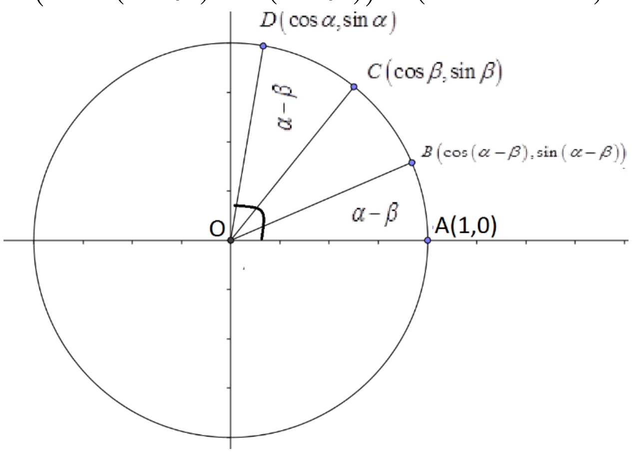
Chapter 10

Trigonometric Identities

Let $m\angle AOD=\alpha$ and $m\angle AOC=\beta$

Then $m\angle COD=\alpha-\beta$ $C\left(-\cos\beta,\sin\beta\right)$

$B\left(-\cos\left(\alpha-\beta\right),\sin\left(\alpha-\beta\right)\right)$ $D\left(-\cos\alpha,\sin\alpha\right)$



If two arcs of a circle a congruent then the corresponding angles/chords are also congruent

i.e. Length of arc AB = Length of arc CD

$$m\angle AOB=m\angle COD$$

Now $\left|AB\right|^2=\left|CD\right|^2$ using distance formula

Since $\left|AB\right|^2=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$ Applying

$$\left(\cos\left(\alpha-\beta\right)-1\right)^2+\left(\sin\left(\alpha-\beta\right)-0\right)^2=\left(\cos\alpha-\cos\beta\right)^2+\left(\sin\alpha-\sin\beta\right)^2$$

$$\cos^2\left(\alpha-\beta\right)+1-2\cos\left(\alpha-\beta\right)+\sin^2\left(\alpha-\beta\right)=\cos^2\alpha+\cos^2\beta$$
$$-2\cos\alpha\cos\beta+\sin^2\alpha+\sin^2\beta-2\sin\alpha\sin\beta$$

$$\cos^2\left(\alpha-\beta\right)+\sin^2\left(\alpha-\beta\right)+1-2\cos\left(\alpha-\beta\right)=\cos^2\alpha+\sin^2\alpha$$
$$+\cos^2\beta+\sin^2\beta-2\cos\alpha\cos\beta-2\sin\alpha\sin\beta$$

$$1+1-2\cos\left(\alpha-\beta\right)=1+1-2\cos\alpha\cos\beta-2\sin\alpha\sin\beta$$

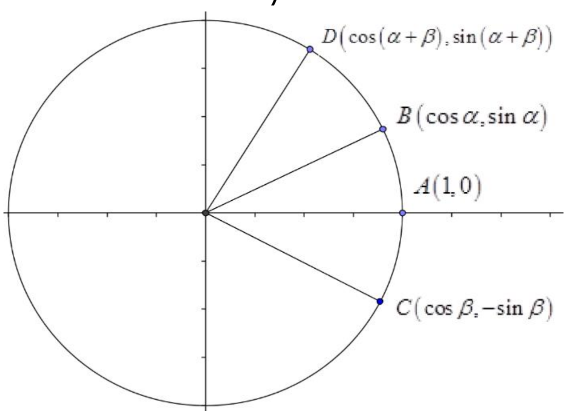
$$2-2\cos\left(\alpha-\beta\right)=2-2\cos\alpha\cos\beta-2\sin\alpha\sin\beta$$

$$-2\cos\left(\alpha-\beta\right)=-2\left(\cos\alpha\cos\beta+\sin\alpha\sin\beta\right)$$

$$\boxed{\cos\left(\alpha-\beta\right)=\cos\alpha\cos\beta+\sin\alpha\sin\beta}$$

Let $m\angle AOB=\alpha$ and $m\angle AOC=\beta$

Then $m\angle AOD=\alpha+\beta$



Now $\left|AD\right|^2=\left|BC\right|^2$ using distance formula

Since $\left|AB\right|^2=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$ Applying

$$\left(\cos\left(\alpha+\beta\right)-1\right)^2+\left(\sin\left(\alpha+\beta\right)-0\right)^2=\left(\cos\alpha-\cos\beta\right)^2+\left(\sin\alpha+\sin\beta\right)^2$$

$$\cos^2\left(\alpha+\beta\right)+1-2\cos\left(\alpha+\beta\right)+\sin^2\left(\alpha+\beta\right)=\cos^2\alpha+\cos^2\beta$$
$$-2\cos\alpha\cos\beta+\sin^2\alpha+\sin^2\beta+2\sin\alpha\sin\beta$$

$$\cos^2\left(\alpha+\beta\right)+\sin^2\left(\alpha+\beta\right)+1-2\cos\left(\alpha+\beta\right)=\cos^2\alpha+\sin^2\alpha$$
$$+\cos^2\beta+\sin^2\beta-2\cos\alpha\cos\beta+2\sin\alpha\sin\beta$$

$$1+1-2\cos\left(\alpha+\beta\right)=1+1-2\cos\alpha\cos\beta+2\sin\alpha\sin\beta$$

$$2-2\cos\left(\alpha+\beta\right)=2-2\cos\alpha\cos\beta+2\sin\alpha\sin\beta$$

$$-2\cos\left(\alpha+\beta\right)=-2\left(\cos\alpha\cos\beta-\sin\alpha\sin\beta\right)$$

$$\boxed{\cos\left(\alpha+\beta\right)=\cos\alpha\cos\beta-\sin\alpha\sin\beta}$$

Exercise 10.1

Q1. Prove that

i). $Sin(\pi+\theta)=-Sin\theta$

Solution; we have to prove $Sin(\pi+\theta)=-Sin\theta$

Take LHS

$$Sin(\pi+\theta)=Sin\pi.Cos\theta+Cos\pi.Sin\theta$$
$$\therefore\cos\pi=-1,\sin\pi=0$$
$$=0.Cos\theta+\left(-1\right).Sin\theta$$
$$=-\sin\theta$$
$$=RHS$$

Hence proved

ii). $\cos(\pi+\theta)=-\cos\theta$

Solution; we have to prove $\cos(\pi+\theta)=-\cos\theta$

Take LHS

$$\cos(\pi+\theta)=\cos\pi.Cos\theta-\sin\pi.Sin\theta$$
$$\therefore\cos\pi=-1,\sin\pi=0$$
$$=\left(-1\right).Cos\theta-0.Sin\theta$$
$$=-\cos\theta$$
$$=RHS$$

Hence proved

iii). $\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$

Solution; we have to prove $\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$

Take LHS $\tan\left(\frac{\pi}{2}+\theta\right)$

$$\frac{Sin\left(\frac{\pi}{2}+\theta\right)}{\cos\left(\frac{\pi}{2}+\theta\right)}=\frac{Sin\frac{\pi}{2}.Cos\theta+Cos\frac{\pi}{2}.Sin\theta}{\cos\frac{\pi}{2}.Cos\theta-\sin\frac{\pi}{2}.Sin\theta}$$
$$\therefore\cos\tfrac{\pi}{2}=0,\sin\tfrac{\pi}{2}=1$$
$$=\frac{1.Cos\theta+0.Sin\theta}{0.Cos\theta-1.Sin\theta}$$
$$=-\frac{Cos\theta}{Sin\theta}$$
$$=-\cot\theta$$
$$=RHS$$

Hence proved

iv). $\cos(270^o+\theta)=+\sin\theta$

Solution; we have to prove

$$\cos(270^o+\theta)=+\sin\theta$$

Take LHS

$$\cos(270^o+\theta)=\cos 270^o.Cos\theta-\sin 270^o.Sin\theta$$
$$\therefore\cos 270^o=0,\sin 270^o=-1$$
$$=0.Cos\theta-\left(-1\right).Sin\theta$$
$$=+\sin\theta=RHS$$

Hence proved

v). $\tan(270^o+\theta)=-\cot\theta$

Solution; we have to prove

$\tan(270^o+\theta)=-\cot\theta$

Take LHS $\tan(270^o+\theta)$

$$\frac{Sin\left(270^o+\theta\right)}{\cos\left(270^o+\theta\right)}=\frac{Sin270^o.Cos\theta+Cos270^o.Sin\theta}{\cos270^o.Cos\theta-\sin270^o.Sin\theta}$$

$$\therefore \cos270^o=0, \sin270^o=-1$$

$$=\frac{-1.Cos\theta+0.Sin\theta}{0.Cos\theta-(-1).Sin\theta}$$

$$=-\frac{Cos\theta}{+Sin\theta}$$

$$=-\cot\theta$$

$$=RHS$$

Hence proved

vi). $Sin(360^o-\theta)=-Sin\theta$

Sol; we have to prove $Sin(360^o-\theta)=-Sin\theta$

Take LHS

$$Sin(360^o-\theta)=Sin360^o.Cos\theta-Cos360^o.Sin\theta$$

$$\therefore \cos360^o=1, \sin360^o=0$$

$$=0.Cos\theta-(1).Sin\theta$$

$$=-\sin\theta$$

$$=RHS$$

Hence proved

vii). $\cot(360^o+\theta)=\cot\theta$

Solution; we have to prove $\cot(360^o+\theta)=\cot\theta$

Take LHS $\cot(360^o+\theta)$

$$\frac{\cos\left(360^o+\theta\right)}{Sin\left(360^o+\theta\right)}=\frac{\cos360^o.Cos\theta-\sin360^o.Sin\theta}{Sin360^o.Cos\theta+Cos360^o.Sin\theta}$$

$$\therefore \cos360^o=1, \sin360^o=0$$

$$=\frac{1.Cos\theta-0.Sin\theta}{0.Cos\theta+1.Sin\theta}$$

$$=\frac{Cos\theta}{Sin\theta}$$

$$=\cot\theta$$

$$=RHS$$

Hence proved

Q2. Show that

i). $Sin(\alpha+\beta)+Sin(\alpha-\beta)=2Sin\alpha.Cos\beta$

Solution; we have to Show

$Sin(\alpha+\beta)+Sin(\alpha-\beta)=2Sin\alpha.Cos\beta$

take LHS $Sin(\alpha+\beta)+Sin(\alpha-\beta)$

$$=\{Sin\alpha.Cos\beta-Cos\alpha.Sin\beta\}+\{Sin\alpha.Cos\beta+Cos\alpha.Sin\beta\}$$

$$=Sin\alpha.Cos\beta+Sin\alpha.Cos\beta-Cos\alpha.Sin\beta+Cos\alpha.Sin\beta$$

$$=2Sin\alpha.Cos\beta=RHS$$

Hence proved

ii). $Cos(\alpha+\beta)-Cos(\alpha-\beta)=-2Sin\alpha.Sin\beta$

Solution; we have to Show

$Cos(\alpha+\beta)-Cos(\alpha-\beta)=-2Sin\alpha.Sin\beta$

take LHS $Cos(\alpha+\beta)-Cos(\alpha-\beta)$

$$=\{Cos\alpha.Cos\beta-Sin\alpha.Sin\beta\}-\{Cos\alpha.Cos\beta+Sin\alpha.Sin\beta\}$$

$$=Cos\alpha.Cos\beta-Sin\alpha.Sin\beta-Cos\alpha.Cos\beta-Sin\alpha.Sin\beta$$

$$=-2Sin\alpha.Sin\beta$$

$$=RHS$$

Hence proved

Q3. Show that

i). $\cos\alpha=2\cos^2\frac{\alpha}{2}-1=1-2\sin^2\frac{\alpha}{2}$

Solution; we have to show

$$\cos\alpha=2\cos^2\frac{\alpha}{2}-1=1-2\sin^2\frac{\alpha}{2}$$

Take LHS

$$\cos\alpha=\cos2\left(\frac{\alpha}{2}\right)=\cos\left(\frac{\alpha}{2}+\frac{\alpha}{2}\right)$$

$$\cos\alpha=\cos\frac{\alpha}{2}\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\sin\frac{\alpha}{2}$$

$$\cos\alpha=\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}\ldots\ldots(1)$$

Since we know that

$$\therefore \cos^2x+\sin^2x=1$$

$$\Rightarrow \cos^2x=1-\sin^2x$$

Putting in equation (1) we get

$$\cos\alpha=1-\sin^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}$$

$$\cos\alpha=1-2\sin^2\frac{\alpha}{2}.........(2)$$

Similarly

$$\therefore \cos^2x+\sin^2x=1$$

$$\Rightarrow \sin^2x=1-\cos^2x$$

Putting in equation (1) we get

$$\cos\alpha=\cos^2\frac{\alpha}{2}-\left(1-\cos^2\frac{\alpha}{2}\right)$$

$$\cos\alpha=\cos^2\frac{\alpha}{2}-1+\cos^2\frac{\alpha}{2}$$

$$\cos\alpha=2\cos^2\frac{\alpha}{2}-1.........(3)$$

From equations (2) and (3)

$$\cos\alpha=2\cos^2\frac{\alpha}{2}-1=1-2\sin^2\frac{\alpha}{2}$$

ii). $\sin(\alpha+\beta)\sin(\alpha-\beta)=\cos^2\beta-\cos^2\alpha$

Solution; we have to prove

$\sin(\alpha+\beta)\sin(\alpha-\beta)=\cos^2\beta-\cos^2\alpha$

Take LHS $Sin(\alpha+\beta).Sin(\alpha-\beta)$

$$=\{Sin\alpha.Cos\beta-Cos\alpha.Sin\beta\}\{Sin\alpha.Cos\beta+Cos\alpha.Sin\beta\}$$

$$\therefore (a+b)(a-b)=a^2-b^2$$

$$=Sin^2\alpha.Cos^2\beta-\cos^2\alpha.\sin^2\beta$$

$$=(1-\cos^2\alpha)Cos^2\beta-\cos^2\alpha.(1-\cos^2\beta)$$

$$=Cos^2\beta-\cos^2\alpha Cos^2\beta-\cos^2\alpha+\cos^2\alpha\cos^2\beta$$

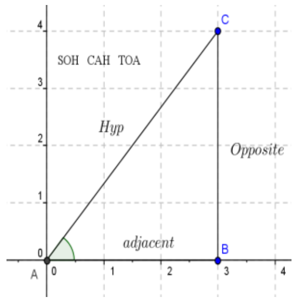
$$=Cos^2\beta-\cos^2\alpha$$

$$=RHS$$

Hence proved

Q4. If $\sin\alpha = \frac{4}{5}$, $\sin\beta = \frac{12}{13}$ and both α and β are measures of first quadrantal angles, than find:

Solution: Take $\sin\alpha = \frac{4}{5}$



AB= Adjacent
BC= Opposite
AC= Hypotenuse

$AC^2 = AB^2 + BC^2$

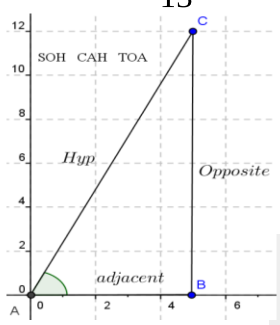
$5^2 = AB^2 + 4^2$

$AB^2 = 25 - 16 = 9$

$AB = 3$

$CAH \Rightarrow \cos\alpha = \frac{3}{5}, TOA \Rightarrow \tan\alpha = \frac{4}{3}$

Take $\sin\beta = \frac{12}{13}$



AB= Adjacent
BC= Opposite
AC= Hypotenuse

$AC^2 = AB^2 + BC^2$

$13^2 = AB^2 + 12^2$

$AB^2 = 169 - 144 = 25$

$AB = 5$

$CAH \Rightarrow \cos\beta = \frac{5}{13}, TOA \Rightarrow \tan\beta = \frac{12}{5}$

i). $Sin(\alpha+\beta)$

Solution; we have to find the value of $Sin(\alpha+\beta)$

$Sin(\alpha+\beta) = Sin\alpha.Cos\beta + Cos\alpha.Sin\beta$

$Sin(\alpha+\beta) = \frac{4}{5}.\frac{5}{13} + \frac{3}{5}.\frac{12}{13}$

$Sin(\alpha+\beta) = \frac{20+36}{65}$

$Sin(\alpha+\beta) = \frac{56}{65}$

ii). $Cos(\alpha+\beta)$

Solution; we have to find the value of $Cos(\alpha+\beta)$

$Cos(\alpha+\beta) = Cos\alpha.Cos\beta - Sin\alpha.Sin\beta$

$Cos(\alpha+\beta) = \frac{3}{5}.\frac{5}{13} - \frac{4}{5}.\frac{12}{13}$

$Cos(\alpha+\beta) = \frac{15-48}{65}$

$Cos(\alpha+\beta) = \frac{-33}{65}$

iii). $\tan(\alpha+\beta)$

Solution; we have to find the value of $\tan(\alpha+\beta)$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$

$\tan(\alpha+\beta) = \left(\frac{4}{3} + \frac{12}{5}\right) \div \left(1 - \frac{4}{3}.\frac{12}{5}\right)$

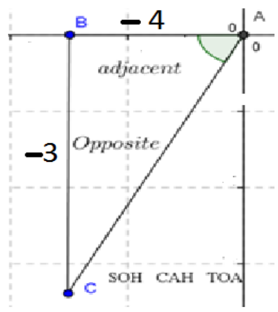
$\tan(\alpha+\beta) = \left(\frac{20+36}{15}\right) \div \left(\frac{15-48}{15}\right)$

$\tan(\alpha+\beta) = \left(\frac{56}{15}\right) \div \left(\frac{-33}{15}\right) = \left(\frac{56}{15}\right) \times \left(\frac{15}{-33}\right)$

$\tan(\alpha+\beta) = \left(\frac{56}{-33}\right)$

Q5. If $\tan\alpha = \frac{3}{4}$, $\sec\beta = \frac{13}{5}$ and neither terminal side of the angle of measure α nor β in the first quadrant, than find:

Solution: Take $\tan\alpha = \frac{3}{4}$ where $\alpha \in III$ Quadrant



AB= Adjacent
BC= Opposite
AC= Hypotenuse

$AC^2 = BC^2 + AB^2$

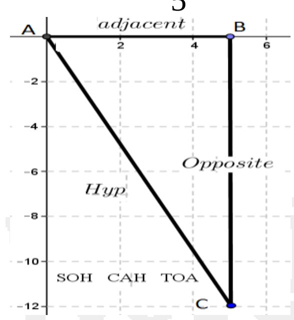
$AC^2 = (-3)^2 + (-4)^2$

$AC^2 = 9 + 16 = 25$

$AC = 5$

$CAH \Rightarrow \cos\alpha = \frac{-4}{5}, SOH \Rightarrow \sin\alpha = \frac{-3}{5}$

Take $\sec\beta = \frac{13}{5} \Rightarrow \cos\beta = \frac{5}{13}$



AB= Adjacent
BC= Opposite
AC= Hypotenuse

$AC^2 = AB^2 + BC^2$

$13^2 = 5^2 + BC^2$

$BC^2 = 169 - 25 = 144$

$BC = -12$

$SOH \Rightarrow \sin\beta = \frac{-12}{13}, TOA \Rightarrow \tan\beta = \frac{-12}{5}$

i). $Sin(\alpha+\beta)$

Solution; we have to find the value of $Sin(\alpha+\beta)$

$Sin(\alpha+\beta) = Sin\alpha.Cos\beta + Cos\alpha.Sin\beta$

$Sin(\alpha+\beta) = \frac{-3}{5}.\frac{5}{13} + \frac{4}{5}.\frac{12}{13}$

$Sin(\alpha+\beta) = \frac{-15+48}{65}$

$Sin(\alpha+\beta) = \frac{33}{65}$

ii). $Cos(\alpha+\beta)$

Solution; we have to find the value of $Cos(\alpha+\beta)$

$Cos(\alpha+\beta) = Cos\alpha.Cos\beta - Sin\alpha.Sin\beta$

$Cos(\alpha+\beta) = \frac{-4}{5}.\frac{5}{13} - \frac{-3}{5}.\frac{-12}{13}$

$Cos(\alpha+\beta) = \frac{-20-36}{65}$

$Cos(\alpha+\beta) = \frac{-56}{65}$

iii). $\tan(\alpha+\beta)$

Solution; we have to find the value of $\tan(\alpha+\beta)$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$

$\tan(\alpha+\beta) = \left(\frac{3}{4} + \frac{-12}{5}\right) \div \left(1 - \frac{3}{4}.\frac{-12}{5}\right)$

$$\tan(\alpha+\beta)=\left(\frac{15-48}{20}\right)\div\left(\frac{20+36}{20}\right)$$

$$\tan(\alpha+\beta)=\left(\frac{-33}{20}\right)\div\left(\frac{56}{20}\right)=\left(\frac{-33}{20}\right)\times\left(\frac{20}{56}\right)$$

$$\tan(\alpha+\beta)=\left(\frac{-33}{56}\right)$$

Q6. Show that

i). $\cot(\alpha+\beta)=\dfrac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$

Solution; we have to prove $\cot(\alpha+\beta)=\dfrac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$

Take LHS $\cot(\alpha+\beta)$

$$\frac{Cos(\alpha+\beta)}{Sin(\alpha+\beta)}=\frac{Cos\alpha.Cos\beta-Sin\alpha.Sin\beta}{Sin\alpha.Cos\beta+Cos\alpha.Sin\beta}$$

$$\cot(\alpha+\beta)=\frac{Sin\alpha.Sin\beta\left(\frac{Cos\alpha.Cos\beta}{Sin\alpha.Sin\beta}-\frac{Sin\alpha.Sin\beta}{Sin\alpha.Sin\beta}\right)}{Sin\alpha.Sin\beta\left(\frac{Sin\alpha.Cos\beta}{Sin\alpha.Sin\beta}+\frac{Cos\alpha.Sin\beta}{Sin\alpha.Sin\beta}\right)}$$

$$\cot(\alpha+\beta)=\frac{\left(\frac{Cos\alpha.Cos\beta}{Sin\alpha.Sin\beta}-1\right)}{\left(\frac{Cos\beta}{Sin\beta}+\frac{Cos\alpha}{Sin\alpha}\right)}$$

$$\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cos\beta+\cot\alpha}=RHS$$

Hence proved

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ii). $\dfrac{Sin(\alpha+\beta)}{Cos\alpha.Cos\beta}=\tan\alpha+\tan\beta$

Solution; we have to prove

$$\frac{Sin(\alpha+\beta)}{Cos\alpha.Cos\beta}=\tan\alpha+\tan\beta$$

Take LHS $\dfrac{Sin(\alpha+\beta)}{Cos\alpha.Cos\beta}$

$$=\frac{Sin\alpha.Cos\beta+Cos\alpha.Sin\beta}{Cos\alpha.Cos\beta}$$

$$=\frac{Sin\alpha.Cos\beta}{Cos\alpha.Cos\beta}+\frac{Cos\alpha.Sin\beta}{Cos\alpha.Cos\beta}$$

$$=\frac{Sin\alpha}{Cos\alpha}+\frac{Sin\beta}{Cos\beta}$$

$$=\tan\alpha+\tan\beta=RHS$$

Hence proved

Q7. Prove that

i). $\tan(\frac{\pi}{4}+\theta)=\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

Solution; we have to prove

$$\tan(\frac{\pi}{4}+\theta)=\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$$

Take LHS $\tan(\frac{\pi}{4}+\theta)$

$$=\frac{\tan\left(\pi/4\right)+\tan\theta}{1-\tan\left(\pi/4\right)\tan\theta}\qquad\therefore\tan\left(\pi/4\right)=1$$

$$=\frac{1+\tan\theta}{1-\tan\theta}\qquad\therefore\tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$=\frac{1+\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin\theta}{\cos\theta}}=\frac{\frac{\cos\theta+\sin\theta}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$=\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}=RHS$$

Hence Proved

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ii). $\tan\left(\dfrac{\pi}{4}-\theta\right)=\dfrac{1-\tan\theta}{1+\tan\theta}$

Solution; we have to prove

$$\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}$$

Take LHS $\tan\left(\frac{\pi}{4}-\theta\right)$

$$=\frac{\tan\left(\frac{\pi}{4}\right)-\tan\theta}{1+\tan\left(\frac{\pi}{4}\right)\tan\theta}\qquad\therefore\tan\left(\frac{\pi}{4}\right)=1$$

$$=\frac{1-\tan\theta}{1+\tan\theta}=RHS$$

Hence Proved

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iii). $\dfrac{\tan(\alpha+\beta)}{\cot(\alpha-\beta)}=\dfrac{\tan^2\alpha-\tan^2\beta}{1-\tan^2\alpha\tan^2\beta}$

Solution; we have to prove

$$\frac{\tan(\alpha+\beta)}{\cot(\alpha-\beta)}=\frac{\tan^2\alpha-\tan^2\beta}{1-\tan^2\alpha\tan^2\beta}$$

Take LHS $\dfrac{\tan(\alpha+\beta)}{\cot(\alpha-\beta)}$

$$=\tan(\alpha+\beta)\frac{1}{\cot(\alpha-\beta)}$$

$$=\tan(\alpha+\beta)\tan(\alpha-\beta)$$

$$=\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta}\right)\left(\frac{\tan\alpha-\tan\beta}{1+\tan\alpha.\tan\beta}\right)$$

$$\therefore(a+b)(a-b)=a^2-b^2$$

$$=\frac{\tan^2\alpha-\tan^2\beta}{1-\tan^2\alpha.\tan^2\beta}=RHS$$

Hence Proved

Q8. Prove that $\dfrac{\sin\theta}{\sec 4\theta}+\dfrac{\cos\theta}{\cos ec4\theta}=\sin 5\theta$

Solution; we have to Prove

$$\frac{\sin\theta}{\sec 4\theta}+\frac{\cos\theta}{\cos ec4\theta}=\sin 5\theta$$

Take LHS

$$\frac{\sin\theta}{\sec 4\theta}+\frac{\cos\theta}{\cos ec4\theta}=\sin\theta.\cos 4\theta+\cos\theta\sin 4\theta$$

Using formula $Sin(\alpha+\beta)=Sin\alpha.Cos\beta+Cos\alpha.Sin\beta$

$$=\sin(\theta+4\theta)$$

$$=\sin 5\theta=RHS$$

Hence proved

Q9. Show that $\dfrac{\sin\left(180^o-\alpha\right)\cos\left(270^o-\alpha\right)}{\sin\left(180^o+\alpha\right)\cos\left(270^o+\alpha\right)}=1$

Solution; we have to show that

$$\frac{\sin\left(180^o-\alpha\right)\cos\left(270^o-\alpha\right)}{\sin\left(180^o+\alpha\right)\cos\left(270^o+\alpha\right)}=1$$

Take LHS

$$\frac{\sin\left(180^o-\alpha\right)\cos\left(270^o-\alpha\right)}{\sin\left(180^o+\alpha\right)\cos\left(270^o+\alpha\right)}$$

$$=\frac{\{Sin180^o Cos\alpha - Cos180^o Sin\alpha\}\{Cos270^o Cos\alpha + Sin270^o Sin\alpha\}}{\{Sin180^o Cos\alpha + Cos180^o Sin\alpha\}\{Cos270^o Cos\alpha - Sin270^o Sin\alpha\}}$$

$\therefore \sin 180^o = 0, \cos 180^o = -1, \sin 270^o = -1, \cos 270^o = 0$

$$=\frac{\{0.Cos\alpha - (-1)Sin\alpha\}\{0.Cos\alpha + (-1)Sin\alpha\}}{\{0.Cos\alpha + (-1)Sin\alpha\}\{0.Cos\alpha - (-1)Sin\alpha\}}$$

$$=\frac{(+Sin\alpha)(-Sin\alpha)}{(-Sin\alpha)(+Sin\alpha)}$$

$$=1=RHS$$

Hence proved

Q10. Express each of following in form of $r\sin(\theta+\phi)$ where terminal ray of θ and ϕ are in first quadrant.

i). $4\sin\theta+3\cos\theta$

Solution: we have $4\sin\theta+3\cos\theta$

Compare the given equation with the expression $a\sin\theta+b\cos\theta$ we get $a=4, b=3$

We know that

$r=\sqrt{a^2+b^2}$ $\quad \cos\phi=\frac{a}{r}$ $\quad \sin\phi=\frac{b}{r}$

$r=\sqrt{4^2+3^2}$ $\quad \cos\phi=\frac{4}{5}$ $\quad \sin\phi=\frac{3}{5}$

$r=\sqrt{16+9}$ $\quad \tan\phi=\frac{\sin\phi}{\cos\phi}$

$r=\sqrt{25}=5$ $\quad \phi=\tan^{-1}\left(\frac{3}{4}\right)$

The reference angle $\sin\phi$ and $\cos\phi$ are positive, the angle χ is lies in the first quadrant

Now $4\sin\theta+3\cos\theta$

Multiply & Divide it by 5 we get $5\left(\frac{4}{5}\sin\theta+\frac{3}{5}\cos\theta\right)$

Substituting $\cos\phi=\frac{4}{5}$, $\sin\phi=\frac{3}{5}$ we get

$5(\cos\phi\sin\theta+\sin\phi\cos\theta)$

$5\sin(\theta+\phi)$

Thus $4\sin\theta+3\cos\theta=5\sin(\theta+\phi)$ Where

$r=5, \quad \cos\phi=\frac{4}{5}, \quad \sin\phi=\frac{3}{5} \quad \Rightarrow \phi=\tan^{-1}\left(\frac{3}{5}\right)$

ii). $15\sin\theta+8\cos\theta$

Solution: we have $15\sin\theta+8\cos\theta$

Compare the given equation with the expression $a\sin\theta+b\cos\theta$ we get $a=15, b=8$

We know that

$r=\sqrt{a^2+b^2}$ $\quad \cos\phi=\frac{a}{r}$ $\quad \sin\phi=\frac{b}{r}$

$r=\sqrt{15^2+8^2}$ $\quad \cos\phi=\frac{15}{17}$ $\quad \sin\phi=\frac{8}{17}$

$r=\sqrt{225+64}$ $\quad \tan\phi=\frac{\sin\phi}{\cos\phi}$

$r=\sqrt{289}=17$ $\quad \phi=\tan^{-1}\left(\frac{8}{15}\right)$

The reference angle $\sin\phi$ and $\cos\phi$ are positive, the angle χ is lies in the first quadrant

Now $15\sin\theta+8\cos\theta$

Multiply and Divide it by 17 we get

$17\left(\frac{15}{17}\sin\theta+\frac{8}{17}\cos\theta\right)$

Substituting $\cos\phi=\frac{15}{17}$, $\sin\phi=\frac{8}{17}$ we get

$17(\cos\phi\sin\theta+\sin\phi\cos\theta)$

$17\sin(\theta+\phi)$

Thus $15\sin\theta+8\cos\theta=17\sin(\theta+\phi)$ Where

$r=17, \quad \cos\phi=\frac{15}{17}, \quad \sin\phi=\frac{8}{17} \quad \Rightarrow \phi=\tan^{-1}\left(\frac{15}{8}\right)$

iii). $2\sin\theta-5\cos\theta$

Solution: we have $2\sin\theta-5\cos\theta$

Compare the given equation with the expression $a\sin\theta+b\cos\theta$ we get $a=2, b=-5$

We know that

$r=\sqrt{a^2+b^2}$ $\quad \cos\phi=\frac{a}{r}$ $\quad \sin\phi=\frac{b}{r}$

$r=\sqrt{2^2+(-5)^2}$ $\quad \cos\phi=\frac{2}{\sqrt{29}}$ $\quad \sin\phi=\frac{-5}{\sqrt{29}}$

$r=\sqrt{4+25}$ $\quad \tan\phi=\frac{\sin\phi}{\cos\phi}$

$r=\sqrt{29}$ $\quad \phi=\tan^{-1}\left(\frac{-5}{2}\right)$

The reference angles $\sin\phi$ is negative and $\cos\phi$ is positive, the angle χ is lies in the fourth quadrant, Now

$2\sin\theta-5\cos\theta$

Multiply and Divide it by $\sqrt{29}$ we get

$\sqrt{29}\left(\frac{2}{\sqrt{29}}\sin\theta-\frac{5}{\sqrt{29}}\cos\theta\right)$

Substituting $\cos\phi=\frac{2}{\sqrt{29}}$, $\sin\phi=\frac{-5}{\sqrt{29}}$ we get

$\sqrt{29}(\cos\phi\sin\theta+\sin\phi\cos\theta)$

$\sqrt{29}\sin(\theta+\phi)$

Thus $2\sin\theta-5\cos\theta=\sqrt{29}\sin(\theta+\phi)$ Where

$r=\sqrt{29}, \quad \cos\phi=\frac{2}{\sqrt{29}}, \quad \sin\phi=\frac{-5}{\sqrt{29}} \quad \Rightarrow \phi=\tan^{-1}\left(\frac{-5}{2}\right)$

iv). $\sin\theta+\cos\theta$

Solution: we have $\sin\theta+\cos\theta$

Compare the given equation with the expression $a\sin\theta+b\cos\theta$ we get $a=1, b=1$

We know that

$r=\sqrt{a^2+b^2}$ $\quad \cos\phi=\frac{a}{r}$ $\quad \sin\phi=\frac{b}{r}$

$r=\sqrt{1^2+1^2}$ $\quad \cos\phi=\frac{1}{\sqrt{2}}$ $\quad \sin\phi=\frac{1}{\sqrt{2}}$

$r=\sqrt{1+1}$ $\quad \tan\phi=\frac{\sin\phi}{\cos\phi}$

$r=\sqrt{2}$ $\quad \phi=\tan^{-1}\left(\frac{1}{1}\right)=\frac{\pi}{4}$

The reference angle $\sin\phi$ and $\cos\phi$ are positive, the angle χ is lies in the first quadrant

Now $\sin\theta+\cos\theta$

Multiply and Divide it by $\sqrt{2}$ we get

$\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta\right)$

Substituting $\cos\phi=\frac{1}{\sqrt{2}}$, $\sin\phi=\frac{1}{\sqrt{2}}$ we get

$\sqrt{2}(\cos\phi\sin\theta+\sin\phi\cos\theta)$

$\sqrt{2}\sin(\theta+\phi)$

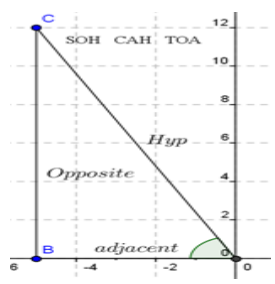
Thus $\sin\theta+\cos\theta=\sqrt{2}\sin(\theta+\phi)$ Where

$r=\sqrt{2}, \quad \cos\phi=\frac{1}{\sqrt{2}}, \quad \sin\phi=\frac{1}{\sqrt{2}} \quad \Rightarrow \phi=\tan^{-1}(1)=\frac{\pi}{4}$

Exercise 10.2

Q1. If $\sin\theta=\frac{5}{13}$ and terminal ray of θ is in the second quadrant, than find:

Solution: we have $\sin\theta=\frac{5}{13}$



AB= Adjacent
BC= Opposite
AC= Hypotenuse

$AC^2=AB^2+BC^2$

$13^2=AB^2+5^2$

$AB^2=169-25=144$

$AB=-12$

$$CAH \Rightarrow \cos\theta=\frac{-12}{13}, TOA \Rightarrow \tan\theta=-\frac{5}{12}$$

i). $\sin 2\theta$

Solution; we have to find the value of $\sin 2\theta$

$$\sin 2\theta=\sin\theta\cos\theta+\cos\theta\sin\theta$$
$$\sin 2\theta=2\sin\theta\cos\theta$$
$$\sin 2\theta=2\left(\frac{5}{13}\right)\left(\frac{-12}{13}\right)$$
$$\sin 2\theta=\frac{-120}{169}$$

ii). $\cos 2\theta$

Solution; we have to find the value of $\cos 2\theta$

$$\therefore \cos 2\theta=\cos\theta\cos\theta-\sin\theta\sin\theta$$
$$\cos 2\theta=\cos^2\theta-\sin^2\theta$$
$$\cos 2\theta=\left(\frac{-12}{13}\right)^2-\left(\frac{5}{13}\right)^2$$
$$\cos 2\theta=\frac{144}{169}-\frac{25}{169}$$
$$\cos 2\theta=\frac{119}{169}$$

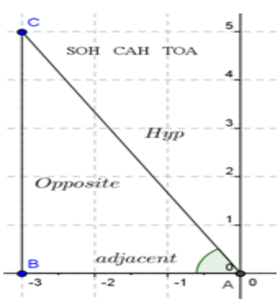
iii). $\tan 2\theta$

Solution; we have to find the value of $\tan 2\theta$

$$\therefore \tan(\theta+\theta)=\frac{\tan\theta+\tan\theta}{1-\tan\theta.\tan\theta}$$
$$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$$
$$\tan 2\theta=2\left(\frac{-5}{12}\right)\div\left\{1-\left(\frac{-5}{12}\right)^2\right\}$$
$$\tan 2\theta=\frac{-5}{6}\div\left\{1-\frac{25}{144}\right\}$$
$$\tan 2\theta=\frac{-5}{6}\div\left\{\frac{144-25}{144}\right\}$$
$$\tan 2\theta=\frac{-5}{6}\times\left\{\frac{144}{119}\right\}$$
$$\tan 2\theta=\frac{-120}{119}$$

Q2. If $\sin\theta=\frac{4}{5}$ and terminal ray of θ is in the second quadrant, than find:

Solution: we have $\sin\theta=\frac{4}{5}$



AB= Adjacent
BC= Opposite
AC= Hypotenuse

$$AC^2=AB^2+BC^2$$
$$5^2=AB^2+4^2$$
$$AB^2=25-16=9$$
$$AB=-3$$
$$CAH\Rightarrow\cos\theta=\frac{-3}{5},TOA\Rightarrow\tan\theta=-\frac{4}{3}$$

i). $\sin 2\theta$

Solution; we have to find the value of $\sin 2\theta$

$$\sin 2\theta=\sin\theta\cos\theta+\cos\theta\sin\theta$$
$$\sin 2\theta=2\sin\theta\cos\theta$$
$$\sin 2\theta=2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right)$$
$$\sin 2\theta=\frac{-24}{25}$$

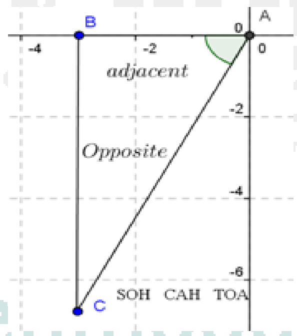
ii). $\cos\frac{\theta}{2}$

Solution; we have to find the value of $\cos\frac{\theta}{2}$

$$\therefore Cos\frac{\theta}{2}=\sqrt{\frac{1+Cos\theta}{2}}$$
$$Cos\frac{\theta}{2}=\sqrt{\frac{1}{2}\left(1-\frac{3}{5}\right)}$$
$$Cos\frac{\theta}{2}=\sqrt{\frac{1}{2}\left(\frac{2}{5}\right)}$$
$$Cos\frac{\theta}{2}=\frac{1}{\sqrt{5}}$$

Q3. If $\cos\theta=-\frac{3}{7}$ and terminal ray of θ is in the third quadrant, than find: $\sin\frac{\theta}{2}$

Solution: we have $\cos\theta=-\frac{3}{7}$



AB= Adjacent
BC= Opposite
AC= Hypotenuse

$$AC^2=AB^2+BC^2$$
$$7^2=(-3)^2+BC^2$$
$$BC^2=49-9=40$$
$$BC=2\sqrt{10}$$
$$SOH\Rightarrow\sin\theta=\frac{-2\sqrt{10}}{5},TOA\Rightarrow\tan\theta=\frac{2\sqrt{10}}{3}$$

Now $Sin\frac{\theta}{2}=\sqrt{\frac{1-Cos\theta}{2}}$

$$Sin\frac{\theta}{2}=\sqrt{\frac{1}{2}\left(1+\frac{3}{7}\right)}$$
$$Sin\frac{\theta}{2}=\sqrt{\frac{1}{2}\left(\frac{7+3}{7}\right)}$$
$$Sin\frac{\theta}{2}=\sqrt{\frac{1}{2}\left(\frac{10}{7}\right)}$$
$$Sin\frac{\theta}{2}=\sqrt{\frac{5}{7}}$$
$$Sin\frac{\theta}{2}=\frac{\sqrt{5}}{\sqrt{7}}$$

Q4. Using double angle identities, Find the values of following.

i). $\sin\frac{2\pi}{3}$

Solution; we have to find the value of $\sin\frac{2\pi}{3}$

$$Sin\frac{2\pi}{3}=Sin\left(\frac{\pi}{3}+\frac{\pi}{3}\right)$$
$$Sin\frac{2\pi}{3}=2\sin\frac{\pi}{3}\cos\frac{\pi}{3}$$

$Sin\dfrac{2\pi}{3}=2\left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}\right)$

$Sin\dfrac{2\pi}{3}=\dfrac{\sqrt{3}}{2}$

i). $\cos\frac{2\pi}{3}$

Solution; we have to find the value of $\cos\frac{2\pi}{3}$

$\cos\dfrac{2\pi}{3}=\cos\left(\dfrac{\pi}{3}+\dfrac{\pi}{3}\right)$

$\cos\dfrac{2\pi}{3}=\cos^2\dfrac{\pi}{3}-\sin^2\dfrac{\pi}{3}$

$\cos\dfrac{2\pi}{3}=\left(\dfrac{1}{2}\right)^2-\left(\dfrac{\sqrt{3}}{2}\right)^2$

$\cos\dfrac{2\pi}{3}=\dfrac{1}{4}-\dfrac{3}{4}$

$\cos\dfrac{2\pi}{3}=\dfrac{-2}{4}$

$\cos\dfrac{2\pi}{3}=\dfrac{-1}{2}$

Prove the following identities;

Q5. $(\sin\theta-\cos\theta)^2=1-\sin 2\theta$

Solution: we have to prove

$(\sin\theta-\cos\theta)^2=1-\sin 2\theta$

Take LHS $(\sin\theta-\cos\theta)^2$

$=\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta$

$=1-2\sin\theta\cos\theta$

$=1-\sin 2\theta \qquad \therefore 2\sin\theta\cos\theta=\sin 2\theta$

=RHS Hence proved

Q6. $\dfrac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta$

Solution: we have to prove that

$\dfrac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta$

Take LHS $\dfrac{2\tan\theta}{1+\tan^2\theta}$

$=\dfrac{2\tan\theta}{1+\tan^2\theta}$

$=\dfrac{2\tan\theta}{\sec^2\theta} \qquad \therefore 1+\tan^2\theta=\sec^2\theta$

$=2\tan\theta\cos^2\theta \qquad \therefore \sec^2\theta=\dfrac{1}{\cos^2\theta}$

$=2\dfrac{\sin\theta}{\cos\theta}(\cos^2\theta)$

$=2\sin\theta\cos\theta$

=RHS Hence proved

Q7. $\dfrac{1}{\sec 2\alpha}=\cos^4\alpha-\sin^4\alpha$

Solution: we have to prove $\dfrac{1}{\sec 2\alpha}=\cos^4\alpha-\sin^4\alpha$

Take LHS $\dfrac{1}{\sec 2\alpha}=\cos 2\alpha$

$\dfrac{1}{\sec 2\alpha}=\cos(\alpha+\alpha)$

$\dfrac{1}{\sec 2\alpha}=\cos^2\alpha-\sin^2\alpha$

$\dfrac{1}{\sec 2\alpha}=(\cos^2\alpha-\sin^2\alpha)\times\dfrac{\cos^2\alpha+\sin^2\alpha}{\cos^2\alpha+\sin^2\alpha}$

$\dfrac{1}{\sec 2\alpha}=\dfrac{(\cos^2\alpha)^2-(\sin^2\alpha)^2}{\cos^2\alpha+\sin^2\alpha}$

$\dfrac{1}{\sec 2\alpha}=\cos^4\alpha-\sin^4\alpha$

=RHS Hence proved

Q8. $\dfrac{1+\cos 2\theta}{\sin 2\theta}=\cot\theta$

Solution: we have to prove $\dfrac{1+\cos 2\theta}{\sin 2\theta}=\cot\theta$

Take LHS $\dfrac{1+\cos 2\theta}{\sin 2\theta}$

$=\dfrac{1+\cos^2\theta-\sin^2\theta}{2\sin\theta\cos\theta}$

$=\dfrac{1-\sin^2\theta+\cos^2\theta}{2\sin\theta\cos\theta}$

$=\dfrac{2\cos^2\theta}{2\sin\theta\cos\theta}$

$=\dfrac{\cos\theta}{\sin\theta}$

$=\cot\theta$

=RHS Hence proved

Q9. $\cos ec2\alpha-\cot 2\alpha=\tan\alpha$

Solution: we have to prove

$\cos ec2\alpha-\cot 2\alpha=\tan\alpha$

Take LHS $\cos ec2\alpha-\cot 2\alpha$

$=\dfrac{1}{\sin 2\alpha}-\dfrac{\cos 2\alpha}{\sin 2\alpha}$

$=\dfrac{1-\cos 2\alpha}{\sin 2\alpha}$

$=\dfrac{1-(\cos^2\alpha-\sin^2\alpha)}{2\sin\alpha\cos\alpha}$

$=\dfrac{1-\cos^2\alpha+\sin^2\alpha}{2\sin\alpha\cos\alpha}$

$=\dfrac{\sin^2\alpha+\sin^2\alpha}{2\sin\alpha\cos\alpha}$

$=\dfrac{2\sin^2\alpha}{2\sin\alpha\cos\alpha}$

$=\dfrac{\sin\alpha}{\cos\alpha}$

$=\tan\alpha$

=RHS Hence proved

Q10. $\dfrac{\sin 3\beta}{\sin\beta}-\dfrac{\cos 3\beta}{\cos\beta}=2$

Solution: we have to prove $\dfrac{\sin 3\beta}{\sin\beta}-\dfrac{\cos 3\beta}{\cos\beta}=2$

Take LHS $\dfrac{\sin 3\beta}{\sin\beta}-\dfrac{\cos 3\beta}{\cos\beta}$

$=\dfrac{\sin 3\beta\cos\beta-\sin\beta\cos 3\beta}{\sin\beta\cos\beta}$

$=\dfrac{\sin(3\beta-\beta)}{\sin\beta\cos\beta}$

$=\dfrac{\sin(2\beta)}{\sin\beta\cos\beta}$

$=\dfrac{2\sin\beta\cos\beta}{\sin\beta\cos\beta}=2$

=RHS Hence proved

Q11. $\dfrac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}=\dfrac{2+\sin 2\theta}{2}$

Solution; we have to prove

$\dfrac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}=\dfrac{2+\sin 2\theta}{2}$

Take LHS $\dfrac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

$=\dfrac{(\cos\theta-\sin\theta)(\cos^2\theta+\cos\theta\sin\theta+\sin^2\theta)}{\cos\theta-\sin\theta}$

$=\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta$

$=1+\cos\theta\sin\theta$

$=\dfrac{2}{2}(1+\cos\theta\sin\theta)$

$=\dfrac{2+2\cos\theta\sin\theta}{2}=\dfrac{2+\sin 2\theta}{2}$

= RHS Hence proved

Q12. $\dfrac{\sin 3\theta}{\cos\theta}+\dfrac{\cos 3\theta}{\sin\theta}=2\cot 2\theta$

Solution: we have to prove

$\dfrac{\sin 3\theta}{\cos\theta}+\dfrac{\cos 3\theta}{\sin\theta}=2\cot 2\theta$

Take LHS $\dfrac{\sin 3\theta}{\cos\theta}+\dfrac{\cos 3\theta}{\sin\theta}$

$=\dfrac{\sin 3\theta\sin\theta+\cos\theta \mathrm{os}\, 3\theta}{\sin\theta\cos\theta}$

$=\dfrac{\cos(3\theta-\theta)}{\sin\theta\cos\theta}$

$=\dfrac{\cos 2\theta}{\sin\beta\cos\beta}$

$=\dfrac{2\cos 2\theta}{2\sin\theta\cos\theta}$

$=\dfrac{2\cos 2\theta}{\sin 2\theta}=2\cot 2\theta$

=RHS Hence proved

Q13. $\tan\theta.\tan\frac{\theta}{2}=\sec\theta-1$

Solution: we have to prove $\tan\theta.\tan\frac{\theta}{2}=\sec\theta-1$

Take LHS $\tan\theta.\tan\frac{\theta}{2}$

$=\dfrac{\sin\theta}{\cos\theta}.\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$=\dfrac{\sin\theta}{\cos\theta}.\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}\times\dfrac{1-\cos\theta}{1-\cos\theta}}$

$=\dfrac{\sin\theta}{\cos\theta}.\sqrt{\dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$=\dfrac{\sin\theta}{\cos\theta}.\sqrt{\dfrac{(1-\cos\theta)^2}{\sin^2\theta}}$

$=\dfrac{\sin\theta}{\cos\theta}.\dfrac{1-\cos\theta}{\sin\theta}$

$=\dfrac{1-\cos\theta}{\cos\theta}$

$=\dfrac{1}{\cos\theta}-\dfrac{\cos\theta}{\cos\theta}$

$=\sec\theta-1$

=RHS Hence proved

Q14. $\dfrac{\sin\alpha+\cos\alpha}{\cos\alpha-\sin\alpha}+\dfrac{\sin\alpha-\cos\alpha}{\cos\alpha+\sin\alpha}=2\tan 2\alpha$

Solution: we have to prove

$\dfrac{\sin\alpha+\cos\alpha}{\cos\alpha-\sin\alpha}+\dfrac{\sin\alpha-\cos\alpha}{\cos\alpha+\sin\alpha}=2\tan 2\alpha$

Take LHS $\dfrac{\sin\alpha+\cos\alpha}{\cos\alpha-\sin\alpha}+\dfrac{\sin\alpha-\cos\alpha}{\cos\alpha+\sin\alpha}$

$=\dfrac{(\sin\alpha+\cos\alpha)}{(\cos\alpha-\sin\alpha)}-\dfrac{(\cos\alpha-\sin\alpha)}{(\cos\alpha+\sin\alpha)}$

$=\dfrac{(\sin\alpha+\cos\alpha)^2-(\cos\alpha-\sin\alpha)^2}{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}$

$\because (a+b)^2-(a-b)^2=4ab$

$=\dfrac{4\sin\alpha\cos\alpha}{\cos^2\alpha-\sin^2\alpha}$

$=\dfrac{2(2\sin\alpha\cos\alpha)}{\cos(\alpha+\alpha)}$

$=\dfrac{2\sin 2\alpha}{\cos 2\alpha}$

$=2\tan 2\alpha$

=RHS Hence proved

Q15. $\dfrac{\cot^2\beta-1}{\cos ee^2\beta}=\cos 2\beta$

Solution: we have to prove $\dfrac{\cot^2\beta-1}{\cos ee^2\beta}=\cos 2\beta$

Take LHS $\dfrac{\cot^2\beta-1}{\cos ee^2\beta}$

$=\sin^2\beta(\cot^2\beta-1)$

$=\sin^2\beta\left(\dfrac{\cos^2\beta}{\sin^2\beta}-1\right)$

$=\sin^2\beta\left(\dfrac{\cos^2\beta-\sin^2\beta}{\sin^2\beta}\right)$

$=\cos(\beta+\beta)$

$=\cos 2\beta$

=RHS Hence proved

Q16. $\sin\theta=\dfrac{2}{\tan\frac{\theta}{2}+\cot\frac{\theta}{2}}$

Solution; we have to prove $\sin\theta=\dfrac{2}{\tan\frac{\theta}{2}+\cot\frac{\theta}{2}}$

Take RHS $\dfrac{2}{\tan\frac{\theta}{2}+\cot\frac{\theta}{2}}$

$$=\dfrac{2}{\dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}+\dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}}$$

$$=\dfrac{2}{\dfrac{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}}{\cos\frac{\theta}{2}\sin\frac{\theta}{2}}}$$

$$=\dfrac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1}$$

$$=\sin 2\left(\tfrac{\theta}{2}\right)$$

$$=\sin\theta$$

=LHS Hence proved

Q17. $\sin^2\dfrac{\theta}{2}=\dfrac{\sin\theta.\tan\frac{\theta}{2}}{2}$

Solution; we have to prove $\sin^2\dfrac{\theta}{2}=\dfrac{\sin\theta.\tan\frac{\theta}{2}}{2}$

Take RHS $\dfrac{\sin\theta.\tan\frac{\theta}{2}}{2}$

$$\frac{1}{2}\sin 2\left(\tfrac{\theta}{2}\right).\tan\tfrac{\theta}{2}$$

$$=\frac{1}{2}\left\{2\sin\tfrac{\theta}{2}\cos\tfrac{\theta}{2}\right\}.\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$=\sin^2\tfrac{\theta}{2}$$

=LHS Hence proved

Q18. $\dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\cos\alpha$

Solution: we have to prove $\dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}=\cos\alpha$

Take LHS $\dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$

$$=\frac{1-\tan^2\frac{\alpha}{2}}{\sec^2\frac{\alpha}{2}}$$

$$=\cos^2\tfrac{\alpha}{2}\left(1-\tan^2\tfrac{\alpha}{2}\right)$$

$$=\cos^2\tfrac{\alpha}{2}\left(1-\frac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}\right)$$

$$=\cos^2\tfrac{\alpha}{2}\left(\frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}\right)$$

$$=\cos 2\left(\tfrac{\alpha}{2}\right)$$

$$=\cos\alpha$$

=RHS Hence proved

Q19. $\tan 2\beta+\sec 2\beta=\dfrac{\cos\beta+\sin\beta}{\cos\beta-\sin\beta}$

Solution: we have to prove

$$\tan 2\beta+\sec 2\beta=\frac{\cos\beta+\sin\beta}{\cos\beta-\sin\beta}$$

Take LHS $\tan 2\beta+\sec 2\beta$

$$=\frac{\sin 2\beta}{\cos 2\beta}+\frac{1}{\cos 2\beta}$$

$$=\frac{\sin 2\beta+1}{\cos 2\beta}$$

$$=\frac{1+\sin 2\beta}{\cos(\beta+\beta)}$$

$$=\frac{\cos^2\beta+\sin^2\beta+2\sin\beta\cos\beta}{\cos^2\beta-\sin^2\beta}$$

$$=\frac{(\cos\beta+\sin\beta)^2}{(\cos\beta+\sin\beta)(\cos\beta-\sin\beta)}$$

$$=\frac{\cos\beta+\sin\beta}{\cos\beta-\sin\beta}$$

=RHS Hence proved

Q20. $\cos^4\theta=\dfrac{3}{8}+\dfrac{1}{8}\cos 2\theta+\dfrac{1}{8}\cos 4\theta$

Solution; we have to prove

$$\cos^4\theta=\frac{3}{8}+\frac{1}{8}\cos 2\theta+\frac{1}{8}\cos 4\theta$$

Take LHS $\cos^4\theta$

$$=\left(\cos^2\theta\right)^2=\left(\frac{1+\cos 2\theta}{2}\right)^2 \quad \because Cos^2\frac{x}{2}=\frac{1+Cosx}{2}$$

$$=\frac{1}{4}\left\{1+\cos^2 2\theta+2\cos 2\theta\right\}$$

$$=\frac{1}{4}\left\{1+\frac{1+\cos 4\theta}{2}+2\cos 2\theta\right\}$$

$$=\frac{1}{4}+\frac{1}{8}(1+\cos 4\theta)+\frac{2}{4}\cos 2\theta$$

$$=\frac{2+1}{8}+\frac{1}{8}\cos 4\theta+\frac{1}{2}\cos 2\theta$$

$$=\frac{3}{8}+\frac{1}{8}\cos 4\theta+\frac{1}{2}\cos 2\theta$$

=LHS Hence proved

Exercise 10.3

Q1. Express the following products as sums or differences.

i). $2\sin 60^o\sin 20^o$

Solution; we have $2\sin 60^o\sin 20^o$

using the formula

$$2Sin\alpha.Sin\beta=Cos(\alpha-\beta)-Cos(\alpha+\beta)$$

Here $\alpha=60^o,\beta=20^o$ then

$$2\sin 60^o\sin 20^o=\cos\left(60^o-20^o\right)-\cos\left(60^o+20^o\right)$$

$$2\sin 60^o\sin 20^o=\cos 40^o-\cos 80^o$$

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ii). $2\cos 8\theta\sin 4\theta$

Solution; we have $2\cos 8\theta\sin 4\theta$

using the formula

$$2Cos\alpha.Sin\beta=Sin(\alpha+\beta)-Sin(\alpha-\beta)$$

Here $\alpha=8\theta,\beta=4\theta$ then

$$2\cos 8\theta\sin 4\theta=\sin\left(8\theta+4\theta\right)-\sin\left(8\theta-4\theta\right)$$

$$2\cos 8\theta\sin 4\theta=\sin 12\theta-\sin 4\theta$$

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iii). $2\cos 75\alpha\sin 25\alpha$

Solution; we have $2\cos 75\alpha\sin 25\alpha$

using the formula

$$2Cos\alpha.Sin\beta=Sin(\alpha+\beta)-Sin(\alpha-\beta)$$

Here $\alpha = 75\alpha, \beta = 25\alpha$ then

$2\cos 75\alpha \sin 25\alpha = \sin(75\alpha + 25\alpha) - \sin(75\alpha - 25\alpha)$

$2\cos 75\alpha \sin 25\alpha = \sin 100\alpha - \sin 50\alpha$

iv). $\sin 32^o \cos 24^o$

Solution; we have $\sin 32^o \cos 24^o$

using the formula

$2Sin\alpha.Cos\beta = Sin(\alpha+\beta) + Sin(\alpha-\beta)$

Here $\alpha = 32^o, \beta = 24^o$ then

$\sin 32^o \cos 24^o = \frac{1}{2}\left\{\sin(32^o + 24^o) - \sin(32^o - 24^o)\right\}$

$\sin 32^o \cos 24^o = \frac{1}{2}\left\{\sin 56^o - \sin 8^o\right\}$

v). $\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

Solution; we have $\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

using the formula

$2Sin\alpha.Cos\beta = Sin(\alpha+\beta) + Sin(\alpha-\beta)$

Here $\alpha = \frac{A+B}{2}, \beta = \frac{A-B}{2}$ then

$\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$= \frac{1}{2}\left\{\sin\left(\frac{A+B}{2} + \frac{A-B}{2}\right) + \sin\left(\frac{A+B}{2} - \frac{A-B}{2}\right)\right\}$

$= \frac{1}{2}\left\{\sin\left(\frac{A+B+A-B}{2}\right) + \sin\left(\frac{A+B-A+B}{2}\right)\right\}$

$= \frac{1}{2}\left\{\sin\left(\frac{2A}{2}\right) + \sin\left(\frac{2B}{2}\right)\right\}$

$= \frac{1}{2}\left\{\sin(A) + \sin(B)\right\}$

vi). $\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$

Solution; we have $\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$

using the formula

$2Cos\alpha.Cos\beta = Cos(\alpha+\beta) + Cos(\alpha-\beta)$

Here $\alpha = \frac{P+Q}{2}, \beta = \frac{P-Q}{2}$ then

$\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$

$= \frac{1}{2}\left\{\cos\left(\frac{P+Q}{2} + \frac{P-Q}{2}\right) + \cos\left(\frac{P+Q}{2} - \frac{P-Q}{2}\right)\right\}$

$= \frac{1}{2}\left\{\cos\left(\frac{P+Q+P-Q}{2}\right) + \cos\left(\frac{P+Q-P+Q}{2}\right)\right\}$

$= \frac{1}{2}\left\{\cos\left(\frac{2P}{2}\right) + \cos\left(\frac{2Q}{2}\right)\right\}$

$= \frac{1}{2}\left\{\cos(P) + \cos(Q)\right\}$

Q2. Convert following sums or differences to products;

i). $\sin 94^o - \sin 86^o$

Solution; we have $\sin 94^o - \sin 86^o$

Using the formula

$Sin\alpha - Sin\beta = 2Cos(\frac{\alpha+\beta}{2})Sin(\frac{\alpha-\beta}{2})$

Here $\alpha = 94^o, \beta = 86^o$ then

$\sin 94^o - \sin 86^o = 2Cos\left(\frac{94^o + 86^o}{2}\right)Sin\left(\frac{94^o - 86^o}{2}\right)$

$\sin 94^o - \sin 86^o = 2Cos\left(\frac{180^o}{2}\right)Sin\left(\frac{8^o}{2}\right)$

$\sin 94^o - \sin 86^o = 2Cos(90^o)Sin(4^o)$

ii). $\cos 86^o + \cos 22^o$

Solution; we have $\cos 86^o + \cos 22^o$

Using the formula

$Cos\alpha + Cos\beta = 2Cos(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$

Here $\alpha = 86^o, \beta = 22^o$ then

$\cos 86^o + \cos 22^o = 2Cos\left(\frac{86^o + 22^o}{2}\right)Cos\left(\frac{86^o - 22^o}{2}\right)$

$\cos 86^o + \cos 22^o = 2Cos\left(\frac{108^o}{2}\right)Cos\left(\frac{64^o}{2}\right)$

$\cos 86^o + \cos 22^o = 2Cos(54^o)Cos(32^o)$

iii). $\cos 95^o - \cos 41^o$

Solution; we have $\cos 95^o - \cos 41^o$

Using the formula

$Cos\alpha - Cos\beta = -2Sin(\frac{\alpha+\beta}{2})Sin(\frac{\alpha-\beta}{2})$

Here $\alpha = 95^o, \beta = 41^o$ then

$\cos 95^o - \cos 41^o = -2Sin\left(\frac{95^o + 41^o}{2}\right)Sin\left(\frac{95^o - 41^o}{2}\right)$

$\cos 95^o - \cos 41^o = -2Sin\left(\frac{136^o}{2}\right)Sin\left(\frac{54^o}{2}\right)$

$\cos 95^o - \cos 41^o = -2Sin(68^o)Sin(27^o)$

vi). $\sin\frac{P+Q}{2} - \sin\frac{P-Q}{2}$

Solution; we have $\sin\frac{P+Q}{2} - \sin\frac{P-Q}{2}$

using the formula

$Sin\alpha - Sin\beta = 2Cos(\frac{\alpha+\beta}{2})Sin(\frac{\alpha-\beta}{2})$

Here $\alpha = \frac{P+Q}{2}, \beta = \frac{P-Q}{2}$ then

$\sin\frac{P+Q}{2} - \sin\frac{P-Q}{2}$

$= 2\left\{\cos\frac{1}{2}\left(\frac{P+Q}{2} + \frac{P-Q}{2}\right)\sin\frac{1}{2}\left(\frac{P+Q}{2} - \frac{P-Q}{2}\right)\right\}$

$= 2\left\{\cos\frac{1}{2}\left(\frac{P+Q+P-Q}{2}\right)\sin\frac{1}{2}\left(\frac{P+Q-P+Q}{2}\right)\right\}$

$= 2\left\{\cos\frac{1}{2}\left(\frac{2P}{2}\right)\sin\frac{1}{2}\left(\frac{2Q}{2}\right)\right\}$

$= 2\left\{\cos\left(\frac{P}{2}\right)\sin\left(\frac{Q}{2}\right)\right\}$

v). $\cos 84^o + \cos 76^o$

Solution; we have $\cos 84^o + \cos 76^o$

Using the formula

$Cos\alpha + Cos\beta = 2Cos(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$

Here $\alpha = 84^o, \beta = 76^o$ then

$\cos 84^o + \cos 76^o = 2Cos\left(\frac{84^o + 76^o}{2}\right)Cos\left(\frac{84^o - 76^o}{2}\right)$

$\cos 84^o + \cos 76^o = 2Cos\left(\frac{160^o}{2}\right)Cos\left(\frac{8^o}{2}\right)$

$\cos 84^o + \cos 76^o = 2Cos\left(80^o\right)Cos\left(4^o\right)$

vi). $\cos\left(\frac{A+B}{2}\right)+\cos\left(\frac{A-B}{2}\right)$

Solution; we have $\cos\left(\frac{A+B}{2}\right)+\cos\left(\frac{A-B}{2}\right)$

Using the formula

$$Cos\alpha + Cos\beta = 2Cos(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

Here $\alpha = \frac{A+B}{2}, \beta = \frac{A-B}{2}$ then

$\cos\left(\frac{A+B}{2}\right)+\cos\left(\frac{A-B}{2}\right)$

$= 2Cos\frac{1}{2}\left(\frac{A+B}{2}+\frac{A-B}{2}\right)Cos\frac{1}{2}\left(\frac{A+B}{2}-\frac{A-B}{2}\right)$

$= 2Cos\frac{1}{2}\left(\frac{2A}{2}\right)Cos\frac{1}{2}\left(\frac{2B}{2}\right)$

$= 2Cos\left(\frac{A}{2}\right)Cos\left(\frac{B}{2}\right)$

Prove the following identities;

Q3. $\frac{Sin\alpha - Sin\beta}{Cos\alpha + Cos\beta} = \tan\left(\frac{\alpha-\beta}{2}\right)$

Solution; we have to prove

$\frac{Sin\alpha - Sin\beta}{Cos\alpha + Cos\beta} = \tan\left(\frac{\alpha-\beta}{2}\right)$

Using formulae

$$Sin\alpha - Sin\beta = 2Cos(\frac{\alpha+\beta}{2})Sin(\frac{\alpha-\beta}{2})$$

$$Cos\alpha + Cos\beta = 2Cos(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

Then LHS $\frac{Sin\alpha - Sin\beta}{Cos\alpha + Cos\beta}$

$$= \frac{2Cos(\frac{\alpha+\beta}{2})Sin(\frac{\alpha-\beta}{2})}{2Cos(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})}$$

$$= \frac{Sin(\frac{\alpha-\beta}{2})}{Cos(\frac{\alpha-\beta}{2})}$$

$$= \tan\left(\frac{\alpha-\beta}{2}\right) = RHS$$

Hence proved

Q4. $\frac{\cos 5\theta + \cos 3\theta}{\sin 5\theta - \sin 3\theta} = \cot\theta$

Solution; we have to prove

$\frac{\cos 5\theta + \cos 3\theta}{\sin 5\theta - \sin 3\theta} = \cot\theta$

Using formulae

$$Cos\alpha + Cos\beta = 2Cos(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

$$Sin\alpha - Sin\beta = 2Cos(\frac{\alpha+\beta}{2})Sin(\frac{\alpha-\beta}{2})$$

Then LHS $\frac{\cos 5\theta + \cos 3\theta}{\sin 5\theta - \sin 3\theta}$

$$= \frac{2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)}{2\cos\left(\frac{5\theta+3\theta}{2}\right)\sin\left(\frac{5\theta-3\theta}{2}\right)}$$

$$= \frac{\cos\left(\frac{5\theta-3\theta}{2}\right)}{\sin\left(\frac{5\theta-3\theta}{2}\right)}$$

$$= \cot\left(\frac{2\theta}{2}\right)$$

$$= \cot\theta = RHS$$

Hence proved

Q5. $\frac{\sin\alpha + \sin 9\alpha}{\cos\alpha + \cos 9\alpha} = \tan 5\alpha$

Solution; we have to prove

$\frac{\sin\alpha + \sin 9\alpha}{\cos\alpha + \cos 9\alpha} = \tan 5\alpha$

Using formulae

$$Sin\alpha + Sin\beta = 2Sin(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

$$Cos\alpha + Cos\beta = 2Cos(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

Then LHS $\frac{\sin\alpha + \sin 9\alpha}{\cos\alpha + \cos 9\alpha}$

$$= \frac{2\sin\left(\frac{9\alpha+\alpha}{2}\right)\cos\left(\frac{9\alpha-\alpha}{2}\right)}{2\cos\left(\frac{9\alpha+\alpha}{2}\right)\cos\left(\frac{9\alpha-\alpha}{2}\right)}$$

$$= \frac{\sin\left(\frac{10\alpha}{2}\right)}{\cos\left(\frac{10\alpha}{2}\right)}$$

$$= \tan\left(\frac{10\theta}{2}\right)$$

$$= \tan 5\alpha = RHS$$

Hence proved

Q6. $\frac{\cos\beta + \cos 3\beta + \cos 5\beta}{\sin\beta + \sin 3\beta + \sin 5\beta} = \cot 3\beta$

Solution; we have to prove

$\frac{\cos\beta + \cos 3\beta + \cos 5\beta}{\sin\beta + \sin 3\beta + \sin 5\beta} = \cot 3\beta$

Using formulae

$$Cos\alpha + Cos\beta = 2Cos(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

$$Sin\alpha + Sin\beta = 2Sin(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

Then LHS $\frac{\cos\beta + \cos 3\beta + \cos 5\beta}{\sin\beta + \sin 3\beta + \sin 5\beta}$

$$=\frac{\cos 5\beta+\cos\beta+\cos 3\beta}{\sin 5\beta+\sin\beta+\sin 3\beta}$$

$$=\frac{2\cos\left(\frac{5\beta+\beta}{2}\right)\cos\left(\frac{5\beta-\beta}{2}\right)+\cos 3\beta}{2\sin\left(\frac{5\beta+\beta}{2}\right)\cos\left(\frac{5\beta-\beta}{2}\right)+\sin 3\beta}$$

$$=\frac{2\cos 3\beta\cos 2\beta+\cos 3\beta}{2\sin 3\beta\cos 2\beta+\sin 3\beta}$$

$$=\frac{2\cos 3\beta(\cos 2\beta+1)}{2\sin 3\beta(\cos 2\beta+1)}$$

$$=\frac{\cos 3\beta}{\sin 3\beta}$$

$$=\cot 3\beta=RHS$$

Hence proved

Q7 $\sin 3\theta+\sin 5\theta+\sin 7\theta+\sin 9\theta=4\cos\theta\sin 6\theta\cos 2\theta$

Solution; we have to prove

$\sin 3\theta+\sin 5\theta+\sin 7\theta+\sin 9\theta=4\cos\theta\sin 6\theta\cos 2\theta$

Using formula

$$Sin\alpha+Sin\beta=2Sin(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

Take LHS $\sin 3\theta+\sin 5\theta+\sin 7\theta+\sin 9\theta$

$$=\sin 9\theta+\sin 3\theta+\sin 7\theta+\sin 5\theta$$

$$=2\sin\left(\frac{9\theta+3\theta}{2}\right)\cos\left(\frac{9\theta-3\theta}{2}\right)+2\sin\left(\frac{7\theta+5\theta}{2}\right)\cos\left(\frac{7\theta-5\theta}{2}\right)$$

$$=2\sin 6\theta\cos 3\theta+2\sin 6\theta\cos\theta$$

$$=2\sin 6\theta(\cos 3\theta+\cos\theta)$$

again using formula

$$Cos\alpha+Cos\beta=2Cos(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

$$=2\sin 6\theta\left\{2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right\}$$

$$=4\sin 6\theta\cos 2\theta\cos\theta=RHS$$

Hence proved

Q8. $\dfrac{\cos 75^o+\cos 15^o}{\sin 75^o-\sin 15^o}=\sqrt{3}$

Solution; we have to prove

$$\frac{\cos 75^o+\cos 15^o}{\sin 75^o-\sin 15^o}=\sqrt{3}$$

Using formulae

$$Cos\alpha+Cos\beta=2Cos(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

$$Sin\alpha-Sin\beta=2Cos(\frac{\alpha+\beta}{2})Sin(\frac{\alpha-\beta}{2})$$

Take LHS $\dfrac{\cos 75^o+\cos 15^o}{\sin 75^o-\sin 15^o}$

$$=\frac{2\cos\left(\frac{75^o+15^o}{2}\right)\cos\left(\frac{75^o-15^o}{2}\right)}{2\cos\left(\frac{75^o+15^o}{2}\right)\sin\left(\frac{75^o-15^o}{2}\right)}$$

$$=\frac{\cos\left(\frac{60^o}{2}\right)}{\sin\left(\frac{60^o}{2}\right)}$$

$$=\frac{\cos 30^o}{\sin 30^o}$$

$$=\cot 30^o$$

$$=\sqrt{3}=RHS$$

Hence proved

Q9 $\cos 3\alpha(1-2\sin\alpha)=\cos 3\alpha-(\sin 4\alpha-\sin 2\alpha)$

Solution; we have to prove

$\cos 3\alpha(1-2\sin\alpha)=\cos 3\alpha-(\sin 4\alpha-\sin 2\alpha)$

Take LHS $\cos 3\alpha(1-2\sin\alpha)$

$$=\cos 3\alpha-2\cos 3\alpha\sin\alpha$$

$$=\cos 3\alpha-\{\sin(3\alpha+\alpha)-\sin(3\alpha-\alpha)\}$$

$$=\cos 3\alpha-\{\sin 4\alpha-\sin 2\alpha\}=RHS$$

Hence proved

Q10 $\cos\beta+\cos 2\beta+\cos 5\beta=\cos 2\beta(1+2\cos 3\beta)$

Solution; we have to prove

$\cos\beta+\cos 2\beta+\cos 5\beta=\cos 2\beta(1+2\cos 3\beta)$

Take LHS $\cos\beta+\cos 2\beta+\cos 5\beta$

$$=\cos 5\beta+\cos\beta+\cos 2\beta$$

Using the formula

$$Cos\alpha+Cos\beta=2Cos(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

$$=2\cos\left(\frac{5\beta+\beta}{2}\right)\cos\left(\frac{5\beta-\beta}{2}\right)+\cos 2\beta$$

$$=2\cos 3\beta\cos 2\beta+\cos 2\beta$$

$$=\cos 2\beta(2\cos 3\beta+1)=RHS$$

Hence proved

Q11. $\sin 5\theta+\sin\theta+2\sin 3\theta=4\sin 3\theta\cos^2\theta$

Solution; we have

$\sin 5\theta+\sin\theta+2\sin 3\theta=4\sin 3\theta\cos^2\theta$

Using the formula

$$Sin\alpha+Sin\beta=2Sin(\frac{\alpha+\beta}{2})Cos(\frac{\alpha-\beta}{2})$$

Take LHS $\sin 5\theta+\sin\theta+2\sin 3\theta$

$$=2\sin\left(\frac{5\theta+\theta}{2}\right)\cos\left(\frac{5\theta-\theta}{2}\right)+2\sin 3\theta$$

$$=2\sin 3\theta\cos 2\theta+2\sin 3\theta$$

$$=2\sin 3\theta(\cos 2\theta+1)$$

$$=2\sin 3\theta(\cos^2\theta-\sin^2\theta+1)$$

$$=2\sin 3\theta(\cos^2\theta+1-\sin^2\theta)$$

$$=2\sin 3\theta(\cos^2\theta+\cos^2\theta)$$

$$=2\sin 3\theta(2\cos^2\theta)$$

$$=4\sin 3\theta\cos^2\theta=RHS$$

Hence proved