Chapter 1

Independent and dependent Variables:

The variable x is called the independent variable of function f, and the variable y is called depended variable of function f

Piece-wise function:

When the function f is defined be some rules,

Example $f(x)=\begin{cases} x & when \quad 0\le x\le 1 \\ x-1 & when \quad 1<x\le 2 \end{cases}$

Function

A function is a rule or correspondence, relating two sets in such a way that each element in first set corresponds to one and only one element in second set.

Function and its domain and range:

A function f from a set X to a set Y is rule or a correspondence that assigns to each element x in X a unique element y in Y. Set X is called **domain** of f set of corresponding elements y in Y is called **range** of f

Identification of function through Graph

Graph provides a visual technique for determining whether the set of points represents a function or not. If a vertical line intersects a graph in more than one point. It is not the graph of function.

How to find domain

We can take all values for the domain **not including**

i). non zero **denominator** i.e., $\frac{1}{x\neq 0}$

ii). positive **radicand** i.e., $\sqrt{x\ge 0}$

iii). log(natural or common) $\ln x$ with $x>0$

How to find Range:

i). Put y in the place of $f(x)$ then separate x

i.e. $y=f(x) \quad \Rightarrow f^{-1}(y)=x$

e.g. $f(x)=x+5 \quad \Rightarrow y=x+5$

$\Rightarrow x=y-5$

ii). we put $f^{-1}(y)$ in the place of x

e.g., $f^{-1}(y)=y-5$

iii).Now we will change the demi variable y with x

e.g., $f^{-1}(y)=y-5 \quad \Rightarrow f^{-1}(x)=x-5$

then we check the domain of $f^{-1}(x)$ which is Range

Absolute value of Numbers:

Absolute value of any number x is denoted by $|x|$ and is defined as $|x|=\begin{cases} x, & x\ge 0 \\ -x, & x<0 \end{cases}$

Exercise 1.1

Q1. Identify the independent and dependent variable for the following problems.

a). $P=64d$

Solution: independent variable is d
dependent variable is P

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b). $F(c)=\frac{9}{5}c+32$

Solution: independent variable is c
dependent variable is F

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c). $C(F)=\frac{5}{9}(F-32)$

Solution: independent variable is F
dependent variable is C

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d). $s=f(r,\theta)=r.\theta$

Solution: independent variables are r,θ
dependent variable is f or s

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e). $F=\theta(m,a)=ma$

Solution: independent variables are m,a
dependent variable is θ or F

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f). $SA=\theta(l,w,h)=2lw+2lh+2wh$

Solution: independent variables are l,w,h
dependent variable is θ or SA

Q2. Evaluate the following function for the indicated independent variables

a). $f(x)=3x^2+7x-5;\ f(3),f(-4),f(a+h)$

Sol: Given $f(x)=3x^2+7x-5$

At $x=3$ $\quad f(3)=3(3)^2+7(3)-5$

$f(3)=3(9)+21-5$

$f(3)=27+16$

$f(3)=43$

At $x=-4$ $\quad f(-4)=3(-4)^2+7(-4)-5$

$f(-4)=3(16)-28-5$

$f(-4)=48-33$

$f(-4)=15$

At $x=a+h$ $\quad f(a+h)=3(a+h)^2+7(a+h)-5$

$f(a+h)=3(a^2+2ah+h^2)+7a+7h-5$

$f(a+h)=3a^2+6ah+3h^2+7a+7h-5$

$f(a+h)=3a^2+3h^2+6ah+7a+7h-5$

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b). $f(t)=\frac{t+5}{t-3};\ f(2),f(7.4),f(-3.7)$

Sol: Given $f(t)=\frac{t+5}{t-3}$ $\quad$ At $t=2$

$f(2)=\frac{2+5}{2-3}$

$f(2)=-7$

At $t=7.4$ $\quad f(7.4)=\frac{7.4+5}{7.4-3}$

$f(7.4)=\frac{12.4}{4.4}$

$f(7.4)=2.81818181....$

$f(7.4)=2.\overline{81}$

At $t=-3.7$ $\quad f(-3.7)=\frac{-3.7+5}{-3.7-3}$

$f(-3.7)=\frac{1.3}{-6.7}$

$f(-3.7)=-0.194029$

c). $g(R)=\frac{R^2-R+6}{R-3};\ g(2), g(3), g\left(\frac{3}{8}\right)$

Sol: Given $g(R)=\frac{R^2-R+6}{R-3}$ At $R=2$

$g(2)=\frac{(2)^2-(2)+6}{(2)-3}$

$g(2)=\frac{4-2+6}{2-3}$

$g(2)=\frac{8}{-1}$

$g(2)=-8$

At $R=3$ $g(3)=\frac{(3)^2-(3)+6}{(3)-3}$

$g(3)=\frac{9-3+6}{3-3}$ **does not exists**

$g(3)=\frac{12}{0}$

At $R=\frac{3}{8}$ $g\left(\frac{3}{8}\right)=\frac{\left(\frac{3}{8}\right)^2-\left(\frac{3}{8}\right)+6}{\left(\frac{3}{8}\right)-3}$

$g\left(\frac{3}{8}\right)=\frac{\frac{9}{64}-\frac{3}{8}+6}{\frac{3}{8}-3}$

$g\left(\frac{3}{8}\right)=\frac{\frac{9}{64}-\frac{3}{8}\times\frac{8}{8}+\frac{6}{1}\times\frac{64}{64}}{\frac{3}{8}-\frac{3}{1}\times\frac{8}{8}}$

$g\left(\frac{3}{8}\right)=\frac{\frac{9}{64}-\frac{24}{64}+\frac{384}{64}}{\frac{3}{8}-\frac{24}{8}}$

$g\left(\frac{3}{8}\right)=\frac{\frac{9-24+384}{64}}{\frac{3-24}{8}}$

$g\left(\frac{3}{8}\right)=\frac{\frac{369}{64}}{\frac{-21}{8}}=\frac{369}{64}\div\frac{-21}{8}$

$g\left(\frac{3}{8}\right)=\frac{369}{64}\times\frac{8}{-21}$

$g\left(\frac{3}{8}\right)=\frac{123}{8\times-7}=-\frac{123}{56}$

$g\left(\frac{3}{8}\right)=-2.19642871$

d). $f(t)=3t^2+2t-\sqrt{t};\ f(3.217), f(5.613), f(\pi)$

Sol: Given $f(t)=3t^2+2t-\sqrt{t}$ At $t=3.217$

$f(3.217)=3(3.217)^2+2(3.217)-\sqrt{3.217}$

$f(3.217)=3(10.349)+6.434-1.794$

$f(3.217)=31.047+4.640$

$f(3.217)=35.687$

At $t=5.613$

$f(5.613)=3(5.613)^2+2(5.613)-\sqrt{5.613}$

$f(5.613)=3(31.506)+11.226-2.369$

$f(5.613)=94.518+8.857$

$f(5.613)=103.375$

At $t=\pi=3.142$

$f(\pi)=3(\pi)^2+2(\pi)-\sqrt{\pi}$

$f(\pi)=3(3.142)^2+2(3.142)-\sqrt{3.142}$

$f(\pi)=3(9.872)+6.284-1.773$

$f(\pi)=29.616+4.511$

$f(\pi)=34.127$

Q3. The circumference of a circle is given be $c(r)=2\pi r$ where r is the length of radius. Find

a). $C(2.34\quad inch)$

Sol: Given $c(r)=2\pi r$ At $r=2.34, \pi=3.14$

$c(2.34)=2(3.14)(2.34)$

$c(2.34)=14.70\ \ inch$

b). $C(6.41\quad inch)$

Sol: Given $c(r)=2\pi r$ At $r=6.41, \pi=3.14$

$c(6.41)=2(3.14)(6.41)$

$c(6.41)=40.25\ \ inch$

c). $C\left(\frac{5}{11}\quad inch\right)$

Sol: Given $c(r)=2\pi r$ At $r=\frac{5}{11}=0.45, \pi=3.14$

$c\left(\frac{5}{11}\right)=2(3.14)\left(\frac{5}{11}\right)$

$c\left(\frac{5}{11}\right)=2.85454\ \ inch$

Q4. The area of a circle is given by $A(r)=\pi r^2$ where r is the length of radius. Find

a). $A(2.34\quad inch)$

Sol: Given $A(r)=\pi r^2$ At $r=2.34, \pi=3.14$

$A(2.34)=(3.14)(2.34)^2$

$A(2.34)=(3.14)(5.48)$

$A(2.34)=17.20\ \ inch^2$

b). $A(6.41\quad inch)$

Sol: Given $A(r)=\pi r^2$ At $r=6.41, \pi=3.14$

$A(6.41)=(3.14)(6.41)^2$

$A(6.41)=129.02\ \ inch^2$

c). $A\left(\frac{5}{11}\quad inch\right)$

Sol: Given $A(r)=\pi r^2$ At $r=\frac{5}{11}=0.45, \pi=3.14$

$A\left(\frac{5}{11}\right)=(3.14)\left(\frac{5}{11}\right)^2$

$A\left(\frac{5}{11}\right)=(3.14)\left(\frac{25}{121}\right)$

$A\left(\frac{5}{11}\right)=(3.14)(0.21)$

$A\left(\frac{5}{11}\right)=0.66\ \ inch^2$

Q5.Total surface area of a cube is given by function $f(s)=6s^2$, where s is length of the side of cube.

a). $f(3.75\,m)$

Sol: Given $f(s)=6s^2$ At $s=3.75$

$$f(3.75)=6(3.75)^2$$
$$f(3.75)=6(14.06)$$
$$f(3.75)=84.36m^2$$

b). $f(6.05\,inch)$

Sol: Given $f(s)=6s^2$ At $s=6.05$

$$f(6.05)=6(6.05)^2$$
$$f(6.05)=6(36.60)$$
$$f(6.05)=219.60\ \ inch^2$$

c). $f(13.42\,mm)$

Sol: Given $f(s)=6s^2$ At $s=13.42$

$$f(13.42)=6(13.42)^2$$
$$f(13.42)=6(180.10)$$
$$f(13.42)=1080.60\ \ mm^2$$

Q6. Measure of angle θ in radian is given by $\theta=f(s,r)=\frac{s}{r}$ where s is length of the arc determined by θ & r is the length of radius of circle. Find

a). $f(4.71,\ 3)$

Sol: Given $\theta=f(s,r)=\frac{s}{r}$ At $s=4.71, r=3$

$$\theta=f(s,r)=\frac{4.71}{3}$$
$$\theta=f(s,r)=1.57\,Radians$$

b). $f(15.71,\ 5)$

Sol: Given $\theta=f(s,r)=\frac{s}{r}$ At $s=15.71, r=5$

$$\theta=f(s,r)=\frac{15.71}{5}$$
$$\theta=f(s,r)=3.142\,Radians$$

Q7. If $f(x)=\begin{cases}x-3, & for \quad x<0\\ 2x+5 & for \quad x\geq 0\end{cases}$, then find

a). $f(-1)$

Sol: Given $f(x)=\begin{cases}x-3, & for \quad x<0\\ 2x+5 & for \quad x\geq 0\end{cases}$

At $x=-1<0$ satisfied so take corresponding function

$$f(-1)=(-1)-3$$
$$f(-1)=-1-3$$
$$f(-1)=-4$$

b). $f(0)$

Sol: Given $f(x)=\begin{cases}x-3, & for \quad x<0\\ 2x+5 & for \quad x\geq 0\end{cases}$

At $x=0\geq 0$ satisfied so take corresponding function

$$f(0)=2(0)+5$$
$$f(0)=0+5$$
$$f(0)=5$$

c). $f(1)$

Sol: Given $f(x)=\begin{cases}x-3, & for \quad x<0\\ 2x+5 & for \quad x\geq 0\end{cases}$

At $x=1\geq 0$ satisfied so take corresponding function

$$f(1)=2(1)+5$$
$$f(1)=2+5$$
$$f(1)=7$$

Q7. If $f(x)=\begin{cases}\frac{x^2-4}{x-2}, & for \quad x\neq 2\\ 4 & for \quad x=2\end{cases}$, then find

a). $f(0)$

Sol: At $x=0\neq 2$ satisfied so take $f(x)=x^2-4$

$$f(0)=\frac{0-4}{0-2}$$
$$f(0)=\frac{-4}{-2}$$
$$f(0)=2$$

b). $f(2)$

Sol: Given $f(x)=\begin{cases}\frac{x^2-4}{x-2}, & for \quad x\neq 2\\ 4 & for \quad x=2\end{cases}$

At $x=2$ satisfied so take corresponding function

$$f(2)=4$$

c). $f(4)$

Sol: Given $f(x)=\begin{cases}\frac{x^2-4}{x-2}, & for \quad x\neq 2\\ 4 & for \quad x=2\end{cases}$

At $x=4\neq 2$ satisfied so take

$$f(4)=\frac{(4)^2-4}{(4)-2}$$
$$f(4)=\frac{16-4}{4-2}$$
$$f(4)=6$$

Q10. Indicate whether each table specifies a function

a).	Domain		Range
$y=f(x)$	3	→	0
	5	→	1
	7	→	2

Sol: Since each element in the Domain set corresponds to one and only one element in Range set so the given table forms a function (bijective function)

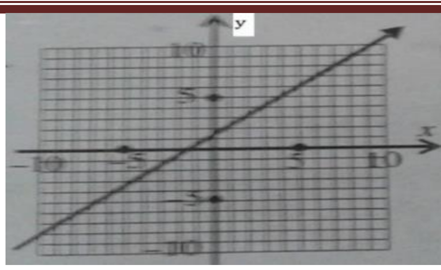
b).	Domain		Range
$y=f(x)$	- 1	→	5
	- 2	→	7
	- 3	→	9

Sol: we have each element in the Domain set corresponds to one and only one element in the Range set so the given table forms a function (bijective function)

c).	Domain		Range
$y=f(x)$	3	→ 5, → 6	5
			6
	4	→ 6, → 7	7
	5	→	8

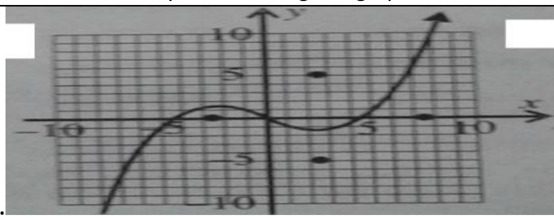
Sol: we have elements 3 and 4 in the Domain set corresponds to two elements 5,6 and 6,7 respectively in Range set so the given table does **not** forms a function

Q10. Indicate whether each graph specifies a function $y=f(x)$



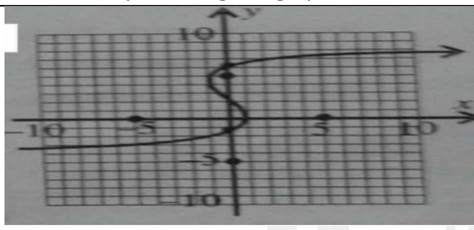
a)

sol: vertical line intersects a graph one point. So it is a function OR each element of the x-axis corresponds to one and only one element of the y-axis so the given graph forms a function



b).

sol: vertical line intersects a graph one point. So it is a function OR each element of the x-axis corresponds to one and only one element of y-axis so given graph forms a function



c).

sol: vertical line intersects a graph more than one point between $-1 \le x \le 1$. So It is a not function OR at $x = 0$ corresponds $y = -2$, $y = 2.5, y = 6$ which is not one to one so given graph does not forms a function

Q11.Determine domain and range of following:

a). $y = 3x + 4$

Sol: Given $y = 3x + 4$

Domain = R {set of real number}

For range separate x

$$y = 3x + 4$$
$$y - 4 = 3x$$
$$\frac{y-4}{3} = x$$

Therefore Range = R {set of real number}

b). $f(t) = t^2 + 5$

Sol: Given $f(t) = t^2 + 5$

Domain = R {set of real number}

For range Let $y = f(t)$ and separate t

$$y = t^2 + 5$$
$$y - 5 = t^2$$
$$t^2 = y - 5$$
$$\sqrt{t^2} = \pm\sqrt{y-5}$$
$$t = \pm\sqrt{y-5}$$
$$\Rightarrow y - 5 \ge 0$$
$$\Rightarrow y \ge 5$$

Therefore Range = $f(t) \ge 5$

c). $SA = f(r) = 4\pi r^2$

Sol: Given $SA = f(r) = 4\pi r^2$

Domain = R {set of real number}

For range separate r

$$SA = 4\pi r^2$$
$$\frac{SA}{4\pi} = r^2$$
$$r^2 = \frac{SA}{4\pi}$$
$$\sqrt{r^2} = \pm\sqrt{\frac{SA}{4\pi}}$$
$$r = \pm\sqrt{\frac{SA}{4\pi}}$$
$$\Rightarrow SA \ge 0$$

Therefore Range = $SA \ge 0$

Q12. Find the composite functions $g\left[f(x)\right]$ and $f\left[g(x)\right]$ of the following functions;

a). $f(x) = x^2 + 1, g(x) = 2x$

Sol: Given $f(x) = x^2 + 1, g(x) = 2x$

First we find $g\left[f(x)\right]$

$$g\left[f(x)\right] = g\left[x^2 + 1\right]$$
$$g\left[f(x)\right] = 2\left(x^2 + 1\right)$$
$$g\left[f(x)\right] = 2x^2 + 2$$

Now we find $f\left[g(x)\right] = f\left[2x\right]$

$$f\left[g(x)\right] = (2x)^2 + 1$$
$$f\left[g(x)\right] = 4x^2 + 1$$

b). $f(x) = \sin x \quad , g(x) = 1 - x^2$

Sol: Given $f(x) = \sin x \quad , g(x) = 1 - x^2$

$$g\left[f(x)\right] = g\left[\sin x\right]$$
$$g\left[f(x)\right] = 1 - (\sin x)^2$$
$$g\left[f(x)\right] = 1 - \sin^2 x$$
$$g\left[f(x)\right] = \cos^2 x$$

Now we find $f\left[g(x)\right] = f\left[1 - x^2\right]$

$$f\left[g(x)\right] = \sin\left(1 - x^2\right)$$

c). $f(t) = \sqrt{t} \quad , g(t) = t^2$

Sol: Given $f(t) = \sqrt{t} \quad , g(t) = t^2$

$$g\left[f(t)\right] = g\left[\sqrt{t}\right]$$
$$g\left[f(t)\right] = \left(\sqrt{t}\right)^2$$
$$g\left[f(t)\right] = t$$

Now we find $f\left[g(t)\right] = f\left[t^2\right]$

$$f\left[g(t)\right] = \sqrt{t^2}$$
$$f\left[g(t)\right] = t$$

d). $f(u) = \dfrac{u-1}{u+1} \quad , g(u) = \dfrac{u+1}{1-u}$

Sol: Given $f(u) = \dfrac{u-1}{u+1} \quad , g(u) = \dfrac{u+1}{1-u}$

$g[f(u)]=g\left[\frac{u-1}{u+1}\right]$

$g[f(u)]=\frac{\frac{u-1}{u+1}+1}{1-\frac{u-1}{u+1}}$

$g[f(u)]=\frac{\frac{u-1+u+1}{u+1}}{\frac{u+1-u+1}{u+1}}$

$g[f(u)]=\frac{2u}{2}$

$g[f(u)]=u$

Now we find $f[g(u)]=f\left[\frac{u+1}{1-u}\right]$

$f[g(u)]=\frac{\frac{u+1}{1-u}-1}{\frac{u+1}{1-u}+1}$

$f[g(u)]=\frac{\frac{u+1-1+u}{1-u}}{\frac{u+1+1-u}{1-u}}$

$f[g(u)]=\frac{2u}{2}$

$f[g(u)]=u$

e). $f(x)=\sin x \quad ,g(x)=2x+3$

Sol: Given $f(x)=\sin x \quad ,g(x)=2x+3$

First we find $g[f(x)]$

$g[f(x)]=g[\sin x]$

$g[f(x)]=2(\sin x)+3$

$g[f(x)]=2\sin x+3$

Now we find $f[g(x)]=f[2x+3]$

$f[g(x)]=\sin(2x+3)$

f). $f(x)=\frac{1}{x} \quad ,g(x)=\tan x$

Sol: Given $f(x)=\frac{1}{x} \quad ,g(x)=\tan x$

First we find $g[f(x)]=g\left[\frac{1}{x}\right]$

$g[f(x)]=\tan\left(\frac{1}{x}\right)$

Now we find $f[g(x)]=f[\tan x]$

$f[g(x)]=\frac{1}{\tan x}$

$f[g(x)]=\cot x$

Q13. Determine the inverse function of each of the following functions:

a). $y=f(x)=x+5$

Sol: Given $y=f(x)=x+5$

Take $y=x+5$ separate x $\quad \because y=f(x)$

$x=y-5 \quad \Rightarrow f^{-1}(y)=x$

Put $x=f^{-1}(y)$ then we get

$f^{-1}(y)=y-5$ put $y=x$

$\Rightarrow f^{-1}(x)=x-5$

b). $y=f(x)=2x+7$

Sol: Given $y=f(x)=2x+7$

Take $y=2x+7$ separate x $\quad \because y=f(x)$

$2x=y-7$

$x=\frac{y-7}{2} \quad \Rightarrow f^{-1}(y)=x$

Put $x=f^{-1}(y)$ then we get

$f^{-1}(y)=\frac{y-7}{2}$ put $y=x$

$\Rightarrow f^{-1}(x)=\frac{x-7}{2}$

c). $y=f(x)=2(x-4)$

Sol: Given $y=f(x)=2(x-4)$

Take $y=2(x-4)$ separate x $\quad \because y=f(x)$

$x-4=\frac{y}{2} \quad \Rightarrow f^{-1}(y)=x$

$x=\frac{y}{2}+4$

Put $x=f^{-1}(y)$ then we get

$f^{-1}(y)=\frac{y}{2}+4$ put $y=x$

$\Rightarrow f^{-1}(x)=\frac{x}{2}+4$

d). $y=f(x)=\frac{x+4}{2}$

Sol: Given $y=f(x)=\frac{x+4}{2}$

Take $y=\frac{x+4}{2}$ separate x $\quad \because y=f(x)$

$x+4=2y \quad \Rightarrow f^{-1}(y)=x$

$x=2y-4$

Put $x=f^{-1}(y)$ then we get

$f^{-1}(y)=2y-4$ put $y=x$

$\Rightarrow f^{-1}(x)=2x-4$

Q14.Graph each of following absolute functions:

a). $y=f(x)=|x-4|$

Sol: Given $y=f(x)=|x-4|$

$$f(x)=\begin{cases} x-4, & x-4\ge 0 \\ -(x-4), & x-4<0 \end{cases}$$

Or $$f(x)=\begin{cases} x-4, & x\ge 4 \\ -x+4, & x<4 \end{cases}$$

$x<4$ means $\quad x\ge 4$ means

$3,2,1,0,\ldots$ $\qquad$ $4,5,6,7,8,\ldots$

And the corresponding values for given function

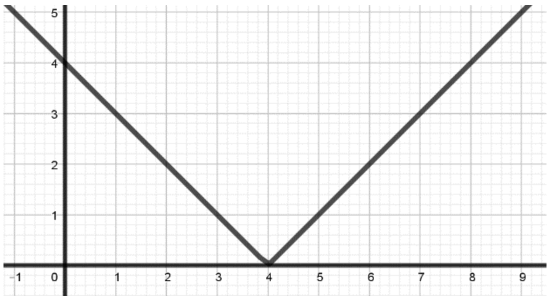
$f(0)=-(0)+4=4$ $\quad$ $f(8)=(8)-4=4$

$f(1)=-(1)+4=3$ $\quad$ $f(7)=(7)-4=3$

$f(2)=-(2)+4=2$ $\quad$ $f(6)=(6)-4=2$

$f(3)=-(3)+4=1$ $\quad$ $f(5)=(5)-4=1$

$f(4)=(4)-4=0$



b). $\quad y=f(x)=|-4-x|$

Sol: Given $y=f(x)=|-4-x|$

$$f(x)=\begin{cases}-4-x, & -4-x\geq 0\\ -(-4-x), & -4-x<0\end{cases}$$

Or $\quad f(x)=\begin{cases}-4-x, & -x\geq 4\\ 4+x, & -x<4\end{cases}$

Or $\quad f(x)=\begin{cases}-4-x, & x\leq -4\\ 4+x, & x>-4\end{cases}$

$x\leq -4$ means $\qquad$ $x>-4$ means

$-4,-5,-6,-7,-8,\ldots$ $\qquad$ $-3,-2,-1,0,\ldots$

And the corresponding values for given function

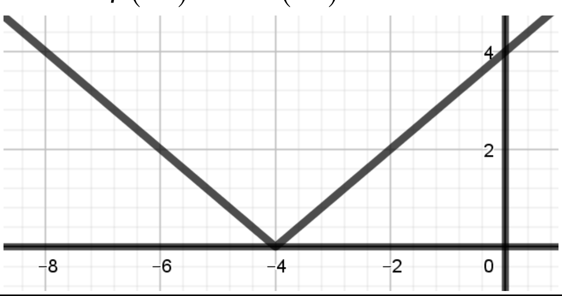
$f(-8)=-4-(-8)=4$ $\quad$ $f(0)=4+(0)=4$

$f(-7)=-4-(-7)=3$ $\quad$ $f(-1)=4+(-1)=3$

$f(-6)=-4-(-6)=2$ $\quad$ $f(-2)=4+(-2)=2$

$f(-5)=-4-(-5)=1$ $\quad$ $f(-3)=4+(-3)=1$

$f(-4)=-4-(-4)=0$



c). $\quad y=f(x)=|2x+5|$

Sol: Given $y=f(x)=|2x+5|$

$$f(x)=\begin{cases}2x+5, & 2x+5\geq 0\\ -(2x+5), & 2x+5<0\end{cases}$$

Or $\quad f(x)=\begin{cases}2x+5, & 2x\geq -5\\ -2x-5, & 2x<-5\end{cases}$

Or $\quad f(x)=\begin{cases}2x+5, & x\geq \frac{-5}{2}\\ -2x-5, & x<\frac{-5}{2}\end{cases}$

$x\geq \frac{-5}{2}$ means $\qquad$ $x<\frac{-5}{2}$ means

$\frac{-5}{2},-2,-1,0,1,\ldots$ $\qquad$ $-3,-4,-5,-6,\ldots$

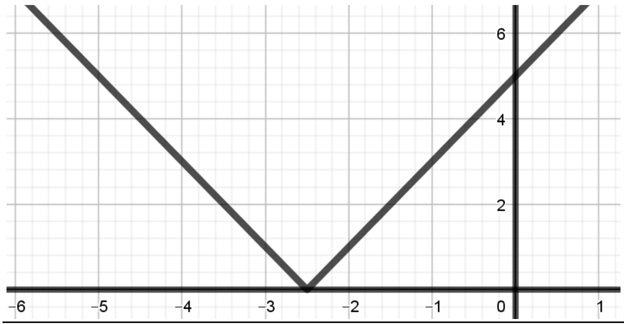
And the corresponding values for given function

$f(0)=2(0)+5=5$ $\quad$ $f(-5)=-2(-5)-5=5$

$f(-1)=2(-1)+5=3$ $\quad$ $f(-4)=-2(-4)-5=3$

$f(-2)=2(-2)+5=1$ $\quad$ $f(-3)=-2(-3)-5=1$

$f\left(\frac{-5}{2}\right)=2\left(\frac{-5}{2}\right)+5=0$



d). $\quad y=f(x)=-|x|$

Sol: Given $y=f(x)=-|x|$

$$f(x)=-\begin{cases}x, & x\geq 0\\ -x, & x<0\end{cases}$$

Or $\quad f(x)=\begin{cases}-x, & x\geq 0\\ x, & x<0\end{cases}$

$x\geq 0$ means $\qquad$ $x<0$ means

$0,1,2,3,4,\ldots$ $\qquad$ $-1,-2,-3,-4,\ldots$

And the corresponding values for given function

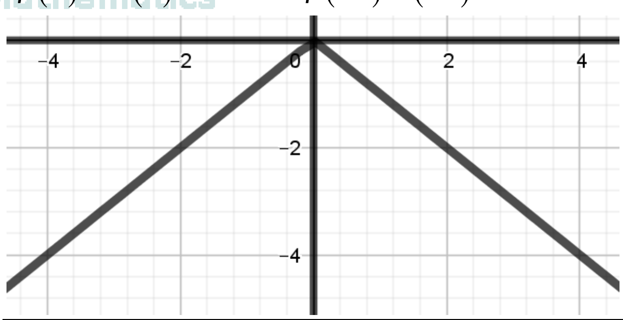
$f(0)=-(0)=0$

$f(1)=-(1)=-1$ $\quad$ $f(-1)=(-1)=-1$

$f(2)=-(2)=-2$ $\quad$ $f(-2)=(-2)=-2$

$f(3)=-(3)=-3$ $\quad$ $f(-3)=(-3)=-3$

$f(4)=-(4)=-4$ $\quad$ $f(-4)=(-4)=-4$



Some rules of powers

1. $x^m\times x^n=x^{m+n}$
2. $(x.y)^n=x^n.y^n$
3. $a^{\frac{m}{n}}=\left(a^m\right)^{\frac{1}{n}}=\sqrt[n]{a^m}$ $\qquad$ or

   $a^{\frac{m}{n}}=\left(a^{\frac{1}{n}}\right)^m=\left(\sqrt[n]{a}\right)^m$
4. $\dfrac{x^m}{x^n}=x^m.x^{-n}=x^{m-n}$ where $m>n$
5. $\left(x^n\right)^m=x^{n\times m}$
6. $a^n=a^m \quad\Rightarrow n=m$
7. $a^n=b^n \quad\Rightarrow a=b$

Logarithm:

$a^y = x, \Rightarrow y = \log_a x.$

Laws of Logarithm:

First Law: $\log_a mn = \log_a m + \log_a n$

Second Law: $\log_a \frac{m}{n} = \log_a m - \log_a n$

Third Law: $\log_a (m)^n = n.\log_a m$

Fourth Law: $\log_b m = \frac{\log_a m}{\log_a b}$

Exercise 1.2

Q1.Simplify the following functions

a). $(4^{3x})^{2y}$

Sol: Given $(4^{3x})^{2y}$

$(4^{3x})^{2y} = 4^{3x\times 2y}$

$(4^{3x})^{2y} = 4^{6xy}$

b). $10^{3x-1}10^{4-x}$

Sol: Given

$10^{3x-1}10^{4-x} = 10^{3x-1+4-x}$

$10^{3x-1}10^{4-x} = 10^{3x-x+3}$

$10^{3x-1}10^{4-x} = 10^{2x+3}$

c). $\frac{e^{x-3}}{e^{x-4}}$

Sol: Given $\frac{e^{x-3}}{e^{x-4}}$

$\frac{e^{x-3}}{e^{x-4}} = e^{x-3-x+4}$

$\frac{e^{x-3}}{e^{x-4}} = e^{x-x-3+4}$

$\frac{e^{x-3}}{e^{x-4}} = e^{1}$

d). $\frac{e^{x}}{e^{3-x}}$

Sol: Given $\frac{e^{x}}{e^{3-x}}$

$\frac{e^{x}}{e^{3-x}} = e^{x-3+x}$

$\frac{e^{x}}{e^{3-x}} = e^{x+x-3}$

$\frac{e^{x}}{e^{3-x}} = e^{2x-3}$

e). $(2e^{1.2t})^3$

Sol: Given $(2e^{1.2t})^3$

$(2e^{1.2t})^3 = 2^3 e^{1.2t\times 3}$

$(2e^{1.2t})^3 = 8e^{3.6t}$

f). $(3e^{-1.4x})^2$

Sol: Given $(3e^{-1.4x})^2$

$(3e^{-1.4x})^2 = 3^2 e^{-1.4x\times 2}$

$(3e^{-1.4x})^2 = 9e^{-2.8x}$

Q2. Solve the following equations

a). $10^{2-3x} = 10^{5x-6}$

Sol: Given $10^{2-3x} = 10^{5x-6}$

$\Rightarrow 2-3x = 5x-6$

$2+6 = 5x+3x$

$8 = 8x$

or $x = 1$

Solution Set $= \{1\}$

b). $5^{3x} = 5^{4x-2}$

Sol: Given $5^{3x} = 5^{4x-2}$

$3x = 4x-2$

$+2 = 4x-3x$

$2 = x$ *or* $x = 2$

Solution Set $= \{2\}$

c). $4^{5x-x^2} = 4^{-6}$

Sol: Given $4^{5x-x^2} = 4^{-6}$

$5x-x^2 = -6$

$0 = x^2-5x-6$

$x^2-5x-6 = 0$

$x^2-6x+1x-6 = 0$

$x(x-6)+1(x-6) = 0$

$(x-6)(x+1) = 0$

$\therefore x-6 = 0$ *or* $x+1 = 0$

$x = 6$ $x = -1$

Solution Set $= \{-1, 6\}$

d). $7^{x^2} = 7^{2x+3}$

Sol: Given $7^{x^2} = 7^{2x+3}$

$x^2 = 2x+3$

$x^2-2x-3 = 0$

$x^2-3x+1x-3 = 0$

$x(x-3)+1(x-3) = 0$

$(x+1)(x-3) = 0$

$\therefore x+1 = 0$ *or* $x-3 = 0$

$x = -1$ $x = 3$

Solution Set $= \{-1, 3\}$

e). $5^3 = (x+2)^3$

Sol: Given $5^3 = (x+2)^3$

$5 = x+2$

$5-2 = x$

$3 = x$ *Or* $x = 3$

Solution Set $=\{3\}$

f). $(1-x)^5=(2x-1)^5$

Sol: Given $(1-x)^5=(2x-1)^5$

$1-x=2x-1$

$1+1=2x+x$

$2=3x$

$or\ \ x=\frac{2}{3}$

Solution Set $=\left\{\frac{2}{3}\right\}$

g). $(x-3)e^x=0$

Sol: Given $(x-3)e^x=0$

$\therefore x-3=0 \qquad \because e^x\neq 0$

$x=3$

Solution Set $=\{3\}$

h). $2xe^{-x}=0$

Sol: Given $2xe^{-x}=0$

$\because 2\neq 0, \qquad x=0 \qquad e^{-x}\neq 0$

$\Rightarrow x=0$

Solution Set $=\{0\}$

i). $3xe^{-x}+x^2e^{-x}=0$

Sol: Given $3xe^{-x}+x^2e^{-x}=0$

$xe^{-x}(3+x)=0$

$\therefore x=0 \quad e^{-x}\neq 0 \quad 3+x=0$

$x=0 \qquad\qquad x=-3$

Solution Set $=\{-3,0\}$

j). $x^2e^x-5xe^x=0$

Sol: Given $x^2e^x-5xe^x=0$

$xe^x(x-5)=0$

$\therefore x=0 \quad e^x\neq 0 \quad x-5=0$

$x=0 \qquad\qquad x=5$

Solution Set $=\{0,5\}$

Q3. Rewrite in equivalent exponential form, the following logarithmic functions

a). $\log_3 27=3$

Sol: Given $\log_3 27=3$

exponential form $3^3=27$

b). $\log_2 32=5$

Sol: Given $\log_2 32=5$

exponential form $2^5=32$

c). $\log_{10} 1=0$

Sol: Given $\log_{10} 1=0$

exponential form $10^0=1$

d). $\log_e 1=0$

Sol: Given $\log_e 1=0$

exponential form $e^0=1$

e). $\log_4 8=\frac{3}{2}$

Sol: Given $\log_4 8=\frac{3}{2}$

exponential form $4^{\frac{3}{2}}=8$

f). $\log_9 27=\frac{3}{2}$

Sol: Given $\log_9 27=\frac{3}{2}$

exponential form $9^{\frac{3}{2}}=27$

Q4. Rewrite in equivalent logarithmic form, the following exponential functions

a). $49=7^2$

Sol: Given $49=7^2$

logarithmic form $\log_7 49=2$

b). $36=6^2$

Sol: Given $36=6^2$

logarithmic form $\log_6 36=2$

c). $8=4^{\frac{3}{2}}$

Sol: Given $8=4^{\frac{3}{2}}$

logarithmic form $\log_4 8=\frac{3}{2}$

d). $9=27^{\frac{2}{3}}$

Sol: Given $9=27^{\frac{2}{3}}$

logarithmic form $\log_{27} 9=\frac{2}{3}$

e). $A=b^u$

Sol: Given $A=b^u$

logarithmic form $\log_b A=u$

f). $M=b^x$

Sol: Given $M=b^x$

logarithmic form $\log_b M=x$

Q6. Find x,y and b without a scientific calculator use

a). $\log_3 x=2$

Sol: Given $\log_3 x=2$

exponential form $x=3^2$

$x=9$

b). $\log_2 x=2$

Sol: Given $\log_2 x=2$

exponential form $x=2^2$

$x=4$

c). $\log_7 49=y$

Sol: Given $\log_7 49=y$

exponential form $7^y=49$

$7^y=7^2$

$\Rightarrow y=2$

d). $\log_b 10^{-4} = -4$

Sol: Given $\log_b 10^{-4} = -4$

exponential form $b^{-4} = 10^{-4}$

$b = 10$

e). $\log_{\frac{1}{3}} 9 = y$

Sol: Given $\log_{\frac{1}{3}} 9 = y$

exponential form $\left(\frac{1}{3}\right)^y = 9$

$3^{-y} = 3^2$

$\Rightarrow -y = 2$

$\Rightarrow y = -2$

f). $\log_b 1000 = \frac{3}{2}$

Sol: Given $\log_b 1000 = \frac{3}{2}$

exponential form $b^{\frac{3}{2}} = 1000$

$b^{\frac{3}{2}} = \left(10^2\right)^{\frac{3}{2}}$

$b^{\frac{3}{2}} = (100)^{\frac{3}{2}}$

$\Rightarrow b = 100$

Q6. Solve following equations for unknown x

a). $\log_b x = \frac{2}{3}\log_b 8 + \frac{1}{2}\log_b 9 - \log_b 6$

sol: $\log_b x = \log_b 8 + \frac{1}{2}\log_b 9 - \log_b 6$

$\log_b x = \log_b 8^{\frac{2}{3}} + \log_b 9^{\frac{1}{2}} - \log_b 6$

$\log_b x = \log_b \left(2^3\right)^{\frac{2}{3}} + \log_b \left(3^2\right)^{\frac{1}{2}} - \log_b 6$

$\log_b x = \log_b 4 + \log_b 3 - \log_b 6$

$\log_b x = \log_b \frac{4\times 3}{6}$

$\log_b x = \log_b \frac{12}{6}$

$\log_b x = \log_b 2$

$\Rightarrow x = 2$

b). $\log_b x = \frac{2}{3}\log_b 27 + 2\log_b 2 - \log_b 3$

Sol: Given $\log_b x = \frac{2}{3}\log_b 27 + 2\log_b 2 - \log_b 3$

$\log_b x = \log_b 27^{\frac{2}{3}} + \log_b 2^2 - \log_b 3$

$\log_b x = \log_b \left(3^3\right)^{\frac{2}{3}} + \log_b 4 - \log_b 3$

$\log_b x = \log_b 3^2 + \log_b 4 - \log_b 3$

$\log_b x = \log_b 9 + \log_b 4 - \log_b 3$

$\log_b x = \log_b \frac{9\times 4}{3}$

$\log_b x = \log_b 3\times 4$

$\log_b x = \log_b 12$

$\Rightarrow x = 12$

c). $\log_b x = \frac{3}{2}\log_b 4 - \frac{2}{3}\log_b 8 + 2\log_b 2$

Sol: Given $\log_b x = \frac{3}{2}\log_b 4 - \frac{2}{3}\log_b 8 + 2\log_b 2$

$\log_b x = \log_b 4^{\frac{3}{2}} - \log_b 8^{\frac{2}{3}} + \log_b 2^2$

$\log_b x = \log_b \left(2^2\right)^{\frac{3}{2}} - \log_b \left(2^3\right)^{\frac{2}{3}} + \log_b 4$

$\log_b x = \log_b 2^3 - \log_b 2^2 + \log_b 4$

$\log_b x = \log_b 8 - \log_b 4 + \log_b 4$

$\log_b x = \log_b 8$

$\Rightarrow \quad x = 8$

d). $\log_b x + \log_b (x-4) = \log_b 21$

Sol: Given $\log_b x + \log_b (x-4) = \log_b 21$

$\log_b x(x-4) = \log_b 21$

$\Rightarrow \quad x(x-4) = 21$

$x^2 - 4x - 21 = 0$

$x^2 - 7x + 3x - 21 = 0$

$x(x-7) + 3(x-7) = 0$

$(x+3)(x-7) = 0$

$\therefore x+3 = 0 \qquad x-7 = 0$

$x = -3 \qquad x = 7$

e). $\log_{10}(x-1) - \log_{10}(x+1) = 1$

Sol: Given $\log_{10}(x-1) - \log_{10}(x+1) = 1$

$\log_{10}\frac{(x-1)}{(x+1)} = \log_{10} 10 \qquad \because \log_{10} 10 = 1$

$\frac{x-1}{x+1} = 10$

$x - 1 = 10(x+1)$

$x - 1 = 10x + 10$

$-10 - 1 = 10x - x$

$-11 = 9x$

$x = \frac{-11}{9}$

f). $\log_{10}(x+6) - \log_{10}(x-3) = 1$

Sol: Given $\log_{10}(x+6) - \log_{10}(x-3) = 1$

$\log_{10}\frac{(x+6)}{(x-3)} = \log_{10} 10 \qquad \because \log_{10} 10 = 1$

$\frac{x+6}{x-3} = 10$

$x + 6 = 10(x-3)$

$x + 6 = 10x - 30$

$+30 + 6 = 10x - x$

$36 = 9x$

$\frac{36}{9} = x$

$4 = x \qquad or \quad x = 4$

Q7. Suppose the sale of a certain product are approximated by $S(t) = 125 + 83\log(5t+1)$

where $S(t)$ is sale in thousands of dollars t years after the product introduced on the market. Find

a). $S(0)$

solution: $S(t)=125+83\log(5t+1)$ At $t=0$

$S(0)=125+83\log(5(0)+1)$

$S(0)=125+83\log(1)$ $\because \log(1)=0$

$S(0)=125+83(0)$

$S(0)=125+0$

$S(0)=125\ thousand\ dollars$

$S(0)=\$\ 125000$

b). $S(2)$

Sol: Given $S(t)=125+83\log(5t+1)$

At $t=2$ $S(2)=125+83\log(5(2)+1)$

$S(2)=125+83\log(10+1)$

$S(2)=125+83\log(11)$

$S(2)=125+83(1.0414)$

$S(2)=125+86.4356$

$S(2)=211.4356\ thousand\ dollars$

$S(2)=\$\ 211435.6$

c). $S(4)$

Sol: Given $S(t)=125+83\log(5t+1)$

At $t=4$ $S(4)=125+83\log(5(4)+1)$

$S(4)=125+83\log(20+1)$

$S(4)=125+83\log(21)$

$S(4)=125+83(1.3222)$

$S(4)=125+109.7442$

$S(4)=234.7442\ thousand\ dollars$

$S(4)=\$\ 234744.2$

d). $S(31)$

Sol: Given $S(t)=125+83\log(5t+1)$

At $t=31$ $S(31)=125+83\log(5(31)+1)$

$S(31)=125+83\log(155+1)$

$S(31)=125+83\log(156)$

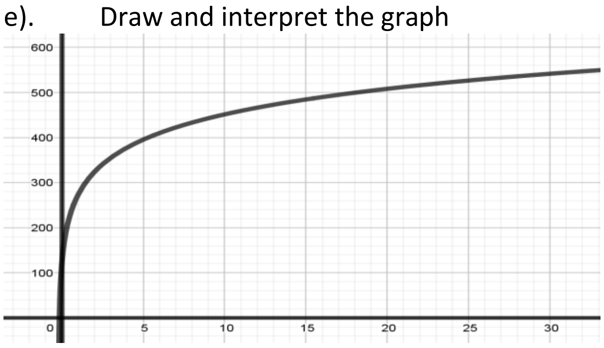
$S(31)=125+83(2.1931)$

$S(31)=125+182.0293$

$S(31)=307.0293\ thousand\ dollars$

$S(31)=\$307029.3$

e). Draw and interpret the graph



Exercise 1.3

Q1. Using calculator and point by point to plot the following logarithmic functions

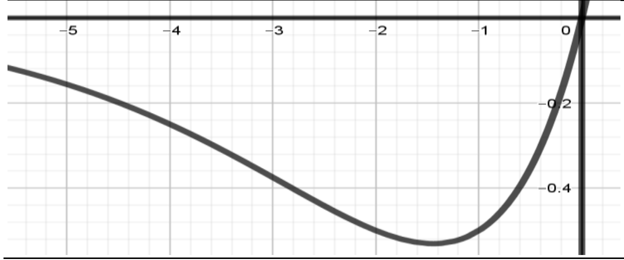
a). $h(x)=x(2^x);$ $[-5,0]$

Sol: Given $h(x)=x(2^x);$

At $x=-5$ $h(-5)=(-5).2^{-5}=-0.15625$

Similarly we can find

x	-5	-4	-3	-2	-1	0
2^x	0.03125	0.0625	0.125	0.25	0.5	1
$h=x\times 2^x$	-0.15625	-0.25	-0.375	-0.5	-.5	0



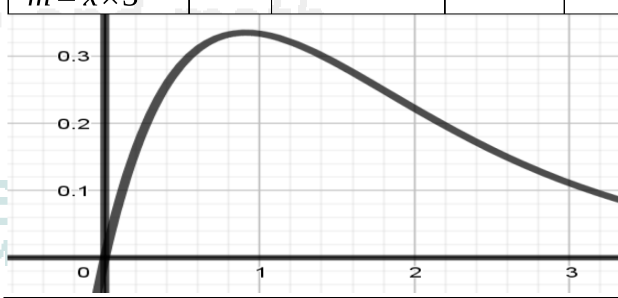
b). $m(x)=x(3^{-x});$ $[0,3]$

Sol: Given $m(x)=x(3^{-x});$

At $x=0$ $m(0)=(0)(3^{-0})=0$

Similarly we can find

x	0	1	2	3
3^{-x}	1	3	1/9	1/27
$m=x\times 3^{-x}$	0	0.333	0.333	0.111



c). $N(t)=\dfrac{100}{1+e^{-t}};$ $[0,5]$

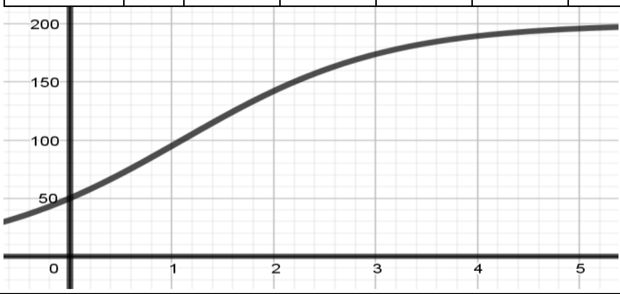
Sol: Given $N(t)=\dfrac{100}{1+e^{-t}};$

t	0	1	2	3	4	5
e^{-t}	1	0.367	0.135	0.049	0.018	0.006
$1+e^{-t}$	2	1.367	1.135	1.049	1.018	1.006
$\dfrac{100}{1+e^{-t}}$	50	73.15	88.11	95.33	98.23	99.4

d). $N(t)=\dfrac{200}{1+3e^{-t}};$ $[0,5]$

Sol: Given $N(t)=\dfrac{200}{1+3e^{-t}};$

t	0	1	2	3	4	5
e^{-t}	1	0.37	0.14	0.049	0.018	0.006
$1+3e^{-t}$	2	2.10	1.41	1.15	1.054	1.02
$\dfrac{200}{1+3e^{-t}}$	50	95.24	141.8	173.9	189.8	196.1

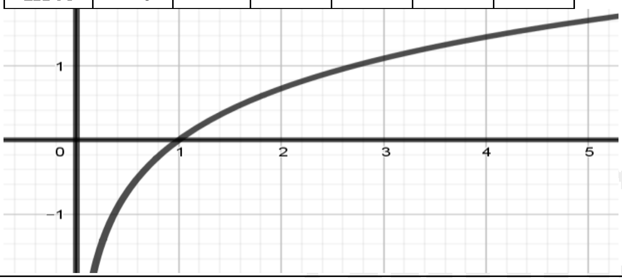


Q2. Using calculator and point by point to plot the following logarithmic functions

a). $y=\ln x$

Sol: Given

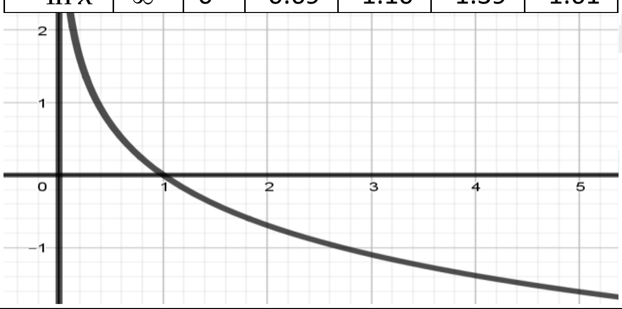
x	0	1	2	3	4	5
$\ln x$	$-\infty$	0	0.69	1.10	1.39	1.61



b). $u=-\ln x$

Sol: Given

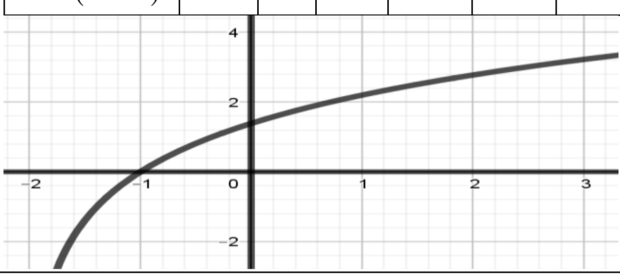
x	0	1	2	3	4	5
$-\ln x$	∞	0	-0.69	-1.10	-1.39	-1.61



c). $y=2\ln(x+2)$

Sol: Given

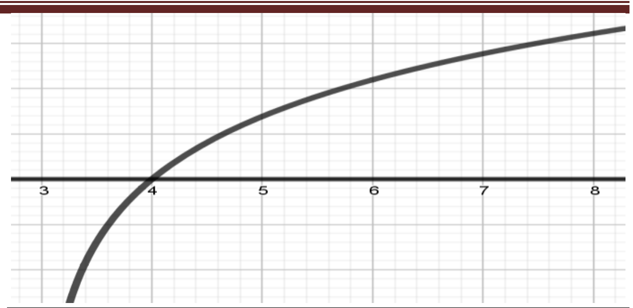
x	-2	-1	0	1	2	3
$\ln(x+2)$	$-\infty$	0	0.69	1.10	1.39	1.61
$2\ln(x+2)$	$-\infty$	0	1.38	2.20	2.78	3.22



d). $y=4\ln(x-3)$

Sol: Given

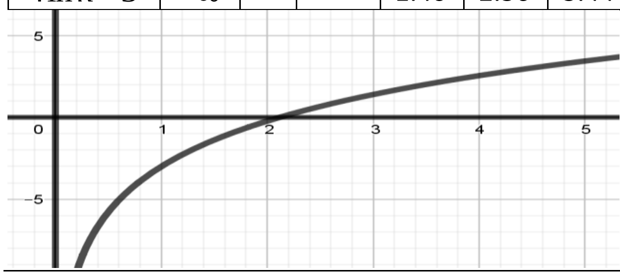
x	3	4	5	6	7	8
$\ln(x-3)$	$-\infty$	0	0.69	1.10	1.39	1.61
$4\ln(x-3)$	$-\infty$	0	2.76	4.40	5.56	6.44



e). $y=4\ln x-3$

Sol: Given

x	0	1	2	3	4	5
$\ln x$	$-\infty$	0	0.69	1.10	1.39	1.61
$4\ln x$	$-\infty$	0	2.76	4.40	5.56	6.44
$4\ln x-3$	$-\infty$	-3	-0.24	1.40	2.56	3.44



Q3. Sketch the following parametric curves:

a). $(x(t),y(t))=(3-t,2t)$

Sol: Given $(x(t),y(t))=(3-t,2t)$

At $t=0$ $\quad (x(0),y(0))=(3-(0),2(0))$

$(x(0),y(0))=(3,0)$

At $t=1$ $\quad (x(1),y(1))=(3-(1),2(1))$

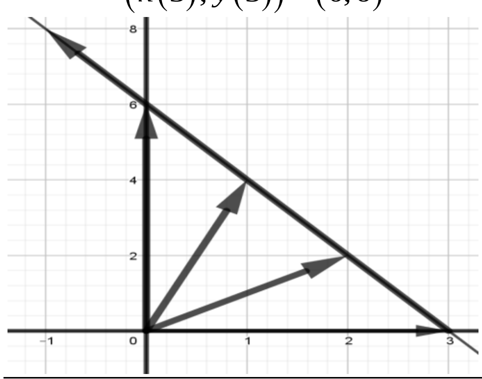
$(x(1),y(1))=(2,2)$

At $t=2$ $\quad (x(2),y(2))=(3-(2),2(2))$

$(x(2),y(2))=(1,4)$

At $t=3$ $\quad (x(3),y(3))=(3-(3),2(3))$

$(x(3),y(3))=(0,6)$



b). $(x(t),y(t))=(4\cos t,3\sin t)$

Sol: Given $(x(t),y(t))=(4\cos t,3\sin t)$

we can write

$x=4\cos t \qquad\qquad y=3\sin t$

$\dfrac{x}{4}=\cos t \qquad\qquad \dfrac{y}{3}=\sin t$

Squaring both sides we get

$\left(\dfrac{x}{4}\right)^2=(\cos t)^2 \qquad\qquad \left(\dfrac{y}{3}\right)^2=(\sin t)^2$

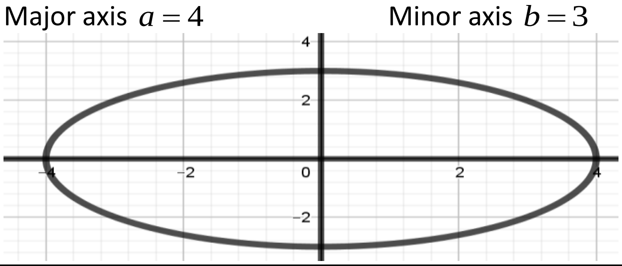
$\dfrac{x^2}{16}=\cos^2 t \qquad\qquad \dfrac{y^2}{9}=\sin^2 t$

Adding $\dfrac{x^2}{16}+\dfrac{y^2}{9}=\cos^2 t+\sin^2 t$

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

Which is equation of ellipse with

Major axis $a=4$ Minor axis $b=3$



c). $\left(x(t),y(t),z(t)\right)=\left(3+2t,5-3t,2-4t\right)$

sol: $\left(x(t),y(t),z(t)\right)=\left(3+2t,5-3t,2-4t\right)$

At $t=0$

$$\left(x(0),y(0),z(0)\right)=\left(3+2(0),5-3(0),2-4(0)\right)$$

$$\left(x(0),y(0),z(0)\right)=\left(3,5,2\right)$$

At $t=1$

$$\left(x(1),y(1),z(1)\right)=\left(3+2(1),5-3(1),2-4(1)\right)$$

$$\left(x(1),y(1),z(1)\right)=\left(3+2,5-3,2-4\right)$$

$$\left(x(1),y(1),z(1)\right)=\left(5,2,-2\right)$$

At $t=2$

$$\left(x(2),y(2),z(2)\right)=\left(3+2(2),5-3(2),2-4(2)\right)$$

$$\left(x(2),y(2),z(2)\right)=\left(3+4,5-6,2-8\right)$$

$$\left(x(2),y(2),z(2)\right)=\left(7,-1,-6\right)$$

At $t=3$

$$\left(x(3),y(3),z(3)\right)=\left(3+2(3),5-3(3),2-4(3)\right)$$

$$\left(x(3),y(3),z(3)\right)=\left(3+6,5-9,2-12\right)$$

$$\left(x(3),y(3),z(3)\right)=\left(9,-4,-10\right)$$

We cannot plot 3d graph on a 2d plane

Indeterminate forms

$$\frac{0}{0},\quad \frac{\infty}{\infty},\quad 0\times\infty,\quad \infty\pm\infty,\ 0^0,\quad 1^\infty\quad and\quad \infty^0$$

L'Hôpital's rule

For evaluating the indeterminate forms $\dfrac{0}{0}$ and $\dfrac{\infty}{\infty}$

$$\lim_{x\to\infty}\frac{f(x)}{g(x)}=\lim_{x\to\infty}\frac{f'(x)}{g'(x)}\ \text{or}\ \lim_{x\to0}\frac{f(x)}{g(x)}=\lim_{x\to0}\frac{f'(x)}{g'(x)}$$

Note that this rule does *not* apply to expressions $\frac{\infty}{0},\frac{1}{0}$ and so on; these expressions are not indeterminate forms

Steps for solution of $\dfrac{0}{0},\quad\dfrac{\infty}{\infty}$

1). Factorize of numerator and denominator

2). L Hopital's Rule

3). Rationalize numerator or denominator

4). Taking Common

Properties of continuous function

1). Polynomial function is always continuous function

2). Rational function is continuous function excepts those values where denominator will be zero

3). Radical function i.e., square root function is continuous only to get positive values

4). When graph is given then make sure that there is no gap/Jump

Exercise 1.4

Q1. Evaluate the following limits;

a). $\lim\limits_{x\to-2}\left(x^2+3x-7\right)$

Sol: Given $\lim\limits_{x\to-2}\left(x^2+3x-7\right)$

$$\lim_{x\to-2}\left(x^2+3x-7\right)=(-2)^2+3(-2)-7$$

$$\lim_{x\to-2}\left(x^2+3x-7\right)=4-6-7$$

$$\lim_{x\to-2}\left(x^2+3x-7\right)=4-13$$

$$\lim_{x\to-2}\left(x^2+3x-7\right)=-9$$

b). $\lim\limits_{x\to3}(x+5)(2x-7)$

Sol: Given $\lim\limits_{x\to3}(x+5)(2x-7)=(3+5)(2(3)-7)$

$$\lim_{x\to3}(x+5)(2x-7)=(8)(6-7)$$

$$\lim_{x\to3}(x+5)(2x-7)=(8)(-1)$$

$$\lim_{x\to3}(x+5)(2x-7)=-8$$

c). $\lim\limits_{z\to1}\dfrac{z^2+z-3}{z+1}$

Sol: Given $\lim\limits_{z\to1}\dfrac{z^2+z-3}{z+1}=\dfrac{1^2+1-3}{1+1}$

$$\lim_{z\to1}\frac{z^2+z-3}{z+1}=\frac{1-2}{2}$$

$$\lim_{z\to1}\frac{z^2+z-3}{z+1}=\frac{-1}{2}$$

d). $\lim\limits_{x\to4}\left(\dfrac{1}{x}+\dfrac{3}{x-5}\right)$

Sol: Given $\lim\limits_{x\to4}\left(\dfrac{1}{x}+\dfrac{3}{x-5}\right)=\dfrac{1}{4}+\dfrac{3}{4-5}$

$$\lim_{x\to4}\left(\frac{1}{x}+\frac{3}{x-5}\right)=\frac{1}{4}+\frac{3}{-1}$$

$$\lim_{x\to4}\left(\frac{1}{x}+\frac{3}{x-5}\right)=\frac{1}{4}-\frac{3}{1}\times\frac{4}{4}$$

$$\lim_{x\to4}\left(\frac{1}{x}+\frac{3}{x-5}\right)=\frac{1}{4}-\frac{12}{4}$$

$$\lim_{x\to4}\left(\frac{1}{x}+\frac{3}{x-5}\right)=\frac{-11}{4}$$

e). $\lim\limits_{x\to1}\left(\dfrac{x^2+3x+2}{x^2+x+2}\right)^2$

sol: Given $\lim\limits_{x\to1}\left(\dfrac{x^2+3x+2}{x^2+x+2}\right)^2=\left(\dfrac{(1)^2+3(1)+2}{(1)^2+(1)+2}\right)^2$

$$\lim_{x\to1}\left(\frac{x^2+3x+2}{x^2+x+2}\right)^2=\left(\frac{1+3+2}{1+1+2}\right)^2$$

$$\lim_{x\to1}\left(\frac{x^2+3x+2}{x^2+x+2}\right)^2=\left(\frac{6}{4}\right)^2$$

$$\lim_{x\to 1}\left(\frac{x^2+3x+2}{x^2+x+2}\right)^2=\left(\frac{3}{2}\right)^2$$

$$\lim_{x\to 1}\left(\frac{x^2+3x+2}{x^2+x+2}\right)^2=\frac{9}{4}$$

f). $\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}$

Sol: Given **Indeterminate** form $\frac{0}{0}$

Rationalize the numerator

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}\times\frac{\sqrt{x}+1}{\sqrt{x}+1}$$

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to 1}\frac{\left(\sqrt{x}\right)^2-(1)^2}{(x-1)\left(\sqrt{x}+1\right)}$$

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to 1}\frac{x-1}{(x-1)\left(\sqrt{x}+1\right)}$$

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to 1}\frac{1}{\sqrt{x}+1}$$

Now putting the limit

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\frac{1}{\sqrt{1}+1}$$

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\frac{1}{1+1}$$

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\frac{1}{2}$$

g). $\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}$

Sol: Given **Indeterminate** form $\frac{0}{0}$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\lim_{x\to 0}\frac{1}{x}\left(\frac{1}{x+3}-\frac{1}{3}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\lim_{x\to 0}\frac{1}{x}\left(\frac{3-(x+3)}{3(x+3)}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\lim_{x\to 0}\frac{1}{x}\left(\frac{3-x-3}{3(x+3)}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\lim_{x\to 0}\frac{1}{x}\left(\frac{-x}{3(x+3)}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\lim_{x\to 0}\left(\frac{-1}{3(x+3)}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\left(\frac{-1}{3(0+3)}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\frac{-1}{9}$$

h). $\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}$

Sol: Given **Indeterminate** form $\frac{0}{0}$

$$\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}=\lim_{x\to 1}\frac{1}{x-1}\left(\frac{1}{x}-1\right)$$

$$\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}=\lim_{x\to 1}\frac{1}{x-1}\left(\frac{1-x}{x}\right)$$

$$\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}=\lim_{x\to 1}\frac{1}{x-1}\left(\frac{-(x-1)}{x}\right)$$

$$\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}=\lim_{x\to 1}\frac{-1}{x}$$

$$\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}=\frac{-1}{1}$$

$$\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}=-1$$

i). $\lim_{x\to 0}\frac{1-\sin x}{\cos^2 x}$

Sol: Given

$$\lim_{x\to 0}\frac{1-\sin x}{\cos^2 x}=\frac{1-\sin(0)}{(\cos(0))^2}$$

$$\lim_{x\to 0}\frac{1-\sin x}{\cos^2 x}=\frac{1-0}{(1)^2}$$

$$\lim_{x\to 0}\frac{1-\sin x}{\cos^2 x}=\frac{1}{1}$$

$$\lim_{x\to 0}\frac{1-\sin x}{\cos^2 x}=1$$

j). $\lim_{x\to 0}\frac{\tan x}{x}$

Sol: Given **Indeterminate** form $\frac{0}{0}$

$$\lim_{x\to 0}\frac{\tan x}{x}=\lim_{x\to 0}\frac{\sin x}{x}.\frac{1}{\cos x}$$

$$\lim_{x\to 0}\frac{\tan x}{x}=\lim_{x\to 0}\frac{\sin x}{x}.\lim_{x\to 0}\frac{1}{\cos x}$$

$$\lim_{x\to 0}\frac{\tan x}{x}=1\;.\lim_{x\to 0}\frac{1}{\cos(0)}$$

$$\lim_{x\to 0}\frac{\tan x}{x}=\frac{1}{1}$$

$$\lim_{x\to 0}\frac{\tan x}{x}=1$$

k). $\lim_{x\to 0}\frac{\sec x-1}{x\sec x}$

Sol: Given **Indeterminate** form $\frac{0}{0}$

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=\lim_{x\to 0}\frac{\frac{1}{\cos x}-1}{\frac{x}{\cos x}}$$

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=\lim_{x\to 0}\frac{\frac{1-\cos x}{\cos x}}{\frac{x}{\cos x}}$$

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=\lim_{x\to 0}\frac{1-\cos x}{x}$$

Again **Indeterminate** form $\frac{0}{0}$

Rationalize the numerator

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=\lim_{x\to 0}\frac{1-\cos x}{x}\times\frac{1+\cos x}{1+\cos x}$$

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=\lim_{x\to 0}\frac{1^2-\cos^2 x}{x(1+\cos x)}$$

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=\lim_{x\to 0}\frac{1-\cos^2 x}{x(1+\cos x)}$$

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=\lim_{x\to 0}\frac{\sin^2 x}{x}.\frac{1}{1+\cos x}$$

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=\lim_{x\to 0}\frac{\sin x}{x}.\lim_{x\to 0}\frac{\sin x}{1+\cos x}$$

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=\quad 1.\quad \lim_{x\to 0}\frac{\sin(0)}{1+\cos(0)}$$

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=\frac{0}{1+1}$$

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=\frac{0}{2}$$

$$\lim_{x\to 0}\frac{\sec x-1}{x\sec x}=0$$

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l). $\lim_{x\to 0}\frac{\tan^2 x}{x}$

Sol: Given **Indeterminate** form $\frac{0}{0}$

$$\lim_{x\to 0}\frac{\tan^2 x}{x}=\lim_{x\to 0}\frac{\sin^2 x}{x\cos^2 x}$$

$$\lim_{x\to 0}\frac{\tan^2 x}{x}=\lim_{x\to 0}\frac{\sin x}{x}.\lim_{x\to 0}\frac{\sin x}{\cos^2 x}$$

$$\lim_{x\to 0}\frac{\tan^2 x}{x}=\quad 1\quad .\frac{\sin(0)}{(\cos(0))^2}$$

$$\lim_{x\to 0}\frac{\tan^2 x}{x}=\frac{0}{(1)^2}$$

$$\lim_{x\to 0}\frac{\tan^2 x}{x}=0$$

Q2. Use algebra and the rule of the limits to evaluate the following limits

a). $\lim_{x\to 4}\frac{-6}{(x-4)^2}$

Sol: Given $\lim_{x\to 4}\frac{-6}{(x-4)^2}$

$$\lim_{x\to 4}\frac{-6}{(x-4)^2}=\frac{-6}{(4-4)^2}$$

$$\lim_{x\to 4}\frac{-6}{(x-4)^2}=\frac{-6}{(0)^2}$$

$$\lim_{x\to 4}\frac{-6}{(x-4)^2}=\frac{-6}{0}=undefined=\infty$$

---

b). $\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}$

Sol: Given **Indeterminate** form $\frac{0}{0}$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\lim_{x\to 0}\frac{1}{x}\left(\frac{1}{x+3}-\frac{1}{3}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\lim_{x\to 0}\frac{1}{x}\left(\frac{3-(x+3)}{3(x+3)}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\lim_{x\to 0}\frac{1}{x}\left(\frac{3-x-3}{3(x+3)}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\lim_{x\to 0}\frac{1}{x}\left(\frac{-x}{3(x+3)}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\lim_{x\to 0}\left(\frac{-1}{3(x+3)}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\left(\frac{-1}{3(0+3)}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}=\frac{-1}{9}$$

---

c). $\lim_{x\to 5}\frac{\sqrt{x}-\sqrt{5}}{x-5}$

Sol: Given **Indeterminate** form $\frac{0}{0}$

Rational the numerator

$$\lim_{x\to 5}\frac{\sqrt{x}-\sqrt{5}}{x-5}=\lim_{x\to 5}\frac{\sqrt{x}-\sqrt{5}}{x-5}\times\frac{\sqrt{x}+\sqrt{5}}{\sqrt{x}+\sqrt{5}}$$

$$\lim_{x\to 5}\frac{\sqrt{x}-\sqrt{5}}{x-5}=\lim_{x\to 5}\frac{(\sqrt{x})^2-(\sqrt{5})^2}{(x-5)(\sqrt{x}+\sqrt{5})}$$

$$\lim_{x\to 5}\frac{\sqrt{x}-\sqrt{5}}{x-5}=\lim_{x\to 5}\frac{x-5}{(x-5)(\sqrt{x}+\sqrt{5})}$$

$$\lim_{x\to5}\frac{\sqrt{x}-\sqrt{5}}{x-5}=\lim_{x\to5}\frac{1}{\sqrt{x}+\sqrt{5}}$$

$$\lim_{x\to5}\frac{\sqrt{x}-\sqrt{5}}{x-5}=\frac{1}{\sqrt{5}+\sqrt{5}}$$

$$\lim_{x\to5}\frac{\sqrt{x}-\sqrt{5}}{x-5}=\frac{1}{2\sqrt{5}}$$

d). $\lim_{x\to25}\frac{\sqrt{x}-5}{x-25}$

Sol: Given **Indeterminate** form $\frac{0}{0}$

Rational the numerator

$$\lim_{x\to25}\frac{\sqrt{x}-5}{x-25}=\lim_{x\to25}\frac{\sqrt{x}-5}{x-25}\times\frac{\sqrt{x}+5}{\sqrt{x}+5}$$

$$\lim_{x\to25}\frac{\sqrt{x}-5}{x-25}=\lim_{x\to25}\frac{\left(\sqrt{x}\right)^2-(5)^2}{(x-25)\left(\sqrt{x}+5\right)}$$

$$\lim_{x\to25}\frac{\sqrt{x}-5}{x-25}=\lim_{x\to25}\frac{x-25}{(x-25)\left(\sqrt{x}+5\right)}$$

$$\lim_{x\to25}\frac{\sqrt{x}-5}{x-25}=\lim_{x\to25}\frac{1}{\sqrt{x}+5}$$

$$\lim_{x\to25}\frac{\sqrt{x}-5}{x-25}=\frac{1}{\sqrt{25}+5}$$

$$\lim_{x\to25}\frac{\sqrt{x}-5}{x-25}=\frac{1}{5+5}=\frac{1}{10}$$

Q3. Find limit of convergent of following sequence:

a). $\left\{\frac{5n}{n+7}\right\}$

Sol: Given $\left\{\frac{5n}{n+7}\right\}$

$$\lim_{n\to\infty}\frac{5n}{n+7}=\lim_{n\to\infty}\frac{5n}{n+\frac{7}{1}.\frac{n}{n}}$$

$$\lim_{n\to\infty}\frac{5n}{n+7}=\lim_{n\to\infty}\frac{5n}{n\left(1+\frac{7}{n}\right)}$$

$$\lim_{n\to\infty}\frac{5n}{n+7}=\lim_{n\to\infty}\frac{5}{1+\frac{7}{n}}$$

$$\lim_{n\to\infty}\frac{5n}{n+7}=\frac{5}{1+\frac{7}{\infty}}=\frac{5}{1+0}$$

$$\lim_{n\to\infty}\frac{5n}{n+7}=5$$

b). $\left\{\frac{4-7n}{8+n}\right\}$

Sol: Given $\left\{\frac{4-7n}{8+n}\right\}$

$$\lim_{n\to\infty}\frac{4-7n}{8+n}=\lim_{n\to\infty}\frac{4.\frac{n}{n}-7n}{8\frac{n}{n}+n}$$

$$\lim_{n\to\infty}\frac{4-7n}{8+n}=\lim_{n\to\infty}\frac{n\left(\frac{4}{n}-7\right)}{n\left(\frac{8}{n}+1\right)}$$

$$\lim_{n\to\infty}\frac{4-7n}{8+n}=\lim_{n\to\infty}\frac{\frac{4}{n}-7}{\frac{8}{n}+1}$$

$$\lim_{n\to\infty}\frac{4-7n}{8+n}=\frac{\frac{4}{\infty}-7}{\frac{8}{\infty}+1}$$

$$\lim_{n\to\infty}\frac{4-7n}{8+n}=\frac{0-7}{0+1}$$

$$\lim_{n\to\infty}\frac{4-7n}{8+n}=\frac{-7}{1}=-7$$

c). $\left\{\frac{8n-500\sqrt{n}}{2n+800\sqrt{n}}\right\}$

sol: Given $\lim_{n\to\infty}\frac{8n-500\sqrt{n}}{2n+800\sqrt{n}}=\lim_{n\to\infty}\frac{8n-500\sqrt{n}\,\frac{\sqrt{n}}{\sqrt{n}}}{2n+800\sqrt{n}\,\frac{\sqrt{n}}{\sqrt{n}}}$

$$\lim_{n\to\infty}\frac{8n-500\sqrt{n}}{2n+800\sqrt{n}}=\lim_{n\to\infty}\frac{n\left(8-\frac{500}{\sqrt{n}}\right)}{n\left(2+\frac{800}{\sqrt{n}}\right)}$$

$$\lim_{n\to\infty}\frac{8n-500\sqrt{n}}{2n+800\sqrt{n}}=\lim_{n\to\infty}\frac{8-\frac{500}{\sqrt{n}}}{2+\frac{800}{\sqrt{n}}}$$

$$\lim_{n\to\infty}\frac{8n-500\sqrt{n}}{2n+800\sqrt{n}}=\frac{8-\frac{500}{\infty}}{2+\frac{800}{\infty}}$$

$$\lim_{n\to\infty}\frac{8n-500\sqrt{n}}{2n+800\sqrt{n}}=\frac{8-0}{2+0}=\frac{8}{2}$$

$$\lim_{n\to\infty}\frac{8n-500\sqrt{n}}{2n+800\sqrt{n}}=4$$

Q4. Weekly sales (in rupee) at big store x weeks after end of an advertising campaign are given by

$$S(x)=5000+\frac{3600}{x+2}$$

Find the sale for the indicated weeks limits

a). $S(2)$

Sol: Given $S(x)=5000+\frac{3600}{x+2}$

$$S(2)=5000+\frac{3600}{2+2}$$

$$S(2)=5000+\frac{3600}{4}$$

$$S(2)=5000+900$$

$$S(2)=Rs.\ 5900$$

b). $\lim_{x\to5}S(x)$

Sol: Given $S(x)=5000+\dfrac{3600}{x+2}$

$$\lim_{x\to5} S(x)=\lim_{x\to5}\left\{5000+\frac{3600}{x+2}\right\}$$

$$\lim_{x\to5} S(x)=5000+\lim_{x\to5}\frac{3600}{x+2}$$

$$\lim_{x\to5} S(x)=5000+\frac{3600}{5+2}$$

$$\lim_{x\to5} S(x)=5000+\frac{3600}{7}$$

$$\lim_{x\to5} S(x)=5000+514\frac{2}{7}$$

$$\lim_{x\to5} S(x)=5514\frac{2}{7}$$

$$\lim_{x\to5} S(x)=Rs.\ 5514.285$$

c). $\lim_{x\to16} S(x)$

Sol: Given $S(x)=5000+\dfrac{3600}{x+2}$

$$\lim_{x\to16} S(x)=\lim_{x\to16}\left\{5000+\frac{3600}{x+2}\right\}$$

$$\lim_{x\to16} S(x)=5000+\lim_{x\to16}\frac{3600}{x+2}$$

$$\lim_{x\to16} S(x)=5000+\frac{3600}{16+2}$$

$$\lim_{x\to16} S(x)=5000+\frac{3600}{18}$$

$$\lim_{x\to16} S(x)=5000+200$$

$$\lim_{x\to16} S(x)=Rs.\ 5200$$

Q5. Use properties of continuous functions to test the continuity and discontinuity of the following functions

a). $f(x)=2x-3$

Sol: Given $f(x)$ is polynomial function so it is continuous for all values of x

b). $g(x)=3-5x$

Sol: Given $f(x)$ is polynomial function so it is continuous for all values of x

c). $h(x)=\dfrac{2}{x-5}$

Sol: Given rational function $h(x)$ with denominator $x-5$ we take

$x-5=0 \quad \Rightarrow x=5$

i.e., $h(x)$ is continuous expects $x=5$

d). $k(x)=\dfrac{x}{x+3}$

Sol: Given rational function $k(x)$ with denominator $x+3$ we take

$x+3=0 \quad \Rightarrow x=-3$

i.e., $k(x)$ is continuous expects $x=-3$

e). $g(x)=\dfrac{x-5}{(x-3)(x+2)}$

Sol: Given rational function $g(x)$ with denominator $x-3$ and $x+2$ we take

$x-3=0 \quad x+2=0$

$\Rightarrow x=3 \quad x=-2$

i.e., $g(x)$ is continuous expects $x=3$ and $x=-2$

f). $F(x)=\dfrac{1}{x(x+7)}$

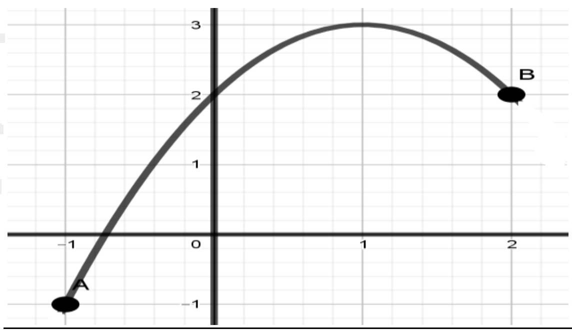
Sol: Given rational function $F(x)$ with denominator x and $x+7$ we take

$x=0 \quad \Rightarrow x+7=0$

$\Rightarrow x=-7$

i.e., $F(x)$ is continuous expects $x=0$ and $x=-7$

Q7. Use the graph of the function g(x) to answer the following questions. $g(x)=-x^2+2x+2$



a). Is g(x) continuous on the open interval (-1,2)?
solution: Yes, graph between $x=-1$ and $x=2$ is **continuous** because there is no gap between them.

b). Is g(x) continuous from the right at x= -1? Is $\lim_{x\to-1^+} g(x)=g(-1)$

solution: Yes, there is no gap between graph at $x=-1$ and right hand from $x=-1$ is **continuous**.

i.e., there is no gap between graph at $g(-1)$ and right hand from $\lim_{x\to-1^+} g(x)$ is **continuous or** $\lim_{x\to-1^+} g(x)=g(-1)$

c). Is g(x) continuous from the left at x= 2? Is $\lim_{x\to2^-} g(x)=g(2)$

Solution: Yes, there is no gap between graph at $g(2)$ and left hand from $\lim_{x\to2^-} g(x)$ is **continuous or** $\lim_{x\to2^-} g(x)=g(2)$

d). Is g(x) continuous on the closed interval [-1, 2]
solution: Yes,graph between and including $x=-1$ and $x=2$ is **continuous** because there is no gap between them.