Chapter 12

Trigonometry and their graph

Exercise 12.1

Find the domain of each function;

Q1. $\sin 2x$

Solution; we have $\sin 2x$ Let $\theta = 2x$

Domain of $\sin\theta = \Re$

Where $\theta = 2x \in \Re$

$x \in \Re$

Thus Domain of $\sin 2x = \Re$

Q2. $4\cos x$

Solution; we have $4\cos x$ Let $\theta = x$

Domain of $\cos\theta = \Re$

Where $\theta = x \in \Re$

Thus Domain of $4\cos x = \Re$

Q3. $3\sin 3x$

Solution; We have $3\sin 3x$ Let $\theta = 3x$

Domain of $\sin\theta = \Re$

Where $\theta = 3x \in \Re$

$x \in \Re$

Thus Domain of $3\sin 3x = \Re$

Q4. $\sec 2x$

Solution; we have $\sec 2x$ Let $\theta = 2x$

Domain of $\sec\theta = \Re - (2n+1)\frac{\pi}{2}, n \in Z$

Where $\theta = 2x \in \Re - (2n+1)\frac{\pi}{2}, n \in Z$

$x \in \Re - (2n+1)\frac{\pi}{4}, n \in Z$

Domain of sec2x= $R - \left\{ x/x = (2n+1)\frac{\pi}{4}, n \in Z \right\}$

Q5. $\tan\frac{1}{2}x$

Solution; we have $\tan\frac{1}{2}x$ Let $\theta = \frac{1}{2}x$

Domain of $\tan\theta = \Re - (2n+1)\frac{\pi}{2}, n \in Z$

Where $\theta = \frac{1}{2}x \in \Re - (2n+1)\frac{\pi}{2}, n \in Z$

$x \in \Re - (2n+1)\pi, n \in Z$

Domain of $\tan\frac{1}{2}x$ = $R - \{x/x = (2n+1)\pi, n \in Z\}$

Q6. $\cos ec2x$

Sol; we have $\cos ec2x$ Let $\theta = 2x$

Domain of $\cos ec\theta = \Re - n\pi, n \in Z$

Where $\theta = 2x \in \Re - n\pi, n \in Z$

$x \in \Re - \frac{n\pi}{2}, n \in Z$

Domain of s $\cos ec2x$ = $R - \left\{ x/x = \frac{n\pi}{2}, n \in Z \right\}$

Q7. $3\cos 2x$

Solution; we have $3\cos 2x$ Let $\theta = x$

Domain of $\cos\theta = \Re$

Where $\theta = 2x \in \Re$

$x \in \Re$

Thus Domain of $3\cos 2x = \Re$

Q8. $6\sec\frac{1}{2}x$

Solution; we have $6\sec\frac{1}{2}x$ Let $\theta = \frac{1}{2}x$

Domain of $\sec\theta = \Re - (2n+1)\frac{\pi}{2}, n \in Z$

Where $\theta = \frac{1}{2}x \in \Re - (2n+1)\frac{\pi}{2}, n \in Z$

$x \in \Re - (2n+1)\pi, n \in Z$

Domain of $6\sec\frac{1}{2}x$ = $R - \{x/x = (2n+1)\pi, n \in Z\}$

Q9. $5\tan 3x$

Solution; we have $5\tan 3x$ Let $\theta = 3x$

Domain of $\tan\theta = \Re - (2n+1)\frac{\pi}{2}, n \in Z$

Where $\theta = 3x \in \Re - (2n+1)\frac{\pi}{2}, n \in Z$

$x \in \Re - (2n+1)\frac{\pi}{6}, n \in Z$

Domain of $5\tan 3x$ = $R - \left\{ x/x = (2n+1)\frac{\pi}{6}, n \in Z \right\}$

Q10. $5\sin 5x$

Solution; we have $5\sin 5x$ Let $\theta = 5x$

Domain of $\sin\theta = \Re$

So $\theta = 5x \in \Re$

$x \in \Re$

Thus Domain of $5\sin 5x = \Re$

Find the range of the following functions;

Q11. $\sin 2x$

Solution; we have $\sin 2x$

we know that Range of sine θ,

$-1 \le \sin\theta \le 1$ Here θ = 2 x

Then $-1 \le \sin 2x \le 1$

Hence the range of $\sin 2x$ is $[-1, 1]$

$S.S = \{y/y \in R \wedge -1 \le y \le 1\}$

Q12. $\cos 4x$

Solution; we have $\cos 4x$

We know that Range of cosine θ,

$-1 \le \cos\theta \le 1$ Here θ = 4 x

Then $-1 \le \cos 4x \le 1$

Hence the range of $\cos 4x$ is $[-1, 1]$

$S.S = \{y/y \in R \wedge -1 \le y \le 1\}$

Q13. $2\sin 3x$

Solution; we have $2\sin 3x$

We know that Range of sine θ,

$-1 \le \sin\theta \le 1$ Here θ = 3 x

Then $-1 \le \sin 3x \le 1$

Then $-2 \le 2\sin 3x \le 2$

Hence the range of $\sin 2x$ is $[-2, 2]$

$S.S = \{y/y \in R \wedge -2 \le y \le 2\}$

Q14. $5\cos x$

Solution; we have $5\cos x$

we know that Range of cosine θ,

$-1 \le \cos\theta \le 1$ Here θ = x

Then $-1 \le \cos x \le 1$

Then $-5 \le 5\cos x \le 5$

Hence the range of $5\cos x$ is $[-5,5]$

$S.S = \{y/y \in R \wedge -5 \le y \le 5\}$

Q15. $3\cot x$

Solution; we have $3\cot x$

We know that Range of cot θ,

$-\infty \le \cot\theta \le \infty$ Here θ = x

Then $-\infty \le \cot x \le \infty$

Then $-\infty \le 3\cot x \le \infty$

Hence the range of $3\cot x$ is R $S.S = R$

Q16. $2\sec 2x$

Solution; we have $2\sec 2x$

We know that Range of sec θ,

$\sec\theta \ge 1, \quad \sec\theta \le -1$ Here θ = 2 x

Then $\sec 2x \ge 1, \quad \sec 2x \le -1$

Then $2\sec 2x \ge 2, \quad 2\sec 2x \le -2$

$S.S = R - \{y/y \in R \wedge y \ge 2, \quad y \le -2\}$

Q17. $\cos ec 2x$

Solution; we have $\cos ec 2x$

We know that Range of cosec θ,

$\cos ec\theta \ge 1, \quad \cos ec\theta \le -1$ Here θ = 2 x

Then $\cos ec 2x \ge 1, \quad \cos ec 2x \le -1$

$S.S = R - \{y/y \in R \wedge y \ge 1, \quad y \le -1\}$

Q18. $\sin \pi x$

Solution; we have $\sin \pi x$

We know that Range of sine θ,

$-1 \le \sin\theta \le 1$ Here θ = π x

Then $-1 \le \sin \pi x \le 1$

Hence the range of $\sin \pi x$ is $[-1,1]$

$S.S = \{y/y \in R \wedge -1 \le y \le 1\}$

Q19. $\tan \frac{\pi}{4} x$

Solution; we have $\tan \frac{\pi}{4} x$

We know that Range of tan θ,

$-\infty \le \tan\theta \le \infty$ Here θ = $\frac{\pi}{4}$ x

Then $-\infty \le \tan \frac{\pi}{4} x \le \infty$

Hence the range of $\tan \frac{\pi}{4} x$ is R

$S.S = R$

Q20. $\sec(2\pi x + 3)$

Solution; we have $\sec(2\pi x + 3)$

We know that Range of sec θ,

$\sec\theta \ge 1, \quad \sec\theta \le -1$ Here θ = $2\pi x + 3$

Then $\sec(2\pi x + 3) \ge 1, \quad \sec(2\pi x + 3) \le -1$

$S.S = R - \{y/y \in R \wedge y \ge 1, \quad y \le -1\}$

Exercise 12.2

Find the value of each function

Q1. $\sin(-\pi)$

Solution we have $\sin(-\pi)$

$\sin(-\pi) = -\sin\pi$

$= -0$

$= 0$

Q2. $\cos\left(-\frac{\pi}{4}\right)$

Solution: we have $\cos\left(-\frac{\pi}{4}\right)$

$\cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$

Q3. $\tan\left(-\frac{\pi}{4}\right)$

Solution we have $\tan\left(-\frac{\pi}{4}\right)$

$\tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4}$

$= -1$

Q4. $\cot\left(-\frac{3\pi}{2}\right)$

Solution we have $\cot\left(-\frac{3\pi}{2}\right)$

$\cot\left(-\frac{3\pi}{2}\right) = \cot\frac{3\pi}{2}$

$= 0$

Q5. $\cos ec\left(-\frac{\pi}{4}\right)$

Solution we have $\cos ec\left(-\frac{\pi}{4}\right)$

$\cos ec\left(-\frac{\pi}{4}\right) = -\cos ec\frac{\pi}{4}$

$= -\sqrt{2}$

Q6. $\sec(-\pi)$

Solution we have $\sec(-\pi)$

$\sec(-\pi) = \sec\pi$

$= -1$

Find the period of each function

Q7. $2\sin x$

Solution: we have $2\sin x$

Period of $2\sin x = \frac{\text{period of } \sin x}{1}$

Period of $2\sin x = \frac{2\pi}{1} = 2\pi$

Q8. $3\tan x$

Solution we have $3\tan x$

Period of $3\tan x = \frac{\text{period of } \tan x}{1}$

Period of $3\tan x = \frac{\pi}{1} = \pi$

Q9. $5\cos 3x$

Solution we have $5\cos 3x$

Period of $5\cos 3x = \frac{\text{period of } \cos x}{3}$

Period of $5\cos 3x = \frac{2\pi}{3}$

Q10. $\frac{1}{2}\sec x$

Solution we have $\frac{1}{2}\sec x$

Period of $\frac{1}{2}\sec x = \frac{\text{period of } \sec x}{1}$

Period of $\frac{1}{2}\sec x = \frac{2\pi}{1} = 2\pi$

Q11. $-2\operatorname{cosec} \pi x$

Solution: we have $-2\operatorname{cosec} \pi x$

Period of $-2\operatorname{cosec} \pi x = \frac{\text{period of } \operatorname{cosec} x}{\pi}$

Period of $-2\operatorname{cosec} \pi x = \frac{2\pi}{\pi} = 2$

Q12. $\frac{7}{2}\cot\frac{2\pi x}{3}$

Solution we have $\frac{7}{2}\cot\frac{2\pi x}{3}$

Period of $\frac{7}{2}\cot\frac{2\pi x}{3} = \frac{\text{period of } \cot x}{\frac{2\pi}{3}}$

Period of $\frac{7}{2}\cot\frac{2\pi x}{3} = \frac{\pi}{\frac{2\pi}{3}} = \frac{3}{2}$

Q13. $3\operatorname{cosec}\frac{\pi x}{2}$

Solution we have $3\operatorname{cosec}\frac{\pi x}{2}$

Period of $3\operatorname{cosec}\frac{\pi x}{2} = \frac{\text{period of } \operatorname{cosec} x}{\frac{\pi}{2}}$

Period of $3\operatorname{cosec}\frac{\pi x}{2} = \frac{2\pi}{\frac{\pi}{2}} = 4$

Q14. $-\cot\frac{x}{2\pi}$

Solution we have $-\cot\frac{x}{2\pi}$

Period of $-\cot\frac{x}{2\pi} = \frac{\text{period of } \cot x}{\frac{1}{2\pi}}$

Period of $-\cot\frac{x}{2\pi} = \frac{\pi}{\frac{1}{2\pi}} = 2\pi^2$

Q15. $-\frac{2}{5}\sec\frac{3x}{\pi}$

Solution we have $-\frac{2}{5}\sec\frac{3x}{\pi}$

Period of $-\frac{2}{5}\sec\frac{3x}{\pi} = \frac{\text{period of } \sec x}{\frac{3}{\pi}}$

Period of $-\frac{2}{5}\sec\frac{3x}{\pi} = \frac{2\pi}{\frac{3}{\pi}} = \frac{2}{3}\pi^2$

Q16. $\frac{7}{9}\sec\frac{2x}{\theta}$

Solution we have $\frac{7}{9}\sec\frac{2x}{\theta}$

Period of $\frac{7}{9}\sec\frac{2x}{\theta} = \frac{\text{period of } \sec x}{\frac{2}{\theta}}$

Period of $\frac{7}{9}\sec\frac{2x}{\theta} = \frac{2\pi}{\frac{2}{\theta}} = \pi\theta$

Exercise 12.3

Draw graph of following functions in indicated interval.

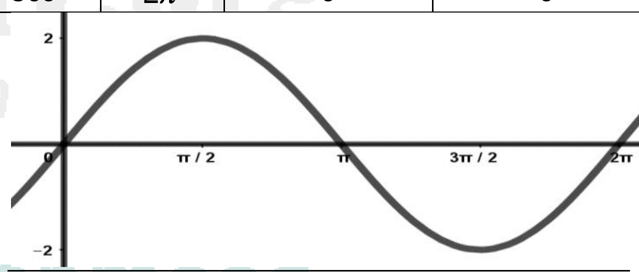
Q1. $y = 2\sin x$ $\quad 0 \le x \le 2\pi$

Solution: we have $y = 2\sin x$

Given domain for the function $0 \le x \le 2\pi$

To plot the given function we will find few values of the function within domain

x	x	$\sin x$	$2\sin x$
0^o	0	0	0
30^o	$\frac{\pi}{6}$	$\frac{1}{2} = 0.5$	1
45^o	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}} = 0.707$	$\sqrt{2} = 1.414$
60^o	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2} = 0.866$	$\sqrt{3} = 1.732$
90^o	$\frac{\pi}{2}$	1	2
120^o	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2} = 0.866$	$\sqrt{3} = 1.732$
135^o	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}} = 0.707$	$\sqrt{2} = 1.414$
150^o	$\frac{5\pi}{6}$	$\frac{1}{2} = 0.5$	1
180^o	π	0	0
210^o	$\frac{7\pi}{6}$	$\frac{-1}{2} = -0.5$	-1
225^o	$\frac{5\pi}{4}$	$\frac{-1}{\sqrt{2}} = -0.707$	$-\sqrt{2} = -1.414$
240^o	$\frac{4\pi}{3}$	$\frac{-\sqrt{3}}{2} = -0.866$	$-\sqrt{3} = -1.732$
270^o	$\frac{3\pi}{2}$	-1	-2
300^o	$\frac{5\pi}{3}$	$\frac{-\sqrt{3}}{2} = -0.866$	$-\sqrt{3} = -1.732$
315^o	$\frac{7\pi}{4}$	$\frac{-1}{\sqrt{2}} = -0.707$	$-\sqrt{2} = -1.414$
330^o	$\frac{11\pi}{6}$	$\frac{-1}{2} = -0.5$	-1
360^o	2π	0	0



Q2. $y = -\cos x \quad 0 \le x \le 2\pi$

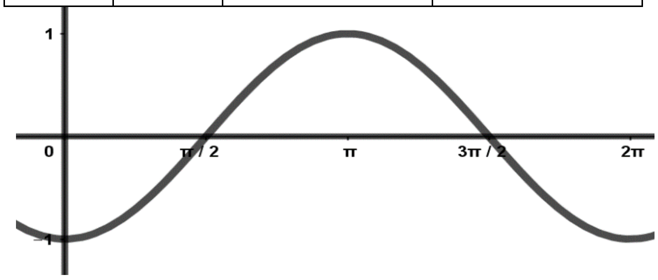
Solution: we have $y = -\cos x$

Given the domain for the function $0 \le x \le 2\pi$

To plot the given function we will find few values of the function within domain

x	x	$\cos x$	$-\cos x$
0^o	0	1	-1
30^o	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2} = 0.866$	$\frac{-\sqrt{3}}{2} = -0.866$
45^o	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}} = 0.707$	$\frac{-1}{\sqrt{2}} = -0.707$
60^o	$\frac{\pi}{3}$	$\frac{1}{2} = 0.5$	$\frac{-1}{2} = -0.5$
90^o	$\frac{\pi}{2}$	0	0
120^o	$\frac{2\pi}{3}$	$\frac{-1}{2} = -0.5$	$\frac{1}{2} = 0.5$
135^o	$\frac{3\pi}{4}$	$\frac{-1}{\sqrt{2}} = -0.707$	$\frac{1}{\sqrt{2}} = 0.707$
150^o	$\frac{5\pi}{6}$	$\frac{-\sqrt{3}}{2} = -0.866$	$\frac{\sqrt{3}}{2} = 0.866$
180^o	π	-1	1
210^o	$\frac{7\pi}{6}$	$\frac{-\sqrt{3}}{2} = -0.866$	$\frac{\sqrt{3}}{2} = 0.866$
225^o	$\frac{5\pi}{4}$	$\frac{-1}{\sqrt{2}} = -0.707$	$\frac{1}{\sqrt{2}} = 0.707$
240^o	$\frac{4\pi}{3}$	$\frac{-1}{2} = -0.5$	$\frac{1}{2} = 0.5$
270^o	$\frac{3\pi}{2}$	0	0
300^o	$\frac{5\pi}{3}$	$\frac{1}{2} = 0.5$	$\frac{-1}{2} = -0.5$

315°	$\frac{7\pi}{4}$	$\frac{1}{\sqrt{2}}=0.707$	$\frac{-1}{\sqrt{2}}=-0.707$
330°	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}=0.866$	$\frac{-\sqrt{3}}{2}=-0.866$
360°	2π	1	-1



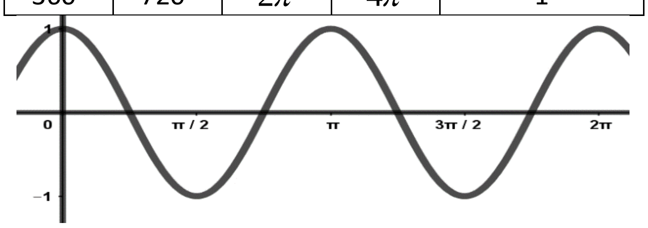
Q3. $y=\cos 2x$ $\quad 0\le x\le 2\pi$

Solution: we have $y=\cos 2x$

Given the domain for the function $0\le x\le 2\pi$

To plot the given function we will find few values of the function within domain

x	2 x	x	$2x$	$\cos 2x$
0°	0°	0	0	1
30°	60°	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{1}{2}=0.5$
45°	90°	$\frac{\pi}{4}$	$\frac{\pi}{2}$	0
60°	120°	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{-1}{2}=-0.5$
90°	180°	$\frac{\pi}{2}$	π	-1
120°	240°	$\frac{2\pi}{3}$	$\frac{4\pi}{3}$	$\frac{-1}{2}=-0.5$
135°	270°	$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	0
150°	300°	$\frac{5\pi}{6}$	$\frac{5\pi}{3}$	$\frac{1}{2}=0.5$
180°	360°	π	$\frac{\pi}{2}$	0
210°	420°	$\frac{7\pi}{6}$	$\frac{7\pi}{3}$	$\frac{1}{2}=0.5$
225°	450°	$\frac{5\pi}{4}$	$\frac{5\pi}{2}$	0
240°	480°	$\frac{4\pi}{3}$	$\frac{8\pi}{3}$	$\frac{-1}{2}=-0.5$
270°	540°	$\frac{3\pi}{2}$	3π	-1
300°	600°	$\frac{5\pi}{3}$	$\frac{10\pi}{3}$	$\frac{-1}{2}=-0.5$
315°	630°	$\frac{7\pi}{4}$	$\frac{7\pi}{2}$	0
330°	660°	$\frac{11\pi}{6}$	$\frac{11\pi}{3}$	$\frac{1}{2}=0.5$
360°	720°	2π	4π	1



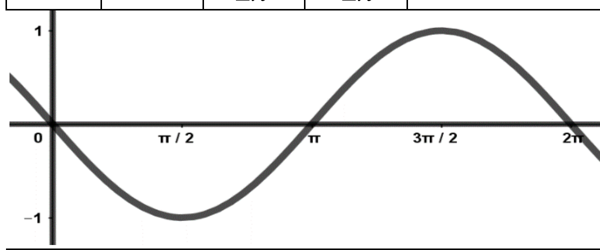
Q4. $y=\sin(-x)$ $\quad 0\le x\le 2\pi$

Solution: we have $y=\sin(-x)$

Given the domain for the function $0\le x\le 2\pi$

To plot the given function we will find few values of the function within domain

x	- x	x	- x	$\sin(-x)$
0°	-0°	0	0	0
30°	-30°	$\frac{\pi}{6}$	- $\frac{\pi}{6}$	$\frac{-1}{2}=-0.5$
45°	-45°	$\frac{\pi}{4}$	- $\frac{\pi}{4}$	$\frac{-1}{\sqrt{2}}=-0.707$
60°	-60°	$\frac{\pi}{3}$	- $\frac{\pi}{3}$	$\frac{-\sqrt{3}}{2}=-0.866$
90°	-90°	$\frac{\pi}{2}$	- $\frac{\pi}{2}$	-1
120°	-120°	$\frac{2\pi}{3}$	- $\frac{2\pi}{3}$	$\frac{-\sqrt{3}}{2}=-0.866$
135°	-135°	$\frac{3\pi}{4}$	- $\frac{3\pi}{4}$	$\frac{-1}{\sqrt{2}}=-0.707$
150°	-150°	$\frac{5\pi}{6}$	- $\frac{5\pi}{6}$	$\frac{-1}{2}=-0.5$
180°	-180°	π	- π	0
210°	-210°	$\frac{7\pi}{6}$	- $\frac{7\pi}{6}$	$\frac{1}{2}=0.5$
225°	-225°	$\frac{5\pi}{4}$	- $\frac{5\pi}{4}$	$\frac{1}{\sqrt{2}}=0.707$
240°	-240°	$\frac{4\pi}{3}$	- $\frac{4\pi}{3}$	$\frac{\sqrt{3}}{2}=0.866$
270°	-270°	$\frac{3\pi}{2}$	- $\frac{3\pi}{2}$	1
300°	-300°	$\frac{5\pi}{3}$	- $\frac{5\pi}{3}$	$\frac{\sqrt{3}}{2}=0.866$
315°	-315°	$\frac{7\pi}{4}$	- $\frac{7\pi}{4}$	$\frac{1}{\sqrt{2}}=0.707$
330°	-330°	$\frac{11\pi}{6}$	- $\frac{11\pi}{6}$	$\frac{1}{2}=0.5$
360°	-360°	2π	- 2π	0



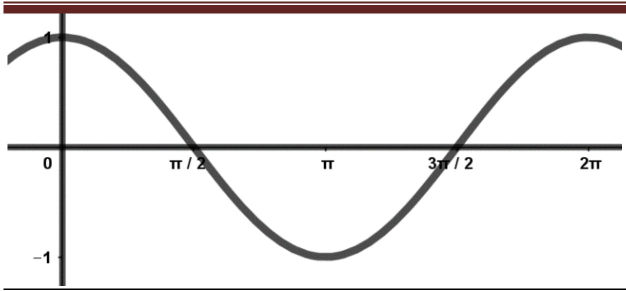
Q5. $y=\sin\left(x+\frac{\pi}{2}\right)$ $\quad 0\le x\le 2\pi$

Solution: we have $y=\sin\left(x+\frac{\pi}{2}\right)$

Given the domain for the function $0\le x\le 2\pi$

To plot the given function we will find few values of the function within domain

x	x + 90°	x	$x+\frac{\pi}{2}$	$\sin\left(x+\frac{\pi}{2}\right)$
0°	90°	0	$\frac{\pi}{2}$	1
30°	120°	$\frac{\pi}{6}$	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}=0.866$
45°	135°	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}=0.707$
60°	150°	$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{1}{2}=0.5$
90°	180°	$\frac{\pi}{2}$	π	0
120°	210°	$\frac{2\pi}{3}$	$\frac{7\pi}{6}$	$\frac{-1}{2}=-0.5$
135°	225°	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{-1}{\sqrt{2}}=-0.707$
150°	240°	$\frac{5\pi}{6}$	$\frac{4\pi}{3}$	$\frac{-\sqrt{3}}{2}=-0.866$
180°	270°	π	$\frac{3\pi}{2}$	-1
210°	300°	$\frac{7\pi}{6}$	$\frac{5\pi}{3}$	$\frac{-\sqrt{3}}{2}=-0.866$
225°	315°	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$	$\frac{-1}{\sqrt{2}}=-0.707$
240°	330°	$\frac{4\pi}{3}$	$\frac{11\pi}{6}$	$\frac{-1}{2}=-0.5$
270°	360°	$\frac{3\pi}{2}$	2π	0
300°	390°	$\frac{5\pi}{3}$	$\frac{13\pi}{6}$	$\frac{1}{2}=0.5$
315°	405°	$\frac{7\pi}{4}$	$\frac{9\pi}{4}$	$\frac{1}{\sqrt{2}}=0.707$
330°	420°	$\frac{11\pi}{6}$	$\frac{7\pi}{3}$	$\frac{\sqrt{3}}{2}=0.866$
360°	450°	2π	$\frac{5\pi}{2}$	1



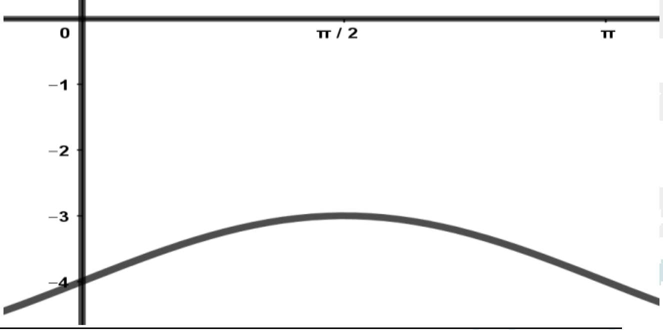
Q6. $y=-4+\sin x \qquad 0\le x\le\pi$

Solution: we have $y=-4+\sin x$

Given the domain for the function $0\le x\le\pi$

To plot the given function we will find few values of the function within domain

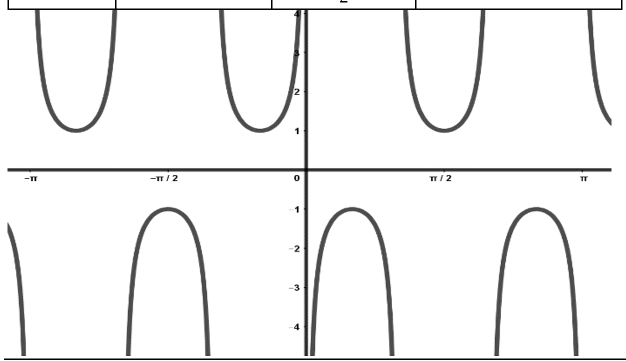
x		$\sin x$	$-4+\sin x$
0°	0	0	-4
30°	$\frac{\pi}{6}$	$\frac{1}{2}=0.52$	-3.5
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}=0.707$	-3.293
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}=0.866$	-3.134
90°	$\frac{\pi}{2}$	1	-3
120°	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}=0.866$	-3.134
135°	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}=0.707$	-3.293
150°	$\frac{5\pi}{6}$	$\frac{1}{2}=0.5$	-3.5
180°	π	0	-4



Q7. $y=\sec\left(3x+\frac{\pi}{2}\right) \qquad -\pi\le x\le\pi$

Solution: we have $y=\sec\left(3x+\frac{\pi}{2}\right)$ Given the domain for the function $-\pi\le x\le\pi$, To plot the given function we will find few values of the function within domain

x	x	$3x+\dfrac{\pi}{2}$	$\sec\left(3x+\dfrac{\pi}{2}\right)$
-180°	$-\pi$	$\frac{-5\pi}{2}$	∞
-150°	$-\frac{5\pi}{6}$	-2π	1
-135°	$-\frac{3\pi}{4}$	$\frac{-7\pi}{4}$	$\sqrt{2}=1.414$
-120°	$-\frac{2\pi}{3}$	$\frac{-3\pi}{2}$	∞
-90°	$-\frac{\pi}{2}$	$-\pi$	-1
-60°	$-\frac{\pi}{3}$	$-\frac{\pi}{2}$	∞
-45°	$-\frac{\pi}{4}$	$-\frac{\pi}{4}$	$\sqrt{2}=1.414$
-30°	$-\frac{\pi}{6}$	0	1
0°	0	$\frac{\pi}{2}$	∞
30°	$\frac{\pi}{6}$	π	-1
0°	0	$\frac{\pi}{2}$	∞
30°	$\frac{\pi}{6}$	π	-1
45°	$\frac{\pi}{4}$	$\frac{5\pi}{4}$	$-\sqrt{2}=-1.414$
60°	$\frac{\pi}{3}$	$\frac{3\pi}{2}$	∞
90°	$\frac{\pi}{2}$	2π	1
120°	$\frac{2\pi}{3}$	$\frac{5\pi}{2}$	∞
135°	$\frac{3\pi}{4}$	$\frac{11\pi}{4}$	$-\sqrt{2}=-1.414$
150°	$\frac{5\pi}{6}$	3π	-1
180°	π	$\frac{7\pi}{2}$	∞



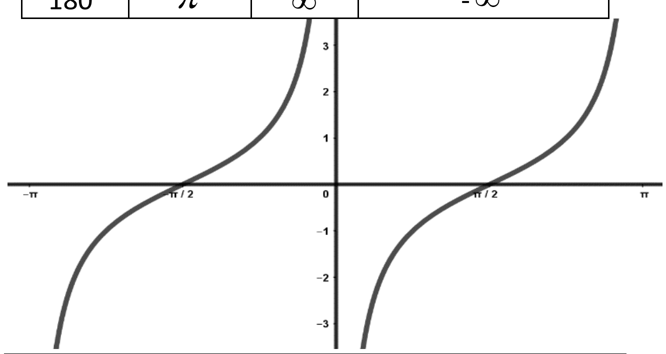
Q8. $y=-\cot x \qquad -\pi\le x\le\pi$

Solution: we have $y=-\cot x$

Given the domain for the function $-\pi\le x\le\pi$

To plot the given function we will find few values of the function within domain

x	x	$\cot x$	$y=-\cot x$
-180°	$-\pi$	$-\infty$	∞
-150°	$-\frac{5\pi}{6}$	$\sqrt{3}$	$-\sqrt{3}=-1.732$
-135°	$-\frac{3\pi}{4}$	1	-1
-120°	$-\frac{2\pi}{3}$	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}=-0.577$
-90°	$-\frac{\pi}{2}$	$-\infty$	∞
-60°	$-\frac{\pi}{3}$	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}=0.577$
-45°	$-\frac{\pi}{4}$	-1	1
-30°	$-\frac{\pi}{6}$	$-\sqrt{3}$	$\sqrt{3}=1.732$
0°	0	∞	$-\infty$
30°	$\frac{\pi}{6}$	$\sqrt{3}$	$-\sqrt{3}=-1.732$
45°	$\frac{\pi}{4}$	1	-1
60°	$\frac{\pi}{3}$	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}=-0.577$
90°	$\frac{\pi}{2}$	∞	$-\infty$
120°	$\frac{2\pi}{3}$	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}=0.577$
135°	$\frac{3\pi}{4}$	-1	1
150°	$\frac{5\pi}{6}$	$-\sqrt{3}$	$\sqrt{3}=1.732$
180°	π	∞	$-\infty$



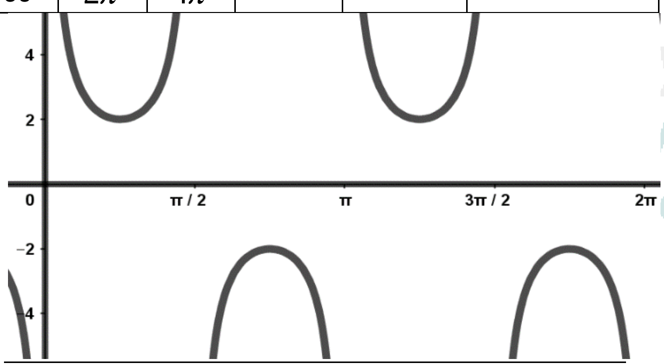
Q9. $y = 2\operatorname{cosec} 2x$ $\quad 0 \le x \le 2\pi$

Solution: we have $y = 2\operatorname{cosec} 2x$

Given the domain for the function $0 \le x \le 2\pi$

To plot the given function we will find few values of the function within domain

x	x	$2x$	Sin 2x	$\operatorname{cosec} 2x$	$2\operatorname{cosec} 2x$
0°	0	0	0	∞	∞
30°	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$	$\frac{4}{\sqrt{3}} = 2.309$
45°	$\frac{\pi}{4}$	$\frac{\pi}{2}$	1	1	2
60°	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$	$\frac{4}{\sqrt{3}} = 2.309$
90°	$\frac{\pi}{2}$	π	0	∞	∞
120°	$\frac{2\pi}{3}$	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{2}{\sqrt{3}}$	$\frac{-4}{\sqrt{3}} = -2.309$
135°	$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	-1	-1	-2
150°	$\frac{5\pi}{6}$	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{2}{\sqrt{3}}$	$\frac{-4}{\sqrt{3}} = -2.309$
180°	π	$\frac{\pi}{2}$	1	1	2
210°	$\frac{7\pi}{6}$	$\frac{7\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$	$\frac{4}{\sqrt{3}} = 2.309$
225°	$\frac{5\pi}{4}$	$\frac{5\pi}{2}$	1	1	2
240°	$\frac{4\pi}{3}$	$\frac{8\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}$	$\frac{4}{\sqrt{3}} = 2.309$
270°	$\frac{3\pi}{2}$	3π	0	∞	∞
300°	$\frac{5\pi}{3}$	$\frac{10\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{2}{\sqrt{3}}$	$\frac{-4}{\sqrt{3}} = -2.309$
315°	$\frac{7\pi}{4}$	$\frac{7\pi}{2}$	-1	-1	-2
330°	$\frac{11\pi}{6}$	$\frac{11\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{2}{\sqrt{3}}$	$\frac{-4}{\sqrt{3}} = -2.309$
360°	2π	4π	0	∞	∞



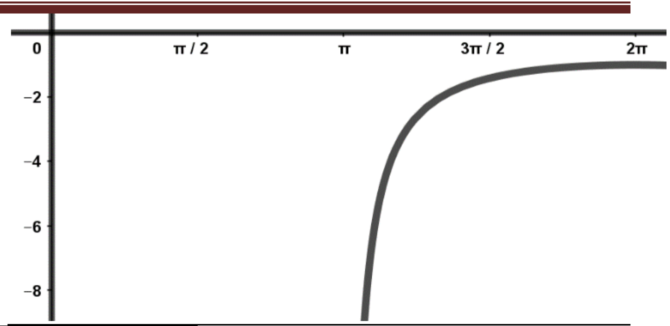
Q10. $y = \sec \frac{1}{2}x$ $\quad \pi \le x \le 2\pi$

Solution: we have $y = \sec \frac{1}{2}x$

Given the domain for the function $\pi \le x \le 2\pi$

To plot the given function we will find few values of the function within domain

x Degree	$\frac{x}{2}$ Degree	x Radians	$\frac{x}{2}$	$\sec \frac{1}{2}x$
180°	90°	π	$\frac{\pi}{2}$	∞
210°	105°	$\frac{7\pi}{6}$	$\frac{7\pi}{12}$	$-\sqrt{6}-\sqrt{2} = -3.864$
225°	112.5°	$\frac{5\pi}{4}$	$\frac{5\pi}{8}$	-0.383
240°	120°	$\frac{4\pi}{3}$	$\frac{2\pi}{3}$	-2
270°	135°	$\frac{3\pi}{2}$	3π	-1
300°	150°	$\frac{5\pi}{3}$	$\frac{5\pi}{6}$	$-\frac{2}{\sqrt{3}} = -1.155$
315°	157.5°	$\frac{7\pi}{4}$	$\frac{7\pi}{8}$	-0.924
330°	115°	$\frac{11\pi}{6}$	$\frac{11\pi}{12}$	$-\sqrt{6}+\sqrt{2} = -1.035$
360°	180°	2π	π	-1



Periodic Property

$$\sin(\theta \pm 2\pi) = \sin\theta$$
$$\cos(\theta \pm 2\pi) = \cos\theta$$
$$\tan(\theta \pm \pi) = \tan\theta$$

Translation Property

$$\sin(\theta - \pi) = -\sin\theta$$
$$\sin(\pi - \theta) = \sin\theta$$
$$\cos(\theta - \pi) = -\cos\theta$$
$$\cos(\pi - \theta) = -\cos\theta$$
$$\tan(\theta - \pi) = \tan\theta$$
$$\tan(\pi - \theta) = -\tan\theta$$

Odd Property

$$\sin(-\theta) = -\sin\theta$$
$$\tan(-\theta) = -\tan\theta$$

Even Property

$$\cos(-\theta) = \cos\theta$$

Exercise 12.4

Q1. Without drawing, guess graph of each of following functions. Also find its period, frequency and amplitude.

i). $y = \cos 2\theta$

Solution: we have $y = \cos 2\theta$

Period of $\cos x$ is 2π

Here $x = 2\theta$, means that there are 2 periods in the interval of length 2π

Graph of $\cos 2\theta$ will repeated two time in length of interval $[0, 2\pi]$

Now Period of $\cos 2\theta = \dfrac{\text{period of } \cos x}{2}$

Period of $\cos 2\theta = \dfrac{2\pi}{2} = \pi$

And frequency of $\cos 2\theta = \dfrac{2}{\text{period of } \cos x}$

frequency of $\cos 2\theta = \dfrac{2}{2\pi} = \dfrac{1}{\pi}$

And amplitude of $\cos 2\theta = 1$

ii). $y = \sin 6\theta$

Sol: we have $y = \sin 6\theta$ Period of $\sin x$ is 2π

Here $x = 6\theta$, means that there are 6 periods in the interval of length 2π

Graph of $\sin 6\theta$ will repeated six time in length of interval $[0, 2\pi]$

Now Period of $\sin 6\theta = \dfrac{\text{period of } \cos x}{6}$

Period of $\sin 6\theta = \frac{2\pi}{6} = \frac{\pi}{3}$

And frequency of $\sin 6\theta = \frac{6}{\text{period of } \cos x}$

frequency of $\sin 6\theta = \frac{6}{2\pi} = \frac{3}{\pi}$

And amplitude of $\sin 6\theta = 1$

iii). $y = \sin \pi\theta$

Solution we have $y = \sin \pi\theta$

Period of $\sin x$ is 2π

Here $x = \pi\theta$, means that there are π periods in the interval of length 2π

Graph of $\sin \pi\theta$ will repeated π time in length of interval $[0, 2\pi]$

Now Period of $\sin \pi\theta = \frac{\text{period of } \cos x}{\pi}$

Period of $\sin \pi\theta = \frac{2\pi}{\pi} = 2$

And frequency of $\sin \pi\theta = \frac{\pi}{\text{period of } \cos x}$

frequency of $\sin \pi\theta = \frac{\pi}{2\pi} = \frac{1}{2}$

And amplitude of $\sin \pi\theta = 1$

iv). $y = \cos \frac{\pi}{2}\theta$

Solution we have $y = \cos \frac{\pi}{2}\theta$

Period of $\cos x$ is 2π

Here $x = \pi\theta$, means that there are π periods in the interval of length 2π

Graph of $\cos \frac{\pi}{2}\theta$ will repeated π time in length of interval $[0, 2\pi]$

Now Period of $\cos \frac{\pi}{2}\theta = \frac{\text{period of } \cos x}{\frac{\pi}{2}}$

Period of $\cos \frac{\pi}{2}\theta = \frac{2\pi}{\frac{\pi}{2}} = 4$

And frequency of $\cos \frac{\pi}{2}\theta = \frac{\frac{\pi}{2}}{\text{period of } \cos x}$

frequency of $\cos \frac{\pi}{2}\theta = \frac{\frac{\pi}{2}}{2\pi} = \frac{1}{4}$

And amplitude of $\cos \pi\theta = 1$

Q2. Use symmetric and periodic properties of the sine, cosine and tangent functions to establish the following identities.

i). $\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$

Solution: we have $\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$

Take LHS $\sin\left(\frac{\pi}{2} + \theta\right) = \sin\left(\theta + \frac{\pi}{2}\right)$

$\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$

ii). $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$

Solution: we have $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$

Take LHS $\cos\left(\frac{\pi}{2} + \theta\right)$

$\cos\left(\frac{\pi}{2} + \theta\right) = \cos\left(\theta + \frac{\pi}{2}\right)$

$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$

iii). $\sin(\pi - \theta) = \sin\theta$

Solution: we have $\sin(\pi - \theta) = \sin\theta$

Take LHS $\sin(\pi - \theta)$

$\sin(\pi - \theta) = \sin\theta$ Translation $\sin(\pi - \theta) = \sin\theta$

=RHS Hence proved

iv). $\cos(\pi - \theta) = -\cos\theta$

Solution: we have $\cos(\pi - \theta) = -\cos\theta$

Take LHS $\cos(\pi - \theta)$

$\cos(\pi - \theta) = -\cos\theta$ Translation $\cos(\pi - \theta) = -\cos\theta$

=RHS Hence proved

v). $\sin(\pi + \theta) = -\sin\theta$

Solution: we have $\sin(\pi + \theta) = -\sin\theta$

Take LHS $\sin(\pi + \theta)$

$\sin(\pi + \theta) = \sin(\theta + \pi + \pi - \pi)$

$\sin(\pi + \theta) = \sin(\theta + 2\pi - \pi)$

$\sin(\pi + \theta) = \sin(\theta - \pi)$ Periodic $\sin(\theta + 2\pi) = \sin\theta$

$\sin(\pi + \theta) = -\sin\theta$ Translation $\sin(\theta - \pi) = -\sin\theta$

= RHS hence proved

vi). $\cos(\pi + \theta) = -\cos\theta$

Solution: we have $\cos(\pi + \theta) = -\cos\theta$

Take LHS $\cos(\pi + \theta)$

$\cos(\pi + \theta) = \cos(\pi + \pi + \theta - \pi)$

$\cos(\pi + \theta) = \cos(2\pi + \theta - \pi)$

$\cos(\pi + \theta) = \cos(\theta - \pi)$ Periodic $\cos(2\pi + \theta) = \cos\theta$

$\cos(\pi + \theta) = -\cos\theta$ Translation $\cos(\theta - \pi) = -\cos\theta$

= RHS hence proved

vii). $\tan(\pi - \theta) = -\tan\theta$

Solution: we have $\tan(\pi - \theta) = -\tan\theta$

Take LHS $\tan(\pi - \theta)$

$\tan(\pi - \theta) = -\tan\theta$ Translation $\tan(\pi - \theta) = -\tan\theta$

=RHS Hence proved

viii). $\tan(2\pi-\theta)=-\tan\theta$

Solution: we have $\tan(2\pi-\theta)=-\tan\theta$

Take LHS $\tan(2\pi-\theta)$

$\tan(2\pi-\theta)=\tan(-\theta+2\pi)$

$\tan(2\pi-\theta)=\tan\{-(\theta-2\pi)\}$

$\tan(2\pi-\theta)=-\tan(\theta-2\pi)$ Odd $\tan(-\theta)=-\tan\theta$

$\tan(2\pi-\theta)=-\tan(\theta-\pi-\pi)$

$\tan(2\pi-\theta)=-\tan(\theta-\pi)$ Periodic $\tan(\theta-\pi)=\tan\theta$

$\tan(2\pi-\theta)=-\tan\theta$ Again Periodic $\tan(\theta-\pi)=\tan\theta$

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ix). $\sin\left(\frac{3\pi}{2}+\theta\right)=-\cos\theta$

Solution: we have $\sin\left(\frac{3\pi}{2}+\theta\right)=-\cos\theta$

Take LHS $\sin\left(\frac{3\pi}{2}+\theta\right)$

$\sin\left(\frac{3\pi}{2}+\theta\right)=\sin\left(\frac{3\pi}{2}+\frac{\pi}{2}+\theta-\frac{\pi}{2}\right)$

$\sin\left(\frac{3\pi}{2}+\theta\right)=\sin\left(2\pi+\theta-\frac{\pi}{2}\right)$

$\sin\left(\frac{3\pi}{2}+\theta\right)=\sin\left(\theta-\frac{\pi}{2}\right)$ Periodic $\sin(2\pi+\theta)=\sin\theta$

$\sin\left(\frac{3\pi}{2}+\theta\right)=-\cos\theta$

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x). $\cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta$

Solution: we have $\cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta$

Take LHS $\cos\left(\frac{3\pi}{2}+\theta\right)$

$\cos\left(\frac{3\pi}{2}+\theta\right)=\cos\left(\frac{3\pi}{2}+\frac{\pi}{2}+\theta-\frac{\pi}{2}\right)$

$\cos\left(\frac{3\pi}{2}+\theta\right)=\cos\left(2\pi+\theta-\frac{\pi}{2}\right)$

$\cos\left(\frac{3\pi}{2}+\theta\right)=\cos\left(\theta-\frac{\pi}{2}\right)$ Periodic $\cos(2\pi+\theta)=\cos\theta$

$\cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta$

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xi). $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$

Solution: we have $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$

Take LHS $\sin\left(\frac{\pi}{2}-\theta\right)$

$\sin\left(\frac{\pi}{2}-\theta\right)=\sin\left(-\theta+\frac{\pi}{2}\right)$

$\sin\left(\frac{\pi}{2}-\theta\right)=\sin\left\{-\left(\theta-\frac{\pi}{2}\right)\right\}$

$\sin\left(\frac{\pi}{2}-\theta\right)=-\sin\left(\theta-\frac{\pi}{2}\right)$ Odd $\sin(-\theta)=-\sin\theta$

$\sin\left(\frac{\pi}{2}-\theta\right)=-(-\cos\theta)$

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$

---

xii). $\sin\left(-\theta-\frac{\pi}{2}\right)=-\cos\theta$

Solution: we have $\sin\left(-\theta-\frac{\pi}{2}\right)=-\cos\theta$

Take LHS $\sin\left(-\theta-\frac{\pi}{2}\right)$

$\sin\left(-\theta-\frac{\pi}{2}\right)=\sin\left\{-\left(\theta+\frac{\pi}{2}\right)\right\}$

$\sin\left(-\theta-\frac{\pi}{2}\right)=-\sin\left(\theta+\frac{\pi}{2}\right)$ Odd $\sin(-\theta)=-\sin\theta$

$\sin\left(-\theta-\frac{\pi}{2}\right)=-\cos\theta$

Q3. For any integer k, deduce that

i). $\sin(\theta+2k\pi)=\sin\theta$

Sol; we have $\sin(\theta+2k\pi)=\sin\theta$ Let K=1

$\sin(\theta+2\pi)=\sin\theta$

∵ Period of sine is 2π

Or Take LHS

$\sin(\theta+2\pi)$

$=\sin\theta\cos 2\pi+\cos\theta\sin 2\pi$

$=\sin\theta.1+\cos\theta.0$ $\because \cos 2\pi=1, \sin 2\pi=0$

$=\sin\theta=RHS$

Hence proved

---

ii). $\cos(\theta+2k\pi)=\cos\theta$

Sol; we have $\cos(\theta+2k\pi)=\cos\theta$ Let K=1

$\cos(\theta+2\pi)=\cos\theta$

∵ Period of cosine is 2π

Or Take LHS

$\cos(\theta+2\pi)$

$=\cos\theta\cos 2\pi-\sin\theta\sin 2\pi$

$=\cos\theta.1+\sin\theta.0$ $\because \cos 2\pi=1, \sin 2\pi=0$

$=\cos\theta=RHS$

Hence proved

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iii). $\tan(\theta+2k\pi)=\tan\theta$

Sol; we have $\tan(\theta+2k\pi)=\tan\theta$ Let K=1

$\tan(\theta+2\pi)=\tan(\overline{\theta+\pi}+\pi)$ let $\varphi=\theta+\pi$

$\tan(\overline{\theta+\pi})=\tan\theta$

$\because$ Period of tangent is π

Or Take LHS

$$\tan(\theta+2\pi)=\frac{\tan\theta+\tan2\pi}{1-\tan\theta\tan2\pi}$$

$$=\frac{\tan\theta+0}{1-(\tan\theta).(0)}=\frac{\tan\theta}{1} \qquad \because \tan2\pi=0$$

$$=\tan\theta=RHS$$

Hence proved

iv). $\cot(\theta+2k\pi)=\cot\theta$

Sol; we have $\cot(\theta+2k\pi)=\cot\theta$ Let K=1

$$\cot(\theta+2\pi)=\frac{1}{\tan(\theta+2\pi)}$$

$$=\frac{1}{\tan(\overline{\theta+\pi}+\pi)} \qquad \text{let } \varphi=\theta+\pi$$

$$=\frac{1}{\tan(\overline{\theta+\pi})}$$

$$=\frac{1}{\tan\theta}$$

$$=\cot\theta$$

$\because$ Period of tangent is π

Or Take LHS

$$\cot(\theta+2\pi)=\frac{\cos(\theta+2\pi)}{\sin(\theta+2\pi)}$$

$$=\frac{\cos\theta\cos2\pi-\sin\theta\sin2\pi}{\sin\theta\cos2\pi+\cos\theta\sin2\pi} \qquad \because \sin2\pi=0$$

$$=\frac{\cos\theta.1-\sin\theta.0}{\sin\theta.1+\cos\theta.0} \qquad \because \cos2\pi=1$$

$$=\frac{\cos\theta}{\sin\theta}$$

$$=\tan\theta=RHS$$

Hence proved

v). $\sec(\theta+2k\pi)=\sec\theta$

Sol; we have $\sec(\theta+2k\pi)=\sec\theta$ Let K=1

$$\sec(\theta+2\pi)=\frac{1}{\cos(\theta+2\pi)}$$

$$=\frac{1}{\cos\theta}$$

$$=\sec\theta$$

$\because$ Period of cosine is 2π

Or Take LHS

$$\sec(\theta+2\pi)=\frac{1}{\cos(\theta+2\pi)}$$

$$=\frac{1}{\cos\theta\cos2\pi-\sin\theta\sin2\pi}$$

$$=\frac{1}{\cos\theta.1+\sin\theta.0} \qquad \because \cos2\pi=1,\sin2\pi=0$$

$$=\frac{1}{\cos\theta}$$

$$=\sec\theta=RHS$$

Hence proved

vi). $\cos ec(\theta+2k\pi)=\cos ec\theta$

Sol; we have $\cos ec(\theta+2k\pi)=\cos ec\theta$ Let K=1

$$\cos ec(\theta+2\pi)=\frac{1}{\sin(\theta+2\pi)}$$

$$=\frac{1}{\sin\theta}$$

$$=\cos ec\theta$$

$\because$ Period of sine is 2π

Or Take LHS

$$\cos ec(\theta+2\pi)=\frac{1}{\sin(\theta+2\pi)}$$

$$=\frac{1}{\sin\theta\cos2\pi+\cos\theta\sin2\pi}$$

$$=\frac{1}{\sin\theta.1+\cos\theta.0} \qquad \because \cos2\pi=1,\sin2\pi=0$$

$$=\frac{1}{\sin\theta}$$

$$=\cos ec\theta=RHS$$

Hence proved

Q4. Find the maximum and minimum of each of the following functions;

i). $y=-2+\frac{1}{2}\sin\left(\frac{1}{3}\theta+2\right)$

Solution; we have $y=-2+\frac{1}{2}\sin\left(\frac{1}{3}\theta+2\right)$

Compare with the expression

$a+b\sin\theta$ We get $a=-2, b=\frac{1}{2}$

The maximum value of the function

$$y=M=a+|b|$$

$$M=-2+\frac{1}{2}=\frac{-3}{2}$$

$\because$ The maximum value of sine is +1

The minimum value of the function $y=m=a-|b|$

$$m=-2-\frac{1}{2}=\frac{-5}{2}$$

ii). $y=5-4\sin(\theta+30)$

Solution; we have $y=5-4\sin(\theta+30)$

Compare with the expression

$a+b\sin\theta$ We get $a=5, b=-4$

The maximum value of the function

$$y=M=a+|b|$$

$$M=5+4=9$$

$\because$ The maximum value of sine is +1

The minimum value of the function $y=m=a-|b|$

$$m=5-4=1$$

iii). $y=\frac{1}{19-10\sin(3\theta-45)}$

Solution; we have $y=\frac{1}{19-10\sin(3\theta-45)}$

Compare with the expression

$a+b\sin\theta$ We get $a=19, b=10$

The maximum value of the function

$$y=M'=\frac{1}{m}=\frac{1}{a-|b|}$$

$$M'=\frac{1}{19-10}=\frac{1}{9}$$

∵ The maximum value of sine is +1

The minimum value of the function

$$y=m'=\frac{1}{M}=\frac{1}{a+|b|}$$

$$m'=\frac{1}{19+10}=\frac{1}{29}$$

---

iv). $y=\dfrac{1}{4\cos 2\pi\theta}$

Solution; we have $y=\dfrac{1}{4\cos 2\pi\theta}$

Compare with the expression

$a+b\sin\theta$ We get $a=0, b=4$

The maximum value of the function

$$y=M'=\frac{1}{m}=\frac{1}{a-|b|}$$

$$M'=\frac{1}{0-(-4)}=\frac{1}{4}$$

∵ The minimum value of cosine is -1

The minimum value of the function

$$y=m'=\frac{1}{M}=\frac{1}{a+|b|}$$

$$m'=\frac{1}{0-(4)}=-\frac{1}{4}$$

Exercise 12.5

Find all the solution of the trigonometric functions.

Q1. $\sin\theta=\frac{\sqrt{2}}{2}$

Solution; we have $\sin\theta=\frac{\sqrt{2}}{2}$

Since sine is positive in 1st and 2nd quadrants

In 1st quadrant	2nd quadrant
$\sin\theta=\frac{\sqrt{2}}{2}$	$\sin\theta=\frac{\sqrt{2}}{2}$
$\theta=\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$	$\theta=\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$
$\theta=\frac{\pi}{4}$	$\theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$

Since period of sine is 2π

$$S.S=\left\{\frac{\pi}{4}+2k\pi\right\} or \left\{\frac{3\pi}{4}+2k\pi\right\}, k\in Z$$

Q2. $\cos\theta=\dfrac{-\sqrt{3}}{2}$

Solution; we have $\cos\theta=\dfrac{-\sqrt{3}}{2}$

Since cosine is negative in 2nd and 3rd quadrants

In 2nd quadrant	3rd quadrant
$\cos\theta=\frac{-\sqrt{3}}{2}$	$\cos\theta=\frac{-\sqrt{3}}{2}$
$\theta=\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$	$\theta=\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$
$\theta=\pi-\frac{\pi}{6}$	$\theta=\pi+\frac{\pi}{6}$
$\theta=\frac{5\pi}{6}$	$\theta=\frac{7\pi}{6}$

Since period of cosine is 2π

$$S.S=\left\{\frac{5\pi}{6}+2k\pi\right\} or \left\{\frac{7\pi}{6}+2k\pi\right\}, k\in Z$$

Q3. $\tan\theta=\sqrt{3}$

Solution; we have $\tan\theta=\sqrt{3}$

Since tangent is positive in 1st and 3rd quadrants

In 1st quadrant	3rd quadrant
$\tan\theta=\sqrt{3}$	$\tan\theta=\sqrt{3}$
$\theta=\tan^{-1}\left(\sqrt{3}\right)$	$\theta=\tan^{-1}\left(\sqrt{3}\right)$
$\theta=\frac{\pi}{3}$	$\theta=\pi+\frac{\pi}{3}=\frac{4\pi}{3}$

Since period of tangent is π

$$S.S=\left\{\frac{\pi}{3}+k\pi\right\}, k\in Z$$

Q4. $\cos\theta=-1$

Solution; we have $\cos\theta=-1$

Since this value of cosine is at 180° or π

$$\cos\theta=-1$$

$$\theta=\cos^{-1}(-1)$$

$$\theta=\pi$$

$$S.S=\{(2k+1)\pi\}, k\in Z$$

Q5. $\tan\theta=-1$

Solution; we have $\tan\theta=-1$

Since tangent is negative in 2nd and 4th quadrants

In 2nd quadrant	4th quadrant
$\tan\theta=-1$	$\tan\theta=-1$
$\theta=\tan^{-1}(-1)$	$\theta=\tan^{-1}(-1)$
$\theta=\pi-\frac{\pi}{4}$	$\theta=2\pi-\frac{\pi}{4}$
$\theta=\frac{3\pi}{4}$	$\theta=\frac{7\pi}{4}$

Since period of tangent is π

$$S.S=\left\{\frac{3\pi}{4}+k\pi\right\}, k\in Z$$

Q6. $\cos\theta=\dfrac{\sqrt{2}}{2}$

Solution; we have $\cos\theta=\dfrac{\sqrt{2}}{2}$

Since cosine is positive in 1st and 4th quadrants

In 1st quadrant	4th quadrant
$\cos\theta=\frac{\sqrt{2}}{2}$	$\cos\theta=\frac{\sqrt{2}}{2}$

$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ $\qquad$ $\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\theta = \frac{\pi}{4}$ $\qquad$ $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

Since period of cosine is 2π

$S.S = \left\{\frac{\pi}{4} + 2k\pi\right\} or \left\{\frac{7\pi}{4} + 2k\pi\right\}, k \in Z$

Q7. $\tan\theta = 0$

Solution; we have $\tan\theta = 0$

The value of the tangent is at the two points 0° or 180°

$\tan\theta = 0$ $\qquad$ $\tan\theta = 0$

$\theta = \tan^{-1}(0)$ $\qquad$ $\theta = \tan^{-1}(0)$

$\theta = 0$ $\qquad$ $\theta = \pi$

Since period of tangent is π

$S.S = \{k\pi\}, k \in Z$

Q8. $\tan\theta = \frac{\sqrt{3}}{3}$

Solution; we have $\tan\theta = \frac{\sqrt{3}}{3}$

Since tangent is positive in 1st and 3rd quadrants

In 1st quadrant $\qquad$ 3rd quadrant

$\tan\theta = \frac{\sqrt{3}}{3}$ $\qquad$ $\tan\theta = \frac{\sqrt{3}}{3}$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ $\qquad$ $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$\theta = \frac{\pi}{6}$ $\qquad$ $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

Since period of tangent is π

$S.S = \left\{\frac{\pi}{6} + k\pi\right\}, k \in Z$

Q9. $\sin\theta = -\frac{\sqrt{2}}{2}$

Solution; we have $\sin\theta = -\frac{\sqrt{2}}{2}$

Since sine is negative in 3rd and 4th quadrants

In 3rd quadrant $\qquad$ 4th quadrant

$\sin\theta = -\frac{\sqrt{2}}{2}$ $\qquad$ $\sin\theta = -\frac{\sqrt{2}}{2}$

$\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $\qquad$ $\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$ $\qquad$ $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

Since period of sine is 2π

$S.S = \left\{\frac{5\pi}{4} + 2k\pi\right\} or \left\{\frac{7\pi}{4} + 2k\pi\right\}, k \in Z$

Q10. $\cos\theta = 0$

Solution; we have $\cos\theta = 0$

The value of cosine is at the two points 90° or 270°

$\cos\theta = 0$ $\qquad$ $\cos\theta = 0$

$\theta = \cos^{-1}(0)$ $\qquad$ $\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}$ $\qquad$ $\theta = \frac{3\pi}{2}$

Since period of cosine is 2π

$S.S = \left\{\frac{\pi}{2} + 2k\pi\right\}, k \in Z$

In problems 11 –14, use the graph to estimate the solution of each equation.

Q11. $2\sin\theta - \theta = 0$

Solution; we have $2\sin\theta - \theta = 0$

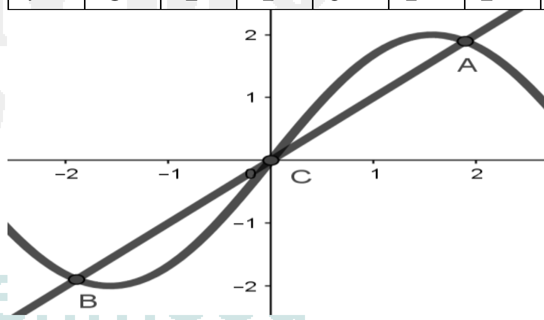
OR $\qquad 2\sin\theta = \theta$

Let $y = 2\sin\theta \;\&\; y = \theta$

x	$\sin x$	$2\sin x$
$-\pi$	0	0
$\frac{-3\pi}{4}$	$\frac{-1}{\sqrt{2}} = -0.707$	$-\sqrt{2} = -1.414$
$-\frac{\pi}{2}$	-1	-2
$-\frac{\pi}{4}$	$\frac{-1}{\sqrt{2}} = -0.707$	$-\sqrt{2} = -1.414$
0	0	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}} = 0.707$	$\sqrt{2} = 1.414$
$\frac{\pi}{2}$	1	2
$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}} = 0.707$	$\sqrt{2} = 1.414$
π	0	0

And $y = \theta$

θ	-3	-2	-1	0	1	2	3
Y	-3	-2	-1	0	1	2	3



From the graph the solution set

$S.S = \{(1.9, 1.9), (-1.9, -1.9)\}$

Q12. $\tan\theta = 2\theta$

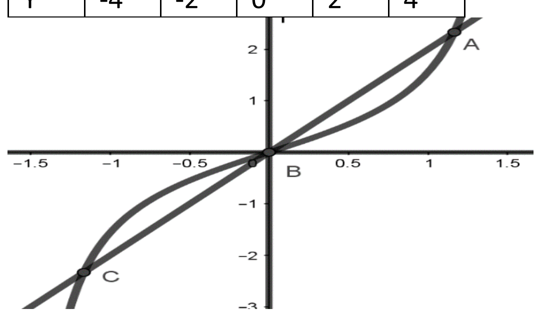
Solution; we have $\tan\theta = 2\theta$

Let $y = \tan\theta$

x	$\tan\theta$
$\frac{-\pi}{2}$	-∞
$\frac{-\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	∞

And $y = 2\theta$

θ	-2	-1	0	1	2
Y	-4	-2	0	2	4



From the graph the solution set

$S.S=\{(1.17,2.33),(-1.17,-2.33)\}$

Q13. $\cos\theta=\theta^2$

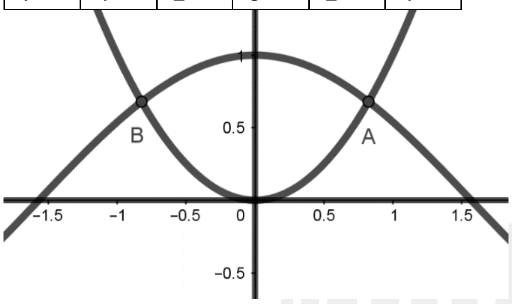
Solution; we have $\cos\theta=\theta^2$

Let $y=\cos\theta$

x	$\cos\theta$
$\frac{-\pi}{2}$	0
$\frac{-\pi}{4}$	$\frac{\sqrt{2}}{2}$
0	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{2}$	0

$y=\theta^2$

θ	-2	-1	0	1	2
Y	4	1	0	1	4



From the graph the solution set

$S.S=\{(0.84,0.67),(-0.84,0.67)\}$

Q14. $\tan\theta=1+\theta$

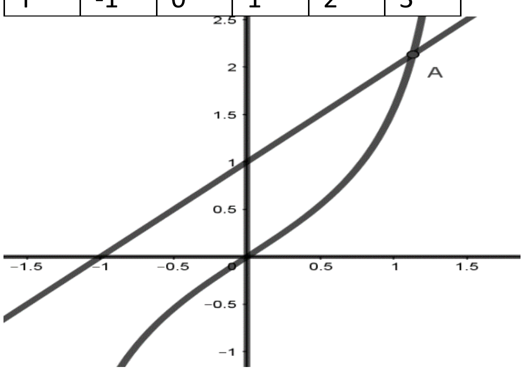
Solution; we have $\tan\theta=1+\theta$

Let $y=\tan\theta$

x	$\tan\theta$
$\frac{-\pi}{2}$	-∞
$\frac{-\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	∞

And $y=1+\theta$

θ	-2	-1	0	1	2
Y	-1	0	1	2	3



From the graph the solution set

$S.S=\{(1.13,2.13)\}$

Exercise 12.6

Q. Evaluate the following expressions without using tables or calculators.

i). Arc $\sin(-1)$

Solution: we have Arc $\sin(-1)$

Let $y=\sin^{-1}(-1)$

$\Rightarrow \sin y=(-1) \quad ; y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\Rightarrow y=-\frac{\pi}{2}$

Hence $y=\sin^{-1}(-1)=-\frac{\pi}{2}$

ii). Arc $\cos(-1)$

Solution: Let $y=\cos^{-1}(-1)$

$\Rightarrow \cos y=(-1) \quad ; y\in[-\pi,\pi]$

$\Rightarrow y=\pi$

Hence $y=\cos^{-1}(-1)=\pi$

iii). Arc $\tan(-1)$

Solution: Let $y=\tan^{-1}(-1)$

$\Rightarrow \tan y=(-1) \quad ; y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\Rightarrow y=-\frac{\pi}{4}$

$y=\tan^{-1}(-1)=-\frac{\pi}{4}$

iv). Arc $\sin\left(\frac{1}{2}\right)$

Solution: we have Arc $\sin\left(\frac{1}{2}\right)$

Let $y=\sin^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \sin y=\left(\frac{1}{2}\right) \quad ; y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\Rightarrow y=\frac{\pi}{6}$

Hence $y=\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

v). $\cos ec^{-1}(-\sqrt{2})$

Solution: we have $\cos ec^{-1}(-\sqrt{2})$

Let $y=\cos ec^{-1}(-\sqrt{2})$

$\Rightarrow \cos ecy=(-\sqrt{2}) \quad ; y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\Rightarrow y=-\frac{\pi}{4}$

Hence $y=\sin^{-1}(-1)=-\frac{\pi}{4}$

vi). $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Solution: we have $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Let $y=\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$\Rightarrow \sec y=\left(\frac{2}{\sqrt{3}}\right) \quad ; y\in[0,\pi]$

$\Rightarrow y=\frac{\pi}{6}$

Hence $y=\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=\frac{\pi}{6}$

Q2. Evaluate the following inverse relations of general trigonometric functions.

i). Arc $\sin(-1)$

Solution: we have Arc $\sin(-1)$

Let $y=\sin^{-1}(-1)$

$\Rightarrow \sin y=(-1) \quad ;\Rightarrow y=-\frac{\pi}{2}$ or $y=\frac{3\pi}{2}$

Since period of sin x is 2π

$So \quad y\in\left\{-\frac{\pi}{2}+2k\pi\right\}, k\in Z$ Or

$S.S=\left\{\frac{3\pi}{2}+2k\pi\right\}, k\in Z$

---

ii). Arc $\cos(1)$

Solution: we have Arc $\cos(1)$

Let $y=\cos^{-1}(1)$

$\Rightarrow \cos y=(1)$

$\Rightarrow y=0, \pm 2\pi, \ldots$

Since period of cos x is 2π

$So \quad y\in\{2k\pi\}, k\in Z$

---

iii). Arc $\cos\left(-\frac{\sqrt{2}}{2}\right)$

Solution: we have Arc $\cos\left(-\frac{\sqrt{2}}{2}\right)$

Let $y=\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\Rightarrow \cos y=\left(-\frac{\sqrt{2}}{2}\right)$

$\Rightarrow y=\frac{3\pi}{4}, \frac{5\pi}{4}, \ldots$

Since period of cos x is 2π

$So \quad y\in\left\{\frac{3\pi}{4}+2k\pi\right\}\cup\left\{\frac{5\pi}{4}+2k\pi\right\}, k\in Z$

---

iv). Arc $\tan(0)$

Solution: we have Arc $\tan(0)$

Let $y=\tan^{-1}(0)$

$\Rightarrow \tan y=(0)$

$\Rightarrow y=0, \pm\pi, \pm 2\pi, \ldots$

Since period of tan x is π

$So \quad y\in\{k\pi\}, k\in Z$

---

v). Arc $\tan\left(-\frac{\sqrt{3}}{3}\right)$

Solution: we have Arc $\tan\left(-\frac{\sqrt{3}}{3}\right)$

Let $y=\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

$\Rightarrow \tan y=\left(-\frac{\sqrt{3}}{3}\right)$

$\Rightarrow y=\frac{-\pi}{6}, \frac{5\pi}{6}, \ldots$

Since period of tan x is π

$So \quad y\in\left\{\frac{-\pi}{6}+k\pi\right\}\cup\left\{\frac{5\pi}{6}+k\pi\right\}, k\in Z$

---

vi). Arc $\cos\left(-\frac{\sqrt{3}}{2}\right)$

Solution: we have Arc $\cos\left(-\frac{\sqrt{3}}{2}\right)$

Let $y=\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \cos y=\left(-\frac{\sqrt{3}}{2}\right)$

$\Rightarrow y=-\frac{\pi}{6}, \frac{5\pi}{6}, \ldots$

Since period of cos x is 2π

$So \quad y\in\left\{-\frac{\pi}{6}+2k\pi\right\}\cup\left\{\frac{5\pi}{6}+2k\pi\right\}, k\in Z$

Q3. Use a calculator to find the approximate measures in radians of the inverse functions.

i). $\sin^{-1}(0.1)=0.1002$

ii). $\cos^{-1}(0.6)=0.9273$

iii). $\tan^{-1}(5)=1.3734$

iv). $\tan^{-1}(0.2)=0.1974$

v). $\cos^{-1}\left(\frac{7}{8}\right)=0.5054$

vi). $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)=1.07999$

Q4. Find the exact value of each expression.

i). $\cos\left[\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right]$

Solution: we have $\cos\left[\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right]$

Let $y=\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)=\frac{\pi}{4}$

So $\cos\left[\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right]=\cos\left(\frac{\pi}{4}\right)$

$=\frac{\sqrt{2}}{2}$

---

ii). $\tan\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

Solution: we have $\tan\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

Let $y=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$

So $\tan\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$=\tan\left(\frac{\pi}{6}\right)$

$=\frac{\sqrt{3}}{3}$

---

iii). $\sec\left[\cos^{-1}\left(\frac{1}{2}\right)\right]$

Solution: we have $\sec\left[\cos^{-1}\left(\frac{1}{2}\right)\right]$

Let $y=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

So $\sec\left[\cos^{-1}\left(\frac{1}{2}\right)\right]$

$= \sec\left(\frac{\pi}{3}\right)$

$= 2$

iv). $\operatorname{cosec}\left[\tan^{-1}(1)\right]$

Solution: we have $\operatorname{cosec}\left[\tan^{-1}(1)\right]$

Let $y = \tan^{-1}(1) = \frac{\pi}{4}$

So $\operatorname{cosec}\left[\tan^{-1}(1)\right]$

$= \operatorname{cosec}\left(\frac{\pi}{4}\right)$

$= \sqrt{2}$

v). $\sin\left[\tan^{-1}(-1)\right]$

Solution: we have $\sin\left[\tan^{-1}(-1)\right]$

Let $y = \tan^{-1}(-1) = -\frac{\pi}{4}$

So $\sin\left[\tan^{-1}(-1)\right]$

$= \sin\left(-\frac{\pi}{4}\right)$

$= -\frac{\sqrt{2}}{2}$

iv). $\sec\left[\sin^{-1}\left(\frac{1}{2}\right)\right]$

Solution: we have $\sec\left[\sin^{-1}\left(\frac{1}{2}\right)\right]$

Let $y = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

So $\sec y = \sec\left[\sin^{-1}\left(\frac{1}{2}\right)\right]$

$= \sec\left(\frac{\pi}{6}\right)$

$= \frac{2}{\sqrt{3}}$

Q5. Compute the following expressions which involve principal as well as general trigonometric functions and their inverses.

i). $\sin\left[\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$

Solution: we have $\sin\left[\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$

Let $y = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

And the period of tan is π

So $y = \frac{\pi}{6} + k\pi; k \in Z$

So $\sin y = \sin\left[\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$

$= \sin\left(\frac{\pi}{6} + k\pi\right)$

$= \pm\sin\frac{\pi}{6}$

$= \pm\frac{1}{2}$

$S.S = \left\{-\frac{1}{2}, \frac{1}{2}\right\}$

ii). $\sin\left[Tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$

Solution: we have $\sin\left[Tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$

Let $y = Tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

So $\sin y = \sin\left[Tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$

$= \sin\left(\frac{\pi}{6}\right)$

$= \frac{1}{2}$

$S.S = \left\{\frac{1}{2}\right\}$

iii). $\sin\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$

Solution: we have $\sin\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$

Let $y = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

$y = \frac{5\pi}{6}, \frac{7\pi}{6}$

And the period of cos is 2π

$y = \left\{\frac{5\pi}{6} + 2k\pi\right\} \cup \left\{\frac{7\pi}{6} + 2k\pi\right\}; k \in Z$

So $\sin y = \sin\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$

$= \sin\left(\frac{5\pi}{6} + 2k\pi\right)$

$= \sin\frac{5\pi}{6}$

$= \frac{1}{2}$

And $\sin y = \sin\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$

$= \sin\left(\frac{7\pi}{6} + 2k\pi\right)$

$= \sin\frac{7\pi}{6}$

$= -\frac{1}{2}$

$S.S = \left\{-\frac{1}{2}, \frac{1}{2}\right\}$

iv). $\cos^{-1}\left[\tan\left(\frac{3\pi}{4}\right)\right]$

Solution we have $\cos^{-1}\left[\tan\left(\frac{3\pi}{4}\right)\right]$

Let $y = \tan\left(\frac{3\pi}{4}\right) = -1$

So $\cos^{-1}\left[\tan\left(\frac{3\pi}{4}\right)\right] = \cos^{-1}(-1) = \pi$

Period of cos x is 2 π

So $\cos^{-1}(-1) = \{\pi + 2k\pi\}, k \in Z$

v). $Tan^{-1}\left[\tan\left(\frac{3\pi}{4}\right)\right]$

Solution: we have $Tan^{-1}\left[\tan\left(\frac{3\pi}{4}\right)\right]$

Let $y = \tan\left(\frac{3\pi}{4}\right) = -1$

So $Tan^{-1}\left[\tan\left(\frac{3\pi}{4}\right)\right]=Tan^{-1}(-1)=-\frac{\pi}{4}$

Period of Tan x is π

So $Tan^{-1}(-1)=\left\{-\frac{\pi}{4}+k\pi\right\}, k\in Z$

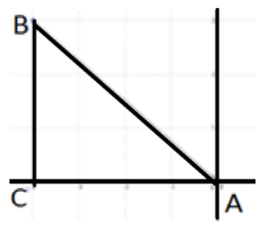
---

vi). $\tan\left[\cos^{-1}\left(\frac{-4}{5}\right)\right]$

Sol: $\tan\left[\cos^{-1}\left(\frac{-4}{5}\right)\right]$

Let $y=\cos^{-1}\left(\frac{-4}{5}\right)$...(1)

$\cos y=\frac{-4}{5}$



Using Pythagoras theorem

$AB^2=AC^2+BC^2$

$5^2=(-4)^2+BC^2$

$BC^2=25-16$

$BC^2=9$

$BC=\pm 3$

Take BC=3

$\tan y=\frac{-3}{4}$

$\Rightarrow y=\tan^{-1}\left(\frac{-3}{4}\right)$.........................(1)

Than $\tan\left[\tan^{-1}\left(\frac{-3}{4}\right)\right]=\frac{-3}{4}$

Exercise 12.7

Q1. Find x, if

i). $\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}-x$

Solution; we have $\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}-x$

$\left(\frac{1}{2}\right)=\sin\left(\frac{\pi}{2}-x\right)$

$\frac{1}{2}=\sin\frac{\pi}{2}\cos x-\cos\frac{\pi}{2}\sin x$

$\frac{1}{2}=1.\cos x-0.\sin x$

$\cos x=\frac{1}{2}\Rightarrow x=\cos^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow x=\frac{\pi}{3}$

---

ii). $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{2}-\sin^{-1}x$

Solution; we have $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{2}-\sin^{-1}x$

$\frac{\sqrt{3}}{2}=\cos\left(\frac{\pi}{2}-\sin^{-1}x\right)$

$\frac{\sqrt{3}}{2}=\cos\frac{\pi}{2}\cos.\left(\sin^{-1}x\right)+\sin\frac{\pi}{2}\sin\left(\sin^{-1}x\right)$

$\frac{\sqrt{3}}{2}=0.\cos\left(\sin^{-1}x\right)+1.\sin\left(\sin^{-1}x\right)$

$\Rightarrow x=\frac{\sqrt{3}}{2}$ using $\left(\sin.\sin^{-1}\theta=\theta\right)$

---

iii). $\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}-x$

Solution; we have $\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}-x$

$\left(\frac{1}{2}\right)=\sin\left(\frac{\pi}{2}-x\right)$

$\frac{1}{2}=\sin\frac{\pi}{2}\cos x-\cos\frac{\pi}{2}\sin x$

$\frac{1}{2}=1.\cos x-0.\sin x$

$\cos x=\frac{1}{2}$

$\Rightarrow x=\cos^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow x=\frac{\pi}{3},\frac{5\pi}{3}$

As period of cos x is 2π

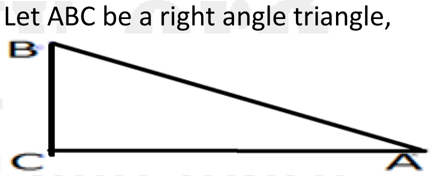
So $x\in\left\{\frac{\pi}{3}+2k\pi\right\}\cup\left\{\frac{5\pi}{3}+2k\pi\right\}, k\in Z$

Q2. Show that

i). $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

solution; we have $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

Let ABC be a right angle triangle,



angles α,β are the complementary angles

i.e., $\alpha+\beta=\frac{\pi}{2}$.........(1)

$\Rightarrow\beta=\frac{\pi}{2}-\alpha$

$\Rightarrow\sin\beta=\sin\left(\frac{\pi}{2}-\alpha\right)$

$\sin\beta=\sin\frac{\pi}{2}\cos\alpha+\cos\frac{\pi}{2}\sin\alpha$

$\sin\beta=1.\cos\alpha+0.\sin\alpha$

$\sin\beta=\cos\alpha$

$\Rightarrow\sin\beta=\cos\alpha=x$

$\Rightarrow\sin\beta=x,\quad \cos\alpha=x$

$\Rightarrow\beta=\sin^{-1}x,\ \alpha=\cos^{-1}x$

Putting the values in equation (1)

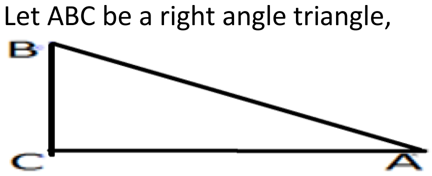
$\cos^{-1}x+\sin^{-1}x=\frac{\pi}{2}$ Hence proved

---

ii). $\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)=\frac{\pi}{2}$

solution; we have $\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)=\frac{\pi}{2}$

Let ABC be a right angle triangle,



angles α,β are the complementary angles

i.e., $\alpha+\beta=\frac{\pi}{2}$.........(1)

$\Rightarrow \beta=\frac{\pi}{2}-\alpha$

$\Rightarrow \tan\beta=\tan\left(\frac{\pi}{2}-\alpha\right)$

$\tan\beta=\frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\cos\left(\frac{\pi}{2}-\alpha\right)}$

$\tan\beta=\frac{\sin\frac{\pi}{2}\cos\alpha+\cos\frac{\pi}{2}\sin\alpha}{\cos\frac{\pi}{2}\cos\alpha-\sin\frac{\pi}{2}\sin\alpha}$

$\tan\beta=\frac{1.\cos\alpha-0.\sin\alpha}{0.\cos\alpha+1.\sin\alpha}$

$\tan\beta=\frac{\cos\alpha}{\sin\alpha}$

$\Rightarrow \tan\beta=\cot\alpha=x$

$\Rightarrow \tan\beta=x,\quad \cot\alpha=x$

$\Rightarrow \beta=\sin^{-1}x,\ \tan\alpha=\frac{1}{x}$

$\alpha=\tan^{-1}\left(\frac{1}{x}\right)$

Putting the values in equation (1)

$\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)=\frac{\pi}{2}$ Hence proved

---

Q3. Show that $\sec(Arc\tan x)=\sqrt{1+x^2}$

Solution; we have $\sec(Arc\tan x)=\sqrt{1+x^2}$

Let $\theta=\tan^{-1}x \qquad \Rightarrow x=\tan\theta$.................(1)

Taking LHS $\sec(Arc\tan x)$

$=\sec(\theta)$

$=\sqrt{\sec^2\theta}$

$=\sqrt{1+\tan^2\theta}$

$=\sqrt{1+x^2}$ using $\tan\theta=x$

= RHS, Hence proved

Q4. Show that $\tan(Arc\sin x)=\frac{x}{\sqrt{1-x^2}}$

Solution; we have $\tan(Arc\sin x)=\frac{x}{\sqrt{1-x^2}}$

Let $\theta=\sin^{-1}x\Rightarrow x=\sin\theta$....(1)

Taking LHS $\tan(Arc\sin x)$

$=\tan(\theta)$

$=\frac{\sin\theta}{\cos\theta}$

$=\frac{\sin\theta}{\sqrt{\cos^2\theta}}$

$=\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

$=\frac{x}{\sqrt{1-x^2}}$

= RHS, Hence proved

Q5. Show that $\tan(Arc\sec x)=\sqrt{x^2-1}$

Solution; we have $\tan(Arc\sec x)=\sqrt{x^2-1}$

Let $\theta=\sec^{-1}x\Rightarrow x=\sec\theta$....(1)

Taking LHS

$\tan(Arc\sec x)=\tan(\theta)$

$=\sqrt{\tan^2\theta}$

$=\sqrt{\sec^2\theta-1}$

$=\sqrt{x^2-1}$

= RHS, Hence proved

Q6. Evaluate

i). $\sin\left[\frac{\pi}{2}-\cos^{-1}\frac{4}{5}\right]$

Solution: we have $\sin\left[\frac{\pi}{2}-\cos^{-1}\frac{4}{5}\right]$

Take LHS $\sin\left[\frac{\pi}{2}-\cos^{-1}\frac{4}{5}\right]$

$=\sin\frac{\pi}{2}\cos\left(\cos^{-1}\frac{4}{5}\right)-\cos\frac{\pi}{2}\sin\left(\cos^{-1}\frac{4}{5}\right)$

$=1.\left(\frac{4}{5}\right)-0.\sin\left(\cos^{-1}\frac{4}{5}\right)$

$=\frac{4}{5}$

---

ii). $\sin\left[\cos^{-1}\frac{\pi}{2}+\pi\right]$

Solution; we have $\sin\left[\cos^{-1}\frac{\pi}{2}+\pi\right]$

$=\sin\left(\cos^{-1}\frac{\pi}{2}\right)\cos\pi+\sin\pi\cos\left(\cos^{-1}\frac{\pi}{2}\right)$

But $\cos^{-1}\frac{\pi}{2}$ does not exists.

Q7. Show that

$\cos\left(\sin^{-1}x-\sin^{-1}y\right)=\sqrt{(1-x^2)(1-y^2)}+xy$

Solution; we have

$\cos\left(\sin^{-1}x-\sin^{-1}y\right)=\sqrt{(1-x^2)(1-y^2)}+xy$

Let $\sin^{-1}x=A\Rightarrow x=\sin A$

$\sin^{-1}y=B\Rightarrow y=\sin B$

Now taking LHS $\cos\left(\sin^{-1}x-\sin^{-1}y\right)$

$=\cos(A-B)$

$=\cos A\cos B+\sin A\sin B$

$=\sqrt{\cos^2 A\cos^2 B}+\sin A\sin B$

$=\sqrt{(1-\sin^2 A)(1-\sin^2 B)}+\sin A\sin B$

$=\sqrt{(1-x^2)(1-y^2)}+xy$

=RHS Hence proved

Q8. Show that

i). $\cos\left(2\sin^{-1}x\right)=1-2x^2$

Solution; we have $\cos\left(2\sin^{-1}x\right)=1-2x^2$

Let $\theta=\sin^{-1}x\Rightarrow x=\sin\theta$

Taking LHS $\cos\left(2\sin^{-1}x\right)$

$=\cos(2\theta)$

$=\cos^2\theta-\sin^2\theta$

$=1-\sin^2\theta-\sin^2\theta$

$=1-2\sin^2\theta$

$=1-2x^2$

=RHS Hence proved.

ii). $2\cos^{-1}x=\cos^{-1}\left(2x^2-1\right)$

Solution; we have $2\cos^{-1}x=\cos^{-1}\left(2x^2-1\right)$

Let $\theta=\cos^{-1}x \qquad \Rightarrow x=\cos\theta$

Given that

$$2\cos^{-1}x=\cos^{-1}\left(2x^2-1\right)$$
$$\cos\left(2\cos^{-1}x\right)=\left(2x^2-1\right)$$
$$\cos\left(2\theta\right)=\left(2x^2-1\right)$$
$$\cos^2\theta-\sin^2\theta=2x^2-1$$
$$\cos^2\theta-\left(1-\cos^2\theta\right)=2x^2-1$$
$$2\cos^2\theta-1=2x^2-1$$
$$2x^2-1=2x^2-1$$
Hence proved.

iii). $\cos\left(\tan^{-1}x\right)=\dfrac{1}{\sqrt{1+x^2}}$

Solution; we have $\cos\left(\tan^{-1}x\right)=\dfrac{1}{\sqrt{1+x^2}}$

Let $\theta=\tan^{-1}x \qquad \Rightarrow x=\tan\theta$

Taking LHS $\cos\left(\tan^{-1}x\right)=\cos\left(\theta\right)$

$$=\frac{1}{\sec\theta}$$
$$=\frac{1}{\sqrt{\sec^2\theta}}$$
$$=\frac{1}{\sqrt{1+\tan^2\theta}}$$
$$=\frac{1}{\sqrt{1+x^2}}$$

=RHS Hence proved.

Q9. Evaluate the following expressions without using tables or calculator;

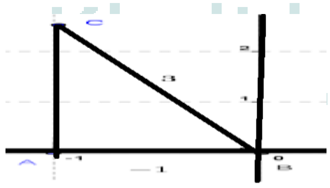
i). $\tan\left[\sec^{-1}(-3)\right]$

Sol: $\tan\left[\sec^{-1}(-3)\right]$

Let $\theta=\sec^{-1}(-3)$

$\Rightarrow \sec\theta=-3$

$\cos\theta=\dfrac{1}{-3}$



Using Pythagoras theorem

$$BC^2=AC^2+AB^2$$
$$3^2=AB^2+(-1)^2$$
$$AB^2=9-1=8$$
$$AB=\pm2\sqrt{2}$$

Take $AB=2\sqrt{2}$

Then $\tan\theta=\dfrac{2\sqrt{2}}{-1}=-2\sqrt{2}$

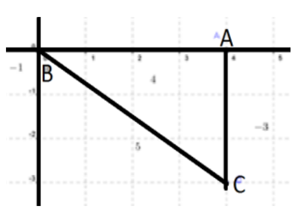
$\theta=\tan^{-1}\left(-2\sqrt{2}\right)$ Then

$$\tan\left[\sec^{-1}(-3)\right]=\tan\left[\theta\right]$$
$$=\tan\left[\tan^{-1}\left(-2\sqrt{2}\right)\right]=-2\sqrt{2}$$

ii). $\cos\left[\tan^{-1}\left(\dfrac{-3}{4}\right)\right]$

Sol: $\cos\left[\tan^{-1}\left(\dfrac{-3}{4}\right)\right]$

Let $\theta=\tan^{-1}\left(\dfrac{-3}{4}\right)$



$\Rightarrow \tan\theta=\dfrac{-3}{4}$

Using Pythagoras theorem

$$BC^2=AC^2+AB^2$$
$$BC^2=4^2+(-3)^2$$
$$BC^2=16+9=25$$
$$BC=\pm5$$

Take $BC=5$

Then $\cos\theta=\dfrac{4}{5}$

$\theta=\cos^{-1}\left(\dfrac{4}{5}\right)$

Then $\cos\left[\tan^{-1}\left(\dfrac{-3}{4}\right)\right]=\cos\left(\theta\right)$

$$=\cos\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$$
$$=\frac{4}{5}$$

iii). $\sin\left(\sin^{-1}\dfrac{4}{5}-\cos^{-1}\dfrac{3}{5}\right)$

Solution; we have $\sin\left(\sin^{-1}\dfrac{4}{5}-\cos^{-1}\dfrac{3}{5}\right)$

Let $\sin^{-1}\dfrac{4}{5}=A \qquad \sin^{-1}\dfrac{3}{5}=B$

$\Rightarrow \dfrac{4}{5}=\sin A \qquad \Rightarrow \dfrac{3}{5}=\sin B$

Now taking LHS $\sin\left(\sin^{-1}\dfrac{4}{5}-\cos^{-1}\dfrac{3}{5}\right)$

$$=\sin\left(A-B\right)$$
$$=\sin A\cos B-\cos A\sin B$$
$$=\sin A\cos B-\sqrt{\cos^2 A\sin^2 B}$$
$$=\sin A\cos B-\sqrt{\left(1-\sin^2 A\right)\left(1-\cos^2 B\right)}$$
$$=\frac{4}{5}\frac{3}{5}-\sqrt{\left(1-\left(\frac{4}{5}\right)^2\right)\left(1-\left(\frac{3}{5}\right)^2\right)}$$
$$=\frac{4}{5}.\frac{3}{5}-\sqrt{\frac{25-16}{25}.\frac{25-9}{25}}$$
$$=\frac{4}{5}.\frac{3}{5}-\sqrt{\frac{9}{25}.\frac{16}{25}}$$
$$=\frac{4}{5}.\frac{3}{5}-\frac{3}{5}.\frac{4}{5}$$
$$=0$$

Hence $\sin\left(\sin^{-1}\dfrac{4}{5}-\cos^{-1}\dfrac{3}{5}\right)=0$

Q10. Express the following in term of $\tan^{-1}x$

i). $\sin^{-1}x$

solution: we have $\sin^{-1}x$

Let $\theta=\sin^{-1}x\Rightarrow x=\sin\theta$

$\therefore \cos\theta=\sqrt{1-\sin^2\theta}$

than $\tan\theta=\dfrac{\sin\theta}{\cos\theta}=\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

$$\tan\theta=\frac{x}{\sqrt{1-x^2}}$$
$$\Rightarrow \theta=\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$
$$\Rightarrow \sin^{-1}x=\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

ii). $\cos^{-1}x$

Solution; we have $\cos^{-1}x$

Let $\theta=\cos^{-1}x$

$\Rightarrow x=\cos\theta$

$\therefore \sin\theta=\sqrt{1-\cos^2\theta}$

than $\tan\theta=\dfrac{\sin\theta}{\cos\theta}=\dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}$

$\tan\theta=\dfrac{\sqrt{1-x^2}}{x}$

$\Rightarrow \theta=\tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right)$

$\Rightarrow \cos^{-1}x=\tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right)$

---

iii). $\cot^{-1}x$

Solution; given that $\cot^{-1}x$

Let $\theta=\cot^{-1}x$

$\Rightarrow x=\cot\theta$

$\Rightarrow \tan\theta=\dfrac{1}{x}$

$\Rightarrow \theta=\tan^{-1}\left(\dfrac{1}{x}\right)$

$\Rightarrow \cot^{-1}x=\tan^{-1}\left(\dfrac{1}{x}\right)$

Q11. Verify that

i). $2Tan^{-1}\left(\dfrac{1}{2}\right)+Tan^{-1}\left(-\dfrac{1}{7}\right)=\dfrac{\pi}{4}$

Solution: we have $2Tan^{-1}\left(\dfrac{1}{2}\right)+Tan^{-1}\left(-\dfrac{1}{7}\right)=\dfrac{\pi}{4}$

Take LHS $2Tan^{-1}\left(\dfrac{1}{2}\right)+Tan^{-1}\left(-\dfrac{1}{7}\right)$

$=Tan^{-1}\left(\dfrac{1}{2}\right)+Tan^{-1}\left(\dfrac{1}{2}\right)+Tan^{-1}\left(-\dfrac{1}{7}\right)$

$=Tan^{-1}\left(\dfrac{\frac{1}{2}+\frac{1}{2}}{1-\frac{1}{2}.\frac{1}{2}}\right)+Tan^{-1}\left(-\dfrac{1}{7}\right)$

$=Tan^{-1}\left(\dfrac{1}{\frac{4-1}{4}}\right)+Tan^{-1}\left(-\dfrac{1}{7}\right)$

$=Tan^{-1}\left(\dfrac{4}{3}\right)+Tan^{-1}\left(-\dfrac{1}{7}\right)$

$=Tan^{-1}\left(\dfrac{\frac{4}{3}-\frac{1}{7}}{1+\frac{4}{3}.\frac{1}{7}}\right)$

$=Tan^{-1}\left(\dfrac{\frac{28-3}{21}}{\frac{21+4}{21}}\right)$

$=Tan^{-1}\left(\dfrac{\frac{25}{21}}{\frac{25}{21}}\right)$

$=Tan^{-1}(1)$

$=\dfrac{\pi}{4}$

=RHS Hence proved.

---

ii). $\sin^{-1}\left(\dfrac{77}{85}\right)-\sin^{-1}\left(\dfrac{3}{5}\right)=\cos^{-1}\left(\dfrac{15}{17}\right)$

Solution: we have $\sin^{-1}\left(\dfrac{77}{85}\right)-\sin^{-1}\left(\dfrac{3}{5}\right)=\cos^{-1}\left(\dfrac{15}{17}\right)$

Let $\sin^{-1}\left(\dfrac{77}{85}\right)=A$ $\qquad \sin^{-1}\left(\dfrac{3}{5}\right)=B$

$\Rightarrow \sin A=\dfrac{77}{85}$ $\qquad \Rightarrow \sin B=\dfrac{3}{5}$

$\sin^{-1}\left(\dfrac{77}{85}\right)-\sin^{-1}\left(\dfrac{3}{5}\right)=\cos^{-1}\left(\dfrac{15}{17}\right)$

$A-B=\cos^{-1}\left(\dfrac{15}{17}\right)$

$\cos(A-B)=\dfrac{15}{17}$..............................(1)

Take LHS of eq (1)

$\cos(A-B)=\cos A\cos B-\sin A\sin B$

$=\sqrt{1-\sin^2 A}.\sqrt{1-\sin^2 B}-\sin A\sin B$

$=\sqrt{1-\left(\dfrac{77}{85}\right)^2}.\sqrt{1-\left(\dfrac{3}{5}\right)^2}-\left(\dfrac{77}{85}\right)\left(\dfrac{3}{5}\right)$

$=\sqrt{\dfrac{85^2-77^2}{85^2}}.\sqrt{\dfrac{5^2-3^2}{5^2}}-\left(\dfrac{77}{85}\right)\left(\dfrac{3}{5}\right)$

$=\sqrt{\dfrac{36^2}{85^2}}.\sqrt{\dfrac{4^2}{5^2}}-\left(\dfrac{77}{85}\right)\left(\dfrac{3}{5}\right)$

$=\left(\dfrac{36}{85}\right)\left(\dfrac{4}{5}\right)-\left(\dfrac{77}{85}\right)\left(\dfrac{3}{5}\right)$

$=\dfrac{144+231}{85\times 5}$

$=\dfrac{375}{85\times 5}$

$=\dfrac{15}{17}$

=RHS of eq (1). Hence proved.

Q12 Express $\dfrac{\pi}{4}-Tan^{-1}\left(-\dfrac{1}{7}\right)$ as single inverse tangent.

Solution: we have $\dfrac{\pi}{4}-Tan^{-1}\left(-\dfrac{1}{7}\right)$ $\therefore Tan^{-1}(1)=\dfrac{\pi}{4}$

Take $\dfrac{\pi}{4}-Tan^{-1}\left(\dfrac{1}{11}\right)$

$=Tan^{-1}(1)-Tan^{-1}\left(\dfrac{1}{11}\right)$

$=Tan^{-1}\left(\dfrac{1-\frac{1}{11}}{1+1.\frac{1}{11}}\right)$

$=Tan^{-1}\left(\dfrac{\frac{11-1}{11}}{\frac{11+1}{11}}\right)$

$=Tan^{-1}\left(\frac{10}{12}\right)$

$=Tan^{-1}\left(\frac{5}{6}\right)$

Exercise 12.8

Solve each equation giving general solutions.

Q1. $\sin 2\theta=\frac{1}{2}$

Solution: we have $\sin 2\theta=\frac{1}{2}$

$\sin 2\theta=\dfrac{1}{2}$

$\Rightarrow 2\theta=\sin^{-1}\left(\frac{1}{2}\right)$

Sin is positive in 1st and 2nd quadrants

$2\theta=\dfrac{\pi}{6},\dfrac{5\pi}{6}$

And period of $\sin\theta$ is 2π

$\Rightarrow 2\theta=\dfrac{\pi}{6}+2k\pi,$ $\qquad 2\theta=\dfrac{5\pi}{6}+2k\pi$

$\Rightarrow \theta=\dfrac{\pi}{12}+k\pi,$ $\qquad \theta=\dfrac{5\pi}{12}+k\pi$

$S.S=\left\{\frac{\pi}{12}+k\pi\right\}\cup\left\{\frac{5\pi}{12}+k\pi\right\},k\in Z$

Q2. $\tan\theta=-\frac{1}{\sqrt{3}}$

Solution; we have $\tan\theta=-\frac{1}{\sqrt{3}}$

$\Rightarrow\theta=\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

tan is negative in 2nd and 4th quadrants

$\theta=-\frac{\pi}{6},\frac{5\pi}{6}$

And period of $\tan\theta$ is π

$S.S=\left\{\frac{5\pi}{6}+k\pi\right\},k\in Z$ Or $S.S=\left\{-\frac{\pi}{6}+k\pi\right\},k\in Z$

Q3. $\cos\theta=-\frac{\sqrt{3}}{2}$

Solution: we have $\cos\theta=-\frac{\sqrt{3}}{2}$

$\cos\theta=-\frac{\sqrt{3}}{2}\Rightarrow\theta=\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Cosine is negative in 2nd and 3rd quadrants

$\theta=\frac{5\pi}{6},\frac{7\pi}{6}$

And period of $\cos\theta$ is 2π

$S.S=\left\{\frac{5\pi}{6}+2k\pi\right\}\cup\left\{\frac{7\pi}{6}+2k\pi\right\},k\in Z$

Q4. $\cos\left(2\theta-\frac{\pi}{2}\right)=-1$

Solution: we have $\cos\left(2\theta-\frac{\pi}{2}\right)=-1$

$\cos\left(2\theta-\frac{\pi}{2}\right)=-1$

$\cos2\theta\cos\frac{\pi}{2}+\sin2\theta\sin\frac{\pi}{2}=-1$

$\cos2\theta.0+\sin2\theta.1=-1$

$\sin2\theta=-1\Rightarrow2\theta=\sin^{-1}(-1)$

sine is negative in 2nd and 3rd quadrants

$2\theta=-\frac{\pi}{2},\frac{3\pi}{2}$

And period of $\sin\theta$ is 2π

$2\theta=-\frac{\pi}{2}+2k\pi,\frac{3\pi}{2}+2k\pi$

$\Rightarrow\theta=-\frac{\pi}{4}+k\pi,\frac{3\pi}{4}+k\pi$

$S.S=\left\{-\frac{\pi}{4}+k\pi\right\}\cup\left\{\frac{3\pi}{4}+k\pi\right\},k\in Z$

Q5. $\sec\frac{3\theta}{2}=-2$

Solution: we have $\sec\frac{3\theta}{2}=-2$

$\sec\frac{3\theta}{2}=-2$

$\Rightarrow\cos\frac{3\theta}{2}=-\frac{1}{2}$

$\frac{3\theta}{2}=\cos^{-1}\left(\frac{-1}{2}\right)$

cosine is negative in 2nd and 3rd quadrants

$\frac{3\theta}{2}=\frac{2\pi}{3},\frac{4\pi}{3}$

And period of $\cos\theta$ is 2π

$\frac{3\theta}{2}=\frac{2\pi}{3}+2k\pi,\frac{4\pi}{3}+2k\pi$

$\Rightarrow\theta=\frac{4\pi}{9}+\frac{4}{3}k\pi,\frac{8\pi}{9}+\frac{4}{3}k\pi$

$S.S=\left\{\frac{4\pi}{9}+\frac{4}{3}k\pi\right\}\cup\left\{\frac{8\pi}{9}+\frac{4}{3}k\pi\right\},k\in Z$

Q6. $4\cos^2\theta-1=0$

Solution: we have $4\cos^2\theta-1=0$

$4\cos^2\theta-1=0$

$\Rightarrow\cos^2\theta=\frac{1}{4}$

$\cos\theta=\pm\frac{1}{2}$

When $\cos\theta=-\frac{1}{2}$

$\cos\theta$ is negative in 2nd and 3rd quadrants

$\theta=\frac{2\pi}{3},\frac{4\pi}{3}$

When $\cos\theta=\frac{1}{2}$

$\cos\theta$ is positive in 1st and 4th quadrants

$\theta=\frac{\pi}{3},\frac{5\pi}{3}$

And period of $\cos\theta$ is 2π

$S.S=\left\{\frac{\pi}{3}+2k\pi\right\}\cup\left\{\frac{5\pi}{3}+2k\pi\right\}\cup\left\{\frac{2\pi}{3}+2k\pi\right\}\cup\left\{\frac{4\pi}{3}+2k\pi\right\},k\in Z$

Solve each of equation in problems 7 – 10. Use exact values in the given interval.

Q7. $(\sin x)(\cos x)=0\quad;0^o\le x\le360^o$

Solution: we have $(\sin x)(\cos x)=0$

Either or

$\sin x=0$ $\quad$ $\cos x=0$

$x=\sin^{-1}(0)$ $\quad$ $x=\cos^{-1}(0)$

$x=0,\pi$ $\quad$ $x=\frac{\pi}{2},\frac{3\pi}{2}$

$S.S=\left\{0,\pi,\frac{\pi}{2},\frac{3\pi}{2}\right\}$

Q8. $(\sin x)(\cot x)=0\quad;0\le x\le2\pi$

Solution: we have $(\sin x)(\cot x)=0$

$(\sin x)\left(\frac{\cos x}{\sin x}\right)=0$

$\Rightarrow\cos x=0$

$\cos x=0$

$x=\cos^{-1}(0)$

$x=\frac{\pi}{2},\frac{3\pi}{2}$ $\quad$ $S.S=\left\{\frac{\pi}{2},\frac{3\pi}{2}\right\}$

Q9. $(\sec x-2)(2\sin x-1)=0\quad;0\le x\le2\pi$

Solution: we have $(\sec x-2)(2\sin x-1)=0$

Either or

$\sec x-2=0$ $\quad$ $2\sin x-1=0$

$\sec x=2$ $\quad$ $2\sin x=1$

$\cos x=\frac{1}{2}$ $\quad$ $\sin x=\frac{1}{2}$

$x=\cos^{-1}\left(\frac{1}{2}\right)$ $\quad$ $x=\sin^{-1}\left(\frac{1}{2}\right)$

$x=\frac{\pi}{3},\frac{5\pi}{3}$ $\quad$ $x=\frac{\pi}{6},\frac{5\pi}{6}$

$S.S=\left\{\frac{\pi}{3},\frac{5\pi}{3},\frac{\pi}{6},\frac{5\pi}{6}\right\}$

Q10. $(\operatorname{cosec}x-2)(2\cos x-1)=0\ ;0\le x\le 2\pi$

Solution: we have $(\operatorname{cosec}x-2)(2\cos x-1)=0$

Either or

$\operatorname{cosec}x-2=0$ $2\cos x-1=0$

$\operatorname{cosec}x=2$ $2\cos x=1$

$\sin x=\frac{1}{2}$ $\cos x=\frac{1}{2}$

$x=\sin^{-1}\left(\frac{1}{2}\right)$ $x=\cos^{-1}\left(\frac{1}{2}\right)$

$x=\frac{\pi}{6},\frac{5\pi}{6}$ $x=\frac{\pi}{3},\frac{5\pi}{3}$

$S.S=\left\{\frac{\pi}{3},\frac{5\pi}{3},\frac{\pi}{6},\frac{5\pi}{6}\right\}$

Use the trigonometric identities to solve the problems 11 – 16 giving general solutions.

Q11. $\cos\theta=\sin\theta$

Solution: we have $\cos\theta=\sin\theta$

Dividing both sides by $\cos\theta$

$1=\frac{\sin\theta}{\cos\theta}\Rightarrow\tan\theta=1\Rightarrow\theta=\tan^{-1}(1)$

$\tan\theta$ is positive in 1st and 3rd quadrants

$\theta=\frac{\pi}{4},\frac{5\pi}{4}$

Period of $\tan\theta$ is π .so

$S.S=\left\{\frac{\pi}{4}+k\pi\right\}\cup\left\{\frac{5\pi}{4}+k\pi\right\},k\in Z$

Q12. $\tan\theta=2\sin\theta$

Solution: we have $\tan\theta=2\sin\theta$

Dividing both sides by $\sin\theta$

$\Rightarrow\sin\theta=0\Rightarrow\theta=\sin^{-1}0=0,\pi$

$\frac{1}{\cos\theta}=2$

$\Rightarrow\cos\theta=\frac{1}{2}$

$\Rightarrow\theta=\cos^{-1}\left(\frac{1}{2}\right)$

$\cos\theta$ is positive in 1st and 4th quadrants

$\theta=\frac{\pi}{3},\frac{5\pi}{3}$

Period of $\cos\theta$ is 2 π .so

$S.S=\{k\pi\}\cup\left\{\frac{\pi}{3}+2k\pi\right\}\cup\left\{\frac{5\pi}{3}+2k\pi\right\},k\in Z$

Q13. $\sin\theta=\operatorname{cosec}\theta$

Solution: we have $\sin\theta=\operatorname{cosec}\theta$

$\Rightarrow\sin\theta=\frac{1}{\sin\theta}$

$\Rightarrow\theta=\sin^2\theta=1$

$\Rightarrow\sin\theta=\pm 1$

When $\sin\theta=-1$

$\sin\theta$ is negative in 3rd and 4th quadrants

$\theta=\frac{3\pi}{2}$

When $\sin\theta=1$

$\sin\theta$ is positive in 1st and 2nd quadrants

$\theta=\frac{\pi}{2}$

And period of $\sin\theta$ is 2π

$S.S=\left\{\frac{3\pi}{2}+2k\pi\right\}\cup\left\{\frac{\pi}{2}+2k\pi\right\},k\in Z$

Q14. $4\cos^2\frac{\theta}{2}-3=0$

Solution: we have $4\cos^2\frac{\theta}{2}-3=0$

$\Rightarrow\cos^2\frac{\theta}{2}=\frac{3}{4}$

$\Rightarrow\cos\frac{\theta}{2}=\pm\frac{\sqrt{3}}{2}$

When $\frac{\theta}{2}=\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\cos\theta$ is negative in 2nd and 3rd quadrants

$\frac{\theta}{2}=\frac{5\pi}{6}+2k\pi,$ $\frac{7\pi}{6}+2k\pi$

$\Rightarrow\theta=\frac{5\pi}{3}+4k\pi,$ $\frac{7\pi}{3}+4k\pi$

When $\frac{\theta}{2}=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Cosine is positive in 1st and 4th quadrants

$\frac{\theta}{2}=\frac{\pi}{6}+2k\pi,$ $\frac{11\pi}{6}+2k\pi$

$\Rightarrow\theta=\frac{\pi}{3}+4k\pi,$ $\frac{11\pi}{3}+4k\pi$

And period of $\cos\theta$ is 2π

$S.S=\left\{\frac{\pi}{3}+4k\pi\right\}\cup\left\{\frac{5\pi}{3}+4k\pi\right\}\cup\left\{\frac{7\pi}{3}+4k\pi\right\}\cup\left\{\frac{11\pi}{3}+4k\pi\right\},k\in Z$

Q15. $\cos 2\theta=\cos\theta$

Solution: we have $\cos 2\theta=\cos\theta$

$2\cos^2\theta-1=\cos\theta$

$2\cos^2\theta-\cos\theta-1=0$

$2\cos^2\theta-2\cos\theta+\cos\theta-1=0$

$2\cos\theta(\cos\theta-1)+1(\cos\theta-1)=0$

$(\cos\theta-1)(2\cos\theta+1)=0$

Either or

$\cos\theta-1=0$ $2\cos\theta+1=0$

$\cos x=1$ $2\cos x=-1$

$x=\cos^{-1}(1)$ $x=\cos^{-1}\left(-\frac{1}{2}\right)$

$x=0,2\pi$ $x=\frac{2\pi}{3},\frac{4\pi}{3}$

$S.S=\{2k\pi\}\cup\left\{\frac{2\pi}{3}+2k\pi\right\}\cup\left\{\frac{4\pi}{3}+2k\pi\right\},k\in Z$

Q16. $\sin 2\theta+\sin\theta=0$

Solution: we have $\sin 2\theta+\sin\theta=0$

$2\sin\theta\cos\theta+\sin\theta=0$

$\sin\theta(2\cos\theta+1)=0$

Either or

$\sin\theta=0$ $2\cos\theta+1=0$

$2\cos x=-1$

$\theta=\sin^{-1}(0)$ $x=\cos^{-1}\left(-\frac{1}{2}\right)$

$x=\pi,2\pi$ $x=\frac{2\pi}{3},\frac{4\pi}{3}$

$S.S=\{k\pi\}\cup\left\{\frac{2\pi}{3}+2k\pi\right\}\cup\left\{\frac{4\pi}{3}+2k\pi\right\},k\in Z$

Use quadratic formula or factorization to solve the problem 17 – 24.

Q17. $2\sin^2 x-3\sin x+1=0$

Solution: we have $2\sin^2 x-3\sin x+1=0$

$2\sin^2 x-3\sin x+1=0$

$2\sin^2 x-2\sin x-\sin x+1=0$

$2\sin x(\sin x-1)-1(\sin x-1)=0$

$(2\sin x-1)(\sin x-1)=0$

Either or

$2\sin x-1=0$ $\quad$ $\sin x-1=0$

$2\sin x=1$ $\quad$ $\sin x=1$

$\sin x=\frac{1}{2}$

$x=\sin^{-1}\left(\frac{1}{2}\right)$ $\quad$ $x=\sin^{-1}(1)$

$x=\frac{\pi}{6},\frac{5\pi}{6}$ $\quad$ $x=\frac{\pi}{2}$

$S.S=\left\{\frac{\pi}{2}+2k\pi\right\}\cup\left\{\frac{\pi}{6}+2k\pi\right\}\cup\left\{\frac{5\pi}{6}+2k\pi\right\},k\in Z$

Q18. $3\cos x+3=2\sin^2 x$

Solution: we have $3\cos x+3=2\sin^2 x$

$3\cos x+3=2\sin^2 x$

$3\cos x+3=2\left(1-\cos^2 x\right)$

$3\cos x+3=2-2\cos^2 x$

$2\cos^2 x+3\cos x+1=0$

$2\cos^2 x+2\cos x+\cos x+1=0$

$2\cos x(\cos x+1)+1(\cos x+1)=0$

$(2\cos x+1)(\cos x+1)=0$

Either $\quad$ or

$2\cos x+1=0$ $\quad$ $\cos x+1=0$

$2\cos x=-1$ $\quad$ $\cos x=-1$

$\cos x=\frac{-1}{2}$

$x=\cos^{-1}\left(\frac{-1}{2}\right)$ $\quad$ $x=\cos^{-1}(-1)$

$x=\frac{2\pi}{3},\frac{4\pi}{3}$ $\quad$ $x=\pi$

$S.S=\{\pi+2k\pi\}\cup\left\{\frac{2\pi}{3}+2k\pi\right\}\cup\left\{\frac{4\pi}{3}+2k\pi\right\},k\in Z$

Q19. $\cos^2 x\sin x=2$

Solution: we have $\cos^2 x\sin x=2$

$\left(1-\sin^2 x\right)\sin x=2$

$\sin x-\sin^3 x=2$

$\sin^3 x-\sin x+2=0$

No real solution exists

Q20. $\cos^2 x-\sin^2 x=\sin x$

Solution: we have $\cos^2 x-\sin^2 x=\sin x$

$1-\sin^2 x-\sin^2 x=\sin x$

$1-2\sin^2 x=\sin x$

$2\sin^2 x+\sin x-1=0$

$2\sin^2 x+2\sin x-\sin x-1=0$

$2\sin x(\sin x+1)-1(\sin x+1)=0$

$(2\sin x-1)(\sin x+1)=0$

Either $\quad$ or

$2\sin x-1=0$ $\quad$ $\sin x+1=0$

$2\sin x=1$ $\quad$ $\sin x=-1$

$\sin x=\frac{1}{2}$

$x=\sin^{-1}\left(\frac{1}{2}\right)$ $\quad$ $x=\sin^{-1}(-1)$

$x=\frac{\pi}{6},\frac{5\pi}{6}$ $\quad$ $x=\frac{3\pi}{2}$

$S.S=\left\{\frac{\pi}{6}+2k\pi\right\}\cup\left\{\frac{5\pi}{6}+2k\pi\right\}\cup\left\{\frac{3\pi}{2}+2k\pi\right\},k\in Z$

Q21. $\cos 2x+\cos x+1=0$

Solution: we have $\cos 2x+\cos x+1=0$

$2\cos^2 x-1+\cos x+1=0$

$2\cos^2 x+\cos x=0$

$\cos x(2\cos x+1)=0$

Either $\quad$ or

$\cos x=0$ $\quad$ $2\cos x+1=0$

$\cos x=\frac{-1}{2}$

$x=\cos^{-1}(0)$ $\quad$ $x=\cos^{-1}\left(\frac{-1}{2}\right)$

$x=\frac{\pi}{2},\frac{3\pi}{2}$ $\quad$ $x=\frac{2\pi}{3},\frac{4\pi}{3}$

$S.S=\left\{\frac{\pi}{2}+2k\pi\right\}\cup\left\{\frac{2\pi}{3}+2k\pi\right\}\cup\left\{\frac{4\pi}{3}+2k\pi\right\},k\in Z$

Q22. $1+\sin x=2\cos^2 x$

Solution: we have $1+\sin x=2\cos^2 x$

$1+\sin x=2\left(1-\sin^2 x\right)$

$1+\sin x=2-2\sin^2 x$

$2\sin^2 x+\sin x-1=0$

$2\sin^2 x+2\sin x-\sin x-1=0$

$2\sin x(\sin x+1)-1(\sin x+1)=0$

$(\sin x+1)(2\sin x-1)=0$

Either $\quad$ or

$\sin x+1=0$ $\quad$ $2\sin x-1=0$

$\sin x=-1$ $\quad$ $\sin x=\frac{1}{2}$

$x=\sin^{-1}(-1)$ $\quad$ $x=\sin^{-1}\left(\frac{1}{2}\right)$

$x=\frac{3\pi}{2}$ $\quad$ $x=\frac{\pi}{6},\frac{5\pi}{6}$

$S.S=\left\{\frac{3\pi}{2}+2k\pi\right\}\cup\left\{\frac{\pi}{6}+2k\pi\right\}\cup\left\{\frac{5\pi}{6}+2k\pi\right\},k\in Z$

Q23. $\tan^2 x=\frac{3}{2}\sec x$

Solution: we have $\tan^2 x=\frac{3}{2}\sec x$

$\tan^2 x=\frac{3}{2}\sec x$

$\sec^2 x-1=\frac{3}{2}\sec x$

$\sec^2 x-\frac{3}{2}\sec x-1=0$

$2\sec^2 x-3\sec x-2=0$

$2\sec^2 x-4\sec x+\sec x-2=0$

$2\sec x(\sec x-2)+1(\sec x-2)=0$

$(\sec x-2)(2\sec x+1)=0$

Either $\quad$ or

$\sec x-2=0$ $\quad$ $2\sec x+1=0$

$\sec x=2$ $\quad$ $\sec x=\frac{-1}{2}$

$\cos x=\frac{1}{2}$ $\quad$ $\cos x=-2$

$x=\cos^{-1}\left(\frac{1}{2}\right)$ $\quad$ does not exists

$x=\frac{\pi}{3},\frac{5\pi}{3}$

$S.S=\left\{\frac{\pi}{3}+2k\pi\right\}\cup\left\{\frac{5\pi}{3}+2k\pi\right\},k\in Z$

Q24. $3-\sin x=\cos 2x$

Solution: we have $3-\sin x=\cos 2x$

$3-\sin x=1-2\sin^2 x$

$2\sin^2 x-\sin x+2=0$

Above equation is quadratic equation in $\sin x$

Compare with the general equation, we get

$a=2,b=-1,c=2$, Using quadratic formula

$\sin x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ putting the values

$$\sin x = \frac{-(-1)\pm\sqrt{(-1)^2-4(2)(2)}}{2(2)}$$

$$\sin x = \frac{1\pm\sqrt{1-16}}{4} = \frac{1\pm\sqrt{-15}}{4}$$

$$\sin x = \frac{1\pm i\sqrt{15}}{4}$$

Not exists, having no real solutions

Exercise 12.9

Use reduction identity to solve the problems 1—5

Q1. $\sin\theta+\cos\theta=1$

Solution: we have $\sin\theta+\cos\theta=1$

Compare the given equation with the expression $a\sin\theta+b\cos\theta$ we get $a=1,b=1$

We know that

$r=\sqrt{a^2+b^2}$	$\cos\alpha=\frac{a}{r}$	$\sin\alpha=\frac{b}{r}$
$r=\sqrt{1^2+1^2}$	$\cos\alpha=\frac{1}{\sqrt2}$	$\sin\alpha=\frac{1}{\sqrt2}$
$r=\sqrt2$	$\alpha=\cos^{-1}\left(\frac{1}{\sqrt2}\right)$	$\alpha=\sin^{-1}\left(\frac{1}{\sqrt2}\right)$
	$\alpha=\frac{\pi}{4}$	$\alpha=\frac{\pi}{4}$

The reference angle $\sin\alpha$ and $\cos\alpha$ are positive, the angle x is lies in the first quadrant

Thus $\alpha=\frac{\pi}{4}+2k\pi;k\in Z$

Now $\sin\theta+\cos\theta=1$

Dividing it by $\sqrt2$ we get

$$\frac{1}{\sqrt2}\sin\theta+\frac{1}{\sqrt2}\cos\theta=\frac{1}{\sqrt2}$$

Substituting $\cos\alpha=\frac{1}{\sqrt2},\sin\alpha=\frac{1}{\sqrt2}$ we get

$$\cos\alpha\sin\theta+\sin\alpha\cos\theta=\frac{1}{\sqrt2}$$

$$\sin(\alpha+\theta)=\frac{1}{\sqrt2}$$

$$\frac{\pi}{4}+\theta=\sin^{-1}\left(\frac{1}{\sqrt2}\right)$$

Either	or
$\frac{\pi}{4}+\theta=\frac{\pi}{4},$	$\frac{\pi}{4}+\theta=\frac{3\pi}{4}$
$\theta=\frac{\pi}{4}-\frac{\pi}{4},$	$\theta=\frac{3\pi}{4}-\frac{\pi}{4}$
$\theta=0,$	$\theta=\frac{\pi}{2}$

Hence the solution set

$$S.S=\{2k\pi\}\cup\left\{\frac{\pi}{2}+2k\pi\right\},k\in z$$

Q2. $\sin\theta+\cos\theta=0$

Solution: we have $\sin\theta+\cos\theta=0$

Compare the given equation with the expression $a\sin\theta+b\cos\theta$ we get $a=1,b=1$

We know that

$r=\sqrt{a^2+b^2}$	$\cos\alpha=\frac{a}{r}$	$\sin\alpha=\frac{b}{r}$
$r=\sqrt{1^2+1^2}$	$\cos\alpha=\frac{1}{\sqrt2}$	$\sin\alpha=\frac{1}{\sqrt2}$
$r=\sqrt2$	$\alpha=\cos^{-1}\left(\frac{1}{\sqrt2}\right)$	$\alpha=\sin^{-1}\left(\frac{1}{\sqrt2}\right)$
	$\alpha=\frac{\pi}{4}$	$\alpha=\frac{\pi}{4}$

The reference angle $\sin\alpha$ and $\cos\alpha$ are positive, the angle x is lies in the first quadrant

Thus $\alpha=\frac{\pi}{4}+2k\pi;k\in Z$ Now

$\sin\theta+\cos\theta=0$

Dividing it by $\sqrt2$ we get

$$\frac{1}{\sqrt2}\sin\theta+\frac{1}{\sqrt2}\cos\theta=\frac{0}{\sqrt2}$$

Substituting $\cos\alpha=\frac{1}{\sqrt2},\sin\alpha=\frac{1}{\sqrt2}$ we get

$$\cos\alpha\sin\theta+\sin\alpha\cos\theta=0$$

$$\sin(\alpha+\theta)=0$$

$$\frac{\pi}{4}+\theta=\sin^{-1}(0)$$

$$\frac{\pi}{4}+\theta=\pi,2\pi$$

$$\theta=\pi-\frac{\pi}{4},2\pi-\frac{\pi}{4}$$

$$\theta=\frac{3\pi}{4},\frac{7\pi}{4}$$

Hence the solution set

$$S.S=\left\{\frac{3\pi}{4}+2k\pi\right\}\cup\left\{\frac{7\pi}{4}+2k\pi\right\},k\in z$$

Q3. $\sqrt3\sin\theta+\cos\theta=1$

Solution: we have $\sqrt3\sin\theta+\cos\theta=1$

Compare the given equation with the expression $a\sin\theta+b\cos\theta$ we get $a=\sqrt3,b=1$, We know that

$r=\sqrt{a^2+b^2}$	$\cos\alpha=\frac{a}{r}$	$\sin\alpha=\frac{b}{r}$
$r=\sqrt{3+1^2}$	$\cos\alpha=\frac{\sqrt3}{2}$	$\sin\alpha=\frac{1}{2}$
$r=\sqrt4$	$\alpha=\cos^{-1}\left(\frac{\sqrt3}{2}\right)$	$\alpha=\sin^{-1}\left(\frac{1}{2}\right)$
$r=2$	$\alpha=\frac{\pi}{6}$	$\alpha=\frac{\pi}{6}$

The reference angle $\sin\alpha$ and $\cos\alpha$ are positive, the angle x is lies in the first quadrant

Thus $\alpha=\frac{\pi}{6}+2k\pi;k\in Z$

Now $\sqrt3\sin\theta+\cos\theta=1$

Dividing it by 2 we get

$$\frac{\sqrt3}{2}\sin\theta+\frac{1}{2}\cos\theta=\frac{1}{2}$$

Substituting $\cos\alpha=\frac{\sqrt3}{2},\sin\alpha=\frac{1}{2}$ we get

$$\cos\alpha\sin\theta+\sin\alpha\cos\theta=\frac{1}{2}$$

$$\sin(\alpha+\theta)=\frac{1}{2}$$

$$\frac{\pi}{6}+\theta=\sin^{-1}\left(\frac{1}{2}\right)$$

$$\frac{\pi}{6}+\theta=\frac{\pi}{6},\frac{5\pi}{6}$$

$$\theta=\frac{\pi}{6}-\frac{\pi}{6},\frac{5\pi}{6}-\frac{\pi}{6}$$

$$\theta=0,\frac{2\pi}{3}$$

Hence the solution set

$$S.S=\{2k\pi\}\cup\left\{\frac{2\pi}{3}+2k\pi\right\},k\in z$$

Q4. $\sqrt3\cos\theta-\sin\theta=\frac{1}{2}$

Solution: we have $\sqrt3\cos\theta-\sin\theta=\frac{1}{2}$

Compare the given equation with the expression $a\sin\theta+b\cos\theta$ we get $a=-1,b=\sqrt3$ We know that

$r=\sqrt{a^2+b^2}$ $\cos\alpha=\frac{a}{r}$ $\sin\alpha=\frac{b}{r}$

$r=\sqrt{1+3}$ $\cos\alpha=\frac{-1}{2}$ $\sin\alpha=\frac{\sqrt{3}}{2}$

$r=\sqrt{4}$ $\alpha=\cos^{-1}\left(\frac{-1}{2}\right)$ $\alpha=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$r=2$ $\alpha=\frac{2\pi}{3}$ $\alpha=\frac{2\pi}{3}$

Reference angle $\sin\alpha$ is positive and $\cos\alpha$ is negative, angle x is lies in the second quadrant

Thus $\alpha=\frac{2\pi}{3}+2k\pi;k\in Z$ Now

$\sqrt{3}\cos\theta-\sin\theta=\frac{1}{2}$

Dividing it by 2 we get

$\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta=\frac{1}{4}$

Substituting $\cos\alpha=\frac{-1}{2},\sin\alpha=\frac{\sqrt{3}}{2}$ we get

$\sin\alpha\cos\theta+\cos\alpha\sin\theta=\frac{1}{4}$

$\sin(\alpha+\theta)=\frac{1}{4}$

$\frac{2\pi}{3}+\theta=\sin^{-1}\left(\frac{1}{4}\right)$

$\frac{2\pi}{3}+\theta=\sin^{-1}\left(\frac{1}{4}\right)$

$\theta=\sin^{-1}\left(\frac{1}{4}\right)-\frac{2\pi}{3}$

Q5. $\sqrt{3}\cos\theta+\sin\theta=1$

Solution: we have $\sqrt{3}\cos\theta+\sin\theta=1$

Compare the given equation with the expression $a\sin\theta+b\cos\theta$ we get $a=1,b=\sqrt{3}$

We know that

$r=\sqrt{a^2+b^2}$ $\cos\alpha=\frac{a}{r}$ $\sin\alpha=\frac{b}{r}$

$r=\sqrt{1+3}$ $\cos\alpha=\frac{1}{2}$ $\sin\alpha=\frac{\sqrt{3}}{2}$

$r=\sqrt{4}$ $\alpha=\cos^{-1}\left(\frac{1}{2}\right)$ $\alpha=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$r=2$ $\alpha=\frac{\pi}{3}$ $\alpha=\frac{\pi}{3}$

The reference angle $\sin\alpha$ and $\cos\alpha$ are positive, the angle x is lies in the first quadrant

Thus $\alpha=\frac{\pi}{6}+2k\pi;k\in Z$

Now

$\sqrt{3}\cos\theta+\sin\theta=1$

Dividing it by 2 we get

$\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta=\frac{1}{2}$

Substituting $\cos\alpha=\frac{1}{2},\sin\alpha=\frac{\sqrt{3}}{2}$ we get

$\sin\alpha\cos\theta+\cos\alpha\sin\theta=\frac{1}{2}$

$\sin(\alpha+\theta)=\frac{1}{2}$

$\frac{\pi}{3}+\theta=\sin^{-1}\left(\frac{1}{2}\right)$

Either or

$\frac{\pi}{3}+\theta=\frac{\pi}{6},$ $\frac{\pi}{3}+\theta=\frac{5\pi}{6}$

$\theta=\frac{\pi}{6}-\frac{\pi}{3}$ $\theta=\frac{5\pi}{6}-\frac{\pi}{3}$

$\theta=-\frac{\pi}{6},$ $\theta=\frac{\pi}{2}$

Hence the solution set

$S.S=\left\{-\frac{\pi}{6}+2k\pi\right\}\cup\left\{\frac{\pi}{2}+2k\pi\right\},k\in z$

Solve the following equations containing principal trigonometric function giving exact values in their respective restricted domains.

Q6. $4\sin^2 x=1$

Solution: Given that $4\sin^2 x=1$

$\sin^2 x=\frac{1}{4}$

$\Rightarrow\sin x=\pm\frac{1}{2}$

Either or

$\sin x=\frac{1}{2}$ $\sin x=-\frac{1}{2}$

$x=\sin^{-1}\left(\frac{1}{2}\right)$ $x=\sin^{-1}\left(-\frac{1}{2}\right)$

$x=\frac{\pi}{6}$ $x=-\frac{\pi}{6}$

Hence S.S= $\left\{-\frac{\pi}{6},\frac{\pi}{6}\right\}$

Q7. $2\sqrt{2}\cos^2 x+\left(2-\sqrt{2}\right)\cos x-1=0$

Solution: Given that

$2\sqrt{2}\cos^2 x+\left(2-\sqrt{2}\right)\cos x-1=0$

$2\sqrt{2}\cos^2 x+\left(2-\sqrt{2}\right)\cos x-1=0$

$2\sqrt{2}\cos^2 x+2\cos x-\sqrt{2}\cos x-1=0$

$2\cos x\left(\sqrt{2}\cos x+1\right)-1\left(\sqrt{2}\cos x+1\right)=0$

$\left(\sqrt{2}\cos x+1\right)\left(2\cos x-1\right)=0$

Either or

$\sqrt{2}\cos x+1=0$ $2\cos x-1=0$

$\sqrt{2}\cos x=-1$ $2\cos x=1$

$\cos x=\frac{-1}{\sqrt{2}}$ $\cos x=\frac{1}{2}$

$x=\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ $x=\cos^{-1}\left(\frac{1}{2}\right)$

$x=\frac{3\pi}{4}$ $x=\frac{\pi}{3}$

Hence S.S= $\left\{\frac{3\pi}{4},\frac{\pi}{3}\right\}$

Q8. $\cot^2 x+\left(\sqrt{3}-1\right)\cot x-\sqrt{3}=0$

Solution: Given that $\cot^2 x+\left(\sqrt{3}-1\right)\cot x-\sqrt{3}=0$

$\cot^2 x+\left(\sqrt{3}-1\right)\cot x-\sqrt{3}=0$

$\cot^2 x+\sqrt{3}\cot x-\cot x-\sqrt{3}=0$

$\cot x\left(\cot x+\sqrt{3}\right)-1\left(\cot x+\sqrt{3}\right)=0$

$(\cot x-1)\left(\cot x+\sqrt{3}\right)=0$

Either or

$\cot x-1=0$ $\cot x+\sqrt{3}=0$

$\cot x=1$ $\cot x=-\sqrt{3}$

$\tan x=1$ $\tan x=\frac{-1}{\sqrt{3}}$

$x=\tan^{-1}(1)$ $x=\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$x=\frac{\pi}{4}$ $x=-\frac{\pi}{6}$

Hence S.S= $\left\{\frac{\pi}{4},-\frac{\pi}{6}\right\}$

Q9. $4\cot^2 x+2\left(\sqrt{3}-1\right)\cot x-\sqrt{3}=0$

Solution: Given that $4\cot^2 x+2(\sqrt{3}-1)\cot x-\sqrt{3}=0$

$4\cot^2 x+2(\sqrt{3}-1)\cot x-\sqrt{3}=0$

$4\cot^2 x+2\sqrt{3}\cot x-2\cot x-\sqrt{3}=0$

$2\cot x(2\cot x+\sqrt{3})-1(2\cot x+\sqrt{3})=0$

$(2\cot x-1)(2\cot x+\sqrt{3})=0$

Either

$2\cot x-1=0$, $2\cot x=1$, $\cot x=\frac{1}{2}$, $\tan x=2$, $x=\tan^{-1}(2)$, $x=63^o26^/$

or

$2\cot x+\sqrt{3}=0$, $2\cot x=-\sqrt{3}$, $\cot x=\frac{-\sqrt{3}}{2}$, $\tan x=\frac{-2}{\sqrt{3}}$, $x=\tan^{-1}\left(\frac{-2}{\sqrt{3}}\right)$, $x=-49^o6^/$

Hence S.S= $\{63^o26^/, -49^o6^/\}$

Q10. $4\cos^2 x+2(\sqrt{3}-1)\cos x-\sqrt{3}=0$

Sol: Given that $4\cos^2 x+2(\sqrt{3}-1)\cos x-\sqrt{3}=0$

$4\cos^2 x+2(\sqrt{3}-1)\cos x-\sqrt{3}=0$

$4\cos^2 x+2\sqrt{3}\cos x-2\cos x-\sqrt{3}=0$

$2\cos x(2\cos x+\sqrt{3})-1(2\cos x+\sqrt{3})=0$

$(2\cos x-1)(2\cos x+\sqrt{3})=0$

Either

$2\cos x-1=0$, $2\cos x=1$, $\cos x=\frac{1}{2}$, $x=\cos^{-1}\left(\frac{1}{2}\right)$, $x=\frac{\pi}{3}$

or

$2\cos x+\sqrt{3}=0$, $2\cos x=-\sqrt{3}$, $\cos x=\frac{-\sqrt{3}}{2}$, $x=\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$, $x=\frac{5\pi}{6}$

Hence S.S= $\left\{\frac{\pi}{3}, \frac{5\pi}{6}\right\}$

Q11. $4\cos^2 x-4\cos x-3=0$

Solution; Given that $4\cos^2 x-4\cos x-3=0$

$4\cos^2 x-4\cos x-3=0$

$4\cos^2 x-6\cos x+2\cos x-3=0$

$2\cos x(2\cos x-3)+1(2\cos x-3)=0$

$(2\cos x-3)(2\cos x+1)=0$

Either

$2\cos x-3=0$, $2\cos x=3$, $\cos x=\frac{3}{2}$, $x=\cos^{-1}\left(\frac{3}{2}\right)$, Not possible

or

$2\cos x+1=0$, $2\cos x=-1$, $\cos x=\frac{-1}{2}$, $x=\cos^{-1}\left(\frac{-1}{2}\right)$, $x=\frac{2\pi}{3}$

Hence S.S= $\left\{\frac{2\pi}{3}\right\}$

Q12. $\sin 4x+\sin 2x=0$

Solution: Given that $\sin 4x+\sin 2x=0$

$2\sin 2x\cos 2x+\sin 2x=0$

$\sin 2x(2\cos 2x+1)=0$

Either

$\sin 2x=0$, $2x=\sin^{-1}(0)$, $2x=0,\pi$, $x=0,\frac{\pi}{2}$

or

$2\cos 2x+1=0$, $2\cos 2x=-1$, $\cos 2x=\frac{-1}{2}$, $2x=\cos^{-1}\left(\frac{-1}{2}\right)$, $2x=\frac{2\pi}{3}$, $x=\frac{\pi}{3}$

Hence S.S = $\left\{0, \frac{\pi}{2}, \frac{\pi}{3}\right\}$

Use inverse trigonometric function to find the solutions in the given intervals correct to four decimal places.

Q13. $2\tan^2 x+9\tan x+3=0$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Solution; we have $2\tan^2 x+9\tan x+3=0$

Above eq is quadratic equation in $\tan x$

Compare with general eq, we get $a=2, b=9, c=3$

Using quadratic formula

$$\tan x=\frac{-9\pm\sqrt{9^2-4(2)(3)}}{2(2)}$$

$$\tan x=\frac{-9\pm\sqrt{81-24}}{4}$$

$$\tan x=\frac{-9\pm\sqrt{57}}{4}$$

Either

$\tan x=\frac{-9+\sqrt{57}}{4}$, $\tan x=-0.3625$, $x=\tan^{-1}(-0.3625)$, $x=-19^o55^/$, $x=-0.3478 radian$

or

$\tan x=\frac{-9-\sqrt{57}}{4}$, $\tan x=-4.1375$, $x=\tan^{-1}(-4.1375)$, $x=76^o24^/$, $x=-1.3336 Radian$

$S.S=\{-19^o55^/, 76^o24^/\}$ OR $S.S=\{-0.3478, -1.3336\}$ radian

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Q14. $3\sin^2 x+7\sin x+3=0$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Solution: we have $3\sin^2 x+7\sin x+3=0$

Above equation is quadratic equation in $\sin x$

Compare with the general equation, we get $a=3, b=7, c=3$

Using quadratic formula

$$\sin x=\frac{-7\pm\sqrt{7^2-4(3)(3)}}{2(3)}$$

$$\sin x=\frac{-7\pm\sqrt{49-36}}{6}$$

$$\sin x=\frac{-7\pm\sqrt{13}}{6}$$

Either

$\sin x=\frac{-7+\sqrt{13}}{6}$, $\sin x=-0.5657$, $x=\sin^{-1}(-0.5657)$, $x=-36^o58^/$, $x=-0.6013 radian$

or

$\sin x=\frac{-7-\sqrt{13}}{6}$, $\sin x=-1.7676$, not defined

$S.S=\{-36^o58^/\}=\{-0.6013 radian\}$

Q15. $15\cos^4 x-14\cos^2 x+3=0$ $[0,\pi]$

Solution: we have $15\cos^4 x-14\cos^2 x+3=0$

Above equation is quadratic equation in $\cos^2 x$

Compare with general eq, $a=15, b=-14, c=3$

Using quadratic formula

$$\cos^2 x=\frac{-(-14)\pm\sqrt{(-14)^2-4(15)(3)}}{2(15)}$$
$$\cos^2 x=\frac{14\pm\sqrt{196-180}}{30}$$
$$\cos^2 x=\frac{14\pm\sqrt{16}}{30}$$
$$\cos^2 x=\frac{14\pm 4}{30}$$
$$\cos^2 x=\frac{7\pm 2}{15}$$

Either

$$\cos^2 x=\frac{7+2}{15}=\frac{9}{15}$$
$$\cos^2 x=\frac{3}{5}=0.6$$
$$\cos x=\pm\sqrt{\frac{3}{5}}$$
$$x=\cos^{-1}\left(\pm\sqrt{\frac{3}{5}}\right)$$

Or

$$\cos^2 x=\frac{7-2}{15}=\frac{5}{15}$$
$$\cos^2 x=\frac{1}{3}=0.3333$$
$$\cos x=\pm\sqrt{\frac{1}{3}}$$
$$x=\cos^{-1}\left(\pm\sqrt{\frac{1}{3}}\right)$$

When

$$x=\cos^{-1}\left(\sqrt{\frac{3}{5}}\right) \qquad x=\cos^{-1}\left(\sqrt{\frac{1}{3}}\right)$$
$$x=0.6847\,radian \qquad x=0.9553\,radian$$

When

$$x=\cos^{-1}\left(-\sqrt{\tfrac{3}{5}}\right) \qquad x=\cos^{-1}\left(-\sqrt{\tfrac{1}{3}}\right)$$
$$x=2.4568\,radian \qquad x=2.1863\,radian$$
$$S.S=\{0.6847,\ 0.9553,\ 2.4568,\ 2.1863\}\,radian$$

Use a calculator to solve each of the problems in 16 – 18, given appropriate answers to nearest multiples of ten minutes in the interval [0°, 360°]

Q16. $\sin^2 t-4\sin t+1=0$

Solution: Given that $\sin^2 t-4\sin t+1=0$

Above equation is quadratic equation in $\sin t$

Compare with the general equation, we get

$a=1, b=-4, c=1$

Using quadratic formula

$$\sin t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$\sin t=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$$
$$\sin t=\frac{4\pm\sqrt{16-12}}{2}$$
$$\sin t=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm\sqrt{4\times 3}}{2}$$
$$\sin t=\frac{4\pm 2\sqrt{3}}{2}$$
$$\sin t=2\pm\sqrt{3}$$

Either

$$\sin t=2-\sqrt{3}$$
$$\sin t=0.2679$$
$$t=\sin^{-1}(0.2679)$$

or

$$\sin t=2+\sqrt{3}$$
$$\sin t=3.7321$$
not possible

Sin is positive in 1st and 2nd quadrants

$$x=15^o32',180^o-15^o32'=164^o28'$$
$$S.S=\{15^o32',164^o28'\}$$

Q17. $5\sin^2\alpha+3\cos\alpha-2=0$

Solution: Given that $5\sin^2\alpha+3\cos\alpha-2=0$

$$5(1-\cos^2\alpha)+3\cos\alpha-2=0$$
$$5-5\cos^2\alpha+3\cos\alpha-2=0$$
$$5\cos^2\alpha-3\cos\alpha-3=0$$

Above equation is quadratic equation in $\cos\alpha$

Compare with the general equation, we get

$a=5, b=-3, c=-3$

Using quadratic formula

$$\cos\alpha=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$\cos\alpha=\frac{-(-3)\pm\sqrt{(-3)^2-4(5)(-3)}}{2(5)}$$
$$\cos\alpha=\frac{3\pm\sqrt{9+60}}{10}$$
$$\cos\alpha=\frac{3\pm\sqrt{69}}{10}$$

Either

$$\cos\alpha=\frac{3+\sqrt{69}}{10}$$
$$\cos\alpha=1.13066$$
not possible

or

$$\cos\alpha=\frac{3-\sqrt{69}}{10}$$
$$\cos\alpha=-0.53066$$
$$\alpha=\cos^{-1}(-0.53066)$$
$$\alpha=122^o3'$$

$$S.S=\{122^o3'\}$$

Q18. $2\sin^3\beta+\sin^2\beta-2\sin\beta-1=0$

Solution: Given that $2\sin^3\beta+\sin^2\beta-2\sin\beta-1=0$

$$2\sin^3\beta+\sin^2\beta-2\sin\beta-1=0$$
$$\sin^2\beta(2\sin\beta+1)-1(2\sin\beta+1)=0$$
$$(\sin^2\beta-1)(2\sin\beta+1)=0$$

Either

$$\sin^2\beta-1=0$$
$$\sin^2\beta=1$$
$$\sin\beta=\pm 1$$
$$\beta=90^o,270^o$$

or

$$2\sin\beta+1=0$$
$$2\sin\beta=-1$$
$$\sin\beta=\tfrac{-1}{2}$$
$$\beta=210^o,330^o$$

$$S.S=\{90^o,210^o,270^o,330^o\}$$