Chapter11

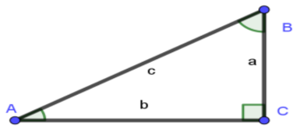
Application of Trigonometry

Exercise 11.1

In problems 1—4 solve right triangles in which $\gamma = 90^o$ &

Q1. $a = 14, \beta = 28^o$

Solution; we have $a = 14, \beta = 28^o$



$\therefore \alpha + \beta + \gamma = 180^o$

$\alpha = 180^o - 28^o - 90^o$

$\alpha = 62^o$

$\tan\beta = \dfrac{b}{a}$

$b = a\tan\beta$

$b = 14.\tan 28^o$

$b = 7.4439$

$\cos\alpha = \dfrac{b}{c}$

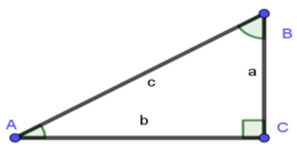
$\Rightarrow c = \dfrac{b}{\cos\alpha}$

$c = \dfrac{7.44}{\cos 62^o}$

$c = 15.8476$

Q2. $b = 8.9, \beta = 21.5^o$

Solution; we have $b = 8.9, \beta = 21.5^o$



$\therefore \alpha + \beta + \gamma = 180^o$

$\alpha = 180^o - 21.5^o - 90^o$

$\alpha = 68.5^o$

$\tan\beta = \dfrac{b}{a}$

$\Rightarrow a = \dfrac{b}{\tan\beta}$

$a = \dfrac{8.9}{\tan 21.5^o}$

$a = 22.5939$

$\sin\beta = \dfrac{b}{c}$

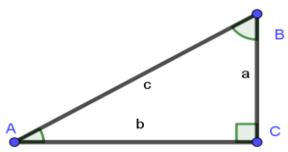
$\Rightarrow c = \dfrac{b}{\sin\beta}$

$c = \dfrac{8.9}{\sin 21.5^o}$

$c = 24.2837$

Q3. $a = 250, \alpha = 42^o25' = 42.42^o$

Solution; we have $a = 250, \alpha = 42^o25' = 42.42^o$



$\therefore \alpha + \beta + \gamma = 180^o$

$\beta = 180^o - 42^o25' - 90^o$

$\beta = 47^o35'$

Now

$\tan\beta = \dfrac{b}{a}$

$\Rightarrow b = a\tan\beta$

$b = 250\tan 47^o35'$

$b = 273.6250$

And

$\sin\beta = \dfrac{b}{c}$

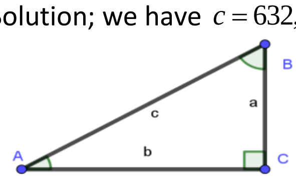
$\Rightarrow c = \dfrac{b}{\sin\beta}$

$c = \dfrac{273.6250}{\sin 47^o35'}$

$c = 370.6355$

Q4. $c = 632, b = 240$

Solution; we have $c = 632, b = 240$



$\sin\beta = \dfrac{b}{c} = \dfrac{240}{632}$

$\beta = \sin^{-1}(0.37975)$

$\beta = 22^o20'$

Now

$\cos\beta = \dfrac{a}{c}$

$\Rightarrow a = c.\cos\beta$

$a = 632.\cos(22^o20')$

$a = 584.5929$

and

$\therefore \alpha + \beta + \gamma = 180^o$

$\alpha = 180^o - 22^o20' - 90^o$

$\alpha = 67^o40'$

Q5. A ladder 32ft long leans against a building and makes an angle of 65° with the ground. What is the distance from the base of the building to the foot of the ladder? How far it is from the ground to the top of the ladder?

Solution; Length of the ladder c = 32ft

Angle with the building = 65°

Distance from ladder to building = a

Height of the building = b

$\cos\beta = \dfrac{a}{c}$

$\Rightarrow a = c.\cos\beta$

$a = 32.\cos(65^o)$

$a = 13.5238$

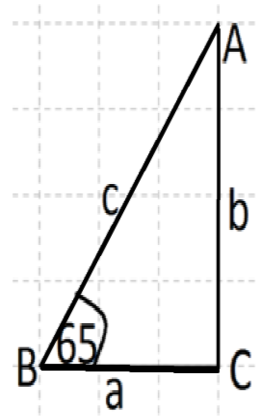
Now

$\sin\beta = \dfrac{b}{c}$

$b = c.\sin\beta$

$b = 32\sin(65)$

$b = 29.0018$

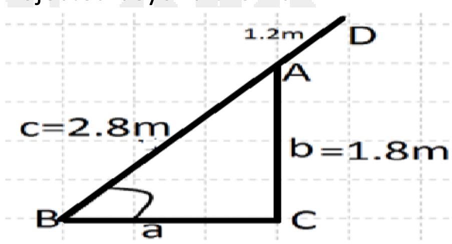


Q6. A 4 meter plank rests against a wall 1.8m high so that 1.2m of it project beyond the wall and find the angle of plank makes with the wall?

Solution; Length of the plank = 4m

Height of the wall b = 1.8m

Projected beyond the wall = 1.2m



Remaining length of plank c = $4m - 1.2m = 2.8m$

Or

BD=AB+AD

4 m = AB+1.2 m

AB = 4 m – 1.2 m

AB = 2.8 m

$\sin\beta = \dfrac{b}{c}$

$\beta = \sin^{-1}\left(\dfrac{1.8}{2.8}\right)$

$\beta = 40^o$

Q7. An isosceles triangle has a vertical angle of 108° and a base 20 cm long calculate its altitude.

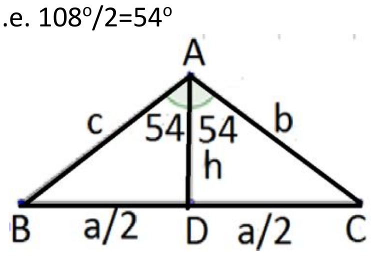
Solution; Vertical angle = 108°

Isosceles triangle having AB =AC

Than opposite angles must be equal i.e., $\beta = \gamma$

And AD=h is an altitude or perpendicular on BD which divide an angle α into two equal parts.

i.e. 108°/2=54°



Than in triangle ABD

$\beta + 90 + 54 = 180$

$\beta = 180 - 144$

$\beta = 36$

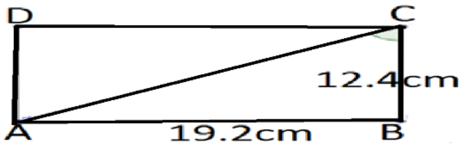
$\tan 36 = \dfrac{h}{10}$

$h = 10 \times \tan 36$

$h = 7.2654cm$

Q8. Length and width of the rectangle are 19.2 cm and 12.4 cm respectively. Find the angle between a diagonal

and the shorter side of the rectangle.
Solution; Length of the rectangle AB = 19.2cm
Width of the rectangle BC = 12.4cm



Suppose $\angle ACB = \alpha$

$\tan\alpha = \frac{AB}{BC}$

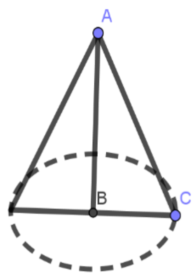
$\tan\alpha = \frac{19.2}{12.4}$

$\tan\alpha = 1.5484$

$\alpha = \tan^{-1}(1.5484)$

$\alpha = 57.14^o$

Q9. If a cone is 8.4cm high and has a vertical angle of 72°, calculate the diameter of its base.
Solution; height of the cone AB = 8.4cm
Vertical angle at A =72°
Height AB/ Perpendicular bisects vertical angle into equal parts i.e. 72°/2=36°
As wall as Height AB/ Perpendicular bisects diameter into equal parts i.e. diameter/2 = radius = BC
In triangle ABC



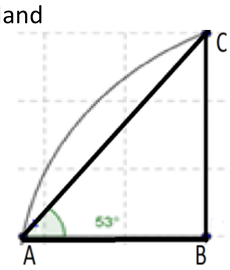
$\tan 36^o = \frac{BC}{8.4}$

$BC = 8.4\tan 36^0$

$BC = 6.10296$

Than diameter = 2(radius)
Diameter = 2 (6.10296)
Diameter = 12.20592 cm

Q10. A kite has 120m of string attached to it when it files at an elevation of 53°. How far is it above hand holding it?
Solution; Let A represents Hand
Length of string = AC
Height of the kite = BC
Angle of elevation = 53°



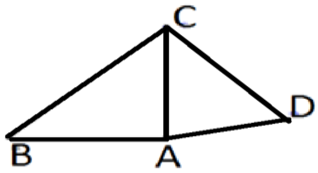
$\sin 53 = \frac{BC}{120}$

$BC = 120.\sin 53$

$BC = 95.8363m$

Exercise 11.2

Q1. An aerial mast is supported by two wires attached to points on the ground each 57 m away from the foot of the mast. If each wire makes an angle of 32° with the horizontal, find the height of the mast.
Solution: Let Distance from the mast AB = AD = 57m
Let height of the mast = AC and $m\angle B = m\angle D = 32^o$
In ΔABC

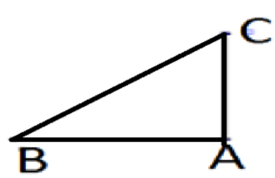


$\tan 32^o = \frac{AC}{57}$

$AC = 57.\tan 32^o$

$AC = 35.618m$

Q2. The angle of elevation of the top of a post from a point on level ground 38m away is 33.23°. Find the height of the post.
Solution: Angle of elevation $m\angle B = 33.23^o$
Distance from the post AB = 38m
Height of the post = AC
In ΔABC

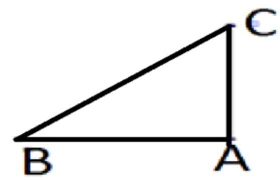


$\tan 33.23^o = \frac{AC}{38}$

$AC = 38.\tan 33.23^o$

$AC = 24.8949m$

Q3. A mosque minar 82meters high casts a shadow of 62meter long. Find the angle of elevation of the sun at that moment.
Solution: Let Height of minar AC=82m,
Length of shadow AB=62m
Angle of elevation $m\angle B = \beta$
In ΔABC

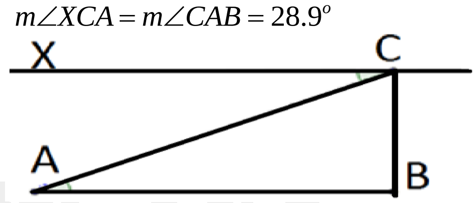


$\tan\beta = \frac{82}{62}$

$\Rightarrow \beta = \tan^{-1}\left(\frac{82}{62}\right)$

$\beta = 52.9072^o$

Q4. The angle of depression of a boat 65.7m from the base of a cloff is 28.9°. How high is the cliff?
Solution: Point A represent baot,
distance from a boat to cliff = AC
Height of a cliff = BC
Angle of depression $m\angle XCA = 28.9^o$
By definition of alternate angles
$m\angle XCA = m\angle CAB = 28.9^o$



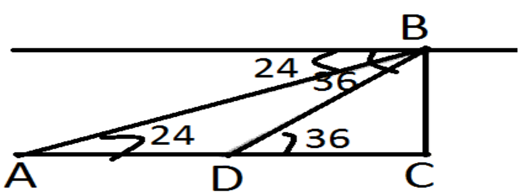
In $\triangle ABC$

$\tan 28.9^o = \frac{BC}{65.7}$

$BC = 65.7\tan 28.9^o$

$BC = 36.2683m$

Q5. From a top of a cliff 52m high the angles of depression of two ships due east of it are 36° and 24° respectively. Find distance between the ships.
Soltuion: Let Height of a cliff = 52m
A and D are two boats
i.e. A,D and C are collinear
Angle of depression are 36° and 24°



From the figure $m\angle A = 24^o, m\angle D = 36^o$
In ΔABC

$\tan 24^o = \frac{52}{AC}$

$AC = \frac{52}{\tan 24^o}$

$AC = 116.79m$

In ΔBCD

$\tan 36^o = \frac{52}{DC}$

$DC = \frac{52}{\tan 36^o}$

$DC = 71.57m$

Now distance between boats
AC=AD+DC
AD=AC-DC
AD= 116.79 m– 71.57m
AD= 45.22m

Q6. Two masts are 20m and 12m high. If the line joining their tops makes an angle of 35° with the horizontal; find their distance apart.

Solution: Let Two masts AE=20m and CD=12m

Line joining their tops makes an angle = 35°

Distance apart AD = ?

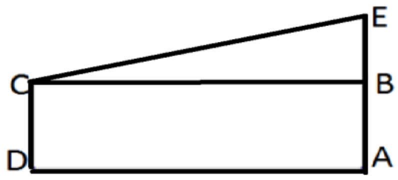
Let ABCD is a rectangle

And AE=AB+BE (using AB=CD)

20=12+BE

BE=20-12

BE=8m



In ΔCBE

$$\tan 35^o = \frac{8}{BC}$$

$$BC = \frac{8}{\tan 35^o}$$

$$BC = 11.43m$$

Hence AD = BC = 11.43m

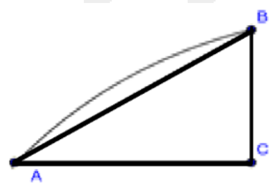
Q7. Measure of the angle of elevation of a kite is 35^o. String of a kite is 340 meter long. If the sag is in the string is 10 meter, find the height of the kite.

Solution: we have angle of elevation $m\angle A = 35^o$

Length of the string or arc length AB =340m

And sag is 10m so the straigt length after subtracting the sag, we have AB=330

In ΔABC



$$\sin 35^o = \frac{BC}{330}$$

$$BC = 330\sin 35^o$$

$$BC = 189.28m$$

Q8. A parachutist is descending vertically. How far does parachutist fall as the angle of elevation changes from 50° to 30°. which observes from a point 100m away from feet of parachutist where he touches the ground.

Solution: parachutist fall from the point A

From point D angle of elevation at A = 50°

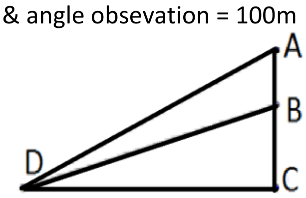
i.e. $m\angle ADC = 50^o$

From point D angle of elevation at B = 30°

i.e. $m\angle BDC = 30^o$

Distance from falling point & angle obsevation = 100m

In ΔADC



$$\tan 50^o = \frac{AC}{100}$$

$$AC = 100\tan 50^o$$

$$AC = 119.18m$$

In ΔBCD

$$\tan 30^o = \frac{BC}{100}$$

$$BC = 100\tan 30^o$$

$$BC = 57.74m$$

AC=AB+BC

AB = AC - BC

AB=119.18-57.74

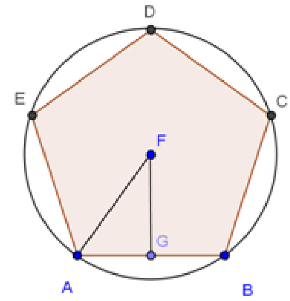
AB=61.44m

Q9. A regular pentagon is incribed I a circle of radius 5 centimeters. Find length of a side of pentagon.

Solution: Let ABCDE is a regular pentagon

i.e., AB=BC=CD=DE=EA

and all of its interior angles are equal



with circle of radius AF =5cm

take a perpendicular bisector from F to a chord AB i.e. AG=GB

internal angle of regular polygon $\frac{n-2}{n}\times 180$

Internal angle For pentagon $\frac{5-2}{5}\times 180 = 108^o$

AF bisect internal angle so $m\angle FAG = \frac{108^o}{2} = 54^o$

In ΔAGF

$$\cos 54^o = \frac{AG}{AF}$$

$$AG = 5\cos 54^o$$

$$AG = 2.94cm$$

$$\therefore AB = AG + GB$$

$$AB = 2AG \quad \because AG = GB$$

$$AB = 2(2.94cm)$$

$$AB = 5.88cm$$

Exercise 11.3

Q1. Find the measure of the smallest angle of the triangle whose sides have lengths

a). 4.3,5.1 and 6.3

Solution: Let $a = 4.3, b = 5.1, c = 6.3$

The smallest side $a = 4.3$

Therefore the smallest angle = α

Using law of cosine

$$\cos\alpha = \frac{c^2 + b^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{6.3^2 + 5.1^2 - 4.3^2}{2(6.3)(5.1)} = \frac{47.21}{64.26}$$

$$\alpha = \cos^{-1}\frac{47.21}{64.26}$$

$$\alpha = 42.72^o$$

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b). 3,4.2 and 3.8

Solution: Let $a = 3, b = 4.2, c = 3.8$

The smallest side $a = 3$

Therefore the smallest angle = α

Using law of cosine

$$\cos\alpha = \frac{c^2 + b^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{4.2^2 + 3.8^2 - 3^2}{2(4.2)(3.8)}$$

$$\cos\alpha = \frac{23.08}{31.92}$$

$$\alpha = \cos^{-1}\frac{23.08}{31.92}$$

$$\alpha = 43.69^o$$

Q2. Find the measure of the largest angle of the triangle whose sides have lengths

a). 2.9,3.3 and 4.1

Solution: Let $a=2.9, b=3.3, c=4.1$
The largest side $c=4.1$
Therefore the largest angle = γ

Using law of cosine $\cos\gamma=\dfrac{a^2+b^2-c^2}{2ab}$

$\cos\gamma=\dfrac{2.9^2+3.3^2-4.1^2}{2(2.9)(3.3)}$

$\cos\gamma=\dfrac{2.49}{19.14}$

$\gamma=\cos^{-1}\dfrac{2.49}{19.14}$

$\gamma=82.52^o$

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b). 6.0, 8 and 9.4
Solution: Let $a=6.0,\ b=8,\ c=9.4$
The largest side $c=9.4$
Therefore the largest angle = γ
Using law of cosine

$\cos\gamma=\dfrac{a^2+b^2-c^2}{2ab}$

$\cos\gamma=\dfrac{6^2+8^2-9.4^2}{2(6)(8)}$

$\cos\gamma=\dfrac{11.64}{96}$

$\gamma=\cos^{-1}\dfrac{11.64}{96}$

$\gamma=83.0358^o$

In problem 3 to 9, find the missing parts of $\triangle ABC$

Q3. $a=209, b=120, c=241$
Solution: we have $a=209, b=120, c=241$

Using $\cos\alpha=\dfrac{c^2+b^2-a^2}{2bc}$

$\cos\alpha=\dfrac{241^2+120^2-209^2}{2(241)(120)}$

$\cos\alpha=\dfrac{28800}{57840}$

$\alpha=\cos^{-1}\dfrac{28800}{57840}$

$\alpha=60.1372^o$

Similarly

$\cos\gamma=\dfrac{a^2+b^2-c^2}{2ab}$

$\cos\gamma=\dfrac{209^2+120^2-241^2}{2(209)(120)}$

$\cos\gamma=\dfrac{0}{950160}$

$\gamma=\cos^{-1}0$

$\gamma=90^o$

Now using

$\alpha+\beta+\gamma=180^o$

$\beta=180^o-60^o-90^o$

$\beta=30^o$

Q4. $a=120, b=240, \gamma=32^o$

Solution: we have $a=120, b=240, \gamma=32^o$

To find $\alpha,\quad \beta,\quad c$

$c^2=a^2+b^2-2ab\cos\gamma$

$c^2=120^2+240^2-2(120)(240)\cos 32^o$

$c^2=72000-57600\cos 32^o$

$c^2=23152.42966$

Taking square root on both sides

$c=152.1592\simeq 152$

Similarly $\cos\alpha=\dfrac{c^2+b^2-a^2}{2bc}$

$\cos\alpha=\dfrac{240^2+152^2-120^2}{2(240)(152)}$

$\cos\alpha=\dfrac{66304}{72960}$

$\alpha=\cos^{-1}\dfrac{66304}{72960}$

$\alpha=24.66^o$

$\alpha\simeq 25^o$

Now $\quad \alpha+\beta+\gamma=180^o$

$\beta=180^o-25^o-32^o$

$\beta=123^o$

Q5. $a=24.5, c=43.8, \beta=112^o$

Solution: we have $a=24.5, c=43.8, \beta=112^o$

To find α,γ,b

$b^2=a^2+c^2-2ac\cos\beta$

$b^2=24.5^2+43.8^2-2(24.5)(43.8)\cos 112^o$

$b^2=600.25+1918.44+803.98$

$b^2=332.67$

taking *square* root

$b=57.64$

Similarly

$\cos\alpha=\dfrac{c^2+b^2-a^2}{2bc}$

$\cos\alpha=\dfrac{57.6^2+43.8^2-24.5^2}{2(57.6)(43.8)}$

$\cos\alpha=\dfrac{4635.95}{5045.76}$

$\alpha=\cos^{-1}\dfrac{4635.95}{5045.76}$

$\alpha=23.25^o$

Now

$\alpha+\beta+\gamma=180^o$

$\gamma=180^o-23^o-112^o$

$\gamma=45^o$

Q6. $a=0.7, c=0.8, \beta=141^o30'=141.5^o$

Solution: we have $a=0.7, c=0.8, \beta=141^o30'=141.5^o$

To find α,γ,b take $b^2=a^2+c^2-2ac\cos\gamma$

$b^2=0.7^2+0.8^2-2(0.7)(0.8)\cos\left(141^o30'\right)$

$b^2=0.49+0.64-112(-0.7826)$

$b^2=2.00652$

Taking square root on both sides

$b=1.4165$

Similarly $\cos\alpha=\dfrac{c^2+b^2-a^2}{2bc}$

$\cos\alpha=\dfrac{0.8^2+1.4165^2-0.7^2}{2(0.8)(1.4165)}$

$\cos\alpha=\dfrac{2.1565}{2.2664}$

$\alpha=\cos^{-1}\dfrac{2.1565}{2.2664}$

$\alpha=17.9163^o=17^054'$

Now $\alpha+\beta+\gamma=180^o$

$\gamma = 180^o - 17^o54' - 141^o30'$

$\gamma = 20^o36'$

Q7. $a = 34, b = 23, c = 58$

Solution: we have $a = 34, b = 23, c = 58$

In this triangle sum of two sides is not greater than the third side.

i.e., $a + b \not> c$ or $34 + 23 = 57 \not> 58$

so triangle is not possible.

Q8. $a = 15.6, b = 18, \gamma = 35^o10'$

Solution: we have $a = 15.6, b = 18, \gamma = 35^o10'$

To find α, β, c

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = 15.6^2 + 18^2 - 2(15.6)(18)\cos 35^o10'$

$c^2 = 243.36 + 324 - 49.09698324$

$c^2 = 108.2631676$

taking *square* root

$c = 10.4049588$

$c \simeq 10.4$

Similarly

$\cos\alpha = \dfrac{c^2 + b^2 - a^2}{2bc}$

$\cos\alpha = \dfrac{18^2 + 10.4^2 - 15.6^2}{2(18)(10.4)}$

$\cos\alpha = \dfrac{188.8}{374.4}$

$\alpha = \cos^{-1}\dfrac{188.8}{374.4}$

$\alpha = 59.7168^o$

$\alpha \simeq 59^o43'$

Now

$\alpha + \beta + \gamma = 180^o$

$\beta = 180^o - 59^o43' - 35^o10'$

$\beta = 85^o07'$

Q9. $b = 1.6, c = 3.2, \alpha = 100^o24'$

Solution: we have $b = 1.6, c = 3.2, \alpha = 100^o24'$

To find α, β, a using $c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = 15.6^2 + 18^2 - 2(15.6)(18)\cos 35^o10'$

$c^2 = 243.36 + 324 - 49.09698324$

$c^2 = 108.2631676$

Taking square root

$c = 10.4049588 \simeq 10.4$

Similarly $\cos\beta = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos\beta = \dfrac{3.83^2 + 3.2^2 - 1.6^2}{2(3.83)(3.2)}$

$\cos\beta = \dfrac{22.3489}{24.512}$

$\beta = \cos^{-1}\dfrac{22.3489}{24.512}$

$\beta = 24.2512^o$

$\beta \simeq 24^o15'$

Now $\alpha + \beta + \gamma = 180^o$

$\gamma = 180^o - 24^o15' - 100^o24'$

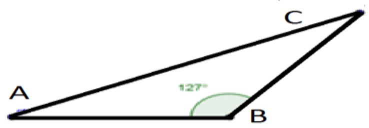
$\gamma = 55^o21'$

Q10. Two planes start from Karachi international Airport at the same time and fly in direction, that makes an angle of 127° with each other. Their speeds are 525km/h. How far they are at the end of 2 hours of flying time?

Sol: Total distance covered in 2 hours = 1050km

i.e. $a = c = 1050km$

Distance between two planes from fig is $AC = b = ?$



$b^2 = a^2 + c^2 - 2ac\cos\gamma$

$b^2 = 1050^2 + 1050^2 - 2(1050)(1050)\cos(127^o)$

$b^2 = 1102500 + 1102500 - 2205000(-0.6018)$

$b^2 = 3532002.126$

taking *square* root

$b = 1879.36km$

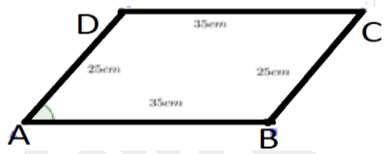
Q11. sides of a parallelogram are 25 cm and 35 cm long and one of its angle is 36°. Find lengths of its diagonals.

Solution: Let ABCD is parallelogram

Then opposite sides are equal and parallel

i.e., $AB = CD = 35cm, AD = BC = 25cm$

And opposite angles are equal $\angle A = \angle C, \quad \angle D = \angle B$



Sum of interior angle of parallelogram are 360°, Then

$\angle A + \angle B + \angle C + \angle D = 360^O$

$2\angle A + 2\angle C = 360^O \quad \because \angle A = \angle C, \angle B = \angle D$

$\angle A + \angle C = 180^O \quad \because \angle A = 36^O$

$\angle C = 180 - 36^O = 144^O$

In triangle ABD

$BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos\angle A$

$BD^2 = 35^2 + 25^2 - 2(35)(25)\cos(36^o)$

$BD^2 = 1225 + 625 - 1982.091636$

$BD^2 = 434.2202598$

Taking square root on both sides

$BD = 20.83795cm$

Similarly In triangle BCD

$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\angle A$

$AC^2 = 35^2 + 25^2 - 2(35)(25)\cos(144^o)$

$AC^2 = 1225 + 625 + 1415.77974$

$AC^2 = 3265.77974$

taking *square* root

$AC = 57.147cm$

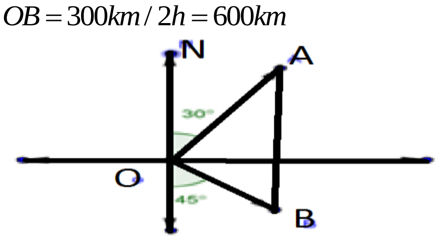
Q12. Two airplanes leave a field at same time on files 30° East of North art 250 km/h, the other 45° East of South at 300 km/h. How far apart are they at end of 2 hours?

Solution: Let total distance of first plane

$OA = 250km/2h = 500km$

And total distance of first plane

$OB = 300km/2h = 600km$



$\angle$NOA+ $\angle$AOB+ $\angle$BOS = 180
$\angle$AOB = 180 – 45 – 30 = 105
In triangle OAB
$AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cos\angle AOB$
$AB^2 = 500^2 + 600^2 - 2(500)(600)\cos(105^o)$
$AB^2 = 250000 + 360000 + 155291.4171$
$AB^2 = 768291.4271$
taking square root
$AB = 874.809km$

Q13. Use law of cosine to prove that

a). $1+\cos\alpha = \dfrac{(b+c+a)(b+c-a)}{2bc}$

Solution: we have to prove
$$1+\cos\alpha = \frac{(b+c+a)(b+c-a)}{2bc}$$
Taking LHS $1+\cos\alpha$
$$= 1+\frac{c^2+b^2-a^2}{2bc}$$
$$= \frac{2bc+c^2+b^2-a^2}{2bc}$$
$$= \frac{c^2+b^2+2bc-a^2}{2bc}$$
$$= \frac{(b+c)^2-a^2}{2bc}$$
$$= \frac{(b+c+a)(b+c-a)}{2bc}$$
= RHS Hence proved

b). $1-\cos\alpha = \dfrac{(a-b+c)(a+b-c)}{2bc}$

Solution: we have to prove
$$1-\cos\alpha = \frac{(a-b+c)(a+b-c)}{2bc}$$
Taking LHS
$1-\cos\alpha$
$$= 1-\frac{c^2+b^2-a^2}{2bc} = \frac{2bc-c^2-b^2+a^2}{2bc}$$
$$= \frac{a^2-c^2-b^2+2bc}{2bc} = \frac{a^2-(b^2+c^2-2bc)}{2bc}$$
$$= \frac{a^2-(b-c)^2}{2bc}$$
$$= \frac{(a-b+c)(a+b-c)}{2bc}$$
= RHS Hence proved

Law of sine
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \quad or \quad \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Exercise 11.4

Solve the triangle with dimensions

Q1. $\alpha = 100^o, c = 345, \gamma = 56.4^o$

Solution: we have $\alpha = 100^o, c = 345, \gamma = 56.4^o$
To find a, b, β Since
$\alpha+\beta+\gamma = 180^o$
$\beta = 180^o - 100^o - 56.4^o$
$\beta = 23.6^o$
Using Law of sine
$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$
$$\frac{a}{\sin 100^o} = \frac{345}{\sin 56.4^o}$$
$$\Rightarrow a = \frac{345\sin 100^o}{\sin 56.4^o}$$
$a = 407.91$
$a \simeq 408$
Again from law of sine
$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$
$$\frac{b}{\sin 23.6^o} = \frac{345}{\sin 56.4^o}$$
$$\Rightarrow b = \frac{345\sin 23.6^o}{\sin 56.4^o}$$
$b = 165.83$
$b \simeq 166$

Q2 . $\alpha = 35^o, \beta = 70^o, c = 115$

Solution: we have $\alpha = 35^o, \beta = 70^o, c = 115$
To find a, b, γ
Since
$\alpha+\beta+\gamma = 180^o$
$\gamma = 180^o - 70^o - 35^o$
$\gamma = 75^o$
Using Law of sine
$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$
$$\frac{a}{\sin 35^o} = \frac{115}{\sin 75^o}$$
$$\Rightarrow a = \frac{115\sin 35^o}{\sin 75^o}$$
$a = 68.2882$
$a \simeq 68$
Again from law of sine $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$
$$\frac{b}{\sin 70^o} = \frac{115}{\sin 75^o}$$
$$\Rightarrow b = \frac{115\sin 70^o}{\sin 75^o}$$
$b = 111.8768$
$b \simeq 112$

Q3 . $\beta = 39^o30', \gamma = 34^o10', a = 240$

Solution: we have $\beta = 39^o30', \gamma = 34^o10', a = 240$
Since $\alpha+\beta+\gamma = 180^o$
$\gamma = 180^o - 39^o30' - 34^o10'$
$\gamma = 106^o20'$
Using Law of sine $\dfrac{b}{\sin\beta} = \dfrac{a}{\sin\alpha}$
$$\frac{b}{\sin 39^o30'} = \frac{240}{\sin 106^o20'}$$
$$\Rightarrow b = \frac{240\sin 39^o30'}{\sin 106^o20'}$$
$b = 159.0789$
$b \simeq 159$
Again from law of sine
$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$
$$\frac{240}{\sin 106^o20'} = \frac{c}{\sin 34^o10'}$$

$\Rightarrow c = \dfrac{240\sin 34^o 10'}{\sin 106^o 20'}$

$c = 140.4529$

$c \simeq 140$

Q4 . $a = 37.5, b = 12.4, \beta = 72^o$

Solution: we have $a = 37.5, b = 12.4, \beta = 72^o$

Using Law of sine $\dfrac{b}{\sin\beta} = \dfrac{a}{\sin\alpha}$

$\dfrac{12.4}{\sin 72^o} = \dfrac{37.5}{\sin\alpha}$

$\Rightarrow \sin\alpha = \dfrac{37.5\sin 72^o}{12.4}$

$\sin\alpha = 2.8762$

Which is not possible
Because Range of sine is [-1,1]
So no triangle is possible

Q5 . $a = 58.4, \beta = 37.2^o, \gamma = 100^o$

Solution: we have $a = 58.4, \beta = 37.2^o, \gamma = 100^o$

Since $\alpha + \beta + \gamma = 180^o$

$\alpha = 180^o - 100^o - 37.2^o$

$\alpha = 42.8^o$

Using Law of sine $\dfrac{b}{\sin\beta} = \dfrac{a}{\sin\alpha}$

$\dfrac{b}{\sin 37.2^o} = \dfrac{58.4}{\sin 42.8^o}$

$\Rightarrow b = \dfrac{58.4\sin 37.2^o}{\sin 42.8^o}$

$b = 51.9671$

$b \simeq 52$

Again from law of sine $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$

$\dfrac{58.4}{\sin 42.8^o} = \dfrac{c}{\sin 100^o}$

$\Rightarrow c = \dfrac{58.4\sin 100^o}{\sin 42.8^o}$

$c = 84.6472$

$c \simeq 84.7$

Q6 . $c = 13.6, \alpha = 30^o 24', \beta = 72^o 6'$

Solution: we have $c = 13.6, \alpha = 30^o 24', \beta = 72^o 6'$

Since $\alpha + \beta + \gamma = 180^o$

$\gamma = 180^o - 72^o 6' - 30^o 24'$

$\gamma = 77^o 30'$

Using Law of sine $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$

$\dfrac{a}{\sin 30^o 24'} = \dfrac{13.6}{\sin 72^o 6'}$

$\Rightarrow a = \dfrac{13.6\sin 30^o 24'}{\sin 77^o 30'}$

$a = 7.0492$

$a \simeq 7.05$

Again from law of sine $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$\dfrac{b}{\sin 72^o 6'} = \dfrac{13.6}{\sin 77^o 30'}$

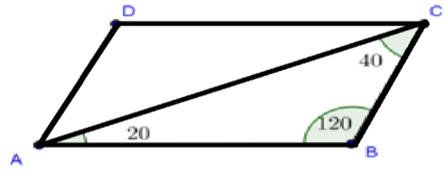
$\Rightarrow b = \dfrac{13.6\sin 72^o 6'}{\sin 77^o 30'}$

$b = 13.2559$

$b \simeq 13.26$

Q7. One diagonal of a parallelogram is 20cm long and at one end forms angles 20^o and 40^o with the sides of the parallelogram. Find length of the sides?

Solution: we have



From fig ABCD is a parallelogram, AC is a diagonal
In ΔABC, AC = b = 20cm, AB = c , BC = a
And $\angle A = \alpha, \angle B = \beta, \angle C = \gamma$ Since

$\alpha + \beta + \gamma = 180^o$

$\beta = 180^o - 20^o - 40^o$

$\beta = 120^o$

Using Law of sine $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$\dfrac{a}{\sin 20^o} = \dfrac{20}{\sin 120^o}$

$\Rightarrow a = \dfrac{20\sin 20^o}{\sin 120^o}$

$a = 7.8986$

$a \simeq 7.9$

Again from law of sine $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$\dfrac{20}{\sin 120^o} = \dfrac{c}{\sin 40^o}$

$\Rightarrow c = \dfrac{20\sin 40^o}{\sin 120^o}$

$c = 14.8445$

$c \simeq 14.8$

Q8. The diagonal of a parallelogram meets the sides at angles of 30^o and 40^o. If the length of the diagonal is 30cm. Then find the perimeter of the parallelogram?

Solution: we have

From fig ABCD is a parallelogram, AC is a diagonal
In ΔABC, AC = b = 30cm, AB = c , BC = a
And $\angle A = \alpha, \angle B = \beta, \angle C = \gamma$ Since

$\alpha + \beta + \gamma = 180^o$

$\beta = 180^o - 30^o - 40^o$

$\beta = 110^o$

Using Law of sine $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$\dfrac{a}{\sin 30^o} = \dfrac{30}{\sin 110^o}$

$\Rightarrow a = \dfrac{30\sin 30^o}{\sin 110^o}$

$a = 15.9627$

$a \simeq 16$

Again from law of sine $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$\dfrac{30}{\sin 110^o} = \dfrac{c}{\sin 40^o}$

$\Rightarrow c = \dfrac{30\sin 40^o}{\sin 110^o}$

$c = 20.5212 \simeq 20.5$

Now perimeter= $= a+a+b+b$

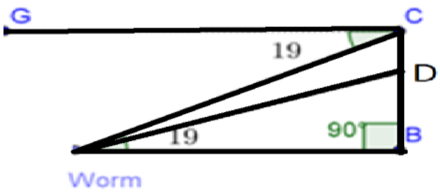
$P = 2a+2b$

$P = 2(16)+2(20.5)$

$P = 32+41 = 73cm$

Q9. A robin on a branch 40ft up in a tree spots a worm at an angle of depression of 14°. From a branch 15ft above the robin, a crow spots same worm at an angle of depression of 19°. How far is each bird from worm?

Sol: Let distance between the crow and worm is DW



distance between the robin and worm is CW

In ΔBDW

$\sin 14^o = \dfrac{BD}{DW}$

$DW = \dfrac{40}{\sin 14^o} = 165.34\ feet$

In ΔCBW

$\sin 19^o = \dfrac{BC}{CW}$

$CW = \dfrac{55}{\sin 19^o}$

$CW = 168.94\ feet$

Q10. angle of elevation of a building is 48° from A and 61° from B if AB is 20m find the height of the building.

Solution: Let Height of the building CD= h

AB= 20m and BC = x

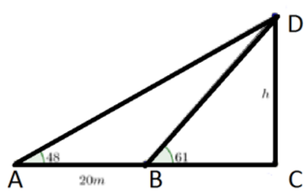
Then AC= AB+BC=20+x

In ΔADC

$\tan 48^o = \dfrac{CD}{AC}$

$h = AC \tan 48^o$

$h = (20+x)\tan 48^o$....(1)



In ΔBCD

$\tan 61^o = \dfrac{CD}{BC}$

$h = BC \tan 61^o$

$h = x\tan 61^o$.................................(2)

Comparing eq (1) and (2)

$x\tan 61^o = (20+x)\tan 48^o$

$1.8x = 1.11(20+x)$

$1.8x = 22.21+1.11x$

$1.8x - 1.11x = 22.21$

$0.69x = 22.21$

$BC = x = \dfrac{22.21}{0.69}$

$BC = x = 32.19m$

Put in (2)

$h = 32.19\tan 61^o$

$h = 58.07m$

Exercise 11.5

Solve the triangle ABC using the law of tangents in problems 1—5

Q1. $a = 48, b = 32,$ and $\gamma = 57^o$

Solution: we have $a = 48, b = 32,$ and $\gamma = 57^o$

To find c, α and β

We know that

$\alpha+\beta+\gamma = 180^o$

$\alpha+\beta = 180^o - \gamma$

$\alpha+\beta = 180^o - 57^o$

$\alpha+\beta = 123^o$.......(1)

$\Rightarrow \dfrac{\alpha+\beta}{2} = 61.5^o$

As $a > b$ $\qquad \dfrac{a+b}{a-b} = \dfrac{\tan\left(\dfrac{\alpha+\beta}{2}\right)}{\tan\left(\dfrac{\alpha-\beta}{2}\right)}$

$\dfrac{48+32}{84-32} = \dfrac{\tan 61.5}{\tan\left(\dfrac{\alpha-\beta}{2}\right)}$

$\dfrac{80}{16} = \dfrac{\tan 61.5}{\tan\left(\dfrac{\alpha-\beta}{2}\right)}$

$\tan\left(\dfrac{\alpha-\beta}{2}\right) = \dfrac{\tan 61.5}{5}$

$\dfrac{\alpha-\beta}{2} = \tan^{-1}\left(\dfrac{1.84177}{5}\right)$

$\dfrac{\alpha-\beta}{2} = 20.22^o$

$\alpha-\beta = 40.44^o$....(2)

Adding equation (1) & (2)

$\alpha+\beta = 123.00^o$

$\alpha-\beta = \ \ 40.44^o$

$2\alpha = 163.44^o$

$\alpha = 81.72^o \qquad \Rightarrow \alpha = 81^o 43'$

Put in (1)

$81.72^o + \beta = 123^o$

$\beta = 123^o - 81.72^o$

$\beta = 41.28^o$

$\beta = 41^o 17'$

Now using Law of sine

$\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$

$\dfrac{48}{\sin 81.72^o} = \dfrac{c}{\sin 57^o}$

$\Rightarrow c = \dfrac{48\sin 57^o}{\sin 81.72^o}$

$c = 40.6802$

$c \simeq 40.68$

Q2. $b = 12.5, c = 23,$ and $\alpha = 38^o 20'$

Solution: we have $b = 12.5, c = 23,$ and $\alpha = 38^o 20'$

To find a, γ and β, We know that

$\alpha+\beta+\gamma = 180^o$

$\beta+\gamma = 180^o - \alpha$

$\beta+\gamma = 180^o - 38^o 20'$

$\beta+\gamma = 141^o 40'$.......(1)

$\Rightarrow \dfrac{\beta+\gamma}{2} = 70^o 50'$

As $c > a$

$\dfrac{c+b}{c-b} = \dfrac{\tan\left(\dfrac{\beta+\gamma}{2}\right)}{\tan\left(\dfrac{\gamma-\beta}{2}\right)}$

$$\frac{23+12.5}{23-12.5}=\frac{\tan 70^o 50^/}{\tan\left(\frac{\gamma-\beta}{2}\right)}$$

$$\frac{35.5}{10.5}=\frac{\tan 70^o 50^/}{\tan\left(\frac{\gamma-\beta}{2}\right)}$$

$$\tan\left(\frac{\gamma-\beta}{2}\right)=\frac{21\tan 70^o 50^/}{71}$$

$$\frac{\gamma-\beta}{2}=\tan^{-1}\left(\frac{21\times 2.87699}{71}\right)$$

$$\frac{\gamma-\beta}{2}=40.39588^o$$

$$\gamma-\beta=80^o 47^/ ....(2)$$

Adding equation (1) & (2)

$$\gamma+\beta=141^o 40^/$$
$$\gamma-\beta=\ \ 80^o 47^/$$
$$2\gamma=222^o 27^/$$
$$\gamma=111^o 13^/ 30^{//}$$

Put in (1)

$$111^o 13^/ 30^{//}+\beta=141^o 40^/$$
$$\beta=141^o 40^/-111^o 13.5^/$$
$$\beta=30^o 26^/ 30^{//}$$

Now using Law of sine

$$\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}$$

$$\frac{a}{\sin\left(38^o 20^/\right)}=\frac{23}{\sin\left(111^o 13^/ 30^{//}\right)}$$

$$\Rightarrow a=\frac{23\sin\left(38^o 20^/\right)}{\sin\left(111^o 13^/ 30^{//}\right)}$$

$$a=15.3035\simeq 15.3$$

Q3. $b=35, c=37,$ and $\alpha=23^o 25^/$

Solution: we have $b=35, c=37,$ and $\alpha=23^o 25^/$

To find a,γ and β

We know that

$$\alpha+\beta+\gamma=180^o$$
$$\beta+\gamma=180^o-\alpha$$
$$\beta+\gamma=180^o-23^o 25^/$$
$$\beta+\gamma=156^o 35^/ ....(1)$$
$$\Rightarrow \frac{\beta+\gamma}{2}=78^o 17^/ 30^{//}$$

As $c>a$

$$\frac{c+b}{c-b}=\frac{\tan\left(\frac{\beta+\gamma}{2}\right)}{\tan\left(\frac{\gamma-\beta}{2}\right)}$$

$$\frac{37+35}{37-35}=\frac{\tan\left(78^o 17^/ 30^{//}\right)}{\tan\left(\frac{\gamma-\beta}{2}\right)}$$

$$\frac{72}{2}=\frac{\tan\left(78^o 17^/ 30^{//}\right)}{\tan\left(\frac{\gamma-\beta}{2}\right)}$$

$$\tan\left(\frac{\gamma-\beta}{2}\right)=\frac{\tan\left(78^o 17^/ 30^{//}\right)}{36}$$

$$\frac{\gamma-\beta}{2}=\tan^{-1}\left(\frac{4.82528}{36}\right)$$

$$\frac{\gamma-\beta}{2}=7.63417^o$$

$$\gamma-\beta=15^o 16^/ ....(2)$$

Adding equation (1) & (2)

$$\gamma+\beta=156^o 35^/$$
$$\gamma-\beta=\ \ 15^o 16^/$$
$$2\gamma=171^o 51^/$$
$$\gamma=85^o 55^/ 30^{//}$$

Put in (1)

$$85^o 55^/ 30^{//}+\beta=156^o 35^/$$
$$\beta=156^o 35^/-85^o 55^/ 30^{//}$$
$$\beta=70^o 39^/ 30^{//}$$

Now using Law of sine

$$\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}$$

$$\frac{a}{\sin\left(23^o 25^/\right)}=\frac{37}{\sin\left(85^o 55^/ 30^{//}\right)}$$

$$\Rightarrow a=\frac{37\sin\left(23^o 25^/\right)}{\sin\left(85^o 55^/ 30^{//}\right)}$$

$$a=14.74161751\simeq 14.74$$

Q4. $a=88, b=48,$ and $\gamma=75^o 51^/$

Solution: we have $a=88, b=48,$ and $\gamma=75^o 51^/$

To find c,α and β, We know that

$$\alpha+\beta+\gamma=180^o$$
$$\alpha+\beta=180^o-\gamma$$
$$\alpha+\beta=180^o-75^o 51^/$$
$$\alpha+\beta=104^o 9^/ ....(1)$$
$$\Rightarrow \frac{\alpha+\beta}{2}=52^o 4^/ 30^{//}$$

As $a>b$ $\quad \frac{a+b}{a-b}=\frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)}$

$$\frac{88+48}{88-48}=\frac{\tan\left(52^o 4^/ 30^{//}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)}$$

$$\frac{136}{40}=\frac{\tan\left(52^o 4^/ 30^{//}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)}$$

$$\tan\left(\frac{\alpha-\beta}{2}\right)=\frac{5\tan\left(52^o 4^/ 30^{//}\right)}{17}$$

$$\frac{\alpha-\beta}{2}=\tan^{-1}\left(\frac{5\times 1.2834}{17}\right)$$

$$\frac{\alpha-\beta}{2}=20.68^o$$

$$\alpha-\beta=41.36011^o=40^o 21^/ 36^{//} ....(2)$$

Adding equation (1) & (2)

$$\alpha+\beta=104^o 9^/$$
$$\alpha-\beta=\ \ 41^o 21^/ 36^{//}$$
$$2\alpha=145^o 30^/ 36^{//}$$
$$\alpha=72^o 45^/ 18^{//}$$

Put in (1)

$$72^o 45^/ 18^{//}+\beta=104^o 9^/$$
$$\beta=104^o 9^/-72^o 45^/ 18^{//}$$
$$\beta=31^o 23^/ 42^{//}$$

Now using Law of sine $\quad \frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}$

$$\frac{88}{\sin\left(72^o 45^/ 18^{//}\right)}=\frac{c}{\sin\left(75^o 51^/\right)}$$

$\Rightarrow c=\dfrac{88\sin(75^o51^/)}{\sin(72^o45^/18^{//})}$

$c=89.346495\simeq 89.35$

Q5. $a=168, c=319,$ and $\beta=110^o22^/$

Solution: we have $a=168, c=319,$ and $\beta=110^o22^/$

To find b, α and γ

We know that

$\alpha+\beta+\gamma=180^o$

$\gamma+\alpha=180^o-\beta$

$\gamma+\alpha=180^o-110^o22^/$

$\gamma+\alpha=69^o38^/$.......(1)

$\Rightarrow \dfrac{\gamma+\alpha}{2}=34^o49^/$

As $c>a$

$\dfrac{c+a}{c-a}=\dfrac{\tan\left(\frac{\gamma+\alpha}{2}\right)}{\tan\left(\frac{\gamma-\alpha}{2}\right)}$

$\dfrac{319+168}{319-168}=\dfrac{\tan(34^o49^/)}{\tan\left(\frac{\gamma-\alpha}{2}\right)}$

$\dfrac{487}{151}=\dfrac{\tan(34^o49^/)}{\tan\left(\frac{\gamma-\alpha}{2}\right)}$

$\tan\left(\dfrac{\gamma-\alpha}{2}\right)=\dfrac{151\tan(34^o49^/)}{487}$

$\dfrac{\gamma-\alpha}{2}=\tan^{-1}\left(\dfrac{151\times 0.695449}{487}\right)$

$\dfrac{\gamma-\alpha}{2}=12.168498^o$

$\gamma-\alpha=24.33699^o$

$\gamma-\alpha=24^o20^/13^{//}$..............................(2)

Adding equation (1) & (2)

$\gamma+\alpha=69^o38^/$

$\gamma-\alpha=24^o20^/13^{//}$

$2\gamma=96^o58^/13^{//}$

$\gamma=46^o59^/6^{//}$

Put in (1)

$46^o59^/6^{//}+\alpha=69^o38^/$

$\alpha=69^o38^/-46^o59^/6^{//}$

$\alpha=22^o38^/54^{//}$

Now using Law of sine $\dfrac{a}{\sin\alpha}=\dfrac{c}{\sin\gamma}$

$\dfrac{168}{\sin(22^o38^/54^{//})}=\dfrac{b}{\sin(110^o22^/)}$

$\Rightarrow b=\dfrac{168\sin(110^o22^/)}{\sin(22^o38^/54^{//})}$

$b=409.005602$

$b\simeq 409$

In problem 6 – 8, find the angle of largest measure

Q6. $a=74, b=52$ and $c=47$

Solution; we have $a=74, b=52$ and $c=47$

Here largest side is $a=74$ so its opposite angle α should be largest

From Law of cosine, we have

$\cos\alpha=\dfrac{c^2+b^2-a^2}{2bc}$

$\cos\alpha=\dfrac{47^2+52^2-74^2}{2(47)(52)}$

$\cos\alpha=\dfrac{-563}{4888}$

$\alpha=\cos^{-1}\dfrac{-563}{4888}$

$\alpha=96.61^o$

$\alpha=96^o36^/50^{//}$

Q7. $a=7, b=9$ and $c=7$

Solution; we have $a=7, b=9$ and $c=7$

Here largest side is $b=9$ so its opposite angle β should be largest

From Law of cosine, we have $\cos\beta=\dfrac{a^2+c^2-b^2}{2ac}$

$\cos\beta=\dfrac{7^2+7^2-9^2}{2(7)(7)}=\dfrac{17}{96}$

$\beta=\cos^{-1}\dfrac{17}{96}$

$\beta=79.8^o$

$\beta\simeq 79^o48^/$

Q8. $a=2.3, b=1.5$ and $c=2.7$

Solution; we have $a=2.3, b=1.5$ and $c=2.7$

Here largest side is $c=2.7$ so its opposite angle γ should be largest

From Law of cosine, we have

$\cos\gamma=\dfrac{a^2+b^2-c^2}{2ab}$

$\cos\gamma=\dfrac{2.3^2+1.5^2-2.7^2}{2(2.3)(1.5)}$

$\cos\gamma=\dfrac{0.25}{6.9}$

$\gamma=\cos^{-1}\left(\dfrac{0.25}{6.9}\right)$

$\gamma=87.9236^o$

$\gamma=87^o55^/25^{//}$

Solve triangle for which length of three sides are given

Q9. $a=9, b=7$ and $c=5$

Solution: we have $a=9, b=7$ and $c=5$

We know that $s=\dfrac{a+b+c}{2}$

$s=\dfrac{9+7+5}{2}=\dfrac{21}{2}$

$s=10.5$

$s-a=10.5-9=1.5$

$s-b=10.5-7=3.5$

Now $\cos\left(\dfrac{\alpha}{2}\right)=\sqrt{\dfrac{s(s-a)}{bc}}$

$\cos\left(\dfrac{\alpha}{2}\right)=\sqrt{\dfrac{10.5(1.5)}{7\times 5}}$

$\cos\left(\dfrac{\alpha}{2}\right)=0.6708$

$\dfrac{\alpha}{2}=\cos^{-1}(0.6708)$

$\dfrac{\alpha}{2}=47.8696^o$

$\alpha=95.7392$

$\alpha\simeq 95.7^o$

Similarly $\cos\left(\frac{\beta}{2}\right)=\sqrt{\frac{s(s-b)}{ca}}$

$\cos\left(\frac{\beta}{2}\right)=\sqrt{\frac{10.5(3.5)}{9\times 5}}$

$\cos\left(\frac{\beta}{2}\right)=0.9037$

$\frac{\beta}{2}=\cos^{-1}(0.9037)$

$\frac{\beta}{2}=25.3418^o$

$\beta=50.7035\simeq 50.7^o$

We know that $\alpha+\beta+\gamma=180^o$

$\gamma=180^o-\alpha-\beta$

$\gamma=180^o-50.7^o-95.7^o$

$\gamma=33.6^o$

Q10. $a=1.2, b=9$ and $c=10$

Solution: we have $a=1.2, b=9$ and $c=10$

To find α, β and γ

We know that $s=\frac{a+b+c}{2}$

$s=\frac{1.2+9+10}{2}=\frac{20.2}{2}$

$s=10.1$

$s-a=10.1-1.2=8.9$

$s-b=10.1-9=1.1$

Now $\cos\left(\frac{\alpha}{2}\right)=\sqrt{\frac{s(s-a)}{bc}}$

$\cos\left(\frac{\alpha}{2}\right)=\sqrt{\frac{10.1(8.9)}{9\times 10}}$

$\cos\left(\frac{\alpha}{2}\right)=0.9994$

$\frac{\alpha}{2}=\cos^{-1}(0.9994)$

$\frac{\alpha}{2}=2.0034^o$

$\alpha=4.0069$

$\alpha\simeq 4^o$

Similarly $\cos\left(\frac{\beta}{2}\right)=\sqrt{\frac{s(s-b)}{ca}}$

$\cos\left(\frac{\beta}{2}\right)=\sqrt{\frac{10.1(1.1)}{1.2\times 10}}$

$\cos\left(\frac{\beta}{2}\right)=0.9622$

$\frac{\beta}{2}=\cos^{-1}(0.9622)$

$\frac{\beta}{2}=15.8033^o$

$\beta=31.6066$

$\beta\simeq 31.6^o$

We know that

$\alpha+\beta+\gamma=180^o$

$\gamma=180^o-\alpha-\beta$

$\gamma=180^o-4^o-31.6^o$

$\gamma=144.4^o$

Q11. $a=6, b=8$ and $c=12$

Solution; we have $a=6, b=8$ and $c=12$

To find α, β and γ We know that $s=\frac{a+b+c}{2}$

$s=\frac{6+8+12}{2}$

$s=\frac{26}{2}=13$

$s-a=13-6=7$

$s-b=13-8=5$

Now $\cos\left(\frac{\alpha}{2}\right)=\sqrt{\frac{s(s-a)}{bc}}$

$\cos\left(\frac{\alpha}{2}\right)=\sqrt{\frac{13(7)}{8\times 12}}$

$\cos\left(\frac{\alpha}{2}\right)=0.9736$

$\frac{\alpha}{2}=\cos^{-1}(0.9736)$

$\frac{\alpha}{2}=13.1922^o$

$\alpha=26.3843$

$\alpha\simeq 26.4^o$

Similarly $\cos\left(\frac{\beta}{2}\right)=\sqrt{\frac{s(s-b)}{ca}}$

$\cos\left(\frac{\beta}{2}\right)=\sqrt{\frac{13(5)}{6\times 12}}$

$\cos\left(\frac{\beta}{2}\right)=0.9501$

$\frac{\beta}{2}=\cos^{-1}(0.9501)$

$\frac{\beta}{2}=18.1680^o$

$\beta=36.331\simeq 36.3^o$

We know that $\alpha+\beta+\gamma=180^o$

$\gamma=180^o-\alpha-\beta$

$\gamma=180^o-26.4^o-36.3^o$

$\gamma=117.3^o$

Q12. A city block is bounded by three streets. If the measure of the sides of the block are 285,375 and 396 meters, find the measure of the angles of the streets make with each other.

Solution; Let $a=285, b=375$ and $c=396$

To find α, β and γ We know that $s=\frac{a+b+c}{2}$

$s=\frac{285+375+396}{2}=\frac{1056}{2}$

$s=528$

$s-a=528-285=243$

$s-b=528-375=153$

Now $\cos\left(\frac{\alpha}{2}\right)=\sqrt{\frac{s(s-a)}{bc}}$

$\cos\left(\frac{\alpha}{2}\right)=\sqrt{\frac{528(243)}{375\times 396}}$

$\cos\left(\frac{\alpha}{2}\right)=0.9295$

$\frac{\alpha}{2}=\cos^{-1}(0.9295)$

$\frac{\alpha}{2}=21.6405^o$

$\alpha=43.2810\simeq 43.3^o=43^o 17'$

Similarly $\cos\left(\frac{\beta}{2}\right)=\sqrt{\frac{s(s-b)}{ca}}$

$\cos\left(\frac{\beta}{2}\right)=\sqrt{\frac{528(153)}{285\times396}}$

$\cos\left(\frac{\beta}{2}\right)=0.8460$

$\frac{\beta}{2}=\cos^{-1}(0.8460)$

$\frac{\beta}{2}=32.2161^o$

$\beta=64.4322$

$\beta\simeq64^o26'$

We know that $\alpha+\beta+\gamma=180^o$

$\gamma=180^o-\alpha-\beta$

$\gamma=180^o-43^o17'-64^o26'$

$\gamma=72^o17'$

Exercise 11.6

Find the area of triangle in each case

Q1. $a=15, b=80,$ and $\gamma=38^o$

Solution: we have $a=15, b=80,$ and $\gamma=38^o$

$\Delta=\frac{1}{2}ab\sin\gamma$

$\Delta=\frac{1}{2}(15)(80)\sin(38^o)$

$\Delta=600\times0.61566$

$\Delta=369.3968$

$\Delta\simeq369.4$ sq unit

Q2. $b=14, c=12$ and $\alpha=82^o$

Solution: we have $b=14, c=12$ and $\alpha=82^o$

$\Delta=\frac{1}{2}bc\sin\alpha$

$\Delta=\frac{1}{2}(14)(12)\sin(82^o)$

$\Delta=84\times0.9945$

$\Delta=83.5398$

$\Delta\simeq83.5$ sq unit

Q3. $a=30, \beta=50^o,$ and $\gamma=100^o$

Solution: we have $a=30, \beta=50^o,$ and $\gamma=100^o$

We know that $\alpha+\beta+\gamma=180^o$

$\alpha=180^o-\beta-\gamma$

$\alpha=180^o-50^o-100^o$

$\alpha=30^o$

$\Delta=\frac{1}{2}a^2\frac{\sin\beta\sin\gamma}{\sin\alpha}$

$\Delta=\frac{1}{2}(30)^2\frac{\sin(50^o)\sin(100^o)}{\sin(30^o)}$

$\Delta=450\times1.5088$

$\Delta=678.9659$

$\Delta\simeq679$ sq unit

Q4. $b=40, \alpha=50^o,$ and $\gamma=60^o$

Solution: we have $b=40, \alpha=50^o,$ and $\gamma=60^o$

We know that $\alpha+\beta+\gamma=180^o$

$\beta=180^o-\alpha-\gamma$

$\beta=180^o-50^o-60^o$

$\beta=70^o$

$\Delta=\frac{1}{2}b^2\frac{\sin\alpha\sin\gamma}{\sin\beta}$

$\Delta=\frac{1}{2}(40)^2\frac{\sin(50^o)\sin(60^o)}{\sin(70^o)}$

$\Delta=800\times0.70599$

$\Delta=564.7923$

$\Delta\simeq565$ sq unit

Q5. $a=7, b=8$ and $c=2$

Solution: we have $a=7, b=8$ and $c=2$

$s=\frac{a+b+c}{2}$

$s=\frac{7+8+2}{2}=\frac{17}{2}$

$s=8.5$

$s-a=8.5-7=1.5$

$s-b=8.5-8=0.5$

$s-c=8.5-2=6.5$

$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$\Delta=\sqrt{8.5(1.5)(0.5)(6.5)}$

$\Delta=6.44$ sq. unit

Q6. $a=11, b=9$ and $c=8$

Solution: we have $a=11, b=9$ and $c=8$

$s=\frac{a+b+c}{2}$

$s=\frac{11+9+8}{2}=\frac{28}{2}$

$s=14$

$s-a=14-11=3$

$s-b=14-9=5$

$s-c=14-8=6$

$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$\Delta=\sqrt{14(3)(5)(6)}$

$\Delta=35.5$ sq. unit

Q7. $b=414, c=485$ and $\alpha=49^o47'$

Solution: we have $b=414, c=485$ and $\alpha=49^o47'$

$\Delta=\frac{1}{2}bc\sin\alpha$

$\Delta=\frac{1}{2}(414)(485)\sin(49^o47')$

$\Delta=100395\times0.7636$

$\Delta=76662.449$

$\Delta\simeq76662$ sq unit

Q8. $a=32, \beta=47^o24',$ and $\gamma=70^o16'$

Solution: we have $a=32, \beta=47^o24',$ and $\gamma=70^o16'$

We know that $\alpha+\beta+\gamma=180^o$

$\alpha=180^o-\beta-\gamma$

$\alpha=180^o-47^o24'-70^o16'$

$\alpha=62^o20'$

$\Delta=\frac{1}{2}a^2\frac{\sin\beta\sin\gamma}{\sin\alpha}$

$\Delta=\frac{1}{2}(32)^2\frac{\sin(47^o24')\sin(70^o16')}{\sin(62^o20')}$

$\Delta=512\times0.7823$

$\Delta=400.5459$

$\Delta\simeq400.5$ sq unit

Q9. $b=47, \alpha=60^o25',$ and $\gamma=41^o35'$

Solution: we have $b=47, \alpha=60^o25',$ and $\gamma=41^o35'$

We know that

$\alpha+\beta+\gamma=180^o$

$\beta=180^o-\alpha-\gamma$

$\beta=180^o-60^o25'-41^o35'$

$\beta=78^o$

$\Delta = \frac{1}{2}b^2 \frac{\sin\alpha \sin\gamma}{\sin\beta}$

$\Delta = \frac{1}{2}(47)^2 \frac{\sin(60^o25')\sin(41^o35')}{\sin(78^o)}$

$\Delta = 1104.5 \times 0.5901$

$\Delta = 651.7448$

$\Delta \simeq 651.7$ sq unit

Q10. $c = 57, \alpha = 23^o24'$, and $\beta = 71^o36'$

Solution: we have $c = 57, \alpha = 23^o24'$, and $\beta = 71^o36'$

We know that $\alpha + \beta + \gamma = 180^o$

$\gamma = 180^o - \alpha - \beta$

$\gamma = 180^o - 23^o24' - 71^o36'$

$\gamma = 85^o$

$\Delta = \frac{1}{2}c^2 \frac{\sin\alpha \sin\beta}{\sin\gamma}$

$\Delta = \frac{1}{2}(57)^2 \frac{\sin(23^o24')\sin(71^o36')}{\sin(85^o)}$

$\Delta = 1624.5 \times 0.3783$

$\Delta = 614.5217$

$\Delta \simeq 614.5$ sq unit

Q11. $a = 925, c = 433$, and $\beta = 42^o17'$

Solution: we have $a = 925, c = 433$, and $\beta = 42^o17'$

$\Delta = \frac{1}{2}ac\sin\beta$

$\Delta = \frac{1}{2}(925)(433)\sin(42^o17')$

$\Delta = 200262.5 \times 0.6728$

$\Delta = 134736.0763$

$\Delta \simeq 134736.1$ sq unit

Q12. $a = 92, b = 71$, and $\gamma = 56^o44'$

Solution: we have $a = 92, b = 71$, and $\gamma = 56^o44'$

$\Delta = \frac{1}{2}ab\sin\gamma$

$\Delta = \frac{1}{2}(92)(71)\sin(56^o44')$

$\Delta = 3266 \times 0.8361$

$\Delta = 2730.7896$

$\Delta = 2730.7$ sq unit

Exercise 11.7

In problem 1 – 4, compute radius of circle inscribed (r) and circumscribed (R) of triangle whose sides are given

Q1. 3,5,6

Solution: Let $a = 3, b = 5$ and $c = 6$

$s = \frac{a+b+c}{2} = \frac{3+5+6}{2} = \frac{14}{2} = 7$

$s - a = 7 - 3 = 4$

$s - b = 7 - 5 = 2$

$s - c = 7 - 6 = 1$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{7(4)(2)(1)}$

$\Delta = 2\sqrt{14} = 7.48$ sq. unit

$r = \frac{\Delta}{s}$

$r = \frac{7.48}{7}$

$r = 1.069 \simeq 1.1$

$R = \frac{abc}{4\Delta}$

$R = \frac{(3)(5)(6)}{4(7.48)}$

$R = 3$

Q2. 21,20,29

Solution: Let $a = 21, b = 20$ and $c = 29$

$s = \frac{a+b+c}{2} = \frac{21+20+29}{2} = \frac{70}{2} = 35$

$s - a = 35 - 21 = 14$

$s - b = 35 - 20 = 15$

$s - c = 35 - 29 = 6$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{35(14)(15)(6)}$

$\Delta = 210$ sq. unit

$r = \frac{\Delta}{s}$

$r = \frac{210}{35}$

$r = 6$

$R = \frac{abc}{4\Delta}$

$R = \frac{(21)(20)(29)}{4(210)}$

$R = 14.5$

Q3. 117,44,125

Solution: Let $a = 117, b = 44$ and $c = 125$

$s = \frac{a+b+c}{2}$

$s = \frac{117+44+125}{2}$

$s = \frac{286}{2} = 143$

$s - a = 143 - 117 = 26$

$s - b = 143 - 44 = 99$

$s - c = 143 - 125 = 18$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{143(26)(99)(18)}$

$\Delta = 2574$ sq. unit

$r = \frac{\Delta}{s}$

$r = \frac{2574}{143}$

$r = 18$

$R = \frac{abc}{4\Delta}$

$R = \frac{(117)(44)(125)}{4(2574)}$

$R = 62.5$

Q4. 20,99,101

Solution: Let $a = 20, b = 99$ and $c = 101$

$s = \frac{a+b+c}{2}$

$s = \frac{20+99+101}{2} = \frac{220}{2}$

$s = 110$

$s - a = 110 - 20 = 90$

$s - b = 110 - 99 = 11$

$s - c = 110 - 101 = 9$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{110(90)(11)(9)}$

$\Delta = 990$ sq. unit

$r = \frac{\Delta}{s}$

$r = \frac{990}{110}$

$r = 9$

$R = \frac{abc}{4\Delta}$

$R = \frac{(20)(99)(101)}{4(990)}$

$R = 50.5$

Q5. Find area of the inscribed circle of the triangle with measure of the sides 55m, 25m and 70m.

Solution: Let $a = 55m, b = 25m$ and $c = 70m$

$s = \frac{a+b+c}{2}$

$s = \frac{55+25+70}{2}$

$s = \frac{150}{2}$

$s = 75$

$s-a = 75-55 = 20$

$s-b = 75-25 = 50$

$s-c = 75-70 = 5$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{75(20)(50)(5)}$

$\Delta = 612.37$ sq. unit

$r = \frac{\Delta}{s}$

$r = \frac{612.37}{75}$

$r = 8.165$

Area of incribed circle $= \pi r^2$

Area of incribed circle $= 3.142(8.165)^2$

Area of incribed circle $= 209.44m^2$

Q6. Measures of sides of a triangle are 20,25 and 30 decimeter. Find radius of the escribed circles.

Solution: Let $a = 20dm, b = 25dm$ and $c = 30dm$

$s = \frac{a+b+c}{2} = \frac{20+25+30}{2}$

$s = \frac{75}{2} = 37.5$

$s-a = 37.5-20 = 17.5$

$s-b = 37.5-25 = 12.5$

$s-c = 37.5-30 = 7.5$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{37.5(17.5)(12.5)(7.5)}$

$\Delta = 248.04\ dm^2$

a). Opposite to larger side

Solution: Larger side c = 30dm

$r_3 = \frac{\Delta}{s-c}$

$r_3 = \frac{248.04}{7.5}$

$r_3 = 33.072dm$

b). opposite to smaller side

Solution: Smaller side a = 20 dm

$r_1 = \frac{\Delta}{s-a}$

$r_1 = \frac{248.04}{17.5}$

$r_1 = 14.174dm$

Q7. Show that $\sqrt{rr_1r_2r_3} = \Delta$ = Area of triangle ABC

Solution: we have to prove $\sqrt{rr_1r_2r_3} = \Delta$

$\therefore r = \frac{\Delta}{s}, r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$

And $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

Taking LHS $\sqrt{rr_1r_2r_3}$

$= \sqrt{\frac{\Delta}{s}\frac{\Delta}{s-a}\frac{\Delta}{s-b}\frac{\Delta}{s-c}}$

$= \sqrt{\frac{\Delta^4}{s(s-a)(s-b)(s-c)}}$

$= \frac{\sqrt{\Delta^4}}{\sqrt{s(s-a)(s-b)(s-c)}}$

$= \frac{\Delta^2}{\Delta}$

$= \Delta = RHS$

Hence proved

Q8 Show that $\frac{abc}{4s}(\sin\alpha + \sin\beta + \sin\gamma) = \Delta$ =Area of ΔABC

Sol: To show that $\frac{abc}{4s}(\sin\alpha + \sin\beta + \sin\gamma) = \Delta$

$\therefore \Delta = \frac{1}{2}bc\sin\alpha, \Delta = \frac{1}{2}ac\sin\beta, \Delta = \frac{1}{2}ab\sin\gamma$

$\Rightarrow \sin\alpha = \frac{2\Delta}{bc}, \sin\beta = \frac{2\Delta}{ac}, \sin\gamma = \frac{2\Delta}{ab}$

Taking LHS $\frac{abc}{4s}(\sin\alpha + \sin\beta + \sin\gamma)$

$= \frac{abc}{4s}\left(\frac{2\Delta}{bc} + \frac{2\Delta}{ac} + \frac{2\Delta}{ab}\right)$

$= \frac{2\Delta abc}{4s}\left(\frac{1}{bc} + \frac{1}{ac} + \frac{1}{ab}\right)$

$= \frac{\Delta abc}{2s}\left(\frac{a+b+c}{abc}\right)$

$= \frac{\Delta abc}{sabc}\left(\frac{a+b+c}{2}\right)$

$= \frac{\Delta}{s}(s)$

$= \Delta = RHS$

Hence Proved

Q9. Prove that for any triangle ABC

$r_1 + r_2 + r_3 - r = 4R$

Solution: we have to prove $r_1 + r_2 + r_3 - r = 4R$

$\therefore r = \frac{\Delta}{s}, r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$

$R = \frac{abc}{4\Delta}, 2s = a+b+c$

Taking LHS $r_1 + r_2 + r_3 - r$

$= \frac{\Delta}{s-a} + \frac{\Delta}{s-b} + \frac{\Delta}{s-c} - \frac{\Delta}{s}$

$= \Delta\left(\frac{1}{s-a} + \frac{1}{s-b}\right) + \Delta\left(\frac{1}{s-c} - \frac{1}{s}\right)$

$= \Delta\left(\frac{s-b+s-a}{(s-a)(s-b)}\right) + \Delta\left(\frac{s-s+c}{s(s-c)}\right)$

$= \Delta\left(\frac{2s-a-b}{(s-a)(s-b)}\right) + \Delta\left(\frac{c}{s(s-c)}\right)$

$= \Delta\left(\frac{a+b+c-a-b}{(s-a)(s-b)}\right) + \Delta\left(\frac{c}{s(s-c)}\right)$

$$= \Delta\left(\frac{c}{(s-a)(s-b)}\right) + \Delta\left(\frac{c}{s(s-c)}\right)$$
$$= \Delta c\left(\frac{1}{(s-a)(s-b)} + \frac{1}{s(s-c)}\right)$$
$$= \Delta c\left(\frac{s(s-c)+(s-a)(s-b)}{\Delta^2}\right)$$
$$= \frac{c}{\Delta}\left(s^2 - sc + s^2 - sb - sa + ab\right)$$
$$= \frac{c}{\Delta}\left(2s^2 - s(a+b+c) + ab\right)$$
$$= \frac{c}{\Delta}\left(2s^2 - 2s^2 + ab\right)$$
$$= \frac{abc}{\Delta}$$
$$= 4\frac{abc}{4\Delta}$$
$$= 4R = RHS$$

Hence Proved

Q10. For a triangle ABC show that $r_1r_2 + r_2r_3 + r_3r_1 = s^2$

Solution: we have to prove $r_1r_2 + r_2r_3 + r_3r_1 = s^2$

$$\therefore r = \frac{\Delta}{s}, r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$$

Taking LHS $r_1r_2 + r_2r_3 + r_3r_1$

$$= \frac{\Delta}{s-a}.\frac{\Delta}{s-b} + \frac{\Delta}{s-b}.\frac{\Delta}{s-c} + \frac{\Delta}{s-c}\frac{\Delta}{s-a}$$
$$= \Delta^2\left(\frac{1}{(s-a)(s-b)} + \frac{1}{(s-b)(s-c)} + \frac{1}{(s-c)(s-a)}\right)$$
$$= \Delta^2\left(\frac{s-c+s-a+s-b}{(s-a)(s-b)(s-c)}\right)$$
$$= \frac{s\Delta^2}{s}\left(\frac{3s-(a+b+c)}{(s-a)(s-b)(s-c)}\right)$$
$$= s\Delta^2\left(\frac{3s-2s}{s(s-a)(s-b)(s-c)}\right) = s\Delta^2\left(\frac{s}{\Delta^2}\right)$$
$$= s^2$$

Q11. For the e—radii r_1, r_2 and r_3 of the triangle ABC Show that $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$

Solution: we have to prove $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$

$$\therefore r = \frac{\Delta}{s}, r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$$
$$\frac{1}{r} = \frac{s}{\Delta}, \frac{1}{r_1} = \frac{s-a}{\Delta}, \frac{1}{r_2} = \frac{s-b}{\Delta}, \frac{1}{r_3} = \frac{s-c}{\Delta}$$

Taking LHS $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

$$= \frac{s-a}{\Delta} + \frac{s-b}{\Delta} + \frac{s-c}{\Delta}$$
$$= \frac{s-a+s-b+s-c}{\Delta}$$
$$= \frac{3s-a-b-c}{\Delta}$$
$$= \frac{3s-(a+b+c)}{\Delta}$$
$$= \frac{3s-2s}{\Delta}$$
$$= \frac{s}{\Delta} = \frac{1}{r} = RHS$$

Hence Proved

Q12. The sides of the triangle are in the ratio 3:7:8. The radius of the inscribed circle is 2m. Find the sides of the triangles.

Solution: The ratios of the sides are 3:7:8

Let the sides are 3x,7x and 8x

$$s = \frac{a+b+c}{2}$$
$$s = \frac{3x+7x+8x}{2}$$
$$s = \frac{18x}{2}$$
$$s = 9x$$

Now

$$s-a = 9x-3x = 6x$$
$$s-b = 9x-7x = 2x$$
$$s-c = 9x-8x = x$$
$$\therefore \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$
$$\Delta = \sqrt{9x(6x)(2x)(x)}$$
$$\Delta = 6x^2\sqrt{3}$$

Given that

$$r = 2m$$
$$\frac{\Delta}{S} = 2m$$
$$\Delta = 2S$$
$$6x^2\sqrt{3} = 2(9x)$$
$$6x^2\sqrt{3} = 18x$$
$$x\sqrt{3} = 3$$
$$x = \frac{3}{\sqrt{3}} = \sqrt{3}$$

The required sides are

$$a = 3x$$
$$a = 3\sqrt{3}m$$
$$b = 7x$$
$$b = 7\sqrt{3}m$$
$$c = 8x$$
$$c = 8\sqrt{3}m$$

Q13. Show that $r_1.r_2.r_3 = rS$

Solution: we have to prove $r_1.r_2.r_3 = rS$

$$\therefore r = \frac{\Delta}{s}, r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$$

Taking LHS $r_1.r_2.r_3$

$$= \frac{\Delta}{s-a}.\frac{\Delta}{s-b}.\frac{\Delta}{s-c}$$
$$= \frac{\Delta^3}{(s-a)(s-b)(s-c)}$$
$$= \frac{s\Delta^3}{s(s-a)(s-b)(s-c)}$$
$$= \frac{S\Delta^3}{\Delta^2}$$
$$= S\Delta$$
$$= \frac{\Delta}{S}S^2$$
$$= rS^2 = RHS$$

Hence proved