta x}=\frac{[9+1]-(9-1)}{6}$$

$$\frac{\Delta y}{\Delta x}=\frac{10-8}{6}=\frac{2}{6}$$

$$\frac{\Delta y}{\Delta x}=\frac{1}{3}$$

c). $s=2t^3-5t+7$ from t=1 to t=3

Solution: We have $s(t)=2t^3-5t+7$

With $\Delta t=t_{\max}-t_{\min}$ then $\Delta x=3-1=2$

$\therefore$ average rate of change $\frac{\Delta s}{\Delta t}=\frac{s(3)-s(1)}{2}$

$$\frac{\Delta s}{\Delta t}=\frac{\left[2(3)^3-5(3)+7\right]-\left[2(1)^3-5(1)+7\right]}{2}$$

$$\frac{\Delta s}{\Delta t}=\frac{\left[2(27)-15+7\right]-(2-5+7)}{2}$$

$$\frac{\Delta s}{\Delta t}=\frac{[54-8]-4}{2}=\frac{42}{2}$$

$$\frac{\Delta s}{\Delta t}=21$$

d). $h=\sqrt{2t}-7$ from t=8 to t=8.5

Solution: We have $h(t)=\sqrt{2t}-7$

$\therefore$ average rate of change $\frac{\Delta h}{\Delta t}=\frac{h(t+\Delta t)-h(t)}{\Delta t}$

With $\Delta t=t_{\max}-t_{\min}$ then $\Delta t=8.5-8=0.5$

Substituting $h(t)=\sqrt{2t}-7$

$$\frac{\Delta h}{\Delta t}=\frac{h(8.5)-h(8)}{0.5}$$

$$\frac{\Delta h}{\Delta t}=\frac{\left(\sqrt{2(8.5)}-7\right)-\left(\sqrt{2(8)}-7\right)}{0.5}$$

$$\frac{\Delta h}{\Delta t}=\frac{\sqrt{17}-\sqrt{16}}{0.5}\simeq 0.246$$

Q2. Use definition to find out average rate of change over specified interval for following functions:

a). $s=2t-3$ from t=2 to t=5

Solution: We have $s(t)=2t-3$

$\therefore$ average rate of change $\frac{\Delta s}{\Delta t}=\frac{s(t+\Delta t)-s(t)}{\Delta t}$

With $\Delta t=t_{\max}-t_{\min}$ then $\Delta t=5-2=3$

$$\frac{\Delta s}{\Delta t}=\frac{s(5)-s(2)}{3}$$

$$\frac{\Delta s}{\Delta t}=\frac{[2(5)-3]-[2(2)-3]}{3}$$

$$\frac{\Delta s}{\Delta t}=\frac{10-3-4+3}{3}$$

$$\frac{\Delta s}{\Delta t}=\frac{6}{3}=2$$

$$\frac{\Delta s}{\Delta t}=2$$

b). $y=x^2-6x+8$ from x=3 to x=3.1

Solution: We have $y=f(x)=x^2-6x+8$

$\therefore$ average rate of change $\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$

With $\Delta x=x_{\max}-x_{\min}$ then $\Delta x=3.1-3=0.1$

$$\frac{\Delta y}{\Delta x}=\frac{f(3.1)-f(3)}{0.1}$$

$$\frac{\Delta y}{\Delta x}=\frac{\left[(3.1)^2-6(3.1)+8\right]-\left[(3)^2-6(3)+8\right]}{0.1}$$

$$\frac{\Delta y}{\Delta x}=\frac{[9.61-18.6+8]-[9-18+8]}{0.1}$$

$$\frac{\Delta y}{\Delta x}=\frac{[-0.99]-[-1]}{0.1}$$

$$\frac{\Delta y}{\Delta x}=\frac{-0.99+1}{0.1}=\frac{0.01}{0.1}=\frac{1}{10}=0.1$$

c). $A=\pi r^2$ from r=2 to r=2.1

Solution: We have $A(r)=\pi r^2$

$\therefore$ average rate of change $\frac{\Delta A}{\Delta r}=\frac{A(r+\Delta r)-A(r)}{\Delta r}$

With $\Delta r=r_{\max}-r_{\min}$ then $\Delta r=2.1-2=0.1$

$$\frac{\Delta A}{\Delta r}=\frac{A(2.1)-A(2)}{0.1}$$

$$\frac{\Delta A}{\Delta r}=\frac{\pi[2.1]^2-\pi[2]^2}{0.1}$$

$$\frac{\Delta A}{\Delta r}=\frac{4.41\pi-4\pi}{0.1}$$

$$\frac{\Delta A}{\Delta r}=\frac{0.41\pi}{0.1}$$
$$\frac{\Delta A}{\Delta r}=4.1\pi\simeq 12.88$$

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d). $h=\sqrt{t}-9$ from t=9 to t=16

Solution: We have $h(t)=\sqrt{t}-9$

With $\Delta t=t_{\max}-t_{\min}$ then $\Delta t=16-9=7$

$\therefore$ average rate of change $\frac{\Delta h}{\Delta t}=\frac{h(16)-h(9)}{7}$

$$\frac{\Delta h}{\Delta t}=\frac{\left[\sqrt{16}-9\right]-\left[\sqrt{9}-9\right]}{7}$$
$$\frac{\Delta h}{\Delta t}=\frac{4-9-3+9}{7}$$
$$\frac{\Delta h}{\Delta t}=\frac{4-3}{7}=\frac{1}{7}$$

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Q3. A ball is thrown straight up. Its height after t seconds is given by the formula $h(t)=-16t^2+80t$ Find the average velocity $\frac{\Delta h}{\Delta t}$ for the specified intervals

a). From t=2 to t=2.1

Solution: We have $h(t)=-16t^2+80t$

With $\Delta t=t_{\max}-t_{\min}$ then $\Delta t=2.1-2=0.1$

$\therefore$ average rate of change $\frac{\Delta h}{\Delta t}=\frac{h(2.1)-h(2)}{0.1}$

$$\frac{\Delta h}{\Delta t}=\frac{\left[-16(2.1)^2+80(2.1)\right]-\left[-16(2)^2+80(2)\right]}{0.1}$$
$$\frac{\Delta h}{\Delta t}=\frac{-16(4.41)+168+16(4)-160}{0.1}$$
$$\frac{\Delta h}{\Delta t}=\frac{-70.56+168+64-160}{0.1}$$
$$\frac{\Delta h}{\Delta t}=\frac{1.44}{0.1}=14.4$$

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b). From t=2 to t=2.01

Solution: We have $h(t)=-16t^2+80t$

With $\Delta t=t_{\max}-t_{\min}$ then $\Delta t=2.01-2=0.01$

$\therefore$ average rate of change $\frac{\Delta h}{\Delta t}=\frac{h(2.1)-h(2)}{0.1}$

$$\frac{\Delta h}{\Delta t}=\frac{\left[-16(2.01)^2+80(2.01)\right]-\left[-16(2)^2+80(2)\right]}{0.01}$$
$$\frac{\Delta h}{\Delta t}=\frac{-16(4.0401)+160.8+16(4)-160}{0.01}$$
$$\frac{\Delta h}{\Delta t}=\frac{-64.6416+160.8+64-160}{0.01}$$
$$\frac{\Delta h}{\Delta t}=\frac{0.1584}{0.01}=15.84$$

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Q4. The rate of change of price is called inflation. price P in rupees after t years is $p(t)=3t^2+t+1$ Find the average rate of change of inflation from t=3 to t=5 years. What the rate of change means? Explain

Solution: We have $p(t)=3t^2+t+1$

$\therefore$ average rate of change $\frac{\Delta p}{\Delta t}=\frac{p(t+\Delta t)-p(t)}{\Delta t}$

With $\Delta t=t_{\max}-t_{\min}$ then $\Delta t=5-3=2$

$$\frac{\Delta p}{\Delta t}=\frac{p(5)-p(3)}{2}$$
$$\frac{\Delta p}{\Delta t}=\frac{\left[3(5)^2+(5)+1\right]-\left[3(3)^2+(3)+1\right]}{2}$$
$$\frac{\Delta p}{\Delta t}=\frac{\left[3(25)+6\right]-\left[3(9)+4\right]}{2}$$
$$\frac{\Delta p}{\Delta t}=\frac{75+6-27-4}{2}$$
$$\frac{\Delta p}{\Delta t}=\frac{50}{2}=25$$

**First principle for derivative**

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

**Slope** when two point $P_1(x_1,y_1),P_2(x_2,y_2)$ are given **Slope** = $m=\frac{y_2-y_1}{x_2-x_1}$

**Differentiation of $y=x^n$ by first principle Rule**

If $f(x)=x^n$ so $f(x+\Delta x)=(x+\Delta x)^n$

Using first principle for derivative of $f(x)$

$$\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$
$$f'(x)=\lim_{\Delta x\to 0}\frac{(x+\Delta x)^n-x^n}{\Delta x}$$

Using binomial theorem

$$(x+\Delta x)^n=x^n+nx^{n-1}\Delta x+\tfrac{n(n-1)}{2!}x^{n-2}(\Delta x)^2+\cdots$$

so $f'(x)=\lim_{\Delta x\to 0}\frac{(x+\Delta x)^n-x^n}{\Delta x}$

$$f'(x)=\lim_{\Delta x\to 0}\frac{x^n+nx^{n-1}\Delta x+\tfrac{n(n-1)}{2!}x^{n-2}(\Delta x)^2+\cdots-x^n}{\Delta x}$$
$$f'(x)=\lim_{\Delta x\to 0}\frac{nx^{n-1}\Delta x+\tfrac{n(n-1)}{2!}x^{n-2}(\Delta x)^2+\cdots}{\Delta x}$$
$$f'(x)=\lim_{\Delta x\to 0}\frac{\Delta x\left[nx^{n-1}+\tfrac{n(n-1)}{2!}x^{n-2}(\Delta x)+\cdots\right]}{\Delta x}$$
$$f'(x)=\lim_{\Delta x\to 0}\left[nx^{n-1}+\tfrac{n(n-1)}{2!}x^{n-2}(\Delta x)+\cdots\right]$$
$$f'(x)=\left[nx^{n-1}+\tfrac{n(n-1)}{2!}x^{n-2}(0)+0+0+\cdots\right]$$
$$f'(x)=nx^{n-1}$$

## Exercise 2.2

Q1. Use first Principle to determine the derivative of the following functions.

a). $f(x)=3x$

Solution: We have $f(x)=3x$

First principle for derivative

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Substituting $f(x)=3x$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{3(x+\Delta x)-3x}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{3x+3\Delta x-3x}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{3\Delta x}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}3=3$$

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b). $f(x)=5x+6$

Solution: We have

First principle for derivative

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Substituting

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{[5(x+\Delta x)+6]-[5x+6]}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{5x+5\Delta x+6-5x-6}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{5\Delta x}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}5$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=5$$

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c). $f(x)=x^2+1$

Solution: We have $f(x)=x^2+1$

First principle for derivative

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Substituting

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{\left[(x+\Delta x)^2+1\right]-\left[x^2+1\right]}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{x^2+(\Delta x)^2+2x\Delta x+1-x^2-1}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{(\Delta x)^2+2x\Delta x}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{\Delta x[\Delta x+2x]}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}[\Delta x+2x]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=[0+2x]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=2x$$

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d). $f(x)=12-x^2$

Solution: We have $f(x)=12-x^2$

First principle for derivative

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Substituting $f(x)=12-x^2$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{\left[12-(x+\Delta x)^2\right]-\left[12-x^2\right]}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{12-\left(x^2+(\Delta x)^2+2x\Delta x\right)-12+x^2}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{12-x^2-(\Delta x)^2-2x\Delta x-12+x^2}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{-(\Delta x)^2-2x\Delta x}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{\Delta x(-\Delta x-2x)}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}[-\Delta x-2x]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=-0-2x$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=-2x$$

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e). $f(x)=16x^2-7x$

Solution: We have $f(x)=16x^2-7x$

First principle for derivative

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Substituting

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{\left[16(x+\Delta x)^2-7(x+\Delta x)\right]-\left[16x^2-7x\right]}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{16\left(x^2+(\Delta x)^2+2x\Delta x\right)-7x-7\Delta x-16x^2+7x}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{16x^2+16(\Delta x)^2+32x\Delta x-7x-7\Delta x-16x^2+7x}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{16(\Delta x)^2+32x\Delta x-7\Delta x}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{\Delta x[16\Delta x+32x-7]}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}[16\Delta x+32x-7]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=[16(0)+32x-7]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=32x-7$$

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f). $f(x)=\frac{7}{x}$

Solution: We have $f(x)=\frac{7}{x}$

First principle for derivative

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Substituting $f(x)=\frac{7}{x}$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{7}{x+\Delta x}-\frac{7}{x}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{7x-7(x+\Delta x)}{x(x+\Delta x)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{7x-7x-7\Delta x}{x(x+\Delta x)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{-7\Delta x}{x(x+\Delta x)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\left[\frac{-7}{x(x+\Delta x)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\left[\frac{-7}{x(x+0)}\right]=\left[\frac{-7}{x(x)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\frac{-7}{x^2}$$

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g). $f(x)=\dfrac{3}{x+3}$

Solution: We have

First principle for derivative

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Substituting $\dfrac{dy}{dx}=\lim_{\Delta x\to 0}\dfrac{1}{\Delta x}\left[\dfrac{3}{(x+\Delta x)+3}-\dfrac{3}{x+3}\right]$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{3}{x+\Delta x+3}-\frac{3}{x+3}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{3(x+3)-3(x+\Delta x+3)}{(x+\Delta x+3)(x+3)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{3x+9-3x-3\Delta x-9}{(x+\Delta x+3)(x+3)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{-3\Delta x}{(x+\Delta x+3)(x+3)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\left[\frac{-3}{(x+\Delta x+3)(x+3)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\left[\frac{-3}{(x+0+3)(x+3)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\left[\frac{-3}{(x+3)(x+3)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\frac{-3}{(x+3)^2}$$

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h). $f(x)=\dfrac{5}{2x-4}$

Solution: We have $f(x)=\dfrac{5}{2x-4}$

First principle for derivative

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Substituting $f(x)=\dfrac{5}{2x-4}$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{5}{2(x+\Delta x)-4}-\frac{5}{2x-4}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{5}{2x+2\Delta x-4}-\frac{5}{2x-4}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{5(2x-4)-5(2x+2\Delta x-4)}{(2x+2\Delta x-4)(2x-4)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{10x-20-10x-10\Delta x+20}{(2x+2\Delta x-4)(2x-4)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{-10\Delta x}{(2x+2\Delta x-4)(2x-4)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\left[\frac{-10}{(2x+2\Delta x-4)(2x-4)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\left[\frac{-10}{(2x+2(0)-4)(2x-4)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\left[\frac{-10}{(2x-4)(2x-4)}\right]$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\frac{-10}{(2x-4)^2}$$

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i). $f(x)=3x^2+4x-9$

Solution: We have $f(x)=3x^2+4x-9$

First principle for derivative

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Substituting $f(x)=3x^2+4x-9$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\left[3(x+\Delta x)^2+4(x+\Delta x)-9\right]-\left[3x^2+4x-9\right]}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{3\left(x^2+(\Delta x)^2+2x\Delta x\right)+4x+4\Delta x-9-3x^2-4x+9}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{3x^2+3(\Delta x)^2+6x\Delta x+4x+4\Delta x-9-3x^2-4x+9}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{3(\Delta x)^2+6x\Delta x+4\Delta x}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta x\left[3\Delta x+6x+4\right]}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\left[3\Delta x+6x+4\right]$$

$$\frac{dy}{dx}=\left[3(0)+6x+4\right]$$

$$\frac{dy}{dx}=6x+4$$

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Q2. Estimate the slope of the tangent line on a curve at a point P(x,y) for each of the following graph

a). Solution: From graph points are $(5,3)$ and $(7,7)$

$$slope=m=\frac{7-3}{7-5}=\frac{4}{2}=2$$

b). Solution: From graph points are $(2,2)$ and $(0,4)$

$$slope=m=\frac{4-2}{0-2}=\frac{2}{-2}=-1$$

c).Solution: From graph points are $(-2,2)$ and $(4,3)$

$$slope=m=\frac{3-2}{4-(-2)}=\frac{1}{6}$$

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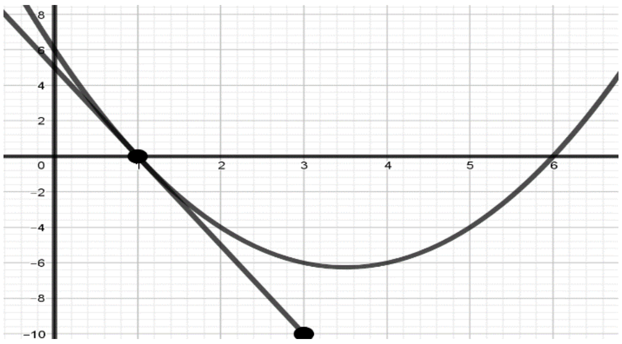
Q3a). Draw a tangent line to the graph of function $f(x)=x^2-7x+6$ at point $(1,0)$ of function and estimate it slope at the same point graphically.

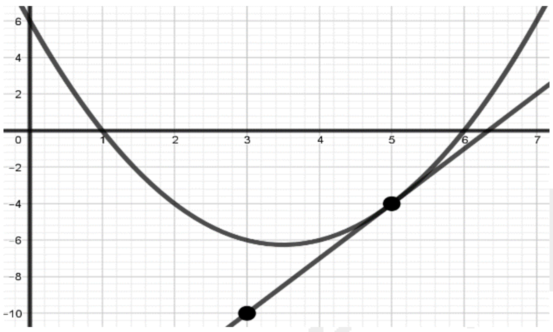
Sol: since tangent line passing through one point of curve therefore draw a tangent line on the curve at point $(1,0)$



Which line passing through the point $(3,-10)$

b). Draw a tangent line to the graph of the function $f(x)=x^2-7x+6$ at point $(5,-4)$ of the function and estimate it slope at same point graphically.

Sol: since tangent line passing through one point of curve therefore draw a tangent line on the curve at point $(1,0)$



Which line passing through the point $(3,-10)$

c). Is your estimate about tangent line at the point $(1,0)$ and $(5,-4)$ equal to actual derivatives of the function at point $(1,0)$ and $(5,-4)$

Solution: we have the function $f(x)=x^2-7x+6$

Differentiating with respect to x

$\frac{d}{dx}f(x)=\frac{d}{dx}x^2-7\frac{d}{dx}x+\frac{d}{dx}6$

$f'(x)=2x-7$

Slope of function at point $(1,0)$ i.e. $x=1$ & $y=0$

$f'(1)=2(1)-7$

$m_1=-5$

So the equation of tangent $y-y_1=m_1(x-x_1)$

$y-0=-5(x-1)$

$y=-5x+5$ ..........................................(1)

Which passing through estimated point $(3,-10)$

i.e. $-10=-5(3)+5$

$-10=-15+5$

$-10=-10$

Slope of $f(x)$ at point $(5,-4)$ i.e. $x=5$ & $y=-4$

$f'(5)=2(5)-7$

$m_2=3$

So the equation of tangent $y-y_1=m_2(x-x_1)$

$y-(-4)=3(x-5)$

$y+4=3x-15$

$y=3x-19$ ......................................(2)

Which passing through estimated point $(3,-10)$

i.e. $-10=3(3)-19$

$-10=9-19$

$-10=-10$

| Name of Rule | Leibniz Notation |
|---|---|
| Constant | $\frac{d}{dx}(c)=0$ |
| Constant multiple | $\frac{d}{dx}(cf)=c\frac{d}{dx}f$ |
| Sum | $\frac{d}{dx}(f+g)=\frac{d}{dx}f+\frac{d}{dx}g$ |
| Difference | $\frac{d}{dx}(f-g)=\frac{d}{dx}f-\frac{d}{dx}g$ |
| Linearity | $\frac{d}{dx}(af+bg)=a\frac{d}{dx}f+b\frac{d}{dx}g$ |
| Product | $\frac{d}{dx}(fg)=f\frac{d}{dx}g+g\frac{d}{dx}f$ |
| Quotient | $\frac{d}{dx}\left(\frac{f}{g}\right)=\frac{g\frac{d}{dx}f-f\frac{d}{dx}g}{g^2}$ |

**Constant Rule**

Let $f(x)=c$ then $f(x+\Delta x)=c$

First principle for derivative

$$f'(x)=\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value

$$f'(x)=\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{c-c}{\Delta x}=0$$

$\Rightarrow f'(x)=0$

**Constant multiple**

First principle for derivative

$$\frac{d}{dx}\{cf(x)\}=\lim_{\Delta x\to 0}\frac{cf(x+\Delta x)-cf(x)}{\Delta x}$$

Putting the value

$$\frac{d}{dx}\{cf(x)\}=c\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$\frac{d}{dx}\{cf(x)\}=c\frac{d}{dx}f(x)$$

**Sum Rule** Let $h(x)=f(x)+g(x)$ then

$h(x+\Delta x)=f(x+\Delta x)+g(x+\Delta x)$

First principle for derivative

$$h'(x)=\frac{d}{dx}h(x)=\lim_{\Delta x\to 0}\frac{h(x+\Delta x)-h(x)}{\Delta x}$$

Putting the value of $h(x)=f(x)+g(x)$

$$\frac{d}{dx}\{f(x)+g(x)\}=\lim_{\Delta x\to 0}\frac{\{f(x+\Delta x)+g(x+\Delta x)\}-\{f(x)+g(x)\}}{\Delta x}$$

$$\frac{d}{dx}\{f(x)+g(x)\}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)+g(x+\Delta x)-g(x)}{\Delta x}$$

$$\frac{d}{dx}\{f(x)+g(x)\}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}+\frac{g(x+\Delta x)-g(x)}{\Delta x}$$

$$\frac{d}{dx}\{f(x)+g(x)\}=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

**Difference Rule** Let $h(x)=f(x)-g(x)$ then

$h(x+\Delta x)=f(x+\Delta x)-g(x+\Delta x)$

First principle for derivative

$$h'(x)=\frac{d}{dx}h(x)=\lim_{\Delta x\to 0}\frac{h(x+\Delta x)-h(x)}{\Delta x}$$

Putting the value of $h(x)=f(x)-g(x)$

$$\frac{d}{dx}\{f(x)-g(x)\}=\lim_{\Delta x\to 0}\frac{\{f(x+\Delta x)-g(x+\Delta x)\}-\{f(x)-g(x)\}}{\Delta x}$$

$$\frac{d}{dx}\{f(x)-g(x)\}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)-g(x+\Delta x)+g(x)}{\Delta x}$$

$$\frac{d}{dx}\{f(x)-g(x)\}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}-\frac{g(x+\Delta x)-g(x)}{\Delta x}$$

$$\frac{d}{dx}\{f(x)-g(x)\}=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

**Linearity Rule** Let $h(x)=af(x)+bg(x)$ then

$$h(x+\Delta x)=af(x+\Delta x)+bg(x+\Delta x)$$

First principle for derivative

$$h'(x)=\frac{d}{dx}h(x)=\lim_{\Delta x\to 0}\frac{h(x+\Delta x)-h(x)}{\Delta x}$$

Putting the value of $h(x)=af(x)+bg(x)$

$$\frac{d}{dx}\{af(x)+bg(x)\}=\lim_{\Delta x\to 0}\frac{\{af(x+\Delta x)+bg(x+\Delta x)\}-\{af(x)+bg(x)\}}{\Delta x}$$

$$\frac{d}{dx}\{af(x)+bg(x)\}=\lim_{\Delta x\to 0}\frac{af(x+\Delta x)-af(x)+bg(x+\Delta x)-bg(x)}{\Delta x}$$

$$\frac{d}{dx}\{af(x)+bg(x)\}=a\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}+b\frac{g(x+\Delta x)-g(x)}{\Delta x}$$

$$\frac{d}{dx}\{af(x)+bg(x)\}=a\frac{d}{dx}f(x)+b\frac{d}{dx}g(x)$$

**Product Rule** Let $h(x)=f(x).g(x)$ then

$$h(x+\Delta x)=f(x+\Delta x)g(x+\Delta x)$$

First principle for derivative

$$h'(x)=\frac{d}{dx}h(x)=\lim_{\Delta x\to 0}\frac{h(x+\Delta x)-h(x)}{\Delta x}$$

Putting the value of $h(x)=f(x).g(x)$

$$\frac{d}{dx}h(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\{f(x+\Delta x)g(x+\Delta x)-f(x)g(x)\}$$

$$h'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left\{\begin{aligned}&f(x+\Delta x)g(x+\Delta x)-g(x+\Delta x)f(x)\\&+g(x+\Delta x)f(x)-f(x)g(x)\end{aligned}\right\}$$

$$h'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left\{\begin{aligned}&g(x+\Delta x)\{f(x+\Delta x)-f(x)\}\\&+f(x)\{g(x+\Delta x)-g(x)\}\end{aligned}\right\}$$

$$h'(x)=\lim_{\Delta x\to 0}g(x+\Delta x)\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}+f(x)\lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x}$$

$$\frac{d}{dx}\{f(x)g(x)\}=g(x)\frac{d}{dx}f(x)+f(x)\frac{d}{dx}g(x)$$

**Quotient Rule** Let $h(x)=\frac{f(x)}{g(x)}$, $g(x)\neq 0$ then

$$h(x+\Delta x)=\frac{f(x+\Delta x)}{g(x+\Delta x)}$$

First principle for derivative

$$h'(x)=\frac{d}{dx}h(x)=\lim_{\Delta x\to 0}\frac{h(x+\Delta x)-h(x)}{\Delta x}$$

Putting the value of $h(x)=\frac{f(x)}{g(x)}$

$$\frac{d}{dx}\{f(x).g(x)\}=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{f(x+\Delta x)}{g(x+\Delta x)}-\frac{f(x)}{g(x)}\right]$$

$$\frac{d}{dx}h(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{g(x)f(x+\Delta x)-f(x)g(x+\Delta x)}{g(x+\Delta x)g(x)}\right]$$

$$h'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{g(x)f(x+\Delta x)-g(x)f(x)-f(x)g(x+\Delta x)+g(x)f(x)}{g(x+\Delta x)g(x)}\right]$$

$$h'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{g(x)\{f(x+\Delta x)-f(x)\}-f(x)\{g(x+\Delta x)-g(x)\}}{g(x+\Delta x)g(x)}\right]$$

$$h'(x)=\lim_{\Delta x\to 0}\left[\frac{g(x)\frac{f(x+\Delta x)-f(x)}{\Delta x}-f(x)\frac{g(x+\Delta x)-g(x)}{\Delta x}}{g(x+\Delta x)g(x)}\right]$$

$$h'(x)=\left[\frac{g(x)\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}-f(x)\lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x}}{\lim_{\Delta x\to 0}g(x+\Delta x)g(x)}\right]$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\left[\frac{g(x)\frac{d}{dx}f(x)-f(x)\frac{d}{dx}g(x)}{g(x)g(x)}\right]$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\frac{d}{dx}f(x)-f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$$

# Exercise 2.3

Q1. Use the product rule to find out the derivative of following functions;

a). $y=(x^2-2)(3x+1)$

Sol: Differentiating $\frac{dy}{dx}=\frac{d}{dx}\{(x^2-2)(3x+1)\}$

Using the product rule

$$\frac{dy}{dx}=(x^2-2)\frac{d}{dx}(3x+1)+(3x+1)\frac{d}{dx}(x^2-2)$$

$$\frac{dy}{dx}=(x^2-2)\left(3\frac{d}{dx}x+\frac{d}{dx}1\right)+(3x+1)\left(\frac{d}{dx}x^2-\frac{d}{dx}2\right)$$

$$y'=(x^2-2)(3(1)+0)+(3x+1)(2x-0)$$

$$y'=3(x^2-2)+2x(3x+1)$$

$$y'=3x^2-6+6x^2+2x$$

$$y'=9x^2+2x-6$$

---

b). $y=(6x^3+2)(5x-3)$

Sol: Differentiating $\frac{dy}{dx}=\frac{d}{dx}\{(6x^3+2)(5x-3)\}$

Using the product rule

$$\frac{dy}{dx}=(6x^3+2)\frac{d}{dx}(5x-3)+(5x-3)\frac{d}{dx}(6x^3+2)$$

$$\frac{dy}{dx}=(6x^3+2)\left(5\frac{d}{dx}x-\frac{d}{dx}3\right)+(5x-3)\left(6\frac{d}{dx}x^3+\frac{d}{dx}2\right)$$

$$y'=(6x^3+2)(5(1)-0)+(5x-3)(6(3x^2)+0)$$

$$y'=5(6x^3+2)+18x^2(5x-3)$$

$$y'=30x^3+10+90x^3-54x^2$$

$$y'=120x^3-54x^2+10$$

---

c). $y=(7x^4+2x)(x^2-4)$

Sol: Differentiating $\frac{dy}{dx}=\frac{d}{dx}\{(7x^4+2x)(x^2-4)\}$

Using the product rule

$$\frac{dy}{dx}=(7x^4+2x)\frac{d}{dx}(x^2-4)+(x^2-4)\frac{d}{dx}(7x^4+2x)$$

$$\frac{dy}{dx}=(7x^4+2x)\left(\frac{d}{dx}x^2-\frac{d}{dx}4\right)+(x^2-4)\left(7\frac{d}{dx}x^4+2\frac{d}{dx}x\right)$$

$$y'=(7x^4+2x)(2x-0)+(x^2-4)(7(4x^3)+2(1))$$

$$y'=2x(7x^4+2x)+(x^2-4)(28x^3+2)$$

$$y'=14x^5+4x^2+28x^5+2x^2-112x^3-8$$

$$y'=14x^5+28x^5-112x^3+4x^2+2x^2-8$$

$$y'=42x^5-112x^3+6x^2-8$$

d). $y=(2x^2+4x-3)(5x^3+x+2)$

Sol: Differentiating $\frac{dy}{dx}=\frac{d}{dx}\{(2x^2+4x-3)(5x^3+x+2)\}$

Using the product rule

$$\frac{dy}{dx}=(2x^2+4x-3)\frac{d}{dx}(5x^3+x+2)+(5x^3+x+2)\frac{d}{dx}(2x^2+4x-3)$$

$$\frac{dy}{dx}=(2x^2+4x-3)\left(5\frac{d}{dx}x^3+\frac{d}{dx}x+\frac{d}{dx}2\right)+(5x^3+x+2)\left(2\frac{d}{dx}x^2+4\frac{d}{dx}x-\frac{d}{dx}3\right)$$

$$y'=(2x^2+4x-3)(5(3x^2)+1+0)+(5x^3+x+2)(2(2x)+4(1)-0)$$

$$y'=(2x^2+4x-3)(15x^2+1)+(5x^3+x+2)(4x+4)$$

$$y'=30x^4+2x^2+60x^3+4x-45x^2-3+20x^4+20x^3+4x^2+4x+8x+8$$

$$y'=30x^4+20x^4+60x^3+20x^3+2x^2-45x^2+4x^2+4x+4x+8x+8-3$$

$$y'=50x^4+80x^3-39x^2+16x+5$$

---

e). $y=(2x-3)(\sqrt{x}-1)$

Sol: Differentiating $\frac{dy}{dx}=\frac{d}{dx}\left\{(2x-3)\left(x^{\frac{1}{2}}-1\right)\right\}$

Using the product rule

$$\frac{dy}{dx}=(2x-3)\frac{d}{dx}\left(x^{\frac{1}{2}}-1\right)+\left(x^{\frac{1}{2}}-1\right)\frac{d}{dx}(2x-3)$$

$$\frac{dy}{dx}=(2x-3)\left(\frac{d}{dx}x^{\frac{1}{2}}-\frac{d}{dx}1\right)+\left(x^{\frac{1}{2}}-1\right)\left(2\frac{d}{dx}x-\frac{d}{dx}3\right)$$

$$\frac{dy}{dx}=(2x-3)\left(\frac{1}{2}x^{\frac{1}{2}-1}-0\right)+\left(x^{\frac{1}{2}}-1\right)(2(1)-0)$$

$$\frac{dy}{dx}=\frac{1}{2}x^{\frac{-1}{2}}(2x-3)+2\left(x^{\frac{1}{2}}-1\right)$$

$$\frac{dy}{dx}=\frac{1}{2}x^{\frac{-1}{2}}(2x)-\frac{3}{2}x^{\frac{-1}{2}}+2x^{\frac{1}{2}}-2$$

$$\frac{dy}{dx}=x^{\frac{-1}{2}+1}-\frac{3}{2}x^{\frac{-1}{2}}+2x^{\frac{1}{2}}-2$$

$$\frac{dy}{dx}=x^{\frac{1}{2}}+2x^{\frac{1}{2}}-\frac{3}{2}x^{\frac{-1}{2}}-2$$

$$\frac{dy}{dx}=3x^{\frac{1}{2}}-\frac{3}{2}x^{\frac{-1}{2}}-2$$

---

f). $y=(-3\sqrt{x}+6)(4\sqrt{x}-2)$

Sol: Differentiating $\frac{dy}{dx}=\frac{d}{dx}\{(-3\sqrt{x}+6)(4\sqrt{x}-2)\}$

Using the product rule

$$\frac{dy}{dx}=\left(-3x^{\frac{1}{2}}+6\right)\frac{d}{dx}\left(4x^{\frac{1}{2}}-2\right)+\left(4x^{\frac{1}{2}}-2\right)\frac{d}{dx}\left(-3x^{\frac{1}{2}}+6\right)$$

$$\frac{dy}{dx}=\left(-3x^{\frac{1}{2}}+6\right)\left(4\frac{d}{dx}x^{\frac{1}{2}}-\frac{d}{dx}2\right)+\left(4x^{\frac{1}{2}}-2\right)\left(-3\frac{d}{dx}x^{\frac{1}{2}}+\frac{d}{dx}6\right)$$

$$\frac{dy}{dx}=\left(-3x^{\frac{1}{2}}+6\right)\left(\frac{4}{2}x^{\frac{1}{2}-1}-0\right)+\left(4x^{\frac{1}{2}}-2\right)\left(\frac{-3}{2}x^{\frac{1}{2}-1}+0\right)$$

$$\frac{dy}{dx}=2x^{\frac{-1}{2}}\left(-3x^{\frac{1}{2}}+6\right)-\frac{3}{2}x^{\frac{-1}{2}}\left(4x^{\frac{1}{2}}-2\right)$$

$$\frac{dy}{dx}=-6x^{\frac{-1}{2}+\frac{1}{2}}+12x^{\frac{-1}{2}}-\frac{3}{2}\times4x^{\frac{-1}{2}+\frac{1}{2}}+\frac{3}{2}\times2x^{\frac{-1}{2}}$$

$$\frac{dy}{dx}=-6x^0+12x^{\frac{-1}{2}}-6x^0+3x^{\frac{-1}{2}}$$

$$\frac{dy}{dx}=12x^{\frac{-1}{2}}+3x^{\frac{-1}{2}}-6-6$$

$$\frac{dy}{dx}=15x^{\frac{-1}{2}}-12$$

---

Q2. Use the quotient rule to find out the derivative of the following functions;

a). $y=\frac{3x-5}{x-4}$

Solution: Differentiating $\frac{dy}{dx}=\frac{d}{dx}\left\{\frac{3x-5}{x-4}\right\}$

Using Quotient rule

$$\frac{dy}{dx}=\frac{(x-4)\frac{d}{dx}(3x-5)-(3x-5)\frac{d}{dx}(x-4)}{(x-4)^2}$$

$$\frac{dy}{dx}=\frac{(x-4)(3\frac{d}{dx}x-\frac{d}{dx}5)-(3x-5)(\frac{d}{dx}x-\frac{d}{dx}4)}{(x-4)^2}$$

$$\frac{dy}{dx}=\frac{(x-4)(3(1)-0)-(3x-5)(1-0)}{(x-4)^2}$$

$$\frac{dy}{dx}=\frac{3(x-4)-1(3x-5)}{(x-4)^2}$$

$$\frac{dy}{dx}=\frac{3x-12-3x+5}{(x-4)^2}$$

$$\frac{dy}{dx}=\frac{-7}{(x-4)^2}$$

---

b). $y=\frac{2}{3x-5}$

Solution: Differentiating $\frac{dy}{dx}=\frac{d}{dx}\left\{\frac{2}{3x-5}\right\}$

Using Quotient rule

$$\frac{dy}{dx}=\frac{(3x-5)\frac{d}{dt}2-2\frac{d}{dt}(3x-5)}{(3x-5)^2}$$

$$\frac{dy}{dx}=\frac{(3x-5).0-2(3\frac{d}{dt}x-\frac{d}{dt}5)}{(3x-5)^2}$$

$$\frac{dy}{dx}=\frac{-2(3(1)-0)}{(3x-5)^2}$$

$$\frac{dy}{dx}=\frac{-6}{(3x-5)^2}$$

---

c). $f(t)=\dfrac{t^2+t}{t-1}$

Solution: Differentiating $\dfrac{d}{dt}f(t)=\dfrac{d}{dt}\left\{\dfrac{t^2+t}{t-1}\right\}$

Using Quotient rule

$$\frac{d}{dt}f(t)=\frac{(t-1)\frac{d}{dt}(t^2+t)-(t^2+t)\frac{d}{dt}(t-1)}{(t-1)^2}$$

$$f'(t)=\frac{(t-1)\left(\frac{d}{dt}t^2+\frac{d}{dt}t\right)-(t^2+t)\left(\frac{d}{dt}t-\frac{d}{dt}1\right)}{(t-1)^2}$$

$$f'(t)=\frac{(t-1)(2t+1)-(t^2+t)(1-0)}{(t-1)^2}$$

$$f'(t)=\frac{(t-1)(2t+1)-1(t^2+t)}{(t-1)^2}$$

$$f'(t)=\frac{2t^2+t-2t-1-t^2-t}{(t-1)^2}$$

$$f'(t)=\frac{2t^2-t^2-t+t-2t-1}{(t-1)^2}$$

$$f'(t)=\frac{t^2-2t-1}{(t-1)^2}$$

d). $y=\dfrac{-x^2+6x}{4x^3+1}$

Solution: Differentiating $\dfrac{dy}{dx}=\dfrac{d}{dx}\left\{\dfrac{-x^2+6x}{4x^3+1}\right\}$

Using Quotient rule

$$\frac{dy}{dx}=\frac{(4x^3+1)\frac{d}{dx}(-x^2+6x)-(-x^2+6x)\frac{d}{dx}(4x^3+1)}{(4x^3+1)^2}$$

$$\frac{dy}{dx}=\frac{(4x^3+1)\left(-\frac{d}{dx}x^2+6\frac{d}{dx}x\right)-(-x^2+6x)\left(4\frac{d}{dx}x^3+\frac{d}{dx}1\right)}{(4x^3+1)^2}$$

$$\frac{dy}{dx}=\frac{(4x^3+1)(-2x+6(1))-(-x^2+6x)(4(3x^2)+0)}{(4x^3+1)^2}$$

$$\frac{dy}{dx}=\frac{(4x^3+1)(-2x+6)-12x^2(-x^2+6x)}{(4x^3+1)^2}$$

$$\frac{dy}{dx}=\frac{-8x^4+24x^3-2x+6+12x^4-72x^3}{(4x^3+1)^2}$$

$$\frac{dy}{dx}=\frac{-8x^4+12x^4+24x^3-72x^3-2x+6}{(4x^3+1)^2}$$

$$\frac{dy}{dx}=\frac{4x^4-48x^3-2x+6}{(4x^3+1)^2}$$

e). $y=\dfrac{5x+6}{\sqrt{x}}$

Solution: Differentiating $\dfrac{dy}{dx}=\dfrac{d}{dx}\left\{\dfrac{5x+6}{\sqrt{x}}\right\}$

Using Quotient rule

$$\frac{dy}{dx}=\frac{1}{(\sqrt{x})^2}\left[(\sqrt{x})\tfrac{d}{dx}(5x+6)-(5x+6)\tfrac{d}{dx}\sqrt{x}\right]$$

$$\frac{dy}{dx}=\frac{1}{x}\left[(\sqrt{x})\left(5\tfrac{d}{dx}x+\tfrac{d}{dx}6\right)-(5x+6)\tfrac{d}{dx}x^{\frac{1}{2}}\right]$$

$$\frac{dy}{dx}=\frac{1}{x}\left[(\sqrt{x})(5(1)+0)-\frac{1}{2}(5x+6)x^{\frac{1}{2}-1}\right]$$

$$\frac{dy}{dx}=\frac{1}{x}\left[5\sqrt{x}-\frac{1}{2}(5x+6)x^{\frac{-1}{2}}\right]$$

$$\frac{dy}{dx}=\frac{1}{x}\left[5\sqrt{x}-\frac{5x+6}{2\sqrt{x}}\right]$$

$$\frac{dy}{dx}=\frac{1}{x}\left[\frac{10x-(5x+6)}{2\sqrt{x}}\right]$$

$$\frac{dy}{dx}=\frac{1}{x}\left[\frac{5x-6}{2\sqrt{x}}\right]$$

$$\frac{dy}{dx}=\frac{5x-6}{2x\sqrt{x}}$$

f). $y=\dfrac{x^2+7x-2}{x-2}$

Solution: Differentiating $\dfrac{dy}{dx}=\dfrac{d}{dx}\left\{\dfrac{x^2+7x-2}{x-2}\right\}$

Using Quotient rule

$$\frac{dy}{dx}=\frac{(x-2)\frac{d}{dx}(x^2+7x-2)-(x^2+7x-2)\frac{d}{dx}(x-2)}{(x-2)^2}$$

$$\frac{dy}{dx}=\frac{(x-2)\left(\frac{d}{dx}x^2+7\frac{d}{dx}x-\frac{d}{dx}2\right)-(x^2+7x-2)\left(\frac{d}{dx}x-\frac{d}{dx}2\right)}{(x-2)^2}$$

$$\frac{dy}{dx}=\frac{(x-2)(2x+7(1)-0)-(x^2+7x-2)(1-0)}{(x-2)^2}$$

$$\frac{dy}{dx}=\frac{(x-2)(2x+7)-1(x^2+7x-2)}{(x-2)^2}$$

$$\frac{dy}{dx}=\frac{2x^2+7x-4x-14-x^2-7x+2}{(x-2)^2}$$

$$\frac{dy}{dx}=\frac{2x^2-x^2+7x-7x-4x-14+2}{(x-2)^2}$$

$$\frac{dy}{dx}=\frac{x^2-4x-12}{(x-2)^2}$$

g). $f(p)=\dfrac{(2p+3)(4p-1)}{(3p+2)}$

Sol: Diff $\dfrac{d}{dp}f(p)=\dfrac{d}{dp}\left\{\dfrac{(2p+3)(4p-1)}{(3p+2)}\right\}$

Using Quotient rule

$$f'(p)=\frac{(3p+2)\frac{d}{dp}\{(2p+3)(4p-1)\}-(2p+3)(4p-1)\frac{d}{dp}(3p+2)}{(3p+2)^2}$$

$$f'(p)=\frac{1}{(3p+2)^2}\left\{\begin{aligned}&(3p+2)\left\{(2p+3)\tfrac{d}{dp}(4p-1)+(4p-1)\tfrac{d}{dp}(2p+3)\right\}\\&-(2p+3)(4p-1)\left(3\tfrac{d}{dp}p+\tfrac{d}{dp}2\right)\end{aligned}\right\}$$

$$f'(p)=\frac{1}{(3p+2)^2}\left\{\begin{matrix}(3p+2)\{(2p+3)(4-0)+(4p-1)(2+0)\}\\ -(2p+3)(4p-1)(3+0)\end{matrix}\right\}$$

$$f'(p)=\frac{1}{(3p+2)^2}\left\{\begin{matrix}(3p+2)\{4(2p+3)+2(4p-1)\}\\ -3(2p+3)(4p-1)\end{matrix}\right\}$$

$$f'(p)=\frac{1}{(3p+2)^2}\left\{\begin{matrix}(3p+2)\{8P+12+8P-2\}\\ -3(8p^2-2p+12p-3)\end{matrix}\right\}$$

$$f'(p)=\frac{(3p+2)(16p+10)-3(8p^2+10p-3)}{(3p+2)^2}$$

$$f'(p)=\frac{48p^2+30p+32p+20-24p^2-30p+9}{(3p+2)^2}$$

$$f'(p)=\frac{48p^2-24p^2+30p-30p+32p+20+9}{(3p+2)^2}$$

$$f'(p)=\frac{24p^2+32p+29}{(3p+2)^2}$$

---

h). $g(x)=\dfrac{x^3+1}{(2x+1)(5x+2)}$

Solution: Differentiating

$$\frac{d}{dx}g(x)=\frac{d}{dx}\left\{\frac{x^3+1}{(2x+1)(5x+2)}\right\}$$

Using Quotient rule

$$g'(x)=\frac{(2x+1)(5x+2)\frac{d}{dx}(x^3+1)-(x^3+1)\frac{d}{dx}\{(2x+1)(5x+2)\}}{\{(2x+1)(5x+2)\}^2}$$

$$g'(x)=\frac{(10x^2+4x+5x+2)(3x^2+0)-(x^3+1)\{(2x+1)\frac{d}{dx}(5x+2)+(5x+2)\frac{d}{dx}(2x+1)\}}{(2x+1)^2(5x+2)^2}$$

$$g'(x)=\frac{3x^2(10x^2+9x+2)-(x^3+1)\{(2x+1)(5+0)+(5x+2)(2+0)\}}{(2x+1)^2(5x+2)^2}$$

$$g'(x)=\frac{3x^2(10x^2+9x+2)-(x^3+1)\{5(2x+1)+2(5x+2)\}}{(2x+1)^2(5x+2)^2}$$

$$g'(x)=\frac{3x^2(10x^2+9x+2)-(x^3+1)\{10x+5+10x+4\}}{(2x+1)^2(5x+2)^2}$$

$$g'(x)=\frac{3x^2(10x^2+9x+2)-(x^3+1)(20x+9)}{(2x+1)^2(5x+2)^2}$$

$$g'(x)=\frac{30x^4+27x^3+6x^2-(20x^4+9x^3+20x+9)}{(2x+1)^2(5x+2)^2}$$

$$g'(x)=\frac{30x^4+27x^3+6x^2-20x^4-9x^3-20x-9}{(2x+1)^2(5x+2)^2}$$

$$g'(x)=\frac{30x^4-20x^4+27x^3-9x^3+6x^2-20x-9}{(2x+1)^2(5x+2)^2}$$

$$g'(x)=\frac{10x^4+18x^3+6x^2-20x-9}{(2x+1)^2(5x+2)^2}$$

Q3. Find an equation of a tangent of a line to the graph of the function at particular point in following problems.

a). $f(x)=3x-7$ at $(3,2)$

Solution: Differentiating $\frac{d}{dx}f(x)=\frac{d}{dx}(3x-7)$

$$f'(x)=3\tfrac{d}{dx}x-\tfrac{d}{dx}7$$

$$f'(x)=3(1)-0$$

$$f'(x)=3$$

Slope of tangent line at $x=3$

$$m=f'(3)=3$$

Since equation of line {tangent} passing through point $P(x_1,y_1)$ and having slope $m$ $y-y_1=m(x-x_1)$

$$y-2=3(x-3)$$

$$y-2=3x-9$$

$$3x-y-9+2=0$$

$$3x-y-7=0$$

---

b). $f(x)=x^3$ at $x=\dfrac{-1}{2}$

Solution: $y_1=f\left(\frac{-1}{2}\right)=\left(\frac{-1}{2}\right)^3=\frac{-1}{8}$

Differentiating $\frac{d}{dx}f(x)=\frac{d}{dx}x^3$

$$f'(x)=3x^2$$

Slope of tangent line at $x=\frac{-1}{2}$

$$m=f'\left(\tfrac{-1}{2}\right)=3\left(\tfrac{-1}{2}\right)^2=\tfrac{3}{4}$$

Since equation of line {tangent} passing through point $P(x_1,y_1)$ and having slope $m$ $y-y_1=m(x-x_1)$

$$y-\left(\tfrac{-1}{8}\right)=\tfrac{3}{4}\left(x-\left(\tfrac{-1}{2}\right)\right)$$

$$y+\frac{1}{8}=\frac{3}{4}\left(x+\frac{1}{2}\right)$$

$$y+\frac{1}{8}=\frac{3}{4}x+\frac{3}{8}$$

$$\frac{3}{4}x-y+\frac{3}{8}-\frac{1}{8}=0$$

$$\frac{3}{4}x-y+\frac{2}{8}=0$$

Multiply each term by 4 we get

$$3x-4y+1=0$$

---

c). $f(x)=\dfrac{1}{x+3}$ at $x=2$

Solution: $y_1=f(2)=\dfrac{1}{2+3}=\dfrac{1}{5}$ Differentiating

$$\tfrac{d}{dx}f(x)=\tfrac{d}{dx}(x+3)^{-1}$$

$$f'(x)=-1(x+3)^{-1-1}\tfrac{d}{dx}(x+3)$$

$$f'(x)=-1(x+3)^{-2}\left(\tfrac{d}{dx}x+\tfrac{d}{dx}3\right)$$

$$f'(x)=\frac{-1}{(x+3)^2}$$

Slope of tangent line at $x=2$

$$m=f'(2)=\frac{-1}{(2+3)^2}=\frac{-1}{25}$$

Since equation of line {tangent} passing through point $P(x_1,y_1)$ and having slope $m$ $y-y_1=m(x-x_1)$

$$y-\frac{1}{5}=\frac{-1}{25}(x-2)$$

$$y-\frac{1}{5}=\frac{-1}{25}x+\frac{2}{25}$$

$$\frac{1}{25}x+y-\frac{1}{5}-\frac{2}{25}=0$$
$$\frac{1}{25}x+y-\frac{5+2}{25}=0$$
Multiply by 25
$$x+25y-7=0$$

---

d). $f(x)=\frac{x}{x-2}$ at $(3,3)$

Solution: Differentiating $\frac{d}{dx}f(x)=\frac{d}{dx}\frac{x}{x-2}$

$$f'(x)=\frac{(x-2)\frac{d}{dx}x-x\frac{d}{dx}(x-2)}{(x-2)^2}$$
$$f'(x)=\frac{(x-2).1-x\left(\frac{d}{dx}x-\frac{d}{dx}2\right)}{(x-2)^2}$$
$$f'(x)=\frac{(x-2).1-x(1-0)}{(x-2)^2}$$
$$f'(x)=\frac{x-2-x}{(x-2)^2}$$
$$f'(x)=\frac{-2}{(x-2)^2}$$

Slope of tangent line at $x=3$
$$m=f'(3)=\frac{-2}{(3-2)^2}=-2$$

Since equation of line {tangent} passing through point $P(x_1,y_1)$ and having slope $m$ $y-y_1=m(x-x_1)$
$$y-3=-2(x-3)$$
$$y-3=-2x+6$$
$$2x+y-3-6=0$$
$$2x+y-9=0$$

---

Q4. A company that manufactures bicycles has determined that a new employee can assemble $M(d)=\frac{200d}{3d+10}$ bicycles per day of on the job training

a). Find $M'(d)$

Solution: Differentiating $\frac{d}{dd}M(d)=\frac{d}{dd}\left(\frac{200d}{3d+10}\right)$

$$M'(d)=\frac{(3d+10)\frac{d}{dd}(200d)-200d\frac{d}{dd}(3d+10)}{(3d+10)^2}$$
$$M'(d)=\frac{(3d+10)(200\frac{d}{dd}d)-200d(3\frac{d}{dd}d+\frac{d}{dd}10)}{(3d+10)^2}$$
$$M'(d)=\frac{(3d+10)(200.1)-200d(3.1+0)}{(3d+10)^2}$$
$$M'(d)=\frac{200(3d+10)-600d}{(3d+10)^2}$$
$$M'(d)=\frac{600d+2000-600d}{(3d+10)^2}$$
$$M'(d)=\frac{2000}{(3d+10)^2}$$

---

b). Find and interpret $M'(2)$ and $M'(5)$

Solution: First we have to find $M'(2)$
$$M'(2)=\frac{2000}{(3(2)+10)^2}$$
$$M'(2)=\frac{2000}{(6+10)^2}$$
$$M'(2)=\frac{2000}{(16)^2}=\frac{2000}{256}=7.8125$$

Now we have to find $M'(5)$
$$M'(5)=\frac{2000}{(3(5)+10)^2}$$
$$M'(5)=\frac{2000}{(15+10)^2}$$
$$M'(5)=\frac{2000}{(25)^2}=\frac{2000}{625}=3.2$$

---

Q5. Suppose that temperature T of food placed in a freezer drops according to the equation

$T(t)=\frac{700}{t^2+4t+10}$ where t is the time in hours

Determine the rate of change of T with respect to t

Sol: Differentiating $\frac{d}{dt}T(t)=700\frac{d}{dt}\left(t^2+4t+10\right)^{-1}$

$$T'(t)=700(-1)\left(t^2+4t+10\right)^{-1-1}\frac{d}{dt}\left(t^2+4t+10\right)$$
$$T'(t)=-700\left(t^2+4t+10\right)^{-2}\left(\frac{d}{dt}t^2+4\frac{d}{dt}t+\frac{d}{dt}10\right)$$
$$T'(t)=\frac{-700(2t+4(1)+0)}{\left(t^2+4t+10\right)^2}$$
$$T'(t)=\frac{-700(2t+4)}{\left(t^2+4t+10\right)^2}$$

---

a). t=1 hour

Solution: We have $T'(t)=\frac{-700(2t+4)}{\left(t^2+4t+10\right)^2}$

$$T'(1)=\frac{-700(2.1+4)}{\left(1^2+4.1+10\right)^2}$$
$$T'(1)=\frac{-700(2+4)}{(1+4+10)^2}$$
$$T'(1)=\frac{-700(6)}{(15)^2}$$
$$T'(1)=\frac{-4200}{225}=-18\frac{2}{3}=-18.67$$

---

b). t= 2 hours

Solution: We have $T'(t)=\frac{-700(2t+4)}{\left(t^2+4t+10\right)^2}$

$$T'(2)=\frac{-700(2(2)+4)}{\left((2)^2+4(2)+10\right)^2}$$
$$T'(1)=\frac{-700(4+4)}{(4+8+10)^2}$$
$$T'(1)=\frac{-700(8)}{(22)^2}$$

$T'(1)=\frac{-5600}{484}=-11\frac{69}{121}=-11.57$

Q6. For a thin lens of constant focal length P, the object distance x and the image distance y are related by the formula $\frac{1}{x}+\frac{1}{y}=\frac{1}{P}$

a). Solve above equation for y in term of x and P

Solution: We have $\frac{1}{x}+\frac{1}{y}=\frac{1}{P}$

$\frac{1}{y}=\frac{1}{P}-\frac{1}{x}$

$\frac{1}{y}=\frac{x-P}{Px}$

Taking reciprocal

$y=\frac{Px}{x-P}$

b). Determine rate of change of y with respect to x

Solution: Differentiating $\frac{d}{dx}y=\frac{d}{dx}\left(\frac{Px}{x-P}\right)$

$y'=\frac{(x-P)P\frac{d}{dx}x-Px\frac{d}{dx}(x-P)}{(x-P)^2}$

$y'=\frac{(x-P)P.1-Px\left(\frac{d}{dx}x-\frac{d}{dx}P\right)}{(x-P)^2}$

$y'=\frac{P(x-P)-Px(1-0)}{(x-P)^2}$

$y'=\frac{Px-P^2-Px}{(x-P)^2}$ $\Rightarrow y'=\frac{-P^2}{(x-P)^2}$

Q7. Suppose you are manger of a trucking firm, and one of your drivers reports that, according to her calculations, her truck burns fuel at the rate of $G(x)=\frac{1}{200}\left(\frac{800}{x}+x\right), G'(x)=\frac{1}{200}\left(-\frac{800}{x^2}+1\right)$ gallons per mile when traveling at x miles per hour on a smooth dry road

Solution: Best speed when fuel burns less

Take $G'(x)=0$ i.e., $\frac{1}{200}\left(\frac{-800}{x^2}+1\right)=0$

$\Rightarrow \frac{-800}{x^2}+1=0$

$1=\frac{800}{x^2}$

$\Rightarrow \quad x^2=800$

Taking square root on both sides

$\Rightarrow \quad \sqrt{x^2}=\sqrt{800}$

$\Rightarrow \quad x=\sqrt{400\times 2}=20\sqrt{2}$

$x=28.28$

Best speed of 28.28 mile per hour

a). If the driver tells you she wants to travel 20 mile per hour, what should you tell her?

Solution: We have $G'(20)=\frac{1}{200}\left(-\frac{800}{20^2}+1\right)$

$G'(20)=\frac{1}{200}\left(-\frac{800}{400}+1\right)$

$G'(20)=\frac{1}{200}(-2+1)$

$G'(20)=\frac{-1}{200}=-0.005$

As first derivative is negative so I will tell to go faster

Or Best speed is 28 mile per hour and driver is at 20 mile per hour so I will tell to driver to go faster

b). If the deriver wants to go 40 miles per hour, what should you say?

Solution: We have $G'(40)=\frac{1}{200}\left(-\frac{800}{40^2}+1\right)$

$G'(40)=\frac{1}{200}\left(-\frac{800}{1600}+1\right)$

$G'(40)=\frac{1}{200}(-0.5+1)$

$G'(40)=\frac{1}{200}(0.5)=\frac{1}{400}=0.0025$

As first derivative is positive so I will tell to go slower

Or Best speed is 28 mile per hour and driver is at 40 mile per hour so I will tell to driver to go slower

**Chain Rule** Let composite function is defined as

$h(x)=f(g(x))$ or $y=h(x)=f[g(x)]$

Where $y=f(u)$, and $u=g(x)$

First principle for derivative

$h'(x)=\frac{d}{dx}h(x)=\lim_{\Delta x\to 0}\frac{h(x+\Delta x)-h(x)}{\Delta x}$

Putting the value of $y=h(x)=f[g(x)]$

$h'(x)=\frac{d}{dx}h(x)=\lim_{\Delta x\to 0}\frac{f[g(x+\Delta x)]-f[g(x)]}{\Delta x}$

$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{f[g(x+\Delta x)]-f[g(x)]}{\Delta x}\cdot\frac{g(x+\Delta x)-g(x)}{g(x+\Delta x)-g(x)}$

$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{f[g(x+\Delta x)]-f[g(x)]}{g(x+\Delta x)-g(x)}\cdot\frac{g(x+\Delta x)-g(x)}{\Delta x}$

put $k=g(x+\Delta x)-g(x)$ so

$u+k=g(x)+g(x+\Delta x)-g(x)$

$u+k=g(x+\Delta x)$

when limit $\Delta x\to 0$ so

$k=[g(x+\Delta x)-g(x)]\to 0$

$\frac{dy}{dx}=\lim_{k\to 0}\frac{f[u+k]-f[u]}{k}\cdot\lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x}$

$\frac{dy}{dx}=f'(u)g'(x)$

$\frac{dy}{dx}=\frac{d}{du}y.\frac{d}{dx}u$

**Differentiation of inverse function**

If $y=f(x)$ is any differential function of x, then it admit an inverse function $x=g(y)$

Small amount of change (increment) in $y$ is $\Delta y$

So small amount of change (increment) in $x$ is $\Delta x$

Increment $\Delta x$ in $x$ corresponds to increment $\Delta y$ in y

$x=g(x)$ $\quad x=g(x)$ is inverse of $y=f(x)$

i.e. $I=\frac{\Delta y}{\Delta x}\cdot\frac{\Delta x}{\Delta y}$ by letting $\Delta x\to 0$ to obtain

$I=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}\frac{\Delta x}{\Delta y}=f'(x)g'(y)$

$I=\frac{dy}{dx}\cdot\frac{dx}{dy}$

$\frac{1}{\frac{dx}{dy}}=\frac{dy}{dx}$

Thus $\frac{dy}{dx}$ and $\frac{dx}{dy}$ are reciprocal of each other

Generalized power rule

Let $y=\left[f(x)\right]^n$ then

$\frac{dy}{dx}=\frac{d}{dx}\left[f(x)\right]^n=n\left[f(x)\right]^{n-1}\frac{d}{dx}f(x)$

# Exercise 2.4

Q1. Determine indicated derivative in each case:

a). $s=5(7-t)^4$ $\qquad \frac{ds}{dt}=?$

Sol: Differentiating $\frac{ds}{dt}=5\frac{d}{dt}(7-t)^4$

$\frac{ds}{dt}=5\times 4(7-t)^{4-1}\frac{d}{dt}(7-t)$

$\frac{ds}{dt}=20(7-t)^3\left(\frac{d}{dt}7-\frac{d}{dt}t\right)$

$\frac{ds}{dt}=20(7-t)^3(0-1)$ $\qquad \Rightarrow \frac{ds}{dt}=-20(7-t)^3$

b). $w=4\left(x^3-4x+2\right)^5$ $\qquad \frac{dw}{dx}=?$

Sol: Differentiating $\frac{dw}{dx}=4\frac{d}{dx}\left(x^3-4x+2\right)^5$

$\frac{dw}{dx}=4\times 5\left(x^3-4x+2\right)^{5-1}\frac{d}{dx}\left(x^3-4x+2\right)$

$\frac{dw}{dx}=20\left(x^3-4x+2\right)^4\left(\frac{d}{dx}x^3-4\frac{d}{dx}x+\frac{d}{dx}2\right)$

$\frac{dw}{dx}=20\left(x^3-4x+2\right)^4\left(3x^2-4(1)+0\right)$

$\frac{dw}{dx}=20\left(x^3-4x+2\right)^4\left(3x^2-4\right)$

c). $x=-3\left(4-11s^2\right)^5$ $\qquad \frac{dx}{ds}=?$

Sol: Differentiating $\frac{dx}{ds}=-3\frac{d}{ds}\left(4-11s^2\right)^5$

$\frac{dx}{ds}=-3\times 5\left(4-11s^2\right)^4\left(\frac{d}{ds}4-11\frac{d}{ds}s^2\right)$

$\frac{dx}{ds}=-15\left(4-11s^2\right)^4\left(0-11(2S)\right)$

$\frac{dx}{ds}=-15\left(4-11s^2\right)^4(-22S)$

$\frac{dx}{ds}=-15\times -22S\left(4-11s^2\right)^4$

$\frac{dx}{ds}=330S\left(4-11s^2\right)^4$

d). $y=\frac{1}{5}\left(4-x^3\right)^{11}$ $\qquad \frac{dy}{dx}=?$

Sol: Differentiating $\frac{dy}{dx}=\frac{1}{5}\frac{d}{dx}\left(4-x^3\right)^{11}$

$\frac{dy}{dx}=\frac{1\times 11}{5}\left(4-x^3\right)^{11-1}\frac{d}{dx}\left(4-x^3\right)$

$\frac{dy}{dx}=\frac{11}{5}\left(4-x^3\right)^{10}\left(\frac{d}{dx}4-\frac{d}{dx}x^3\right)$

$\frac{dy}{dx}=\frac{11}{5}\left(4-x^3\right)^{10}\left(0-3x^2\right)$

$\frac{dy}{dx}=\frac{-33x^2}{5}\left(4-x^3\right)^{10}$

d). $y=\frac{1}{5}\left(4-x^3\right)^{11}$ $\qquad \frac{dy}{dx}=?$

Sol: Differentiating $\frac{dy}{dx}=\frac{1}{5}\frac{d}{dx}\left(4-x^3\right)^{11}$

$\frac{dy}{dx}=\frac{1\times 11}{5}\left(4-x^3\right)^{11-1}\frac{d}{dx}\left(4-x^3\right)$

$\frac{dy}{dx}=\frac{11}{5}\left(4-x^3\right)^{10}\left(\frac{d}{dx}4-\frac{d}{dx}x^3\right)$

$\frac{dy}{dx}=\frac{11}{5}\left(4-x^3\right)^{10}\left(0-3x^2\right)$

$\frac{dy}{dx}=\frac{-33x^2}{5}\left(4-x^3\right)^{10}$

e). $u=\sqrt[3]{1-3t^2}$ $\qquad \frac{du}{dt}=?$

Sol: Differentiating $\frac{du}{dt}=\frac{d}{dt}\left(1-3t^2\right)^{\frac{1}{3}}$

$\frac{du}{dt}=\frac{1}{3}\left(1-3t^2\right)^{\frac{1}{3}-1}\frac{d}{dt}\left(1-3t^2\right)$

$\frac{du}{dt}=\frac{1}{3}\left(1-3t^2\right)^{\frac{-2}{3}}\left(\frac{d}{dt}1-3\frac{d}{dt}t^2\right)$

$\frac{du}{dt}=\frac{1}{3}\left(1-3t^2\right)^{\frac{-2}{3}}\left(0-3(2t)\right)$

$\frac{du}{dt}=\frac{1}{3}\left(1-3t^2\right)^{\frac{-2}{3}}(-6t)$

$\frac{du}{dt}=\left(1-3t^2\right)^{\frac{-2}{3}}(-2t)$

$\frac{du}{dt}=\frac{-2t}{\sqrt[3]{\left(1-3t^2\right)^2}}$

f). $s=\frac{1}{(3t+1)^7}$ $\qquad \frac{ds}{dt}=?$

Sol: Differentiating $\frac{ds}{dt}=\frac{d}{dt}(3t+1)^{-7}$

$\frac{ds}{dt}=-7(3t+1)^{-7-1}\frac{d}{dt}(3t+1)$

$\frac{ds}{dt}=-7(3t+1)^{-8}\left(3\frac{d}{dt}t+\frac{d}{dt}1\right)$

$\frac{ds}{dt}=-7(3t+1)^{-8}\left(3(1)+0\right)$

$\frac{ds}{dt}=\frac{-21}{(3t+1)^8}$

g). $R=\frac{1}{(2x-1)^8}$ $\frac{dR}{dx}=?$

Solution: Differentiating $\frac{dR}{dx}=\frac{d}{dx}(2x-1)^{-8}$

$$\frac{dR}{dx}=-8(2x-1)^{-8-1}\frac{d}{dx}(2x-1)$$
$$\frac{dR}{dx}=-8(2x-1)^{-9}\left(2\frac{d}{dx}x-\frac{d}{dx}1\right)$$
$$\frac{dR}{dx}=\frac{-8}{(2x-1)^9}(2(1)-0)$$
$$\frac{dR}{dx}=\frac{-16}{(2x-1)^9}$$

---

h). $R=\frac{1}{5(4x^2-7)^7}$ $\frac{dR}{dx}=?$

Solution: Differentiating $\frac{dR}{dx}=\frac{1}{5}\frac{d}{dx}(4x^2-7)^{-7}$

$$\frac{dR}{dx}=\frac{-7}{5}(4x^2-7)^{-7-1}\frac{d}{dx}(4x^2-7)$$
$$\frac{dR}{dx}=\frac{-7}{5}(4x^2-7)^{-8}\left(4\frac{d}{dx}x^2-\frac{d}{dx}7\right)$$
$$\frac{dR}{dx}=\frac{-7}{5}(4x^2-7)^{-8}(4(2x)-0)$$
$$\frac{dR}{dx}=\frac{-7}{5(4x^2-7)^8}(8x)$$
$$\frac{dR}{dx}=\frac{-56x}{5(4x^2-7)^8}$$

---

Q2. Determine the derivative $f'(x)$ in each case:

a). $f(x)=(2x-5)^3(5x-7)$

Sol: Differentiating $\frac{d}{dx}f(x)=\frac{d}{dx}\{(2x-5)^3(5x-7)\}$

Using product rule

$$f'(x)=(2x-5)^3\frac{d}{dx}(5x-7)+(5x-7)\frac{d}{dx}(2x-5)^3$$
$$f'(x)=(2x-5)^3\left(5\frac{d}{dx}x-\frac{d}{dx}7\right)+3(5x-7)(2x-5)^{3-1}\frac{d}{dx}(2x-5)$$
$$f'(x)=(2x-5)^3(5(1)-0)+3(5x-7)(2x-5)^2(2(1)-0)$$
$$f'(x)=5(2x-5)^3+3\times2(5x-7)(2x-5)^2$$
$$f'(x)=(2x-5)^2\{5(2x-5)+6(5x-7)\}$$
$$f'(x)=(2x-5)^2\{10x-25+30x-42\}$$
$$f'(x)=(2x-5)^2(40x-67)$$

---

b). $f(x)=\frac{(x+2)^2}{x-1}$

Solution: Differentiating with respect to x

$\frac{d}{dx}f(x)=\frac{d}{dx}\left\{\frac{(x+2)^2}{x-1}\right\}$ Using quotient rule

$$f'(x)=\frac{(x-1)\frac{d}{dx}(x+2)^2-(x+2)^2\frac{d}{dx}(x-1)}{(x-1)^2}$$
$$f'(x)=\frac{2(x-1)(x+2)^{2-1}\frac{d}{dx}(x+2)-(x+2)^2\left(\frac{d}{dx}x-\frac{d}{dx}1\right)}{(x-1)^2}$$
$$f'(x)=\frac{2(x-1)(x+2)^1(1+0)-(x+2)^2(1-0)}{(x-1)^2}$$
$$f'(x)=\frac{2(x-1)(x+2)-(x+2)^2}{(x-1)^2}$$
$$f'(x)=\frac{(x+2)\{2(x-1)-(x+2)\}}{(x-1)^2}$$
$$f'(x)=\frac{(x+2)\{2x-2-x-2\}}{(x-1)^2}$$
$$f'(x)=\frac{(x+2)(x-4)}{(x-1)^2}$$

---

c). $f(x)=\left(\frac{2x-5}{x-4}\right)^4$

Solution: Differentiating with respect to x

$$\frac{d}{dx}f(x)=\frac{d}{dx}\left(\frac{2x-5}{x-4}\right)^4$$

$f'(x)=4\left(\frac{2x-5}{x-4}\right)^{4-1}\frac{d}{dx}\left(\frac{2x-5}{x-4}\right)$ Using quotient rule

$$f'(x)=4\left(\frac{2x-5}{x-4}\right)^3\frac{(x-4)\frac{d}{dx}(2x-5)-(2x-5)\frac{d}{dx}(x-4)}{(x-4)^2}$$
$$f'(x)=4\frac{(2x-5)^3}{(x-4)^3}\frac{(x-4)(2\frac{d}{dx}x-\frac{d}{dx}5)-(2x-5)(\frac{d}{dx}x-\frac{d}{dx}4)}{(x-4)^2}$$
$$f'(x)=4\frac{(2x-5)^3}{(x-4)^3}\frac{(x-4)(2(1)-0)-(2x-5)(1-0)}{(x-4)^2}$$
$$f'(x)=4\frac{(2x-5)^3}{(x-4)^{3+2}}\{2(x-4)-1(2x-5)\}$$
$$f'(x)=4\frac{(2x-5)^3}{(x-4)^5}\{2x-8-2x+5\}$$
$$f'(x)=4\frac{(2x-5)^3}{(x-4)^5}\{-3\}$$
$$f'(x)=\frac{-12(2x-5)^3}{(x-4)^5}$$

---

d). $f(x)=x\sqrt{2x^2+11}$

Sol: Differentiating $\frac{d}{dx}f(x)=\frac{d}{dx}\left\{x\sqrt{2x^2+11}\right\}$

Using product rule

$$f'(x)=x\frac{d}{dx}(2x^2+11)^{\frac{1}{2}}+(2x^2+11)^{\frac{1}{2}}\frac{d}{dx}x$$
$$f'(x)=\tfrac{1}{2}x(2x^2+11)^{\frac{1}{2}-1}\frac{d}{dx}(2x^2+11)+(2x^2+11)^{\frac{1}{2}}.1$$
$$f'(x)=\tfrac{1}{2}x(2x^2+11)^{\frac{-1}{2}}\left(2\frac{d}{dx}x^2+\frac{d}{dx}11\right)+(2x^2+11)^{\frac{1}{2}}$$
$$f'(x)=\tfrac{1}{2}x(2x^2+11)^{\frac{-1}{2}}(2(2x)+0)+(2x^2+11)^{\frac{1}{2}}$$
$$f'(x)=\tfrac{4}{2}x^2(2x^2+11)^{\frac{-1}{2}}+(2x^2+11)^{\frac{1}{2}}$$
$$f'(x)=\frac{2x^2}{(2x^2+11)^{\frac{1}{2}}}+(2x^2+11)^{\frac{1}{2}}$$
$$f'(x)=\frac{2x^2+(2x^2+11)^{\frac{1}{2}+\frac{1}{2}}}{(2x^2+11)^{\frac{1}{2}}}$$

$$f'(x)=\frac{2x^2+\left(2x^2+11\right)^1}{\left(2x^2+11\right)^{\frac{1}{2}}}$$

$$f'(x)=\frac{2x^2+2x^2+11}{\left(2x^2+11\right)^{\frac{1}{2}}}$$

$$f'(x)=\frac{4x^2+11}{\left(2x^2+11\right)^{\frac{1}{2}}}$$

e). $f(x)=3x\sqrt[3]{3x+7}$

Solution: Differentiating with respect to x

$\frac{d}{dx}f(x)=3\frac{d}{dx}\left\{x(3x+7)^{\frac{1}{3}}\right\}$ Using product rule

$$f'(x)=3\left\{x\tfrac{d}{dx}(3x+7)^{\frac{1}{3}}+(3x+7)^{\frac{1}{3}}\tfrac{d}{dx}x\right\}$$

$$f'(x)=3\left\{\tfrac{1}{3}x(3x+7)^{\frac{1}{3}-1}\tfrac{d}{dx}(3x+7)+(3x+7)^{\frac{1}{3}}.1\right\}$$

$$f'(x)=3\left\{\tfrac{1}{3}x(3x+7)^{\frac{-2}{3}}\left(3\tfrac{d}{dx}x+\tfrac{d}{dx}7\right)+(3x+7)^{\frac{1}{3}}\right\}$$

$$f'(x)=3\left\{\tfrac{1}{3}x(3x+7)^{\frac{-2}{3}}(3.1+0)+(3x+7)^{\frac{1}{3}}\right\}$$

$$f'(x)=3\left\{\frac{x}{(3x+7)^{\frac{2}{3}}}+(3x+7)^{\frac{1}{3}}\right\}$$

$$f'(x)=3\left\{\frac{x+(3x+7)^{\frac{1}{3}+\frac{2}{3}}}{(3x+7)^{\frac{2}{3}}}\right\}$$

$$f'(x)=3\left\{\frac{x+(3x+7)^1}{(3x+7)^{\frac{2}{3}}}\right\}$$

$$f'(x)=3\left\{\frac{x+3x+7}{(3x+7)^{\frac{2}{3}}}\right\}=3\left\{\frac{4x+7}{(3x+7)^{\frac{2}{3}}}\right\}$$

f). $f(x)=\dfrac{\sqrt{2x+11}}{(3x-8)^2}$

Sol: Differentiating $\dfrac{d}{dx}f(x)=\dfrac{d}{dx}\left\{\dfrac{\sqrt{2x+11}}{(3x-8)^2}\right\}$

Applying quotient rule

$$f'(x)=\frac{(3x-8)^2\frac{d}{dx}(2x+11)^{\frac{1}{2}}-(2x+11)^{\frac{1}{2}}\frac{d}{dx}(3x-8)^2}{(3x-8)^4}$$

$$f'(x)=\frac{\frac{1}{2}(3x-8)^2(2x+11)^{\frac{1}{2}-1}\frac{d}{dx}(2x+11)-2(2x+11)^{\frac{1}{2}}(3x-8)^{2-1}\frac{d}{dx}(3x-8)}{(3x-8)^4}$$

$$f'(x)=\frac{\frac{1}{2}(3x-8)^2(2x+11)^{\frac{-1}{2}}(2(1)+0)-2(2x+11)^{\frac{1}{2}}(3x-8)^1(3(1)-0)}{(3x-8)^4}$$

$$f'(x)=\frac{(3x-8)^2(2x+11)^{\frac{-1}{2}}-6(2x+11)^{\frac{1}{2}}(3x-8)}{(3x-8)^4}$$

$$f'(x)=\frac{(3x-8)^2(2x+11)^{\frac{-1}{2}}-6(2x+11)^{\frac{1}{2}}(3x-8)}{(3x-8)^4}$$

$$f'(x)=\frac{(3x-8)}{(3x-8)^4}\left\{\frac{(3x-8)}{(2x+11)^{\frac{1}{2}}}-6(2x+11)^{\frac{1}{2}}\right\}$$

$$f'(x)=\frac{1}{(3x-8)^3}\left\{\frac{(3x-8)-6(2x+11)^{\frac{1}{2}+\frac{1}{2}}}{(2x+11)^{\frac{1}{2}}}\right\}$$

$$f'(x)=\frac{(3x-8)-6(2x+11)^1}{(3x-8)^3(2x+11)^{\frac{1}{2}}}$$

$$f'(x)=\frac{3x-8-12x-66}{(3x-8)^3(2x+11)^{\frac{1}{2}}}$$

$$f'(x)=\frac{-9x-74}{(3x-8)^3(2x+11)^{\frac{1}{2}}}$$

g). $f(x)=\left(\dfrac{3x-8}{x+9}\right)^7$

Solution: Differentiating with respect to x

$$\frac{d}{dx}f(x)=\frac{d}{dx}\left(\frac{3x-8}{x+9}\right)^7$$

$$f'(x)=7\left(\frac{3x-8}{x+9}\right)^{7-1}\frac{d}{dx}\left(\frac{3x-8}{x+9}\right)$$

Using quotient rule

$$f'(x)=7\left(\frac{3x-8}{x+9}\right)^6.\frac{(x+9)\frac{d}{dx}(3x-8)-(3x-8)\frac{d}{dx}(x+9)}{(x+9)^2}$$

$$f'(x)=7\frac{(3x-8)^6}{(x+9)^6}.\frac{(x+9)(3.1-0)-(3x-8)(1+0)}{(x+9)^2}$$

$$f'(x)=7\frac{(3x-8)^6}{(x+9)^{6+2}}.\{3(x+9)-1(3x-8)\}$$

$$f'(x)=7\frac{(3x-8)^6}{(x+9)^8}.\{3x+27-3x+8\}$$

$$f'(x)=7\frac{(3x-8)^6}{(x+9)^8}.\{35\}$$

$$f'(x)=245\frac{(3x-8)^6}{(x+9)^8}.$$

h). $f(x)=(2x-9)^2\sqrt{3x+7}$

Solution: Differentiating with respect to x

$\frac{d}{dx}f(x)=\frac{d}{dx}\left\{(2x-9)^2\sqrt{3x+7}\right\}$ Using product rule

$$f'(x)=(2x-9)^2\tfrac{d}{dx}(3x+7)^{\frac{1}{2}}+(3x+7)^{\frac{1}{2}}\tfrac{d}{dx}(2x-9)^2$$

$$f'(x)=\tfrac{1}{2}(2x-9)^2(3x+7)^{\frac{1}{2}-1}\tfrac{d}{dx}(3x+7)+2(3x+7)^{\frac{1}{2}}(2x-9)^{2-1}\tfrac{d}{dx}(2x-9)$$

$$f'(x)=\tfrac{1}{2}(2x-9)^2(3x+7)^{\frac{-1}{2}}(3(1)+0)+2(3x+7)^{\frac{1}{2}}(2x-9)^1(2(1)-0)$$

$$f'(x)=\tfrac{3}{2}(2x-9)^2(3x+7)^{\frac{-1}{2}}+2\times2(3x+7)^{\frac{1}{2}}(2x-9)$$

$$f'(x)=\frac{3(2x-9)^2}{2(3x+7)^{\frac{1}{2}}}+4(3x+7)^{\frac{1}{2}}(2x-9)$$

$$f'(x)=\frac{3(2x-9)^2+8(3x+7)^{\frac{1}{2}+\frac{1}{2}}(2x-9)}{2(3x+7)^{\frac{1}{2}}}$$

$$f'(x)=(2x-9)\left\{\frac{3(2x-9)+8(3x+7)^1}{2(3x+7)^{\frac{1}{2}}}\right\}$$

$$f'(x)=(2x-9)\left\{\frac{6x-27+24x+56}{2(3x+7)^{\frac{1}{2}}}\right\}$$

$$f'(x)=(2x-9)\left\{\frac{30x+29}{2\sqrt{3x+7}}\right\}$$

Q3. Find $\frac{dy}{dx}$ of function in terms of parameter t:

a). $x=1+t^2,\qquad y=t^3+2t^2+1$

Solution: Differentiating with respect to t

$\frac{d}{dt}x=\frac{d}{dt}\left(1+t^2\right)$, $\frac{d}{dt}y=\frac{d}{dt}\left(t^3+2t^2+1\right)$

$\frac{dx}{dt}=\frac{d}{dt}1+\frac{d}{dt}t^2$, $\frac{dy}{dt}=\frac{d}{dt}t^3+2\frac{d}{dt}t^2+\frac{d}{dt}1$

$\frac{dx}{dt}=0+2t$, $\frac{dy}{dt}=3t^2+2\left(2t\right)+0$

$\frac{dx}{dt}=2t$, $\frac{dy}{dt}=3t^2+4t$

$\frac{dt}{dx}=\frac{1}{2t}$

Now using chain rule $\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}$

$$\frac{dy}{dx}=\frac{3t^2+4t}{2t}=\frac{t\left(3t+4\right)}{2t} \quad \Rightarrow \frac{dy}{dx}=\frac{3t+4}{2}$$

---

b). $x=3at^2+2, \quad y=6t^4+9$

Solution: Differentiating with respect to t

$\frac{d}{dt}x=\frac{d}{dt}\left(3at^2+2\right)$, $\frac{d}{dt}y=\frac{d}{dt}\left(6t^4+9\right)$

$\frac{dx}{dt}=3a\frac{d}{dt}t^2+\frac{d}{dt}2$, $\frac{dy}{dt}=6\frac{d}{dt}t^4+\frac{d}{dt}9$

$\frac{dx}{dt}=3a\left(2t\right)+0$, $\frac{dy}{dt}=6\left(4t^3\right)+0$

$\frac{dx}{dt}=6at$, $\frac{dy}{dt}=24t^3$

$\frac{dt}{dx}=\frac{1}{6at}$

Now using chain rule $\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}$

$$\frac{dy}{dx}=\frac{24t^3}{6at}=\frac{4t^2}{a}$$

---

c). $x=\frac{a\left(1-t^2\right)}{1+t^2}, \quad y=\frac{2bt}{1+t^2}$

Solution: Differentiating x with respect to t

$$\frac{d}{dt}x=a\frac{d}{dt}\left(\frac{1-t^2}{1+t^2}\right)$$

$$\frac{dx}{dt}=a\frac{\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right)-\left(1-t^2\right)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}$$

$$\frac{dx}{dt}=a\frac{\left(1+t^2\right)\left(0-2t\right)-\left(1-t^2\right)\left(0+2t\right)}{\left(1+t^2\right)^2}$$

$$\frac{dx}{dt}=a\frac{-2t\left(1+t^2\right)-2t\left(1-t^2\right)}{\left(1+t^2\right)^2}$$

$$\frac{dx}{dt}=a\frac{-2t-2t^3-2t+2t^3}{\left(1+t^2\right)^2}=\frac{-4at}{\left(1+t^2\right)^2}$$

$$\frac{dt}{dx}=\frac{\left(1+t^2\right)^2}{-4at}$$

Differentiating y with respect to t

$$\frac{d}{dt}y=b\frac{d}{dt}\left(\frac{2t}{1+t^2}\right)$$

$$\frac{dy}{dt}=b\frac{\left(1+t^2\right)\frac{d}{dt}\left(2t\right)-\left(2t\right)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}$$

$$\frac{dy}{dt}=b\frac{\left(1+t^2\right)\left(2\right)-\left(2t\right)\left(0+2t\right)}{\left(1+t^2\right)^2}$$

$$\frac{dy}{dt}=b\frac{2+2t^2-4t^2}{\left(1+t^2\right)^2}$$

$$\frac{dy}{dt}=b\frac{2-2t^2}{\left(1+t^2\right)^2}=b\frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2}$$

Now using chain rule $\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}$

$$\frac{dy}{dx}=\frac{2b\left(1-t^2\right)}{\left(1+t^2\right)^2}.\frac{\left(1+t^2\right)^2}{-4at}$$

$$\frac{dy}{dx}=\frac{b\left(1-t^2\right)}{-2at}=\frac{b\left(t^2-1\right)}{2at}$$

---

d). $x=\frac{3at}{1+t^3}, \quad y=\frac{3at^2}{1+t^3}$

Solution: Differentiating x with respect to t

$$\frac{d}{dt}x=3a\frac{d}{dt}\left(\frac{t}{1+t^3}\right)$$

$$\frac{dx}{dt}=3a\frac{\left(1+t^3\right)\frac{d}{dt}\left(t\right)-t\frac{d}{dt}\left(1+t^3\right)}{\left(1+t^3\right)^2}$$

$$\frac{dx}{dt}=3a\frac{\left(1+t^3\right).1-t\left(0+3t^2\right)}{\left(1+t^3\right)^2}$$

$$\frac{dx}{dt}=3a\frac{1+t^3-3t^3}{\left(1+t^3\right)^2}$$

$$\frac{dx}{dt}=3a\frac{1-2t^3}{\left(1+t^3\right)^2}$$

$$\frac{dt}{dx}=\frac{\left(1+t^3\right)^2}{3a\left(1-2t^3\right)}$$

Differentiating y with respect to t

$$\frac{d}{dt}y=3a\frac{d}{dt}\left(\frac{t^2}{1+t^3}\right)$$

$$\frac{dy}{dt}=3a\frac{\left(1+t^3\right)\frac{d}{dt}\left(t^2\right)-\left(t^2\right)\frac{d}{dt}\left(1+t^3\right)}{\left(1+t^3\right)^2}$$

$$\frac{dy}{dt}=3a\frac{\left(1+t^3\right)\left(2t\right)-\left(t^2\right)\left(0+3t^2\right)}{\left(1+t^3\right)^2}$$

$$\frac{dy}{dt}=3a\frac{2t+2t^4-3t^4}{\left(1+t^3\right)^2}$$

$$\frac{dy}{dt}=3a\frac{2t-t^4}{\left(1+t^2\right)^2}$$

Now using chain rule $\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}$

$$\frac{dy}{dx}=\frac{3a\left(2t-t^4\right)}{\left(1+t^3\right)^2}.\frac{\left(1+t^3\right)^2}{3a\left(1-2t^3\right)}$$

$$\frac{dy}{dx}=\frac{2t-t^4}{1-2t^3}$$

---

Q4. Studies show that after 1 hours on the job, the number of items a supermarket cashier can ring up per minute is given by $F\left(t\right)=60-\frac{150}{\sqrt{8+t^2}}$

a). Find $F'\left(t\right)$, rate at which casher's speed is increasing.

Solution: Differentiating with respect to t

$$\frac{d}{dt}F\left(t\right)=\frac{d}{dt}\left\{60-150\left(8+t^2\right)^{\frac{-1}{2}}\right\}$$

$$F'(t)=\frac{d}{dt}60-150\frac{d}{dt}\left(8+t^2\right)^{\frac{-1}{2}}$$

$$F'(t)=\frac{d}{dt}60+\frac{150}{2}\left(8+t^2\right)^{\frac{-1}{2}-1}\frac{d}{dt}\left(8+t^2\right)$$

$$F'(t)=0+75\left(8+t^2\right)^{\frac{-3}{2}}\left(0+2t\right)$$

$$F'(t)=\frac{75(2t)}{\left(8+t^2\right)^{\frac{3}{2}}}=\frac{150t}{\left(8+t^2\right)^{\frac{3}{2}}}$$

b). At what rate is the casher's speed increasing after 5 hours? After 10 hours?

Solution: At t = 5 Hours

$$F'(5)=\frac{150(5)}{\left(8+(5)^2\right)^{\frac{3}{2}}}=\frac{750}{33^{\frac{3}{2}}}=3.9563$$

At t = 10 hours

$$F'(10)=\frac{150(10)}{\left(8+(10)^2\right)^{\frac{3}{2}}}=\frac{1500}{108^{\frac{3}{2}}}=1.3365$$

At t = 20 hours

$$F'(20)=\frac{150(20)}{\left(8+(20)^2\right)^{\frac{3}{2}}}=\frac{3000}{408^{\frac{3}{2}}}=0.364025$$

At t = 40 hours

$$F'(40)=\frac{150(40)}{\left(8+(40)^2\right)^{\frac{3}{2}}}=\frac{6000}{1608^{\frac{3}{2}}}=0.0930551244$$

Q5. At a certain factor, the total cost of manufacturing q units the daily production run is $C(q)=0.2q^2+q+900$ dollars. From experience, it has been determined that approximately $q(t)=t^2+100t$ units are manufactured during the first t hours of a production run. Compute the rate at which the total manufacturing cost is changing with respect to time one hour after production begins.

Solution: Differentiating C with respect to q

$$\frac{d}{dq}C=\frac{d}{dq}\left(0.2q^2+q+900\right)$$

$$\frac{dC}{dq}=0.2\frac{d}{dq}q^2+\frac{d}{dq}q+\frac{d}{dq}900$$

$$\frac{dC}{dq}=0.2(2q)+1+0$$

$$\frac{dC}{dq}=0.4q+1$$

Differentiating q with respect to t

$$\frac{d}{dt}q=\frac{d}{dt}\left(t^2+100t\right)$$

$$\frac{dq}{dt}=\frac{d}{dt}t^2+100\frac{d}{dt}t$$

$$\frac{dq}{dt}=2t+100(1)$$

$$\frac{dq}{dt}=2t+100$$

Now using chain rule

$$\frac{dC}{dt}=\frac{dC}{dq}\frac{dq}{dt}=(0.4q+1)(2t+100)$$

Putting the value of q

$$\frac{d}{dt}C=\left\{0.4\left(t^2+100t\right)+1\right\}(2t+100)$$

At $t=1$ hour

$$\frac{d}{dt}C=\left\{0.4\left(1^2+100\times1\right)+1\right\}(2\times1+100)$$

$$\frac{d}{dt}C=\left\{0.4(1+100)+1\right\}(2+100)$$

$$\frac{d}{dt}C=\left\{0.4(101)+1\right\}(102)$$

$$\frac{d}{dt}C=\left\{40.4+1\right\}(102)$$

$$\frac{d}{dt}C=\left\{41.4\right\}(102)=4222.8$$

Implicit function:
Impossible to explain in term of independent variable
Implicit differentiation: direct differentiate

Explicit function:
Easily explainable in term of dependent variable
Explicit differentiation:
First explain in term of dependent variable then differentiate with respect to independent variable

# Exercise 2.5

Q1. Use implicit differentiation to perform $\frac{dy}{dx}$ for the following functions:

a). $x^2+y^2=25$

Solution: We have $x^2+y^2=25$

Differentiating with respect to x

$$\frac{d}{dx}x^2+\frac{d}{dx}y^2=\frac{d}{dx}25$$

$$2x\frac{d}{dx}x+2y\frac{d}{dx}y=0$$

$$2x(1)+2y\frac{dy}{dx}=0$$

$$2y\frac{dy}{dx}=-2x$$

$$\frac{dy}{dx}=\frac{-2x}{2y}$$

$$\frac{d}{dx}=\frac{-x}{y}$$

b). $xy=25$

Solution: We have $xy=25$

$$\frac{d}{dx}(xy)=\frac{d}{dx}25$$

$$x\frac{d}{dx}y+y\frac{d}{dx}x=0$$

$$x\frac{dy}{dx}+y.1=0$$

$$x\frac{dy}{dx}=-y$$

$$\frac{dy}{dx}=\frac{-y}{x}$$

c). $xy(2x+3y)=2$

Solution: We have $xy(2x+3y)=2$

Differentiating w r t x $\frac{d}{dx}\left\{xy(2x+3y)\right\}=\frac{d}{dx}2$

Using product rule

$$xy\frac{d}{dx}(2x+3y)+(2x+3y)\frac{d}{dx}xy=0$$

$$xy\left(2\frac{d}{dx}x+3\frac{d}{dx}y\right)+(2x+3y)\left(x\frac{d}{dx}y+y\frac{d}{dx}x\right)=0$$

$$xy(2+3y')+(2x+3y)(x\ y'+y)=0$$

$$2xy+3xy\,y'+x\ y'(2x+3y)+y(2x+3y)=0$$

$$2xy+3xy\,y'+x\ y'(2x+3y)+2xy+3y^2=0$$

$$3xy\,y'+x\ y'(2x+3y)+4xy+3y^2=0$$

$$y'\left\{3xy+x(2x+3y)\right\}=-4xy-3y^2$$

$$y'\left\{3xy+2x^2+3xy\right\}=-4xy-3y^2$$

$$y'\left\{2x^2+6xy\right\}=-4xy-3y^2$$

$$y'=\frac{-4xy-3y^2}{2x^2+6xy}$$

d). $x^2+3xy+y^2=15$

Solution: we have $x^2+3xy+y^2=15$

Differentiating with respect to x

$\frac{d}{dx}\left(x^2+3xy+y^2\right)=\frac{d}{dx}15$

$\frac{d}{dx}x^2+3\frac{d}{dx}(xy)+\frac{d}{dx}y^2=0$

$2x+3\left(x\frac{d}{dx}y+y\frac{d}{dx}x\right)+2y\frac{d}{dx}y=0$

$2x+3\left(x\frac{dy}{dx}+y\right)+2y\frac{dy}{dx}=0$

$2x+3x\frac{dy}{dx}+3y+2y\frac{dy}{dx}=0$

$3x\frac{dy}{dx}+2y\frac{dy}{dx}=-(2x+3y)$

$(3x+2y)\frac{dy}{dx}=-(2x+3y)$

$\frac{dy}{dx}=\frac{-(2x+3y)}{3x+2y}$

---

e). $(x+y)^3+3y=3$

Solution: We have $(x+y)^3+3y=3$

Differentiating with respect to x

$\frac{d}{dx}(x+y)^3+3\frac{d}{dx}y=\frac{d}{dx}3$

$3(x+y)^{3-1}\frac{d}{dx}(x+y)+3\frac{dy}{dx}=0$

$3(x+y)^2\left(\frac{d}{dx}x+\frac{d}{dx}y\right)+3\frac{dy}{dx}=0$

$3(x+y)^2\left(1+\frac{dy}{dx}\right)+3\frac{dy}{dx}=0$

$3(x+y)^2+3(x+y)^2\frac{dy}{dx}+3\frac{dy}{dx}=0$

$\left\{3(x+y)^2+3\right\}\frac{dy}{dx}=-3(x+y)^2$

$\frac{dy}{dx}=\frac{-3(x+y)^2}{3\left\{(x+y)^2+1\right\}}=\frac{-(x+y)^2}{(x+y)^2+1}$

---

f). $\frac{1}{y}+\frac{1}{x}=1$

Solution: We have $\frac{1}{y}+\frac{1}{x}=1$

$y^{-1}+x^{-1}=1$

Differentiating with respect to x

$\frac{d}{dx}y^{-1}+\frac{d}{dx}x^{-1}=\frac{d}{dx}1$

$-1y^{-1-1}\frac{d}{dx}y-1x^{-1-1}\frac{d}{dx}x=0$

$-y^{-2}\frac{d}{dx}y-x^{-2}.1=0$

$-\frac{1}{y^2}\frac{d}{dx}y-\frac{1}{x^2}=0$

$-\frac{1}{y^2}\frac{dy}{dx}=\frac{1}{x^2}$

$\frac{d}{dx}y=\frac{-y^2}{x^2}$

---

Q2. Arrange the following function explicitly and implicitly to perform $\frac{dy}{dx}$

a). $x^2y^3+y^3=12$

Solution: differentiate given function implicitly

$\frac{d}{dx}\left(x^2y^3\right)+\frac{d}{dx}y^3=\frac{d}{dx}12$

$x^2\frac{d}{dx}y^3+y^3\frac{d}{dx}x^2+3y^{3-1}\frac{d}{dx}y=0$

$3x^2y^{3-1}\frac{d}{dx}y+2xy^3\frac{d}{dx}x+3y^2\frac{d}{dx}y=0$

$3x^2y^2\frac{dy}{dx}+3y^2\frac{dy}{dx}=-2xy^3\frac{d}{dx}x$

$\left(3x^2y^2+3y^2\right)\frac{dy}{dx}=-2xy^3.1$

$\frac{dy}{dx}=\frac{-2xy^3}{3x^2y^2+3y^2}=\frac{-2xy^3}{3y^2\left(x^2+1\right)}$

$\frac{dy}{dx}=\frac{-2xy}{3\left(x^2+1\right)}$

For explicitly differentiation I will separate y in given function

$x^2y^3+y^3=12$

$y^3\left(x^2+1\right)=12$

$y^3=\frac{12}{x^2+1}$

$y=\left\{\frac{12}{x^2+1}\right\}^{\frac{1}{3}}=12^{\frac{1}{3}}\left(x^2+1\right)^{-\frac{1}{3}}$

Now differentiating

$\frac{dy}{dx}=12^{\frac{1}{3}}\frac{d}{dx}\left(x^2+1\right)^{-\frac{1}{3}}$

$\frac{dy}{dx}=\frac{12^{\frac{1}{3}}}{-3}\left(x^2+1\right)^{-\frac{1}{3}-1}\frac{d}{dx}\left(x^2+1\right)$

$\frac{dy}{dx}=\frac{12^{\frac{1}{3}}}{-3}\left(x^2+1\right)^{-\frac{1}{3}}\left(x^2+1\right)^{-1}(2x+0)$

$\frac{dy}{dx}=\frac{12^{\frac{1}{3}}}{\left(x^2+1\right)^{\frac{1}{3}}}\frac{2x}{-3\left(x^2+1\right)}$

$\frac{dy}{dx}=y\frac{2x}{-3\left(x^2+1\right)}$ $\qquad\therefore y=\frac{12^{\frac{1}{3}}}{\left(x^2+1\right)^{\frac{1}{3}}}$

$\frac{dy}{dx}=\frac{2xy}{-3\left(x^2+1\right)}$

---

b). $xy+2y=x^2$

Solution: differentiate the given function implicitly

$\frac{d}{dx}(xy)+2\frac{d}{dx}y=\frac{d}{dx}x^2$

$x\frac{d}{dx}y+y\frac{d}{dx}x+2\frac{dy}{dx}=2x\frac{d}{dx}x$

$x\frac{dy}{dx}+y.1+2\frac{dy}{dx}=2x.1$

$x\frac{dy}{dx}+2\frac{dy}{dx}=2x-y$

$(x+2)\frac{dy}{dx}=2x-y$

$\frac{dy}{dx}=\frac{2x-y}{x+2}$

For explicitly differentiation I will separate y in given function

$xy+2y=x^2$

$(x+2)y=x^2$

$y=\frac{x^2}{x+2}$

Now differentiating $\frac{dy}{dx}=\frac{d}{dx}\frac{x^2}{x+2}$

$\frac{dy}{dx}=\frac{(x+2)\frac{d}{dx}x^2-x^2\frac{d}{dx}(x+2)}{(x+2)^2}$

$\frac{dy}{dx}=\frac{2x(x+2)\frac{d}{dx}x-x^2\left(\frac{d}{dx}x+\frac{d}{dx}2\right)}{(x+2)^2}$

$$\frac{dy}{dx}=\frac{2x\left(x+2\right).1-x^2\left(1+0\right)}{\left(x+2\right)^2}$$

$$\frac{dy}{dx}=\frac{2x\left(x+2\right)-x^2}{\left(x+2\right)^2}$$

$$\frac{dy}{dx}=\frac{2x^2+4x-x^2}{\left(x+2\right)^2}$$

$$\frac{dy}{dx}=\frac{2x\left(x+2\right)-x^2}{\left(x+2\right)^2}$$

$$\frac{dy}{dx}=\frac{\frac{2x(x+2)-x^2}{x+2}}{x+2}$$

$$\frac{dy}{dx}=\frac{\frac{2x(x+2)}{x+2}-\frac{x^2}{x+2}}{x+2}$$

$$\frac{dy}{dx}=\frac{2x-y}{x+2} \qquad \therefore y=\frac{x^2}{x+2}$$

---

c). $x+\frac{1}{y}=5$

Sol: First differentiate the given function implicitly

$$\tfrac{d}{dx}x+\tfrac{d}{dx}y^{-1}=\tfrac{d}{dx}5$$

$$1-y^{-1-1}\tfrac{d}{dx}y=0$$

$$-y^{-2}\tfrac{dy}{dx}=-1$$

$$y^{-2}\tfrac{dy}{dx}=1$$

$$\tfrac{dy}{dx}=\frac{1}{y^{-2}}=y^2$$

For explicitly differentiation I will separate y in given function

$$x+\frac{1}{y}=5$$

$$\frac{1}{y}=\frac{5-x}{1}$$

Taking Reciprocal

$$y=\frac{1}{5-x}=\left(5-x\right)^{-1}$$

Now differentiating

$$\tfrac{d}{dx}y=\tfrac{d}{dx}\left(5-x\right)^{-1}$$

$$\tfrac{dy}{dx}=-1\left(5-x\right)^{-1-1}\tfrac{d}{dx}\left(5-x\right)$$

$$\tfrac{dy}{dx}=-\left(5-x\right)^{-2}\left(\tfrac{d}{dx}5-\tfrac{d}{dx}x\right)$$

$$\tfrac{dy}{dx}=-\left(5-x\right)^{-2}\left(0-1\right)$$

$$\tfrac{dy}{dx}=\left(5-x\right)^{-2}$$

$$\frac{dy}{dx}=\frac{1}{\left(5-x\right)^2}$$

$$\frac{dy}{dx}=y^2 \qquad y=\frac{1}{5-x}$$

---

d). $xy-x=y+2$

Solution: differentiate the given function implicitly

$$\tfrac{d}{dx}\left(xy\right)-\tfrac{d}{dx}x=\tfrac{d}{dx}y+\tfrac{d}{dx}2$$

$$x\tfrac{d}{dx}y+y\tfrac{d}{dx}x-1=\tfrac{dy}{dx}+0$$

$$x\tfrac{dy}{dx}+y.1-1=\tfrac{dy}{dx}$$

$$x\tfrac{dy}{dx}+y-1=\tfrac{dy}{dx}$$

$$x\tfrac{dy}{dx}-\tfrac{dy}{dx}=1-y$$

$$\left(x-1\right)\tfrac{dy}{dx}=1-y$$

$$\tfrac{dy}{dx}=\frac{1-y}{x-1}$$

For explicitly differentiation I will separate y in given function

$$xy-y=x+2$$

$$y\left(x-1\right)=x+2$$

$$y=\frac{x+2}{x-1}$$

Now differentiating $\frac{dy}{dx}=\frac{d}{dx}\frac{x+2}{x-1}$

$$\frac{dy}{dx}=\frac{\left(x-1\right)\tfrac{d}{dx}\left(x+2\right)-\left(x+2\right)\tfrac{d}{dx}\left(x-1\right)}{\left(x-1\right)^2}$$

$$\frac{dy}{dx}=\frac{\left(x-1\right)\left(\tfrac{d}{dx}x+\tfrac{d}{dx}2\right)-\left(x+2\right)\left(\tfrac{d}{dx}x-\tfrac{d}{dx}1\right)}{\left(x-1\right)^2}$$

$$\frac{dy}{dx}=\frac{\left(x-1\right)\left(1+0\right)-\left(x+2\right)\left(1-0\right)}{\left(x-1\right)^2}$$

$$\frac{dy}{dx}=\frac{\left(x-1\right)-\left(x+2\right)}{\left(x-1\right)^2}$$

$$\frac{dy}{dx}=\frac{\frac{(x-1)-(x+2)}{x-1}}{\left(x-1\right)}$$

$$\frac{dy}{dx}=\frac{\frac{(x-1)}{x-1}-\frac{(x+2)}{x-1}}{\left(x-1\right)}$$

$$\frac{dy}{dx}=\frac{1-y}{\left(x-1\right)} \qquad \therefore y=\frac{x+2}{x-1}$$

---

Q3. Let $\frac{u^2}{a^2}+\frac{v^2}{b^2}=1$ where $a$ & $b$ are nonzero constant.

a). $\frac{du}{dv}$

Solution: We have $\frac{1}{a^2}u^2+\frac{1}{b^2}v^2=1$

Differentiating with respect to v

$$\frac{1}{a^2}\frac{d}{dv}u^2+\frac{1}{b^2}\frac{d}{dv}v^2=\frac{d}{dv}1$$

$$2u\frac{1}{a^2}\frac{d}{dv}u+2v\frac{1}{b^2}\frac{d}{dv}v=0$$

$$2u\frac{1}{a^2}\frac{du}{dv}=-2v\frac{1}{b^2}$$

$$\frac{du}{dv}=-\frac{2v}{2u}\frac{a^2}{b^2}$$

$$\frac{du}{dv}=-\frac{a^2}{b^2}\frac{v}{u}$$

---

b). $\frac{dv}{du}$

Solution: We have $\frac{1}{a^2}u^2+\frac{1}{b^2}v^2=1$

Differentiating with respect to u

$$\frac{1}{a^2}\frac{d}{du}u^2+\frac{1}{b^2}\frac{d}{du}v^2=\frac{d}{du}1$$

$$2u\frac{1}{a^2}\frac{d}{du}u+2v\frac{1}{b^2}\frac{d}{du}v=0$$

$$2u\frac{1}{a^2}+2v\frac{1}{b^2}\frac{dv}{du}=0$$

$$2v\frac{1}{b^2}\frac{dv}{du}=-\frac{2u}{a^2}$$
$$\frac{dv}{du}=-\frac{b^2}{a^2}\frac{2u}{2v}$$
$$\frac{dv}{du}=-\frac{b^2}{a^2}\frac{u}{v}$$

Q4. Let $(a-b)u^3-(a+b)v^2=c$ , a, b, c ∈ R. Find:

a). $\frac{du}{dv}$

Solution: We have $(a-b)u^3-(a+b)v^2=c$

Differentiating with respect to v

$$(a-b)\tfrac{d}{dv}u^3-(a+b)\tfrac{d}{dv}v^2=\tfrac{d}{dv}c$$
$$3u^2(a-b)\tfrac{d}{dv}u-2v(a+b)\tfrac{d}{dv}v=0$$
$$3u^2(a-b)\tfrac{du}{dv}-2v(a+b)=0$$
$$3u^2(a-b)\tfrac{du}{dv}=2v(a+b)$$
$$\frac{du}{dv}=\frac{2v(a+b)}{3u^2(a-b)}$$

b). $\frac{dv}{du}$

Solution: We have $(a-b)u^3-(a+b)v^2=c$

Differentiating with respect to u

$$(a-b)\tfrac{d}{du}u^3-(a+b)\tfrac{d}{du}v^2=\tfrac{d}{du}c$$
$$3u^2(a-b)\tfrac{d}{du}u-2v(a+b)\tfrac{d}{du}v=0$$
$$3u^2(a-b)-2v(a+b)\tfrac{dv}{du}=0$$
$$3u^2(a-b)=2v(a+b)\tfrac{dv}{du}$$
$$\frac{dv}{du}=\frac{3u^2(a-b)}{2v(a+b)}$$

Q5. Determine the slope of the tangent line to the curve $3x^2-7y^2+14y=27$ at point $P(-3,0)$

Solution: we have $3x^2-7y^2+14y=27$

Differentiating with respect to x

$$3\tfrac{d}{dx}x^2-7\tfrac{d}{dx}y^2+14\tfrac{d}{dx}y=\tfrac{d}{dx}27$$
$$3(2x)\tfrac{d}{dx}x-7(2y)\tfrac{d}{dx}y+14\tfrac{dy}{dx}=0$$
$$6x.1-14y\tfrac{dy}{dx}+14\tfrac{dy}{dx}=0$$
$$14\tfrac{dy}{dx}-14y\tfrac{dy}{dx}=-6x$$
$$14(1-y)\tfrac{dy}{dx}=-6x$$
$$\frac{dy}{dx}=\frac{-6x}{14(1-y)}=\frac{-3x}{7(1-y)}$$

Now slope of tangent at the point $P(-3,0)$

$$\left.\frac{dy}{dx}\right|_{(-3,0)}=\frac{-3(-3)}{7(1-0)}=\frac{9}{7}$$

Q6. The graph of $x^3y^3+4y=3x^2$ is a curve that passes through the point $P(2,1)$ what is the slope of the curve at that point?

Solution: we have $x^3y^3+4y=3x^2$

Differentiating with respect to x

$$\tfrac{d}{dx}(x^3y^3)+4\tfrac{d}{dx}y=3\tfrac{d}{dx}x^2$$
$$x^3\tfrac{d}{dx}y^3+y^3\tfrac{d}{dx}x^3+4\tfrac{dy}{dx}=3(2x)\tfrac{d}{dx}x$$
$$3x^3y^2\tfrac{d}{dx}y+3x^2y^3\tfrac{d}{dx}x+4\tfrac{dy}{dx}=6x$$
$$3x^3y^2\tfrac{dy}{dx}+3x^2y^3+4\tfrac{dy}{dx}=6x$$
$$3x^3y^2\tfrac{dy}{dx}+4\tfrac{dy}{dx}=6x-3x^2y^3$$
$$(3x^3y^2+4)\tfrac{dy}{dx}=6x-3x^2y^3$$
$$\frac{dy}{dx}=\frac{6x-3x^2y^3}{3x^3y^2+4}$$

Now slope of tangent at the point $P(2,1)$

$$m=\left.\frac{dy}{dx}\right|_{(2,1)}=\frac{6(2)-3(2)^2(1)^3}{3(2)^3(1)^2+4}$$
$$m=\frac{12-3(4)(1)}{3(8)(1)+4}=\frac{12-12}{24+4}=\frac{0}{28}=0$$

Q7. In biophysics the equation $(L+m)(V+n)=k$ is called the fundamental equation of muscle contraction. Where m,n and k are constants, and V is the velocity of the shortening of muscle fibers for a muscle subjected to a load of L. Find $\frac{dL}{dV}$ using implicit differentiation.

Solution: we have $(L+m)(V+n)=k$

Differentiating with respect to V, Keep m,n & k are constants

$$\tfrac{d}{dV}\{(L+m)(V+n)\}=\tfrac{d}{dV}k$$
$$(L+m)\tfrac{d}{dV}(V+n)+(V+n)\tfrac{d}{dV}(L+m)=0$$
$$(L+m)\left(\tfrac{d}{dV}V+\tfrac{d}{dV}n\right)+(V+n)\left(\tfrac{d}{dV}L+\tfrac{d}{dV}m\right)=0$$
$$(L+m)(1+0)+(V+n)\left(\tfrac{dL}{dV}+0\right)=0$$
$$(L+m)+\tfrac{dL}{dV}(V+n)=0$$
$$\tfrac{dL}{dV}(V+n)=-(L+m)$$
$$\frac{dL}{dV}=-\frac{L+m}{V+n}$$

Derivative of sin x

Let $f(x)=\sin(x)$ then

$f(x+\Delta x)=\sin(x+\Delta x)$ using First principle

$$f'(x)=\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value of $f(x+\Delta x)$ *and* $f(x)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\sin(x+\Delta x)-\sin x}{\Delta x}$$

Using $\sin(a+b)=\sin a\cos b+\cos a\sin b$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\sin x\cos\Delta x+\cos x\sin\Delta x-\sin x}{\Delta x}$$
$$f'(x)=\lim_{\Delta x\to 0}\frac{\sin x\cos\Delta x-\sin x+\cos x\sin\Delta x}{\Delta x}$$
$$f'(x)=\lim_{\Delta x\to 0}\frac{\sin x(\cos\Delta x-1)+\cos x\sin\Delta x}{\Delta x}$$
$$f'(x)=\lim_{\Delta x\to 0}\left\{\sin x\frac{\cos\Delta x-1}{\Delta x}+\cos x\frac{\sin\Delta x}{\Delta x}\right\}$$
$$f'(x)=\sin x\lim_{\Delta x\to 0}\frac{\cos\Delta x-1}{\Delta x}+\cos x\lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x}$$
$$f'(x)=\sin x(0)+\cos x(1)$$
$$\tfrac{d}{dx}\sin x=\cos x$$

**Derivative of sin x** Let $f(x)=\sin(x)$ then

$f(x+\Delta x)=\sin(x+\Delta x)$ Using First principle

$$f'(x)=\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value of $f(x+\Delta x)$ *and* $f(x)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\sin(x+\Delta x)-\sin x}{\Delta x}$$

Using $Sin\alpha-Sin\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{2\cos\left(\frac{x+\Delta x+x}{2}\right)\sin\left(\frac{x+\Delta x-x}{2}\right)}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{2\cos\left(\frac{2x+\Delta x}{2}\right)\sin\left(\frac{\Delta x}{2}\right)}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\cos\left(\frac{2x+\Delta x}{2}\right)\lim_{\frac{\Delta x}{2}\to 0}\frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$f'(x)=\cos\left(\frac{2x+0}{2}\right).1$$

$$\frac{d}{dx}\sin x=\cos x$$

**Derivative of cos x** Let $f(x)=\cos(x)$ then

$f(x+\Delta x)=\cos(x+\Delta x)$ Using First principle

$$f'(x)=\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value of $f(x+\Delta x)$ *and* $f(x)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\cos(x+\Delta x)-\cos x}{\Delta x}$$

Using $\cos\alpha-\cos\beta=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{-2\sin\left(\frac{x+\Delta x+x}{2}\right)\sin\left(\frac{x+\Delta x-x}{2}\right)}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{-2\sin\left(\frac{2x+\Delta x}{2}\right)\sin\left(\frac{\Delta x}{2}\right)}{\Delta x}$$

$$f'(x)=-\lim_{\Delta x\to 0}\sin\left(\frac{2x+\Delta x}{2}\right)\lim_{\frac{\Delta x}{2}\to 0}\frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$f'(x)=-\sin\left(\frac{2x+0}{2}\right).1 \qquad \Rightarrow \frac{d}{dx}\cos x=-\sin x$$

**Derivative of tan x** Let $f(x)=\tan(x)$ then

$f(x+\Delta x)=\tan(x+\Delta x)$ Using First principle

$$f'(x)=\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value of $f(x+\Delta x)$ *and* $f(x)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\tan(x+\Delta x)-\tan x}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\tan(x+\Delta x)-\tan x\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{\sin(x+\Delta x)}{\cos(x+\Delta x)}-\frac{\sin x}{\cos x}\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{\sin(x+\Delta x).\cos x-\sin x.\cos(x+\Delta x)}{\cos(x+\Delta x).\cos x}\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{\sin(x+\Delta x-x)}{\cos(x+\Delta x).\cos x}\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\left[\frac{1}{\cos(x+\Delta x).\cos x}\right]\lim_{\Delta x\to 0}\frac{\sin(\Delta x)}{\Delta x}$$

$$f'(x)=\frac{1}{\cos x.\cos x}\times 1=\frac{1}{\cos^2 x}=\sec^2 x$$

**Derivative of cot x** Let $f(x)=\cot(x)$ then

$f(x+\Delta x)=\cot(x+\Delta x)$

First principle for derivative

$$f'(x)=\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value of $f(x+\Delta x)$ *and* $f(x)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\cot(x+\Delta x)-\cot x}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\cot(x+\Delta x)-\cot x\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{\cos(x+\Delta x)}{\sin(x+\Delta x)}-\frac{\cos x}{\sin x}\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{\sin x.\cos(x+\Delta x)-\sin(x+\Delta x)\cos x}{\sin(x+\Delta x).\sin x}\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{\sin(x-x-\Delta x)}{\sin(x+\Delta x).\sin x}\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\left[\frac{1}{\sin(x+\Delta x).\sin x}\right]\lim_{\Delta x\to 0}\frac{\sin(-\Delta x)}{\Delta x}$$

$$f'(x)=\frac{-1}{\sin x.\sin x}\lim_{\Delta x\to 0}\frac{\sin(\Delta x)}{\Delta x}$$

$$f'(x)=\frac{1}{\sin^2 x}\times 1$$

$$\frac{d}{dx}\cot x=-\cos ec^2 x$$

**Derivative of Sec x** Let $f(x)=\sec(x)$ then

$f(x+\Delta x)=\sec(x+\Delta x)$

First principle for derivative

$$f'(x)=\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value of $f(x+\Delta x)$ *and* $f(x)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\sec(x+\Delta x)-\sec x}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\sec(x+\Delta x)-\sec x\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{1}{\cos(x+\Delta x)}-\frac{1}{\cos x}\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{\cos x-\cos(x+\Delta x)}{\cos(x+\Delta x).\cos x}\right]$$

using $Cos\alpha-Cos\beta=-2Sin\left(\frac{\alpha+\beta}{2}\right)Sin\left(\frac{\alpha-\beta}{2}\right)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{-2\sin\left(\frac{x+x+\Delta x}{2}\right)\sin\left(\frac{x-x-\Delta x}{2}\right)}{\cos(x+\Delta x).\cos x}\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\left[\frac{-\sin\left(\frac{2x+\Delta x}{2}\right)}{\cos(x+\Delta x).\cos x}\right]\lim_{\frac{\Delta x}{2}\to 0}\frac{\sin\left(-\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$f'(x)=\frac{+\sin\left(\frac{2x}{2}\right)}{\cos x.\cos x}\lim_{\frac{\Delta x}{2}\to 0}\frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$f'(x)=\frac{\sin x}{\cos x.\cos x}$$

$$\frac{d}{dx}\sec x=s\,ec\,x.\tan x$$

**Derivative of cosec x** Let $f(x)=\text{cosec}(x)$ then

$f(x+\Delta x)=\text{cosec}(x+\Delta x)$

First principle for derivative

$$f'(x)=\frac{d}{dx}f(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value of $f(x+\Delta x)$ *and* $f(x)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\text{cosec}(x+\Delta x)-\text{cosec}\,x}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\text{cosec}(x+\Delta x)-\text{cosec}\,x\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{1}{\sin(x+\Delta x)}-\frac{1}{\sin x}\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{\sin x-\sin(x+\Delta x)}{\sin(x+\Delta x).\sin x}\right]$$

Using $Sin\alpha-Sin\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\frac{2\cos\left(\frac{x+x+\Delta x}{2}\right)\sin\left(\frac{x-x-\Delta x}{2}\right)}{\sin(x+\Delta x).\sin x}\right]$$

$$f'(x)=\lim_{\Delta x\to 0}\left[\frac{\cos\left(\frac{2x+\Delta x}{2}\right)}{\sin(x+\Delta x).\sin x}\right]\lim_{\frac{\Delta x}{2}\to 0}\frac{\sin\left(-\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$f'(x)=\frac{-\cos x}{\sin x.\sin x}\lim_{\frac{\Delta x}{2}\to 0}\frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$f'(x)=\frac{-\cos x}{\sin x.\sin x}$$

$\frac{d}{dx}\cot x=-\cos ec\,x.\cot x$

**Derivative of $\sin^{-1}x$** Let $y=\sin^{-1}x$ ..............(1)

$\Rightarrow \sin y=x$ Differentiating equation (1) w.r.t. x

$\frac{d}{dx}\sin y=\frac{d}{dx}x$

$\cos y\frac{d}{dx}y=1$

$$\frac{d}{dx}=\frac{1}{\cos y}$$

$$\frac{d}{dx}y=\frac{1}{\sqrt{\cos^2 y}}$$

$$\frac{d}{dx}y=\frac{1}{\sqrt{1-\sin^2 y}} \qquad \therefore \sin^2 x+\cos^2 x=1$$

Putting value of y and using equation (1) we get

$$\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}}$$

**Derivative of $\cos^{-1}x$** Let $y=\cos^{-1}x \Rightarrow$

$\cos y=x$..................(1)

Differentiating equation (1) w.r.t. x

$\frac{d}{dx}\cos y=\frac{d}{dx}x$

$-\sin y\frac{d}{dx}y=1$

$$\frac{d}{dx}=\frac{-1}{\sin y}$$

$$\frac{d}{dx}y=\frac{-1}{\sqrt{\sin^2 y}}$$

$$\frac{d}{dx}y=\frac{-1}{\sqrt{1-\cos^2 y}} \qquad \therefore \sin^2 x+\cos^2 x=1$$

Putting value of y and using equation (1) we get

$$\frac{d}{dx}\cos^{-1}x=\frac{-1}{\sqrt{1-x^2}}$$

**Derivative of $\tan^{-1}x$** Let $y=\tan^{-1}x \Rightarrow$

$\tan y=x$..................(1)

Differentiating equation (1) w.r.t. x

$\frac{d}{dx}\tan y=\frac{d}{dx}x$

$\sec^2 y\frac{d}{dx}y=1$

$$\frac{d}{dx}=\frac{1}{\sec^2 y}$$

$$\frac{d}{dx}y=\frac{1}{1+\tan^2 y} \qquad \therefore \sec^2 x=1+\tan^2 x$$

Putting value of y and using equation (1) we get

$$\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$$

**Derivative of $\cot^{-1}x$** Let $y=\cot^{-1}x$

$\cot y=x$..................(1)

Differentiating equation (1) w.r.t. x

$\frac{d}{dx}\cot y=\frac{d}{dx}x$

$-\cos ec^2 y\frac{d}{dx}y=1$

$$\frac{d}{dx}=\frac{-1}{\text{cosec}^2 y}$$

$$\frac{d}{dx}y=\frac{-1}{1+\cot^2 y} \qquad \therefore \cos ec^2 x=1+\cot^2 x$$

Putting value of y and using equation (1) we get

$$\frac{d}{dx}\cot^{-1}x=\frac{-1}{1+x^2}$$

**Derivative of $\sec^{-1}x$** Let $y=\sec^{-1}x$

$\Rightarrow \sec y=x$..................(1)

Differentiating equation (1) w.r.t. x

$\frac{d}{dx}\sec y=\frac{d}{dx}x$

$\sec y\tan y\frac{d}{dx}y=1$

$$\frac{d}{dx}=\frac{1}{\sec y\tan y}$$

$$\frac{d}{dx}y=\frac{1}{\sec y\sqrt{\tan^2 y}} \qquad \therefore \sec^2 x=1+\tan^2 x$$

$$\frac{d}{dx}y=\frac{1}{\sec y\sqrt{\sec^2 y-1}}$$

Putting value of y and using equation (1) we get

$$\frac{d}{dx}\sec^{-1}x=\frac{1}{x\sqrt{x^2-1}}$$

**Derivative of $\cos ec^{-1}x$** Let $y=\text{cosec}^{-1}x$

$\text{cosec}\,y=x$..................(1)

Differentiating equation (1) w.r.t. x

$\frac{d}{dx}-\text{cosec}\,y=\frac{d}{dx}x$

$-\text{cosec}\,y\cot y\frac{d}{dx}y=1$

$$\frac{d}{dx}y=\frac{-1}{\text{cosec}\,y\cot y}$$

$\frac{d}{dx}y = \frac{-1}{\operatorname{cosec} y\sqrt{\cot^2 y}}$ $\therefore \operatorname{cosec}^2 x = 1+\cot^2 x$

$\frac{d}{dx}y = \frac{-1}{\operatorname{cosec} y\sqrt{\operatorname{cosec}^2 y - 1}}$

Putting value of y and using equation (1) we get

$\frac{d}{dx}\operatorname{cosec}^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$

**Derivative of exponential function i.e. $e^x$**

Let $f(x) = e^x$ so $f(x+\Delta x) = e^{x+\Delta x}$

First principle for derivative

$$f'(x) = \frac{d}{dx}f(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value of $f(x+\Delta x)$ *and* $f(x)$

$$f'(x) = \lim_{\Delta x\to 0}\frac{e^{x+\Delta x}-e^x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{e^x e^{\Delta x}-e^x}{\Delta x}$$

$$f'(x) = e^x\lim_{\Delta x\to 0}\frac{e^{\Delta x}-1}{\Delta x}$$

$f'(x) = e^x(1)$

$f'(x) = e^x$

**Derivative of ln x** Let $f(x) = \ln x$ so

$f(x+\Delta x) = \ln(x+\Delta x)$

First principle for derivative

$$f'(x) = \frac{d}{dx}f(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value of $f(x+\Delta x)$ *and* $f(x)$

$$f'(x) = \lim_{\Delta x\to 0}\frac{\ln(x+\Delta x)-\ln(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\ln(x+\Delta x)-\ln(x)\right]$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{x}{x}\frac{1}{\Delta x}\ln\left(\frac{x+\Delta x}{x}\right)$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{1}{x}\frac{x}{\Delta x}\ln\left(\frac{x}{x}+\frac{\Delta x}{x}\right)$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{1}{x}\ln\left(1+\frac{\Delta x}{x}\right)^{\frac{x}{\Delta x}}$$

When $\Delta x \to 0$ then $\frac{\Delta x}{x}\to 0$

$$f'(x) = \frac{1}{x}\lim_{\Delta x\to 0}\ln\left(1+\frac{\Delta x}{x}\right)^{\frac{x}{\Delta x}}$$

$$f'(x) = \frac{1}{x}\ln(e)$$

$$f'(x) = \frac{1}{x} \qquad \therefore \ln e = 1$$

**Derivative of $\log_a x$** Let $f(x) = \log_a x$ so

$f(x+\Delta x) = \log_a(x+\Delta x)$

First principle for derivative

$$f'(x) = \frac{d}{dx}f(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value of $f(x+\Delta x)$ *and* $f(x)$

$$f'(x) = \lim_{\Delta x\to 0}\frac{\log_a(x+\Delta x)-\log_a(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{1}{\Delta x}\left[\log_a(x+\Delta x)-\log_a(x)\right]$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{x}{x}\frac{1}{\Delta x}\log_a\left(\frac{x+\Delta x}{x}\right)$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{1}{x}\frac{x}{\Delta x}\log_a\left(\frac{x}{x}+\frac{\Delta x}{x}\right)$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{1}{x}\log_a\left(1+\frac{\Delta x}{x}\right)^{\frac{x}{\Delta x}}$$

When $\Delta x \to 0$ then $\frac{\Delta x}{x}\to 0$

$$f'(x) = \frac{1}{x}\lim_{\Delta x\to 0}\log_a\left(1+\frac{\Delta x}{x}\right)^{\frac{x}{\Delta x}}$$

$$f'(x) = \frac{1}{x}\log_a(e)$$

**Derivative of exponential function i.e. $a^x$**

Let $f(x) = a^x$ so $f(x+\Delta x) = a^{x+\Delta x}$

First principle for derivative

$$f'(x) = \frac{d}{dx}f(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Putting the value of $f(x+\Delta x)$ *and* $f(x)$

$$f'(x) = \lim_{\Delta x\to 0}\frac{a^{x+\Delta x}-a^x}{\Delta x}$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{a^x a^{\Delta x}-a^x}{\Delta x}$$

$$f'(x) = a^x\lim_{\Delta x\to 0}\frac{a^{\Delta x}-1}{\Delta x}$$

$f'(x) = a^x\log_e a$

**Derivative of exponential function i.e. $a^x$**

Let $y = a^x$

Taking $\log_e = \ln$ on both sides

$\log_e y = \log_e a^x$

$\log_e y = x\log_e a$

Differentiating with respect to x

$\frac{d}{dx}\log_e y = \log_e a\frac{d}{dx}x$

$\frac{1}{y}\frac{dy}{dx} = \log_e a$

$\frac{dy}{dx} = y.\log_e a$

Putting the value of y

$\frac{dy}{dx} = a^x\log_e a$

# Exercise 2.6

Q1. Use any suitable rule of differentiation to perform $\frac{dy}{dx}$ for the following functions:

a). $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$

Sol: Differentiating the given function with respect to x

$$\frac{d}{dx}y=\frac{d}{dx}\sqrt{\frac{1-\cos x}{1+\cos x}}$$
$$y'=\frac{1}{2}\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{1-\cos x}{1+\cos x}\right)$$
$$y'=\frac{1}{2}\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{-1}{2}}\frac{(1+\cos x)\frac{d}{dx}(1-\cos x)-(1-\cos x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$
$$y'=\frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)^{\frac{1}{2}}\frac{(1+\cos x)(0-\sin x)-(1-\cos x)(0-\sin x)}{(1+\cos x)^2}$$
$$y'=\frac{-\sin x(1+\cos x)+\sin x(1-\cos x)}{2(1-\cos x)^{\frac{1}{2}}(1+\cos x)^{-\frac{1}{2}}(1+\cos x)^2}$$
$$y'=\frac{\sin x\{-1-\cos x+1-\cos x\}}{2(1-\cos x)^{\frac{1}{2}}(1+\cos x)^{2-\frac{1}{2}}}$$
$$y'=\frac{-2\sin x\cos x}{2(1-\cos x)^{\frac{1}{2}}(1+\cos x)^{\frac{3}{2}}}$$
$$y'=\frac{-\sin x\cos x}{(1-\cos x)^{\frac{1}{2}}(1+\cos x)^{\frac{3}{2}}}$$

---

b). $y=\cos\left(x+\frac{\pi}{2}\right)$

Solution: Differentiating the given function with respect to x

$$\frac{d}{dx}y=\frac{d}{dx}\cos\left(x+\frac{\pi}{2}\right)$$
$$y'=-\sin\left(x+\frac{\pi}{2}\right)\frac{d}{dx}\left(x+\frac{\pi}{2}\right)$$
$$y'=-\sin\left(x+\frac{\pi}{2}\right)\left(\frac{d}{dx}x+\frac{d}{dx}\frac{\pi}{2}\right)$$
$$y'=-\sin\left(x+\frac{\pi}{2}\right)(1+0)$$
$$y'=-\sin\left(x+\frac{\pi}{2}\right)$$

---

c). $y=\sin(\sin x)$

Solution: Differentiating the given function with respect to x

$$\frac{d}{dx}y=\frac{d}{dx}\sin(\sin x)$$
$$y'=\cos(\sin x)\frac{d}{dx}\sin x$$
$$y'=\cos(\sin x)\cos x$$

---

d). $y=\sin x\,\cos x$

Solution: Differentiating the given function with respect to x

$$\frac{d}{dx}y=\sin x\frac{d}{dx}\cos x+\cos x\frac{d}{dx}\sin x$$
$$y'=\sin x(-\sin x)\frac{d}{dx}x+\cos x(\cos x)\frac{d}{dx}x$$
$$y'=-\sin^2 x+\cos^2 x$$

---

e). $y=\frac{\sin x}{\cos x}$

Solution: Differentiating the given function with respect to x

$$\frac{d}{dx}y=\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)=\frac{d}{dx}\tan x$$
$$y'=\sec^2 x\frac{d}{dx}x$$
$$y'=\sec^2 x$$

---

f). $y=\sin^3\left(\pi x^2\right)$

Solution: Differentiating the given function with respect to x

$$\frac{d}{dx}y=\frac{d}{dx}\sin^3\left(\pi x^2\right)$$
$$y'=3\sin^2\left(\pi x^2\right)\frac{d}{dx}\sin\left(\pi x^2\right)$$
$$y'=3\sin^2\left(\pi x^2\right)\cos\left(\pi x^2\right)\frac{d}{dx}\left(\pi x^2\right)$$
$$y'=3\sin^2\left(\pi x^2\right)\cos\left(\pi x^2\right)\left(\pi\frac{d}{dx}x^2\right)$$
$$y'=3\sin^2\left(\pi x^2\right)\cos\left(\pi x^2\right)\left(2\pi x\frac{d}{dx}x\right)$$
$$y'=6\pi x\sin^2\left(\pi x^2\right)\cos\left(\pi x^2\right)$$

---

Q2. Use any suitable rule of differentiation to perform $\frac{dy}{dx}$ for the following functions:

a). $y=2\cot 3x$

Solution: Differentiating the given function with respect to x

$$\frac{d}{dx}y=2\frac{d}{dx}\cot 3x$$
$$y'=2\left(-\operatorname{cosec}^2 3x\right)\frac{d}{dx}(3x)$$
$$y'=-2\operatorname{cosec}^2 3x\left(3\frac{d}{dx}x\right)$$
$$y'=-2\operatorname{cosec}^2 3x(3.1)$$
$$y'=-2\times 3\operatorname{cosec}^2 3x$$
$$y'=-6\operatorname{cosec}^2 3x$$

---

b). $y=\sec\pi x$

Solution: Differentiating the given function with respect to x

$$\frac{d}{dx}y=\frac{d}{dx}\sec\pi x$$
$$y'=\sec\pi x\tan\pi x\frac{d}{dx}(\pi x)$$
$$y'=\sec\pi x\tan\pi x\left(\pi\frac{d}{dx}x\right)$$
$$y'=\sec\pi x\tan\pi x(\pi)$$
$$y'=\pi\sec\pi x\tan\pi x$$

---

c). $y=4\operatorname{cosec}2x$

Solution: Differentiating the given function with respect to x

$$\frac{d}{dx}y=4\frac{d}{dx}\operatorname{cosec}2x$$
$$\frac{d}{dx}y=4\left(-\operatorname{cosec}2x\,\cot 2x\right)\frac{d}{dx}(2x)$$
$$y'=-4\operatorname{cosec}2x\,\cot 2x\left(2\frac{d}{dx}x\right)$$
$$y'=-4\operatorname{cosec}2x\,\cot 2x(2)$$
$$y'=-4\times 2\operatorname{cosec}2x\,\cot 2x$$
$$y'=-8\operatorname{cosec}2x\,\cot 2x$$

---

d). $y=2\tan(x+3)^2$

Solution: Differentiating the given function with respect to x

$$\frac{d}{dx}y=2\frac{d}{dx}\tan(x+3)^2$$
$$y'=2\sec^2(x+3)^2\frac{d}{dx}(x+3)^2$$
$$y'=2\times 2(x+3)\sec^2(x+3)^2\frac{d}{dx}(x+3)$$
$$y'=4(x+3)\sec^2(x+3)^2\left(\frac{d}{dx}x+\frac{d}{dx}3\right)$$
$$y'=4(x+3)\sec^2(x+3)^2(1+0)$$
$$y'=4(x+3)\sec^2(x+3)^2$$

---

e). $y=4\cot\sqrt{x^2-1}$

Solution: Differentiating the given function with respect to x

$$\frac{d}{dx}y=4\frac{d}{dx}\cot\sqrt{x^2-1}$$
$$y'=4\left(-\operatorname{cosec}^2\sqrt{x^2-1}\right)\frac{d}{dx}\sqrt{x^2-1}$$
$$y'=-4\operatorname{cosec}^2\sqrt{x^2-1}\frac{d}{dx}\left(x^2-1\right)^{\frac{1}{2}}$$
$$y'=\frac{-4}{2}\left(x^2-1\right)^{\frac{1}{2}-1}\operatorname{cosec}^2\sqrt{x^2-1}\frac{d}{dx}\left(x^2-1\right)$$

$y' = -4x\left(x^2-1\right)^{\frac{-1}{2}}\operatorname{cosec}^2\sqrt{x^2-1}$

f). $y = \sec^2 x^3$

Solution: Differentiating the given function with respect to x

$\frac{d}{dx}y = \frac{d}{dx}\sec^2 x^3$

$y' = 2\sec x^3 \frac{d}{dx}\sec x^3$

$y' = 2\sec x^3 \sec x^3 \tan x^3 \frac{d}{dx}x^3$

$y' = 2\sec^2 x^3 \tan x^3\left(3x^2\right)\frac{d}{dx}x$

$y' = 2\times 3x^2 \sec^2 x^3 \tan x^3$

$y' = 6x^2\sec^2 x^3\tan x^3$

g). $y = 2\operatorname{cosec}^3(x+2)$

Solution: Differentiating the given function with respect to x

$\frac{d}{dx}y = 2\frac{d}{dx}\operatorname{cosec}^3(x+2)$

$y' = 2\times 3\operatorname{cosec}^2(x+2)\frac{d}{dx}\operatorname{cosec}(x+2)$

$y' = 6\operatorname{cosec}^2(x+2)\left\{-\operatorname{cosec}(x+2)\cot(x+2)\right\}\frac{d}{dx}(x+2)$

$y' = -6\operatorname{cosec}^3(x+2)\cot(x+2)\left(\frac{d}{dx}x+\frac{d}{dx}2\right)$

$y' = -6\operatorname{cosec}^3(x+2)\cot(x+2)(1+0)$

$y' = -6\operatorname{cosec}^3(x+2)\cot(x+2)$

h). $y = \dfrac{1+\tan 2x}{\operatorname{cosec}3x}$

Solution: Differentiating the given function with respect to x

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+\tan 2x}{\operatorname{cosec}3x}\right)$

$\frac{dy}{dx} = \frac{\operatorname{cosec}3x\frac{d}{dx}(1+\tan 2x)-(1+\tan 2x)\frac{d}{dx}\operatorname{cosec}x}{(\operatorname{cosec}3x)^2}$

$\frac{dy}{dx} = \frac{\operatorname{cosec}3x\left(\frac{d}{dx}1+\frac{d}{dx}\tan 2x\right)-(1+\tan 2x)(-\operatorname{cosec}3x\cot 3x)\frac{d}{dx}3x}{(\operatorname{cosec}3x)^2}$

$\frac{dy}{dx} = \frac{\operatorname{cosec}3x\left(0+\sec^2 2x\frac{d}{dx}2x\right)+3\operatorname{cosec}3x\cot 3x(1+\tan 2x)}{(\operatorname{cosec}3x)^2}$

$\frac{dy}{dx} = \frac{\operatorname{cosec}3x\left(2\sec^2 2x\right)+3\operatorname{cosec}3x\cot 3x(1+\tan 2x)}{(\operatorname{cosec}3x)^2}$

$\frac{dy}{dx} = \frac{\operatorname{cosec}3x\left(2\sec^2 2x+3\cot 3x(1+\tan 2x)\right)}{(\operatorname{cosec}3x)^2}$

$\frac{dy}{dx} = \frac{2\sec^2 2x+3\cot 3x(1+\tan 2x)}{\operatorname{cosec}3x}$

Q3. Use any suitable rule of differentiation to perform $\frac{dy}{dx}$ for the following functions:

a). $y = \cos^{-1}(x+4)$

Solution: Method 1 Using formula

$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}y = \frac{d}{dx}\cos^{-1}(x+4)$

$y' = \frac{-1}{\sqrt{1-(x+4)^2}}\frac{d}{dx}(x+4)$

$y' = \frac{-1}{\sqrt{1-(x+4)^2}}\left(\frac{d}{dx}x+\frac{d}{dx}4\right)$

$y' = \frac{-1}{\sqrt{1-(x+4)^2}}(1+0)$

$y' = \frac{-1}{\sqrt{1-(x+4)^2}}$

Method 2

$y = \cos^{-1}(x+4)$

$\cos y = x+4$

Differentiating the given function with respect to x

$\frac{d}{dx}\cos y = \frac{d}{dx}(x+4)$

$-\sin y\frac{d}{dx}y = \left(\frac{d}{dx}x+\frac{d}{dx}4\right)$

$-\sin y\frac{d}{dx}y = (1+0)$

$-\sin y\frac{d}{dx}y = 1$

$y' = \frac{-1}{\sin y}$

$y' = \frac{-1}{\sqrt{\sin^2 y}}$

$y' = \frac{-1}{\sqrt{1-\cos^2 y}}$

$y' = \frac{-1}{\sqrt{1-(x+4)^2}}$ $\quad\therefore \cos y = x+4$

b). $y = \tan^{-1}(11x)$

Solution: We have $y = \tan^{-1}(11x)$

$\tan y = 11x$

Differentiating the given function with respect to x

$\frac{d}{dx}\tan y = 11\frac{d}{dx}x$

$\sec^2 y\frac{d}{dx}y = 11(1)$

$\frac{dy}{dx} = \frac{11}{\sec^2 y}$

$\frac{dy}{dx} = \frac{11}{1+\tan^2 y}$

$\frac{dy}{dx} = \frac{11}{1+(11x)^2}$ $\quad\therefore \tan y = 11x$

$\frac{dy}{dx} = \frac{11}{1+121x^2}$

c). $y = \left(\sin^{-1}x\right)^2$

Solution: We have $y = \left(\sin^{-1}x\right)^2$

Differentiating the given function with respect to x

$\frac{d}{dx}y = \frac{d}{dx}\left(\sin^{-1}x\right)^2$

$y' = 2\sin^{-1}x\frac{d}{dx}\sin^{-1}x$

$y' = 2\sin^{-1}x\times\frac{1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

$y' = \frac{2\sin^{-1}x}{\sqrt{1-x^2}}$

d). $y = x^3\sin^{-1}(2x)$

Solution: We have $y = x^3\sin^{-1}(2x)$

Differentiating the given function with respect to x

$$\frac{d}{dx}y=\frac{d}{dx}x^3\sin^{-1}(2x)$$
$$y'=x^3\frac{d}{dx}\sin^{-1}(2x)+\sin^{-1}(2x)\frac{d}{dx}x^3$$
$$y'=\frac{x^3}{\sqrt{1-(2x)^2}}\frac{d}{dx}(2x)+3x^2\sin^{-1}(2x)$$
$$y'=\frac{2x^3}{\sqrt{1-(2x)^2}}+3x^2\sin^{-1}(2x)$$

e). $y=\operatorname{cosec}^{-1}(x+3)$

Solution: We have $y=\operatorname{cosec}^{-1}(x+3)$

$\operatorname{cosec} y=x+3$

Differentiating the given function with respect to x

$$\frac{d}{dx}\operatorname{cosec} y=\frac{d}{dx}x+\frac{d}{dx}3$$
$$-\operatorname{cosec} y\cot y\frac{d}{dx}y=1+0$$
$$\frac{dy}{dx}=\frac{-1}{\operatorname{cosec} y\cot y}$$
$$\frac{dy}{dx}=\frac{-1}{\operatorname{cosec} y\sqrt{\cot^2 y}}$$
$$\frac{dy}{dx}=\frac{-1}{\operatorname{cosec} y\sqrt{\operatorname{cosec}^2 y-1}}$$
$$\frac{dy}{dx}=\frac{-1}{(x+3)\sqrt{(x+3)^2-1}} \qquad \therefore \operatorname{cosec} y=x+3$$

f). $y=\left(1+\cot^{-1}3x\right)^3$

Solution: We have $y=\left(1+\cot^{-1}3x\right)^3$

Differentiating the given function with respect to x

$$\frac{d}{dx}y=\frac{d}{dx}\left(1+\cot^{-1}3x\right)^3$$
$$y'=3\left(1+\cot^{-1}3x\right)^2\frac{d}{dx}\left(1+\cot^{-1}3x\right)$$
$$y'=3\left(1+\cot^{-1}3x\right)^2\left(0+\frac{-1}{1+(3x)^2}\frac{d}{dx}(3x)\right)$$
$$y'=3\left(1+\cot^{-1}3x\right)^2\frac{-3}{1+9x^2}$$
$$y'=\frac{-9\left(1+\cot^{-1}3x\right)^2}{1+9x^2}$$

Q4: Suppose profits on the sale of swimming suits in a departmental store are given approximately by

$$P(t)=5-5\cos\frac{\pi t}{26}, \qquad 0\le t\le 104$$

Where $P(t)$ is profit ( in hundreds of dollars ) for a week of sale t weeks after January first.

a).What is rate of change of profit t weeks after first year?

Sol: Differentiating the given function with respect to x

$$\frac{d}{dx}P(t)=\frac{d}{dx}5-5\frac{d}{dx}\cos\frac{\pi t}{26}$$
$$P'(t)=0-5\left(-\sin\frac{\pi t}{26}\right)\frac{d}{dx}\frac{\pi t}{26}$$
$$P'(t)=\frac{5\pi}{26}\sin\frac{\pi t}{26}$$

b). What is the rate of change of profit 8 weeks after the first year? 26 weeks after the first year? 50 weeks after of the year?

Solution: the rate of change of profit 8 weeks

$$P'(8)=\frac{5\pi}{26}\sin\frac{8\pi}{26}=0.4972 \text{ hundreds of dollars / week}$$
$$P'(8)=\$49.72$$

The rate of change of profit 26 weeks

$$P'(26)=\frac{5\pi}{26}\sin\frac{26\pi}{26}=0 \text{ hundreds of dollars / week}$$
$$P'(26)=\$0 \text{ per week}$$

The rate of change of profit 50 weeks

$$P'(50)=\frac{5\pi}{26}\sin\frac{50\pi}{26}=-0.1446 \text{ hundreds of dollars / week}$$
$$P'(50)=-\$14.46 \text{ per week}$$

Q5. A normal seated adult breathes in and exhales about 0.8 liter of air every 4 seconds. The volume of air $V(t)$ in the lungs t seconds after exhaling is given by $V(t)=0.45-0.35\cos\frac{\pi t}{2}, \quad 0\le t\le 8$

a). What is rate of flow of air t seconds after exhaling?

Solution: Differentiating given function with respect to x

$$\frac{d}{dx}V(t)=\frac{d}{dx}0.45-\frac{d}{dx}0.35\cos\frac{\pi t}{2}$$
$$V'(t)=0-0.35\left(-\sin\frac{\pi t}{2}\right)\frac{d}{dx}\frac{\pi t}{2}$$
$$V'(t)=\frac{0.35\pi}{2}\sin\frac{\pi t}{2}$$

b). What is rate of flow of air 3 seconds after exhaling? 4 seconds after exhaling? 5 seconds after exhaling?

Solution: rate of flow of air 3 seconds after exhaling

$$V'(3)=\frac{0.35\pi}{2}\sin\frac{3\pi}{2}=-0.5498 \text{ liter/ sec}$$

the rate of flow of air 4 seconds after exhaling

$$V'(4)=\frac{0.35\pi}{2}\sin\frac{4\pi}{2}=0 \text{ liter/ sec}$$

the rate of flow of air 5 seconds after exhaling

$$V'(5)=\frac{0.35\pi}{2}\sin\frac{5\pi}{2}=-0.5498 \text{ liter/ sec}$$

**Derivative of hyperbolic functions**

**Derivative of $\sinh x$** Let

$$f(x)=\sinh x=\frac{e^x-e^{-x}}{2}$$

Differentiating with respect to x

$$\frac{d}{dx}f(x)=\frac{d}{dx}\sinh x=\frac{1}{2}\frac{d}{dx}\left[e^x-e^{-x}\right]$$
$$f'(x)=\frac{d}{dx}\sinh x=\frac{1}{2}\left[\frac{d}{dx}e^x-\frac{d}{dx}e^{-x}\right]$$
$$f'(x)=\frac{d}{dx}\sinh x=\frac{1}{2}\left[e^x\frac{d}{dx}(x)-e^{-x}\frac{d}{dx}(-x)\right]$$
$$f'(x)=\frac{d}{dx}\sinh x=\frac{1}{2}\left[e^x(1)-e^{-x}(-1)\right]$$
$$f'(x)=\frac{d}{dx}\sinh x=\frac{e^x+e^{-x}}{2}$$
$$f'(x)=\frac{d}{dx}\sinh x=\cosh x$$

**Derivative of $\cosh x$**

Let

$$f(x)=\cosh x=\frac{e^x+e^{-x}}{2}$$

Differentiating with respect to x

$$\tfrac{d}{dx}f(x)=\tfrac{d}{dx}\cosh x=\frac{1}{2}\tfrac{d}{dx}\left[e^x+e^{-x}\right]$$

$$f'(x)=\tfrac{d}{dx}\cosh x=\frac{1}{2}\left[\tfrac{d}{dx}e^x+\tfrac{d}{dx}e^{-x}\right]$$

$$f'(x)=\tfrac{d}{dx}\cosh x=\frac{1}{2}\left[e^x\tfrac{d}{dx}(x)+e^{-x}\tfrac{d}{dx}(-x)\right]$$

$$f'(x)=\tfrac{d}{dx}\cosh x=\frac{1}{2}\left[e^x(1)+e^{-x}(-1)\right]$$

$$f'(x)=\tfrac{d}{dx}\cosh x=\frac{e^x-e^{-x}}{2}$$

$$f'(x)=\tfrac{d}{dx}\cosh x=\sinh x$$

**Derivative of $\tanh x$**

Let $f(x)=\tanh x=\dfrac{\sinh x}{\cosh x}=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$

Differentiating with respect to x

$$\tfrac{d}{dx}f(x)=\tfrac{d}{dx}\tanh x=\tfrac{d}{dx}\tfrac{\sinh x}{\cosh x}$$

$$f'(x)=\frac{\cosh x\tfrac{d}{dx}\sinh x-\sinh x\tfrac{d}{dx}\cosh x}{\cosh^2 x}$$

$$f'(x)=\frac{\cosh x(\cosh x)-\sinh x(\sinh x)}{\cosh^2 x}$$

$$f'(x)=\frac{\cosh^2 x-\sinh^2 x}{\cosh^2 x}$$

$$f'(x)=\frac{1}{\cosh^2 x}$$

$$\tfrac{d}{dx}\tanh x=\sec h^2 x$$

**Derivative of $\coth x$**

Let $f(x)=\coth x=\dfrac{\cosh x}{\sinh x}=\dfrac{e^x+e^{-x}}{e^x-e^{-x}}$

Differentiating with respect to x

$$\tfrac{d}{dx}f(x)=\tfrac{d}{dx}\coth x=\tfrac{d}{dx}\tfrac{\cosh x}{\sinh x}$$

$$f'(x)=\frac{\sinh x\tfrac{d}{dx}\cosh x-\cosh x\tfrac{d}{dx}\sinh x}{\sinh^2 x}$$

$$f'(x)=\frac{\sinh x(\sinh x)-\cosh x(\cosh x)}{\sinh^2 x}$$

$$f'(x)=\frac{\sinh^2 x-\cosh^2 x}{\sinh^2 x}$$

$$f'(x)=\frac{-\cosh^2 x+\sinh^2 x}{\sinh^2 x}$$

$$f'(x)=\frac{-\left[\cosh^2 x-\sinh^2 x\right]}{\sinh^2 x}$$

$$f'(x)=\frac{-1}{\sinh^2 x}$$

$$\tfrac{d}{dx}\coth x=-\cos ech^2 x$$

**Derivative of $\operatorname{cosech} x$**

Let $f(x)=\operatorname{cosech} x=\dfrac{1}{\sinh x}=\dfrac{2}{e^x-e^{-x}}$

Differentiating with respect to x

$$\tfrac{d}{dx}f(x)=\tfrac{d}{dx}\operatorname{cosech} x=\tfrac{d}{dx}\tfrac{1}{\sinh x}$$

$$f'(x)=\frac{\sinh x\tfrac{d}{dx}(1)-1.\tfrac{d}{dx}\sinh x}{\sinh^2 x}$$

$$f'(x)=\frac{\sinh x(0)-(\cosh x)}{\sinh^2 x}$$

$$f'(x)=\frac{0-\cosh x}{\sinh^2 x}$$

$$f'(x)=\frac{-\cosh x}{\sinh x\sinh x}$$

$$\tfrac{d}{dx}\operatorname{cosech} x=-\cos echx\coth x$$

**Derivative of $\operatorname{sech} x$**

Let $f(x)=\operatorname{sech} x=\dfrac{1}{\cosh x}=\dfrac{2}{e^x+e^{-x}}$

Differentiating with respect to x

$$\tfrac{d}{dx}f(x)=\tfrac{d}{dx}\operatorname{sech} x=\tfrac{d}{dx}\tfrac{1}{\cosh x}$$

$$f'(x)=\frac{\cosh x\tfrac{d}{dx}(1)-(1)\tfrac{d}{dx}\cosh x}{\cosh^2 x}$$

$$f'(x)=\frac{\cosh x(0)-1(\sinh x)}{\cosh^2 x}$$

$$f'(x)=\frac{0-\sinh x}{\cosh^2 x}$$

$$f'(x)=\frac{-\sinh x}{\cosh^2 x}$$

$$\tfrac{d}{dx}\operatorname{sech} x=-\sec hx\tanh x$$

**Derivative of $\sinh^{-1} x$**

Let $y=\sinh^{-1}x$ so

$\sinh y=x$

Differentiating with respect to x

$$\tfrac{d}{dx}\sinh y=\tfrac{d}{dx}x$$

$$\cosh y\tfrac{d}{dx}y=1$$

$$\frac{dy}{dx}=\frac{1}{\cosh y}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{\cosh^2 y}}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1+\sinh^2 y}}$$

Putting the value of $y$ and $\sinh y=x$

$$\frac{d}{dx}\sinh^{-1}x=\frac{1}{\sqrt{1+x^2}}$$

**Derivative of $\cosh^{-1} x$**

Let $y=\cosh^{-1}x$ so

$\cosh y=x$

Differentiating with respect to x

$$\tfrac{d}{dx}\cosh y=\tfrac{d}{dx}x$$

$$\sinh y\tfrac{d}{dx}y=1$$

$$\frac{dy}{dx}=\frac{1}{\sinh y}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{\sinh^2 y}}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{\cosh^2 y-1}}$$

Putting the value of $y$ and $\cosh y = x$

$$\frac{d}{dx}\cosh^{-1}x=\frac{1}{\sqrt{x^2-1}}$$

**Derivative of $\tanh^{-1}x$**

Let $y=\tanh^{-1}x$ so

$\tanh y = x$

Differentiating with respect to x

$\frac{d}{dx}\tanh y=\frac{d}{dx}x$

$\text{sech}^2 y \frac{d}{dx}y=1$

$$\frac{dy}{dx}=\frac{1}{\text{sech}^2 y}$$

$$\frac{dy}{dx}=\frac{1}{1-\tanh^2 x}$$

Putting the value of $y$ and $\tanh y = x$

$$\frac{d}{dx}\tanh^{-1}x=\frac{1}{1-x^2}$$

**Derivative of $\coth^{-1}x$**

Let $y=\coth^{-1}x$ so

$\coth y = x$

Differentiating with respect to x

$\frac{d}{dx}\coth y=\frac{d}{dx}x$

$-\text{cosech}^2 y \frac{d}{dx}y=1$

$$\frac{dy}{dx}=\frac{-1}{\text{cosech}^2 y}$$

$$\frac{dy}{dx}=\frac{-1}{1-\coth^2 x}$$

Putting the value of $y$ and $\coth y = x$

$$\frac{d}{dx}\coth^{-1}x=\frac{-1}{1-x^2}$$

**Derivative of $\text{sech}^{-1}x$**

Let $y=\text{sech}^{-1}x$ so

$\text{sech}\, y = x$

Differentiating with respect to x

$\frac{d}{dx}\text{sech}\, y=\frac{d}{dx}x$

$-\text{sech}\, y\tanh y\frac{d}{dx}y=1$

$$\frac{dy}{dx}=\frac{-1}{\text{sech}\, y\tanh y}$$

$$\frac{dy}{dx}=\frac{-1}{\text{sech}\, y\sqrt{\tanh^2 x}}$$

$$\frac{dy}{dx}=\frac{-1}{\text{sech}\, y\sqrt{1-\text{sech}^2 x}}$$

Putting the value of $y$ and $\text{sech}\, y = x$

$$\frac{d}{dx}\text{sech}^{-1}x=\frac{-1}{x\sqrt{1-x^2}}$$

**Derivative of $\text{cosech}^{-1}x$**

Let $y=\text{cosech}^{-1}x$ so

$\text{cosech}\, y = x$

Differentiating with respect to x

$\frac{d}{dx}\text{cosech}\, y=\frac{d}{dx}x$

$-\text{cosech}\, y\coth y\frac{d}{dx}y=1$

$$\frac{dy}{dx}=\frac{-1}{\text{cosech}\, y\coth y}$$

$$\frac{dy}{dx}=\frac{-1}{\text{cosech}\, y\sqrt{\coth^2 x}}$$

$$\frac{dy}{dx}=\frac{-1}{\text{cosech}\, y\sqrt{\text{cosech}^2 x-1}}$$

Putting the value of $y$ and $\text{sech}\, y = x$

$$\frac{d}{dx}\text{cosech}^{-1}x=\frac{-1}{x\sqrt{x^2-1}}$$

---

Note that: $\log_{10}=\log$ and $\log_e=\ln$

# Exercise 2.7

Q1. Use any suitable rule of differentiation to perform $\frac{dy}{dx}$ for the following functions:

a). $y = x\ln x^2$

Solution: Differentiating the given function with respect to x

$\frac{d}{dx}y=\frac{d}{dx}\left(x\ln x^2\right)$

$\frac{dy}{dx}=x\frac{d}{dx}\ln x^2+\ln x^2\frac{d}{dx}x$

$$y'=\frac{x}{x^2}\frac{d}{dx}x^2+\ln x^2$$

$$y'=\frac{x}{x^2}(2x)+\ln x^2$$

$y'=2+\ln x^2$

---

b). $y=\ln\left(x^2+3x+2\right)$

Solution: Differentiating the given function with respect to x

$\frac{d}{dx}y=\frac{d}{dx}\ln\left(x^2+3x+2\right)$

$$\frac{dy}{dx}=\frac{1}{x^2+3x+2}\frac{d}{dx}\left(x^2+3x+2\right)$$

$$\frac{dy}{dx}=\frac{1}{x^2+3x+2}\left(\frac{d}{dx}x^2+3\frac{d}{dx}x+\frac{d}{dx}2\right)$$

$$\frac{dy}{dx}=\frac{1}{x^2+3x+2}\left(2x\frac{d}{dx}x+3+0\right)$$

$$\frac{dy}{dx}=\frac{1}{x^2+3x+2}\left(2x+3\right)$$

$$\frac{dy}{dx}=\frac{2x+3}{x^2+3x+2}$$

---

c). $y=\frac{\ln 5x}{x^8}$

Solution: Differentiating the given function with respect to x

$\frac{d}{dx}y=\frac{d}{dx}\left(x^{-8}\ln 5x\right)$

$y'=x^{-8}\frac{d}{dx}\ln 5x+\ln 5x\frac{d}{dx}x^{-8}$

$$y'=\frac{x^{-8}}{5x}\frac{d}{dx}5x-8x^{-8-1}\ln 5x\frac{d}{dx}x$$

$$y' = 5\frac{x^{-8}}{5x}\tfrac{d}{dx}x - 8x^{-9}\ln 5x$$
$$y' = x^{-8-1} - 8x^{-9}\ln 5x$$
$$y' = x^{-9} - 8x^{-9}\ln 5x$$
$$y' = x^{-9}\left(1-8\ln 5x\right)$$
$$y' = \frac{1-8\ln 5x}{x^{9}}$$

d). $y = \ln\left(x^2+1\right)^{\frac{1}{2}}$

Solution: We have $y = \ln\left(x^2+1\right)^{\frac{1}{2}}$

$$y = \frac{1}{2}\ln\left(x^2+1\right)$$

Differentiating the given function with respect to x

$$\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\ln\left(x^2+1\right)$$
$$\frac{dy}{dx} = \frac{1}{2\left(x^2+1\right)}\frac{d}{dx}\left(x^2+1\right)$$
$$\frac{dy}{dx} = \frac{1}{2\left(x^2+1\right)}\left(\frac{d}{dx}x^2+\frac{d}{dx}1\right)$$
$$\frac{dy}{dx} = \frac{1}{2\left(x^2+1\right)}\left(2x\frac{d}{dx}x+0\right)$$
$$\frac{dy}{dx} = \frac{2x}{2\left(x^2+1\right)}$$
$$\frac{dy}{dx} = \frac{x}{x^2+1}$$

e). $y = \dfrac{1}{\ln\left(1+x^2\right)}$

Solution: Differentiating the given function with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}\left\{\ln\left(1+x^2\right)\right\}^{-1}$$
$$\frac{dy}{dx} = -1\left\{\ln\left(1+x^2\right)\right\}^{-1-1}\frac{d}{dx}\ln\left(1+x^2\right)$$
$$\frac{dy}{dx} = -1\left\{\ln\left(1+x^2\right)\right\}^{-2}\frac{1}{\left(1+x^2\right)}\frac{d}{dx}\left(1+x^2\right)$$
$$\frac{dy}{dx} = \frac{-1}{\left\{\ln\left(1+x^2\right)\right\}^2\left(1+x^2\right)}\left(\frac{d}{dx}1+\frac{d}{dx}x^2\right)$$
$$\frac{dy}{dx} = \frac{-1}{\left\{\ln\left(1+x^2\right)\right\}^2\left(1+x^2\right)}\left(0+2x\frac{d}{dx}x\right)$$
$$\frac{dy}{dx} = \frac{-2x}{\left\{\ln\left(1+x^2\right)\right\}^2\left(1+x^2\right)}$$

f). $y = \sqrt[3]{\ln\left(1-x^2\right)}$

Solution: Differentiating the given function with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}\left\{\ln\left(1-x^2\right)\right\}^{\frac{1}{3}}$$
$$\frac{dy}{dx} = \frac{1}{3}\left\{\ln\left(1-x^2\right)\right\}^{\frac{1}{3}-1}\frac{d}{dx}\ln\left(1-x^2\right)$$
$$\frac{dy}{dx} = \frac{1}{3\left\{\ln\left(1-x^2\right)\right\}^{\frac{2}{3}}}\frac{1}{\left(1-x^2\right)}\frac{d}{dx}\left(1-x^2\right)$$
$$\frac{dy}{dx} = \frac{1}{3\left\{\ln\left(1-x^2\right)\right\}^{\frac{2}{3}}\left(1-x^2\right)}\left(\frac{d}{dx}1-\frac{d}{dx}x^2\right)$$
$$\frac{dy}{dx} = \frac{1}{3\left\{\ln\left(1-x^2\right)\right\}^{\frac{2}{3}}\left(1-x^2\right)}\left(0-2x\frac{d}{dx}x\right)$$
$$\frac{dy}{dx} = \frac{-2x}{3\left\{\ln\left(1-x^2\right)\right\}^{\frac{2}{3}}\left(1-x^2\right)}$$

Q2. Use any suitable rule of differentiation to perform $\dfrac{dy}{dx}$ for the following functions:

a). $y = 5^{(x+1)}$

Sol: We have $y = 5^{(x+1)}$ Taking ln on both sides

$$\ln y = \ln 5^{(x+1)}$$
$$\ln y = (x+1)\ln 5$$

Differentiating the given function with respect to x

$$\tfrac{d}{dx}\ln y = \ln 5\tfrac{d}{dx}(x+1)$$
$$\frac{1}{y}\frac{d}{dx}y = \ln 5\left(\frac{d}{dx}x+\frac{d}{dx}1\right)$$
$$\frac{dy}{dx} = y\ln 5(1+0)$$
$$\frac{dy}{dx} = 5^{(x+1)}\ln 5 \qquad \therefore y = 5^{(x+1)}$$

b). $y = e^{\sqrt{x}}$

Solution: Differentiating the given function with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}e^{\sqrt{x}}$$
$$\frac{dy}{dx} = e^{\sqrt{x}}\frac{d}{dx}\sqrt{x}$$
$$\frac{dy}{dx} = e^{\sqrt{x}}\frac{d}{dx}x^{\frac{1}{2}}$$
$$\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2}x^{\frac{1}{2}-1}\frac{d}{dx}x$$
$$\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2}x^{\frac{-1}{2}}$$
$$\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2x^{\frac{1}{2}}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

c). $y = \left(e^{-x}+e^{x}\right)^2$

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx}y = \tfrac{d}{dx}\left(e^{-x}+e^{x}\right)^2$$
$$y' = 2\left(e^{-x}+e^{x}\right)^{2-1}\tfrac{d}{dx}\left(e^{-x}+e^{x}\right)$$
$$y' = 2\left(e^{-x}+e^{x}\right)^{1}\left(\tfrac{d}{dx}e^{-x}+\tfrac{d}{dx}e^{x}\right)$$
$$y' = 2\left(e^{-x}+e^{x}\right)\left(e^{-x}\tfrac{d}{dx}(-x)+e^{x}\tfrac{d}{dx}(x)\right)$$
$$y' = 2\left(e^{-x}+e^{x}\right)\left(-e^{-x}+e^{x}\right)$$

$$y' = 2\left(e^{x} + e^{-x}\right)\left(e^{x} - e^{-x}\right)$$
$$y' = 2\left(e^{2x} - e^{-2x}\right)$$

d). $y = \left(e^{3x} - 1\right)^{4}$

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx} y = \tfrac{d}{dx}\left(e^{3x} - 1\right)^{4}$$
$$y' = 4\left(e^{3x} - 1\right)^{4-1} \tfrac{d}{dx}\left(e^{3x} - 1\right)$$
$$y' = 4\left(e^{3x} - 1\right)^{3} \left(\tfrac{d}{dx} e^{3x} - \tfrac{d}{dx} 1\right)$$
$$y' = 4\left(e^{3x} - 1\right)^{3} \left(e^{3x} \tfrac{d}{dx}(3x) - 0\right)$$
$$y' = 4\left(e^{3x} - 1\right)^{3} \left(3e^{3x} \tfrac{d}{dx}(x)\right)$$
$$y' = 4 \times 3e^{3x}\left(e^{3x} - 1\right)^{3}$$
$$y' = 12e^{3x}\left(e^{3x} - 1\right)^{3}$$

e). $y = xe^{x\ln x}$

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx} y = \tfrac{d}{dx}\left(xe^{x\ln x}\right)$$
$$y' = x \tfrac{d}{dx} e^{x\ln x} + e^{x\ln x} \tfrac{d}{dx} x$$
$$y' = xe^{x\ln x} \tfrac{d}{dx}(x\ln x) + e^{x\ln x}$$
$$y' = e^{x\ln x}\left\{x \tfrac{d}{dx}(x\ln x) + 1\right\}$$
$$y' = e^{x\ln x}\left\{x\left(x \tfrac{d}{dx}\ln x + \ln x \tfrac{d}{dx} x\right) + 1\right\}$$
$$y' = e^{x\ln x}\left\{x\left(\frac{x}{x} \tfrac{d}{dx} x + \ln x\right) + 1\right\}$$
$$y' = e^{x\ln x}\left\{x(1 + \ln x) + 1\right\}$$
$$y' = e^{x\ln x}\left\{x + x\ln x + 1\right\}$$

f). $y = 5^{\left(x^2 - x\right)}$

Sol: We have $y = 5^{\left(x^2 - x\right)}$ Taking ln on both sides

$$\ln y = \left(x^2 - x\right)\ln 5$$

Differentiating the given function with respect to x

$$\tfrac{d}{dx} \ln y = \ln 5 \tfrac{d}{dx}\left(x^2 - x\right)$$
$$\frac{1}{y}\frac{dy}{dx} = \ln 5\left(\frac{d}{dx}x^2 - \frac{d}{dx}x\right)$$
$$\frac{dy}{dx} = y\ln 5(2x - 1)$$
$$\frac{dy}{dx} = 5^{\left(x^2 - x\right)}(2x - 1)\ln 5 \qquad \therefore y = 5^{\left(x^2 - x\right)}$$

Q3. Use any suitable rule of differentiation to perform $\dfrac{dy}{dx}$ for the following functions:

a). $y = \log_{10}\left(3x^2 + 7\right)$

Solution: Differentiating the given function with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}\log_{10}\left(3x^2 + 7\right)$$
$$\frac{dy}{dx} = \frac{\log_{10} e}{\left(3x^2 + 7\right)}\frac{d}{dx}\left(3x^2 + 7\right)$$
$$\frac{dy}{dx} = \frac{\log_{10} e}{\left(3x^2 + 7\right)}\left(3\frac{d}{dx}x^2 + \frac{d}{dx}7\right)$$
$$\frac{dy}{dx} = \frac{\log_{10} e}{\left(3x^2 + 7\right)}\left(3 \times 2x\frac{d}{dx}x + 0\right)$$
$$\frac{dy}{dx} = \frac{6x\log_{10} e}{\left(3x^2 + 7\right)}$$

b). $y = \log_{10}\left(x^2 + 3x + 2\right)$

Solution: Differentiating the given function with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}\log_{10}\left(x^2 + 3x + 2\right)$$
$$\frac{dy}{dx} = \frac{\log_{10} e}{\left(x^2 + 3x + 2\right)}\frac{d}{dx}\left(x^2 + 3x + 2\right)$$
$$\frac{dy}{dx} = \frac{\log_{10} e}{\left(x^2 + 3x + 2\right)}\left(\frac{d}{dx}x^2 + 3\frac{d}{dx}x + \frac{d}{dx}2\right)$$
$$\frac{dy}{dx} = \frac{\log_{10} e}{\left(x^2 + 3x + 2\right)}\left(2x\frac{d}{dx}x + 3 + 0\right)$$
$$\frac{dy}{dx} = \frac{(2x + 3)\log_{10} e}{\left(x^2 + 3x + 2\right)}$$

c). $y = \log_{10}\sqrt{x^2 - 7x} + x^3$

Solution: Differentiating the given function with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}\log_{10}\sqrt{x^2 - 7x} + \frac{d}{dx}x^3$$
$$\frac{dy}{dx} = \frac{\log_{10} e}{\sqrt{x^2 - 7x}}\frac{d}{dx}\sqrt{x^2 - 7x} + 3x^2\frac{d}{dx}x$$
$$\frac{dy}{dx} = \frac{\log_{10} e}{\sqrt{x^2 - 7x}}\frac{d}{dx}\left(x^2 - 7x\right)^{\frac{1}{2}} + 3x^2$$
$$\frac{dy}{dx} = \frac{\log_{10} e}{\sqrt{x^2 - 7x}}\frac{1}{2}\left(x^2 - 7x\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(x^2 - 7x\right) + 3x^2$$
$$\frac{dy}{dx} = \frac{\log_{10} e}{2\sqrt{x^2 - 7x}}\left(x^2 - 7x\right)^{\frac{-1}{2}}\left(\frac{d}{dx}x^2 - 7\frac{d}{dx}x\right) + 3x^2$$
$$\frac{dy}{dx} = \frac{\log_{10} e}{2\left(x^2 - 7x\right)^{\frac{1}{2}}}\frac{1}{\left(x^2 - 7x\right)^{\frac{1}{2}}}(2x - 7) + 3x^2$$
$$\frac{dy}{dx} = \frac{(2x - 7)\log_{10} e}{2\left(x^2 - 7x\right)^{\frac{1}{2}+\frac{1}{2}}} + 3x^2$$
$$\frac{dy}{dx} = \frac{(2x - 7)\log_{10} e}{2\left(x^2 - 7x\right)} + 3x^2$$

d). $y = \log\left[\sin(\log x)\right]$

Solution: Differentiating the given function with respect to x

$$\frac{d}{dx}y = \frac{d}{dx}\log\left[\sin(\log x)\right]$$
$$\frac{dy}{dx} = \frac{\log e}{\left[\sin(\log x)\right]}\frac{d}{dx}\left[\sin(\log x)\right]$$
$$\frac{dy}{dx} = \frac{\cos(\log x).\log e}{\left[\sin(\log x)\right]}\frac{d}{dx}(\log x)$$
$$\frac{dy}{dx} = \frac{\cos(\log x).\log e}{\left[\sin(\log x)\right]}\ \frac{\log e}{x}$$

$$\frac{dy}{dx}=\frac{\cos(\log x).(\log e)^2}{x\left[\sin(\log x)\right]}$$

$$\frac{dy}{dx}=\frac{\cot(\log x).(\log e)^2}{x} \qquad \therefore \cot x=\frac{\cos x}{\sin x}$$

e). $y=\log_{10}\left(\sin^{-1}x^2\right)$

Solution: Differentiating the given function with respect to x

$$\frac{dy}{dx}=\frac{d}{dx}\log_{10}\left(\sin^{-1}x^2\right)$$

$$\frac{dy}{dx}=\frac{\log_{10}e}{\left(\sin^{-1}x^2\right)}\frac{d}{dx}\left(\sin^{-1}x^2\right)$$

$$\frac{dy}{dx}=\frac{\log_{10}e}{\left(\sin^{-1}x^2\right)}\frac{1}{\sqrt{1-\left(x^2\right)^2}}\frac{d}{dx}x^2$$

$$\frac{dy}{dx}=\frac{\log_{10}e}{\left(\sin^{-1}x^2\right)}\frac{2x}{\sqrt{1-x^4}}\frac{d}{dx}x$$

$$\frac{dy}{dx}=\frac{2x\log_{10}e}{\left(\sin^{-1}x^2\right)\sqrt{1-x^4}}$$

f). $y=\log\tan\left[\frac{1}{2}x+\frac{1}{4}\pi\right]$

Solution: Differentiating the given function with respect to x

$$\frac{dy}{dx}=\frac{d}{dx}\log\tan\left[\frac{1}{2}x+\frac{1}{4}\pi\right]$$

$$\frac{dy}{dx}=\frac{\log \text{e}}{\tan\left[\frac{1}{2}x+\frac{1}{4}\pi\right]}\frac{d}{dx}\tan\left[\frac{1}{2}x+\frac{1}{4}\pi\right]$$

$$\frac{dy}{dx}=\frac{\log \text{e}.\sec^2\left[\frac{1}{2}x+\frac{1}{4}\pi\right]}{\tan\left[\frac{1}{2}x+\frac{1}{4}\pi\right]}\frac{d}{dx}\left[\frac{1}{2}x+\frac{1}{4}\pi\right]$$

$$\frac{dy}{dx}=\frac{\log \text{e}.\sec^2\left[\frac{1}{2}x+\frac{1}{4}\pi\right]}{\tan\left[\frac{1}{2}x+\frac{1}{4}\pi\right]}\left[\frac{1}{2}\frac{d}{dx}x+\frac{d}{dx}\frac{1}{4}\pi\right]$$

$$\frac{dy}{dx}=\frac{\log \text{e}.\sec^2\left[\frac{1}{2}x+\frac{1}{4}\pi\right]}{\tan\left[\frac{1}{2}x+\frac{1}{4}\pi\right]}\left[\frac{1}{2}+0\right]$$

$$\frac{dy}{dx}=\frac{\log \text{e}.\sec^2\left[\frac{1}{2}x+\frac{1}{4}\pi\right]}{2\tan\left[\frac{1}{2}x+\frac{1}{4}\pi\right]}$$

Q4. Use any suitable rule of differentiation to perform $\frac{dy}{dx}$ for the following functions:

a). $y=\ln\sqrt{\frac{x+1}{x-1}}$

Solution: We have $y=\ln\left(\frac{x+1}{x-1}\right)^{\frac{1}{2}}$

$$y=\frac{1}{2}\left\{\ln(x+1)-\ln(x-1)\right\}$$

Differentiating the given function with respect to x

$$\frac{dy}{dx}=\frac{1}{2}\left\{\frac{d}{dx}\ln(x+1)-\frac{d}{dx}\ln(x-1)\right\}$$

$$\frac{dy}{dx}=\frac{1}{2}\left\{\frac{1}{x+1}\frac{d}{dx}(x+1)-\frac{1}{x-1}\frac{d}{dx}(x-1)\right\}$$

$$\frac{dy}{dx}=\frac{1}{2}\left\{\frac{1}{x+1}\left(\frac{d}{dx}x+\frac{d}{dx}1\right)-\frac{1}{x-1}\left(\frac{d}{dx}x-\frac{d}{dx}1\right)\right\}$$

$$\frac{dy}{dx}=\frac{1}{2}\left\{\frac{1}{x+1}(1+0)-\frac{1}{x-1}(1-0)\right\}$$

$$\frac{dy}{dx}=\frac{1}{2}\left\{\frac{1}{x+1}-\frac{1}{x-1}\right\}$$

$$\frac{dy}{dx}=\frac{1}{2}\left\{\frac{(x-1)-(x+1)}{(x+1)(x-1)}\right\}$$

$$\frac{dy}{dx}=\frac{1}{2}\left\{\frac{x-1-x-1}{(x)^2-(1)^2}\right\}$$

$$\frac{dy}{dx}=\frac{1}{2}\left\{\frac{-2}{x^2-1}\right\}$$

$$\frac{dy}{dx}=\frac{-1}{x^2-1}$$

b). $y=(\cos x)^{\log x}$

Sol: Given $y=(\cos x)^{\log x}$ Taking ln on both sides

$$\ln y=\ln(\cos x)^{\log x}$$

$\ln y=\log x\ln(\cos x)$ Differentiating w r t x

$$\frac{1}{y}y'=\cos x\frac{d}{dx}\log x+\log x\frac{d}{dx}\cos x$$

$$y'=y\left\{\frac{\cos x\log e}{x}-\log x\sin x\right\}$$

$$y'=(\cos x)^{\log x}\left\{\frac{\cos x\log e}{x}-\log x\sin x\right\}$$

c). $y=\left(1+x^{-1}\right)^x$

Sol: We have $y=\left(1+x^{-1}\right)^x$ Taking ln on both sides

$$\ln y=\ln\left(1+x^{-1}\right)^x$$

$$\ln y=x.\ln\left(1+x^{-1}\right)$$

Differentiating the given function with respect to x

$$\frac{d}{dx}\ln y=\frac{d}{dx}\left(x.\ln\left(1+x^{-1}\right)\right)$$

$$\frac{1}{y}\frac{dy}{dx}=x\frac{d}{dx}\ln\left(1+x^{-1}\right)+\ln\left(1+x^{-1}\right)\frac{d}{dx}x$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{x}{\left(1+x^{-1}\right)}\frac{d}{dx}\left(1+x^{-1}\right)+\ln\left(1+x^{-1}\right)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{x}{\left(1+x^{-1}\right)}\left(\frac{d}{dx}1+\frac{d}{dx}x^{-1}\right)+\ln\left(1+x^{-1}\right)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{x}{\left(1+x^{-1}\right)}\left(0-1x^{-1-1}\right)+\ln\left(1+x^{-1}\right)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{-x}{x^{2}\left(1+x^{-1}\right)}+\ln\left(1+x^{-1}\right)$$

$$\frac{dy}{dx}=y\left\{\frac{-1}{x\left(1+x^{-1}\right)}+\ln\left(1+x^{-1}\right)\right\}$$

$$\frac{dy}{dx}=\left(1+x^{-1}\right)^{x}\left\{\frac{-1}{x\left(1+x^{-1}\right)}+\ln\left(1+x^{-1}\right)\right\}$$

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d). $y=\dfrac{(1-x)^{\frac{1}{2}}\left(2-x^{2}\right)^{\frac{2}{3}}}{\left(3-x^{3}\right)^{\frac{3}{4}}\left(4-x^{4}\right)^{\frac{4}{5}}}$

Solution: We have $y=\dfrac{(1-x)^{\frac{1}{2}}\left(2-x^{2}\right)^{\frac{2}{3}}}{\left(3-x^{3}\right)^{\frac{3}{4}}\left(4-x^{4}\right)^{\frac{4}{5}}}$

Taking ln on both sides $\ln y=\ln\dfrac{(1-x)^{\frac{1}{2}}\left(2-x^{2}\right)^{\frac{2}{3}}}{\left(3-x^{3}\right)^{\frac{3}{4}}\left(4-x^{4}\right)^{\frac{4}{5}}}$

$$\ln y=\ln\left\{(1-x)^{\frac{1}{2}}\left(2-x^{2}\right)^{\frac{2}{3}}\right\}-\ln\left\{\left(3-x^{3}\right)^{\frac{3}{4}}\left(4-x^{4}\right)^{\frac{4}{5}}\right\}$$

$$\ln y=\left\{\ln(1-x)^{\frac{1}{2}}+\ln\left(2-x^{2}\right)^{\frac{2}{3}}\right\}-\left\{\ln\left(3-x^{3}\right)^{\frac{3}{4}}+\ln\left(4-x^{4}\right)^{\frac{4}{5}}\right\}$$

$$\ln y=\ln(1-x)^{\frac{1}{2}}+\ln\left(2-x^{2}\right)^{\frac{2}{3}}-\ln\left(3-x^{3}\right)^{\frac{3}{4}}-\ln\left(4-x^{4}\right)^{\frac{4}{5}}$$

$$\ln y=\frac{1}{2}\ln(1-x)+\frac{2}{3}\ln\left(2-x^{2}\right)-\frac{3}{4}\ln\left(3-x^{2}\right)-\frac{4}{5}\ln\left(4-x^{2}\right)$$

Differentiating the given function with respect to x

$$\frac{d}{dx}\ln y=\frac{1}{2}\frac{d}{dx}\ln(1-x)+\frac{2}{3}\frac{d}{dx}\ln\left(2-x^{2}\right)-\frac{3}{4}\frac{d}{dx}\ln\left(3-x^{3}\right)-\frac{4}{5}\frac{d}{dx}\ln\left(4-x^{4}\right)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{2(1-x)}\frac{d}{dx}(1-x)+\frac{2}{3\left(2-x^{2}\right)}\frac{d}{dx}\left(2-x^{2}\right)-\frac{3}{4\left(3-x^{3}\right)}\frac{d}{dx}\left(3-x^{3}\right)-\frac{4}{5\left(4-x^{4}\right)}\frac{d}{dx}\left(4-x^{4}\right)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{2(1-x)}\frac{d}{dx}(-1)+\frac{2}{3\left(2-x^{2}\right)}\frac{d}{dx}(-2x)-\frac{3}{4\left(3-x^{3}\right)}\frac{d}{dx}\left(-3x^{2}\right)-\frac{4}{5\left(4-x^{4}\right)}\frac{d}{dx}\left(-4x^{3}\right)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{-1}{2(1-x)}+\frac{-4x}{3\left(2-x^{2}\right)}-\frac{-9x^{2}}{4\left(3-x^{3}\right)}-\frac{-16x^{3}}{5\left(4-x^{4}\right)}$$

$$\frac{dy}{dx}=y\left\{\frac{-1}{2(1-x)}-\frac{4x}{3\left(2-x^{2}\right)}+\frac{9x^{2}}{4\left(3-x^{3}\right)}+\frac{16x^{3}}{5\left(4-x^{4}\right)}\right\}$$

$$\frac{dy}{dx}=\frac{(1-x)^{\frac{1}{2}}\left(2-x^{2}\right)^{\frac{2}{3}}}{\left(3-x^{3}\right)^{\frac{3}{4}}\left(4-x^{4}\right)^{\frac{4}{5}}}\left\{\frac{-1}{2(1-x)}-\frac{4x}{3\left(2-x^{2}\right)}+\frac{9x^{2}}{4\left(3-x^{3}\right)}+\frac{16x^{3}}{5\left(4-x^{4}\right)}\right\}$$

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e). $y=\dfrac{x\sqrt[3]{x^{2}+4}}{\sqrt{x^{2}+3}}$

Sol: We have $y=\dfrac{x\sqrt[3]{x^{2}+4}}{\sqrt{x^{2}+3}}$ Taking ln on both sides

$$\ln y=\ln\frac{x\left(x^{2}+4\right)^{\frac{1}{3}}}{\left(x^{2}+3\right)^{\frac{1}{2}}}$$

$$\ln y=\ln\left\{x\left(x^{2}+4\right)^{\frac{1}{3}}\right\}-\ln\left(x^{2}+3\right)^{\frac{1}{2}}$$

$$\ln y=\ln x+\ln\left(x^{2}+4\right)^{\frac{1}{3}}-\ln\left(x^{2}+3\right)^{\frac{1}{2}}$$

$$\ln y=\ln x+\frac{1}{3}\ln\left(x^{2}+4\right)-\frac{1}{2}\ln\left(x^{2}+3\right)$$

Differentiating the given function with respect to x

$$\frac{d}{dx}\ln y=\frac{d}{dx}\ln x+\frac{1}{3}\frac{d}{dx}\ln\left(x^{2}+4\right)-\frac{1}{2}\frac{d}{dx}\ln\left(x^{2}+3\right)$$

$$\frac{1}{y}\frac{d}{dx}y=\frac{1}{x}+\frac{1}{3\left(x^{2}+4\right)}\frac{d}{dx}\left(x^{2}+4\right)-\frac{1}{2\left(x^{2}+3\right)}\frac{d}{dx}\left(x^{2}+3\right)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{x}+\frac{1}{3\left(x^{2}+4\right)}(2x)-\frac{1}{2\left(x^{2}+3\right)}(2x)$$

$$\frac{dy}{dx}=y\left\{\frac{1}{x}+\frac{2x}{3\left(x^{2}+4\right)}-\frac{2x}{2\left(x^{2}+3\right)}\right\}$$

$$\frac{dy}{dx}=\frac{x\sqrt[3]{x^{2}+4}}{\sqrt{x^{2}+3}}\left\{\frac{1}{x}+\frac{2x}{3\left(x^{2}+4\right)}-\frac{2x}{2\left(x^{2}+3\right)}\right\}$$

$$\frac{dy}{dx}=\frac{x\sqrt[3]{x^{2}+4}}{\sqrt{x^{2}+3}}\left\{\frac{6\left(x^{2}+3\right)\left(x^{2}+4\right)+4x.x\left(x^{2}+3\right)-6x.x\left(x^{2}+4\right)}{6x\left(x^{2}+4\right)\left(x^{2}+3\right)}\right\}$$

$$\frac{dy}{dx}=\frac{x\left(x^{2}+4\right)^{\frac{1}{3}}}{\left(x^{2}+3\right)^{\frac{1}{2}}}\left\{\frac{6\left(x^{4}+4x^{2}+3x^{2}+12\right)+4x^{2}\left(x^{2}+3\right)-6x^{2}\left(x^{2}+4\right)}{6x\left(x^{2}+4\right)\left(x^{2}+3\right)}\right\}$$

$$\frac{dy}{dx}=\left\{\frac{6\left(x^{4}+4x^{2}+3x^{2}+12\right)+4x^{2}\left(x^{2}+3\right)-6x^{2}\left(x^{2}+4\right)}{6\left(x^{2}+4\right)^{1-\frac{1}{3}}\left(x^{2}+3\right)^{1+\frac{1}{2}}}\right\}$$

$$\frac{dy}{dx}=\frac{6\left(x^{4}+7x^{2}+12\right)+4x^{4}+12x^{2}-6x^{4}-24x^{2}}{6\left(x^{2}+4\right)^{1-\frac{1}{3}}\left(x^{2}+3\right)^{1+\frac{1}{2}}}$$

$$\frac{dy}{dx}=\frac{6x^{4}+42x^{2}+72+4x^{4}+12x^{2}-6x^{4}-24x^{2}}{6\left(x^{2}+4\right)^{\frac{2}{3}}\left(x^{2}+3\right)^{\frac{3}{2}}}$$

$$\frac{dy}{dx}=\frac{6x^{4}-6x^{4}+4x^{4}+42x^{2}+12x^{2}-24x^{2}+72}{6\left(x^{2}+4\right)^{\frac{2}{3}}\left(x^{2}+3\right)^{\frac{3}{2}}}$$

$$\frac{dy}{dx}=\frac{4x^{4}+30x^{2}+72}{6\left(x^{2}+4\right)^{\frac{2}{3}}\left(x^{2}+3\right)^{\frac{3}{2}}}$$

$$\frac{dy}{dx}=\frac{2\left(2x^{4}+15x^{2}+36\right)}{6\left(x^{2}+4\right)^{\frac{2}{3}}\left(x^{2}+3\right)^{\frac{3}{2}}}$$

$$\frac{dy}{dx}=\frac{2x^{4}+15x^{2}+36}{3\left(x^{2}+4\right)^{\frac{2}{3}}\left(x^{2}+3\right)^{\frac{3}{2}}}$$

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f). $y=(\sin x)(\log x)\left(x^{x}\right)$

Solution: We have $y=(\sin x)(\log x)\left(x^{x}\right)$ Taking ln

$$\ln y=\ln\left\{(\sin x)(\log x)\left(x^{x}\right)\right\}$$

$$\ln y=\ln(\sin x)+\ln(\log x)+\ln x^{x}$$

$$\ln y=\ln(\sin x)+\ln(\log x)+x\ln x$$

Differentiating the given function with respect to x

$$\frac{d}{dx}\ln y=\frac{d}{dx}\ln(\sin x)+\frac{d}{dx}\ln(\log x)+\frac{d}{dx}(x\ln x)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{\sin x}\frac{d}{dx}(\sin x)+\frac{1}{\log x}\frac{d}{dx}(\log x)+x\frac{d}{dx}\ln x+\ln x\frac{d}{dx}x$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{\cos x}{\sin x}+\frac{1}{\log x}\frac{\log e}{x}+\frac{x}{x}+\ln x$$

$$\frac{1}{y}\frac{dy}{dx}=\cot x+\frac{\log e}{x\log x}+1+\ln x$$

$$\frac{dy}{dx}=y\left\{\cot x+\frac{\log e}{x\log x}+1+\ln x\right\}$$

$$\frac{dy}{dx}=(\sin x)(\log x)\left(x^x\right)\left\{\cot x+\frac{\log e}{x\log x}+1+\ln x\right\}$$

Q5. Use any suitable rule of differentiation to perform $\frac{dy}{dx}$ for the following functions:

a). $y=\cosh\left(2x^2+3x\right)$

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx}y=\tfrac{d}{dx}\cosh\left(2x^2+3x\right)$$

$$y'=\sinh\left(2x^2+3x\right)\tfrac{d}{dx}\left(2x^2+3x\right)$$

$$y'=\sinh\left(2x^2+3x\right).\left(2\tfrac{d}{dx}x^2+3\tfrac{d}{dx}x\right)$$

$$y'=\sinh\left(2x^2+3x\right).\left(2(2x)+3\right)$$

$$y'=(4x+3)\sinh\left(2x^2+3x\right)$$

b). $y=e^{\sinh^2 x}$

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx}y=\tfrac{d}{dx}e^{\sinh^2 x}$$

$$y'=e^{\sinh^2 x}\tfrac{d}{dx}\sinh^2 x$$

$$y'=e^{\sinh^2 x}\;2\sinh x\tfrac{d}{dx}\sinh x$$

$$y'=e^{\sinh^2 x}\;2\sinh x\cosh x$$

$$y'=e^{\sinh^2 x}\;\sinh 2x$$

c). $y=\log(\cosh x)$

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx}y=\tfrac{d}{dx}\log(\cosh x)$$

$$y'=\frac{\log e}{\cosh x}\tfrac{d}{dx}\cosh x$$

$$y'=\frac{\log e}{\cosh x}\sinh x\tfrac{d}{dx}x$$

$$y'=\frac{\sinh x}{\cosh x}\log e$$

$$y'=\tanh x\;\log e$$

d). $y=\sec h\left(x^2+1\right)+\tanh\left(x^2+1\right)$

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx}y=\tfrac{d}{dx}\sec h\left(x^2+1\right)+\tfrac{d}{dx}\tanh\left(x^2+1\right)$$

$$y'=-\sec h\left(x^2+1\right)\tanh\left(x^2+1\right)\tfrac{d}{dx}\left(x^2+1\right)+\operatorname{sech}^2\left(x^2+1\right)\tfrac{d}{dx}\left(x^2+1\right)$$

$$y'=-\sec h\left(x^2+1\right)\tanh\left(x^2+1\right)(2x+0)+\operatorname{sech}^2\left(x^2+1\right)(2x+0)$$

$$y'=2x\sec h^2\left(x^2+1\right)-2x\sec h\left(x^2+1\right)\tanh\left(x^2+1\right)$$

$$y'=2x\sec h\left(x^2+1\right)\left\{\sec h\left(x^2+1\right)-\tanh\left(x^2+1\right)\right\}$$

e). $y=\operatorname{cosec} h\left(x^3+1\right)$

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx}y=\tfrac{d}{dx}\operatorname{cosec} h\left(x^3+1\right)$$

$$y'=-\operatorname{cosec} h\left(x^3+1\right)\coth\left(x^3+1\right)\tfrac{d}{dx}\left(x^3+1\right)$$

$$y'=-\operatorname{cosec} h\left(x^3+1\right)\coth\left(x^3+1\right)\left(\tfrac{d}{dx}x^3+\tfrac{d}{dx}1\right)$$

$$y'=-\operatorname{cosec} h\left(x^3+1\right)\coth\left(x^3+1\right)\left(3x^2\tfrac{d}{dx}x+0\right)$$

$$y'=-3x^2\operatorname{cosec} h\left(x^3+1\right)\coth\left(x^3+1\right)$$

f). $x\cos hy=y\sinh x+5$

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx}(x\cos hy)=\tfrac{d}{dx}(y\sinh x)+\tfrac{d}{dx}5$$

$$x\tfrac{d}{dx}\cos hy+\cos hy\tfrac{d}{dx}x=y\tfrac{d}{dx}\sinh x+\sinh x\tfrac{d}{dx}y+0$$

$$x\sinh y\tfrac{d}{dx}y+\cos hy=y\cosh x\tfrac{d}{dx}x+\sinh x\tfrac{d}{dx}y$$

$$x\sinh y\tfrac{dy}{dx}+\cos hy=y\cosh x+\sinh x\tfrac{dy}{dx}$$

$$x\sinh y\tfrac{dy}{dx}-\sinh x\tfrac{dy}{dx}=y\cosh x-\cos hy$$

$$(x\sinh y-\sinh x)\tfrac{dy}{dx}=y\cosh x-\cos hy$$

$$\frac{dy}{dx}=\frac{y\cosh x-\cosh y}{x\sinh y-\sinh x}$$

Q6. Use any suitable rule of differentiation to perform $\frac{dy}{dx}$ for the following functions:

a). $y=\tanh^{-1}(\sin x)$

Solution: We have $y=\tanh^{-1}(\sin x)$

$$\tanh y=\sin x$$

Differentiating the given function with respect to x

$$\tfrac{d}{dx}\tanh y=\tfrac{d}{dx}\sin x$$

$$\operatorname{sech}^2 y\tfrac{d}{dx}y=\cos x\tfrac{d}{dx}x$$

$$\operatorname{sech}^2 y\tfrac{d}{dx}y=\cos x$$

$$y'=\frac{\cos x}{\operatorname{sech}^2 y}$$

$$y'=\frac{\cos x}{1-\tanh^2 y}$$

$$y'=\frac{\cos x}{1-\sin^2 x}\qquad\therefore \tanh y=\sin x$$

$$y'=\frac{\cos x}{\cos^2 x}=\frac{1}{\cos x}=\sec x$$

b). $y=\sinh^{-1}(\tan x)$

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx}y=\tfrac{d}{dx}\sinh^{-1}(\tan x)$$

$$y'=\frac{1}{\sqrt{1+(\tan x)^2}}\tfrac{d}{dx}\tan x$$

$$y'=\frac{1}{\sqrt{\sec^2 x}}\sec^2 x$$

$$y' = \frac{\sec^2 x}{\sec x} = \sec x$$

c). $y = \cosh^{-1}(\sec x)$

Solution: We have $y = \cosh^{-1}(\sec x)$

Differentiating the given function with respect to x

$$\tfrac{d}{dx} y = \tfrac{d}{dx}\cosh^{-1}(\sec x)$$

$$y' = \frac{1}{\sqrt{\sec^2 x - 1}}\tfrac{d}{dx}\sec x$$

$$y' = \frac{\sec x \tan x}{\sqrt{\tan^2 x}}$$

$$y' = \sec x$$

d). $y = x\tanh^{-1}(3x)$

Solution: We have $y = x\tanh^{-1}(3x)$

Differentiating the given function with respect to x

$$\tfrac{d}{dx} y = \tfrac{d}{dx}\left\{x\tanh^{-1}(3x)\right\}$$

$$y' = \tanh^{-1}(3x)\tfrac{d}{dx}x + x\tfrac{d}{dx}\tanh^{-1}(3x)$$

$$y' = \tanh^{-1}(3x) + x\left(\frac{1}{1-(3x)^2}\right)\frac{d}{dx}3x$$

$$y' = \tanh^{-1}(3x) + \frac{3x}{1-9x^2}$$

e). $y = x\cosh^{-1} x - \sqrt{x^2-1}$

Solution: Differentiating the given function with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(x\cosh^{-1} x\right) - \frac{d}{dx}\left(x^2-1\right)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = x\frac{d}{dx}\cosh^{-1} x + \cosh^{-1} x\frac{d}{dx}x - \frac{1}{2}\left(x^2-1\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(x^2-1\right)$$

$$\frac{dy}{dx} = x\frac{1}{\sqrt{x^2-1}}\frac{d}{dx}x + \cosh^{-1} x - \frac{1}{2}\left(x^2-1\right)^{\frac{-1}{2}}(2x-0)$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^2-1}} + \cosh^{-1} x - \frac{2x}{2\left(x^2-1\right)^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^2-1}} + \cosh^{-1} x - \frac{x}{\sqrt{x^2-1}}$$

$$\frac{dy}{dx} = \cosh^{-1} x$$

f). $\log\left(\cosh^{-1} x\right) + \sinh^{-1} y = 6$

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx}\log\left(\cosh^{-1} x\right) + \tfrac{d}{dx}\sinh^{-1} y = \tfrac{d}{dx}6$$

$$\frac{\log e}{\cosh^{-1} x}\frac{d}{dx}\cosh^{-1} x + \frac{1}{\sqrt{1+y^2}}\frac{dy}{dx} = 0$$

$$\frac{\log e}{\cosh^{-1} x}\frac{1}{\sqrt{x^2-1}} + \frac{1}{\sqrt{1+y^2}}\frac{dy}{dx} = 0$$

$$\frac{1}{\sqrt{1+y^2}}\frac{dy}{dx} = \frac{-\log e}{\cosh^{-1} x\sqrt{x^2-1}}$$

$$\frac{dy}{dx} = \frac{-\log e\,\sqrt{1+y^2}}{\cosh^{-1} x\,\sqrt{x^2-1}}$$

Q7. A research group ( used hospital records ) developed the approximate mathematical model related to systolic blood pressure and age is

$P(x) = 40 + 25\ln(x+1),\qquad 0 \le x \le 65$ where $P(x)$ is the pressure measured in millimeters of mercury and x is age in years. What is the rate of change of pressure at the end of 10 years? At the end of 30 years? At the end of 60 years?

Solution: Differentiating the given function with respect to x

$$\tfrac{d}{dx}P(x) = \tfrac{d}{dx}40 + 25\tfrac{d}{dx}\ln(x+1)$$

$$P'(x) = 0 + \frac{25}{x+1}\tfrac{d}{dx}(x+1)$$

$$P'(x) = \frac{25}{x+1}\left(\tfrac{d}{dx}x + \tfrac{d}{dx}1\right)$$

$$P'(x) = \frac{25}{x+1}(1+0)$$

$$P'(x) = \frac{25}{x+1}$$

Now we find pressure at the end of 10 years

$P'(10) = \frac{25}{10+1} = 2.27$ mm of mercury per year

Now we find pressure at the end of 30 years

$P'(30) = \frac{25}{30+1} = 0.806$ mm of mercury per year

Now we find pressure at the end of 60 years

$P'(60) = \frac{25}{60+1} = 0.409$ mm of mercury per year

Q8. A single cholera bacterium divides every 0.5 hour to produce two complete cholera bacteria. If we start with a colony of 5000 bacteria, then after t hours there will be a $A(t) = 5000.2^{2t}$ bacteria. Find $A'(t), A'(1)$ and $A'(5)$ . Interpret the results.

Solution: We have $A(t) = 5000.2^{2t}$

Differentiating the given function with respect to x

$$\tfrac{d}{dt}A(t) = 5000.\tfrac{d}{dt}2^{2t}$$

$$A'(t) = 5000.2^{2t}.\ln 2\,\tfrac{d}{dt}(2t)$$

$$A'(t) = 2\times 5000.2^{2t}.\ln 2$$

$$A'(t) = 10\,000.2^{2t}.\ln 2$$

Now we have to find $A'(1)$

$$A'(1) = 10\,000.2^{2(1)}.\ln 2$$

$$A'(1) = 27725.89$$

Now we have to find $A'(5)$

$$A'(5) = 10\,000.2^{2(5)}.\ln 2$$

$$A'(1) = 7\,097\,827.13$$