Chapter 8

Functions and Graphs

Exercise 8.1

Q1: Given that $f(x)=x^2+x-1$

i). find the image of -2,0,2,5

Solution: we have $f(x)=x^2+x-1$

At $x=-2$

$f(-2)=(-2)^2+(-2)-1$

$f(-2)=4-2-1$

$f(-2)=1$

At $x=0$

$f(0)=(0)^2+(0)-1$

$f(0)=0+0-1$

$f(0)=-1$

At $x=2$

$f(2)=(2)^2+(2)-1$

$f(2)=4+2-1$

$f(2)=5$

At $x=5$

$f(5)=(5)^2+(5)-1$

$f(5)=25+5-1$

$f(5)=29$

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ii). If $f(x)=5$ then find the value of x

Solution: we have $f(x)=x^2+x-1$

By given condition $f(x)=5$ We get

$x^2+x-1=5$

$x^2+x-6=0$

$x^2+3x-2x-6=0$

$x(x+3)-2(x+3)=0$

$(x-2)(x+3)=0$

$\therefore x-2=0$ $\quad\therefore x+3=0$

$x=2$ $\quad x=-3$

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iii). Find $f(x+1)$

Solution: we have $f(x)=x^2+x-1$

By given condition $f(x+1)$

We get $f(x+1)=(x+1)^2+(x+1)-1$

$f(x+1)=x^2+2x+1+x+1-1$

$f(x+1)=x^2+3x+1$

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iv). Find $\dfrac{f(x+h)-f(x)}{h}$

Solution: we have $f(x)=x^2+x-1$

By given condition $\dfrac{f(x+h)-f(x)}{h}$ We get

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{1}{h}\left[f(x+h)-f(x)\right]$

$=\dfrac{1}{h}\left[(x+h)^2+(x+h)-1-(x^2+x-1)\right]$

$=\dfrac{1}{h}\left[x^2+2hx+h^2+x+h-1-x^2-x+1\right]$

$=\dfrac{1}{h}\left[2hx+h^2+h\right]$

$=\dfrac{h}{h}\left[2x+h+1\right]$

$=2x+h+1$

Q2 If $f(x)=7x+3, g(x)=\dfrac{2x}{x^2+9}$,

$h(x)=20\sqrt{25-x^2}$ & $k(x)=x^2$, then determine

i). $f(6), g(-1), h(4), k\left(\dfrac{1}{2}\right)$

Solution: we have $f(x)=7x+3$

To find $f(6)$ put $x=6$

$f(6)=7(6)+3$

$f(6)=42+3$

$f(6)=45$

We have $g(x)=\dfrac{2x}{x^2+9}$

At $x=-1$

$g(-1)=\dfrac{2(-1)}{(-1)^2+9}$

$g(-1)=\dfrac{-2}{1+9}$

$g(-1)=\dfrac{-2}{10}$

$g(-1)=\dfrac{-1}{5}$

We have $h(x)=20\sqrt{25-x^2}$

Put $x=4$

$h(4)=20\sqrt{25-4^2}$

$h(4)=20\sqrt{25-16}$

$h(4)=20\sqrt{9}$

$h(4)=20(3)$

$h(4)=60$

We have $k(x)=x^2$

Put $x=\frac{1}{2}$

$k\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^2$

$k\left(\frac{1}{2}\right)=\frac{1}{4}$

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ii). $\dfrac{f(x)-f(2)}{x-2}$

Solution: Since $f(x)=7x+3$

We have to find $\dfrac{f(x)-f(2)}{x-2}$ putting the values

$\dfrac{f(x)-f(2)}{x-2}=\dfrac{7x+3-7(2)-3}{x-2}$

$\dfrac{f(x)-f(2)}{x-2}=\dfrac{7x-14}{x-2}$

$\dfrac{f(x)-f(2)}{x-2}=\dfrac{7(x-2)}{x-2}$

$\dfrac{f(x)-f(2)}{x-2}=7$

Q3.Find domain and range of function $f(x)$

i). $f(x)=2x+1$

Solution: we have $f(x)=2x+1$ linear function

Since linear/polynomial function have domain = $\Re$

For range Let $f(x)=y \quad \Rightarrow x=f^{-1}(y)$

$y=2x+1$

$y-1=2x$

$x=\dfrac{y-1}{2} \quad \because x=f^{-1}(y)$

So $f^{-1}(y)=\dfrac{y-1}{2}$ Or $f^{-1}(x)=\dfrac{x-1}{2}$

Which is linear/polynomial function have Range = $\Re$

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ii). $f(x)=\sqrt{x^2-9}$

Solution: we have $f(x)=\sqrt{x^2-9}$ radical function

For Domain = $x^2-9\ge 0$

$x^2\ge 9$

Taking square root on both sides

$\sqrt{x^2}\ge \pm\sqrt{9}$

$\pm x\ge 3$

Either $x\ge 3$ or $-x\ge 3$

so $x\le -3$ & $x\ge 3$ $\quad x\le -3$

we can put the values of x at $x=3$ & $x=-3$

Domain $=R-(-3,3)$

For range of $f(x)=\sqrt{x^2-9}$

we can put the values of x at $x=3$ & $x=-3$

which gives output of function is zero. And other values for domain function gives output will positive

So Range $=[0,+\infty)$

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iii). $f(x)=\dfrac{x-3}{x+5}$

Solution: we have $f(x)=\dfrac{x-3}{x+5}$ rational function

Take denominator $x+5\ne 0$

or $x\ne -5$

So Domain $=R-\{-5\}$

For range Let $f(x)=y$ So $y=\dfrac{x-3}{x+5}$

By cross multiplication

$y(x+5)=x-3$

$xy+5y=x-3$

$xy-x=-3-5y$

$x(y-1)=-3-5y$

$x=\dfrac{-3-5y}{y-1}$

$x=\dfrac{-(3+5y)}{-(1-y)}$

$x=\dfrac{3+5y}{1-y} \quad \because x=f^{-1}(y)$

So $f^{-1}(y)=\dfrac{3+5y}{1-y}$ Or $f^{-1}(x)=\dfrac{3+5x}{1-x}$

For Range take denominator $1-x\ne 0$

or $x\ne 1$

So Range $=R-\{1\}$

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iv). $f(x)=\dfrac{x}{x^2-16}$ wrong (Range)

Solution: we have $f(x)=\dfrac{x}{x^2-16}$

For Domain take denominator $x^2-16\ne 0$

or $x^2\ne 16$

or $x\ne \pm 4$

So Domain $=R-\{-4,4\}$

For range Let $f(x)=y$ So, $y=\dfrac{x}{x^2-16}$

By cross multiplication

$y(x^2-16)=x$

$x^2y-16y=x$

$x^2y-x-16y=0$ Quadratic equation in x

$x=\dfrac{1\pm\sqrt{1+64y^2}}{2y}$

$f^{-1}(y)=\dfrac{1\pm\sqrt{1+64y^2}}{2y}$ Or $f^{-1}(x)=\dfrac{1\pm\sqrt{1+64x^2}}{2x}$

For Range take denominator $2x\ne 0$

or $x\ne 0$

Either or

$f^{-1}(x)=\dfrac{1+\sqrt{1+64x^2}}{2x}$, $f^{-1}(x)=\dfrac{1-\sqrt{1+64x^2}}{2x}$

Range $=R-\{0\}$ Range $=R$

Q4: Given that $f(x)=2x^3+ax^2+4x-5$

If $f(2)=3$ find the value of a

Solution: we have $f(x)=2x^3+ax^2+4x-5$

According to given condition $f(2)=3$ put $x=2$

$f(2)=2.2^3+a.2^2+4.2-5=3$

$16+4a+8-5=3$

$4a+16+8=3+5$

$4a+24=8$

$4a=8-24$

$4a=-16$

$a=-4$

Q5. Given that $f(x)=x^3-ax^2+bx+1$

If $f(2)=-3,\ f(-1)=0$ Find values of a and b

Solution: we have $f(x)=x^3-ax^2+bx+1$

According to given condition $f(2)=-3$

$f(2)=2^3-a.2^2+b.2+1=-3$

$8-4a+2b+1=-3$

$9+3-4a+2b=0$

$12-4a+2b=0$................................(1)

According to given condition $f(-1)=0$

$f(-1)=(-1)^3-a(-1)^2+b(-1)+1=0$
$-1-a-b+1=0$
$-a-b=0$
$a=-b$ ....(2)

Put in (1) $12-4(-b)+2b=0$
$12+4b+2b=0$
$6b=-12$
$b=-2$

Put in (2) we get $a=-(-2)=2$

Q6 Determine whether given function is even, odd or neither

i). $f(x)=x^2+1$

Solution: we have $f(x)=x^2+1$

$f(-x)=(-x)^2+1$
$f(-x)=x^2+1$
$f(-x)=f(x)$

Hence $f(x)$ is an even function

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ii). $f(x)=(x-2)^2$

Solution: we have $f(x)=(x-2)^2$

$f(-x)=(-x-2)^2$
$f(-x)=(-1)^2(x+2)^2$
$f(-x)=+(x+2)^2$
$f(-x)\neq f(x)$

Neither even nor odd

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iii). $f(x)=x\sqrt{x^2+3}$

Solution: we have $f(x)=x\sqrt{x^2+3}$

$f(-x)=-x\sqrt{(-x)^2+3}$
$f(-x)=-x\sqrt{x^2+3}$
$f(-x)=-f(x)$

$f(x)$ is an odd function

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iv). $f(x)=\frac{x-1}{x+1}$

Solution: we have $f(x)=\frac{x-1}{x+1}$

$f(-x)=\frac{-x-1}{-x+1}$

$f(-x)=\frac{x+1}{x-1}$

$f(-x)\neq f(x)$ Neither

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v). $f(x)=|x|$

$f(-x)=|-x|$
$f(-x)=|x|$
$f(-x)=f(x)$

Even function

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vi). $f(x)=\frac{x^3+x+3}{x^2-2}$

Solution: we have $f(x)=\frac{x^3+x+3}{x^2-2}$

$f(-x)=\frac{(-x)^3+(-x)+3}{(-x)^2-2}$

$f(-x)=\frac{-x^3-x+3}{x^2-2}$

$f(-x)\neq f(x)$

Neither

Q7: Find the inverse of the function

i). $f(x)=2x-3$

Solution: we have $f(x)=2x-3$

Let $f(x)=y$

Then $y=2x-3$

$y+3=2x$

Or $x=\frac{y+3}{2}$ $\because x=f^{-1}(y)$

$f^{-1}(y)=\frac{y+3}{2}$ Or $f^{-1}(x)=\frac{x+3}{2}$

---

ii). $f(x)=\frac{x}{3}-5$

Solution: we have $f(x)=\frac{x}{3}-5$

Let $f(x)=y$

Then $y=\frac{x}{3}-5$ (multiply each term by 5)

$3y=x-15$

or $x=3y+15$ $\because x=f^{-1}(y)$

$f^{-1}(y)=3y+15$ Or $f^{-1}(x)=3x+15$

---

iii). $f(x)=\frac{2x+1}{x-1}$

Solution: we have $f(x)=\frac{2x+1}{x-1}$

Let $f(x)=y$ Then $y=\frac{2x+1}{x-1}$

$y(x-1)=2x+1$
$yx-y=2x+1$
$yx-2x=y+1$
$x(y-2)=y+1$

$x=\frac{y+1}{y-2}$ $\because x=f^{-1}(y)$

$f^{-1}(y)=\frac{y+1}{y-2}$ Or $f^{-1}(x)=\frac{x+1}{x-2}$

---

iv). $f(x)=4+\sqrt{2x}$

Solution: we have $f(x)=4+\sqrt{2x}$

Let $f(x)=y$

Then $y=4+\sqrt{2x}$

$y-4=\sqrt{2x}$ Squaring

$(y-4)^2 = 2x$

$x = \dfrac{(y-4)^2}{2}$ $\qquad \because x = f^{-1}(y)$

$f^{-1}(y) = \dfrac{(y-4)^2}{2}$ Or $f^{-1}(x) = \dfrac{(x-4)^2}{2}$

Q8. If $f(x) = x^3 - 2$ find

i). $f^{-1}(x)$

Solution: we have $f(x) = x^3 - 2$

Let $f(x) = y$

Then $y = x^3 - 2$

$y + 2 = x^3$

$x = \sqrt[3]{y+2}$ $\qquad \because x = f^{-1}(y)$

$f^{-1}(y) = \sqrt[3]{y+2}$ Or $f^{-1}(x) = \sqrt[3]{x+2}$

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ii). $f^{-1}(3)$

Solution: we have $f^{-1}(x) = \sqrt[3]{x+2}$

$f^{-1}(3) = \sqrt[3]{3+2}$

$f^{-1}(3) = \sqrt[3]{5}$

Q9. Without finding inverse, determine domain of f^{-1}

i). $f(x) = \dfrac{1}{x+2}$ $\qquad x \neq -2$

Solution: we have $f(x) = \dfrac{1}{x+2}$ $\qquad x \neq -2$

Since Domain of f^{-1} = Range of f

Function have the domain $x \neq -2$ or $R - \{-2\}$

Function have output set of real numbers except $f(x) = 0$

So Range of $f = R - \{0\}$

Hence domain of $f^{-1} = R - \{0\}$

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ii). $f(x) = \sqrt{x+3}$

Solution: we have $f(x) = \sqrt{x+3}$

Since Domain of f^{-1} = Range of f

Function have the domain $[-3, +\infty)$

Function have output set of real numbers

So Range of $f = R$

Hence domain of $f^{-1} = R$

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iii). $f(x) = \dfrac{x-1}{x-2}$

Solution: we have $f(x) = \dfrac{x-1}{x-2}$

Since Domain of f^{-1} = Range of f

Function have the domain $x = 2$ or $R - \{2\}$

Function have output set of real numbers except

$f(x) \neq 1$

So Range of $f = R - \{1\}$

Hence domain of $f^{-1} = R - \{1\}$

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iv). $f(x) = (x-7)^2$

Solution: we have $f(x) = (x-7)^2$

Since Domain of f^{-1} = Range of f

Function is linear/polynomial so domain $\Re$

Function have output set of positive real numbers

So Range of $f = \{y \mid y \in R \wedge y \geq 0\}$

Hence domain of $f^{-1} = \{x \mid x \in R \wedge x \geq 0\}$

Exercise 8.2

Q1. Sketch the graph of the given function

i). $f(x) = 2x + 3$

Solution: we have $f(x) = 2x + 3$

Put $x = 0$

$f(0) = 2(0) + 3$

$f(0) = 0 + 3$

$f(0) = 3$

Put $x = -1$

$f(-1) = 2(-1) + 3$

$f(-1) = -2 + 3$

$f(-1) = 1$

Put $x = -2$

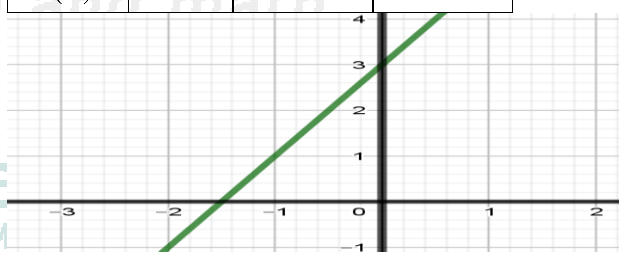
$f(-2) = 2(-2) + 3$

$f(-2) = -4 + 3$

$f(-2) = -1$

So the table

x	0	-1	-2
$f(x)$	3	1	-1



ii). $f(x) = 4x - 5$

Solution: we have $f(x) = 4x - 5$

Put $x = 0$

$f(0) = 4(0) - 5$

$f(0) = 0 - 5$

$f(0) = -5$

Put $x = 1$

$f(1) = 4(1) - 5$

$f(1) = 4 - 5$

$f(1) = -1$

Put $x = 2$

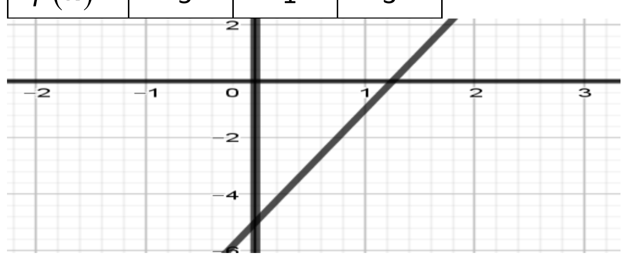
$f(2) = 4(2) - 5$

$f(2) = 8 - 5$

$f(2) = 3$

So the table

x	0	1	2
$f(x)$	-5	-1	3



iii). $f(x)=4-|x|$

Solution: we have $f(x)=4-|x|$

At $x=-2$

$f(-2)=4-|-2|$

$f(-2)=4-2$

$f(-2)=2$

At $x=-1$

$f(-1)=4-|-1|$

$f(-1)=4-1$

$f(-1)=3$

At $x=0$

$f(0)=4-|0|$

$f(0)=4-0$

$f(0)=4$

At $x=1$

$f(1)=4-|1|$

$f(1)=4-1$

$f(1)=3$

At $x=2$

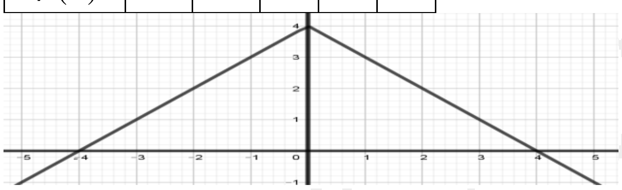
$f(2)=4-|2|$

$f(2)=4-2$

$f(2)=2$

So the table

x	-2	-1	0	1	2
$f(x)$	2	3	4	3	2



Q2.Sketch the graph of the following functions

i). $f(x)=x^2+1$

Solution: we have $f(x)=x^2+1$

At $x=-2$

$f(-2)=(-2)^2+1$

$f(-2)=4+1$

$f(-2)=5$

At $x=-1$

$f(-1)=(-1)^2+1$

$f(-1)=1+1$

$f(-1)=2$

At $x=0$

$f(0)=(0)^2+1$

$f(0)=0+1$

$f(0)=1$

At $x=1$

$f(1)=(1)^2+1$

$f(1)=1+1$

$f(1)=2$

At $x=2$

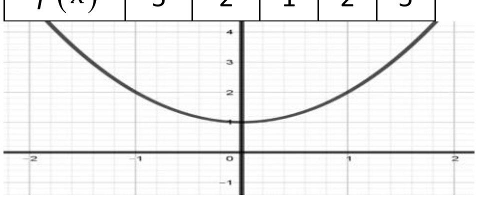
$f(2)=(2)^2+1$

$f(2)=4+1$

$f(2)=5$

So the table

x	-2	-1	0	1	2
$f(x)$	5	2	1	2	5



ii). $f(x)=-x^2+1$

Solution: we have $f(x)=-x^2+1$

At $x=-2$

$f(-2)=-(-2)^2+1$

$f(-2)=-(4)+1$

$f(-2)=-3$

At $x=-1$

$f(-1)=-(-1)^2+1$

$f(-1)=-(1)+1$

$f(-1)=0$

At $x=0$

$f(0)=-(0)^2+1$

$f(0)=-(0)+1$

$f(0)=1$

At $x=1$

$f(1)=-(1)^2+1$

$f(1)=-(1)+1$

$f(1)=0$

At $x=2$

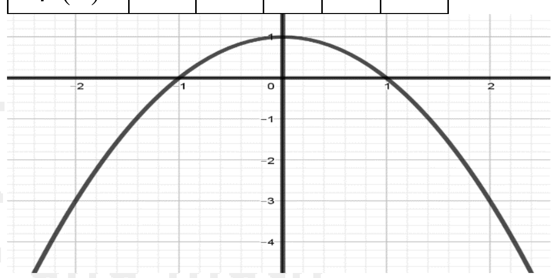
$f(2)=-(2)^2+1$

$f(2)=-(4)+1$

$f(4)=-3$

So the table

x	-2	-1	0	1	2
$f(x)$	-3	0	1	0	-3



iii). $f(x)=x^2+2x+1$

Solution: we have $f(x)=x^2+2x+1$

At $x=-3$

$f(-3)=(-3)^2+2(-3)+1$

$f(-3)=9-6+1$

$f(-3)=4$

At $x=-2$

$f(-2)=(-2)^2+2(-2)+1$

$f(-2)=4-4+1$

$f(-2)=1$

At $x=-1$

$f(-1)=(-1)^2+2(-1)+1$

$f(-1)=1-2+1$

$f(-1)=0$

At $x=0$

$f(0)=(0)^2+2(0)+1$

$f(0)=0+0+1$

$f(0)=1$

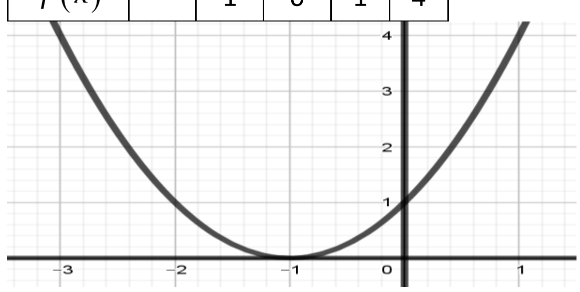
At $x=1$

$f(1)=(1)^2+2(1)+1$

$f(1)=1+2+1$

$f(1)=4$

x	-3	-2	-1	0	1
$f(x)$	4	1	0	1	4



iv). $f(x) = -x^2 + 2x + 1$

Solution: we have $f(x) = -x^2 + 2x + 1$

At $x = -1$

$f(-1) = -(-1)^2 + 2(-1) + 1$

$f(-1) = -1 - 2 + 1$

$f(-1) = -2$

At $x = 0$

$f(0) = -(0)^2 + 2(0) + 1$

$f(0) = -0 + 0 + 1$

$f(0) = 1$

At $x = 1$

$f(1) = -(1)^2 + 2(1) + 1$

$f(1) = -1 + 2 + 1$

$f(1) = 3$

At $x = 2$

$f(2) = -(2)^2 + 2(2) + 1$

$f(2) = -4 + 4 + 1$

$f(2) = 1$

At $x = 3$

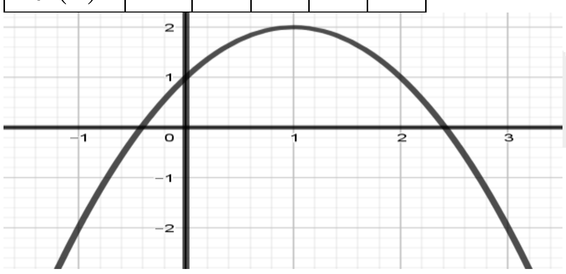
$f(3) = -(3)^2 + 2(3) + 1$

$f(3) = -9 + 6 + 1$

$f(3) = -2$

So the table

x	-1	0	1	2	3
$f(x)$	-2	1	2	1	-2



Q3 Without graphing, find vertex, all intercepts if any and axis of graph of the following function. Also determine whether graphs open upwards or downward.

i). $f(x) = \frac{3}{4}x^2$

Solution: we have $f(x) = \frac{3}{4}x^2$

Compare with the general equation

$f(x) = a(x-h)^2 + k$

$a = \frac{3}{4}, \quad h = 0, \quad k = 0$

$a = \frac{3}{4} > 0$ Opens Up ward

Vertex $(h,k) = (0,0)$

X-Intercept Put $f(x) = 0$

$\frac{3}{4}x^2 = 0$

$x^2 = 0$

$\Rightarrow x = 0$

Thus x-intercept $x = 0$

Y-Intercept $x = 0$

$f(0) = \frac{3}{4}(0)^2$

$f(0) = 0$

Thus y-intercept $y = 0$

Axis of symmetry $x = h$

Thus Axis of symmetry $x = 0$

ii). $f(x) = x^2 + 1$

Solution: we have $f(x) = x^2 + 1$

Compare with the general equation

$f(x) = a(x-h)^2 + k$

$a = 1, \quad h = 0, \quad k = 1$

$a = 1 > 0$ Opens Up ward

Vertex $(h,k) = (0,1)$

X-Intercept Put $f(x) = 0$

$x^2 + 1 = 0$

$x^2 = -1$

$\Rightarrow x = \pm\sqrt{-1}$

Thus X-Intercept= Not exists

Y-Intercept $x = 0$

$f(0) = (0)^2 + 1$

$f(0) = 0 + 1$

$f(0) = 1$

Thus Y-intercept $y = 1$

Axis of symmetry $x = h$

Thus Axis of symmetry $x = 0$

iii). $f(x) = -2x^2 + 8$

Solution: we have $f(x) = -2x^2 + 8$

Compare with the general equation

$f(x) = a(x-h)^2 + k$

$a = -2, \quad h = 0, \quad k = 8$

$a = -2 < 0$ Opens Down ward

Vertex $(h,k) = (0,8)$

X-Intercept Put $f(x) = 0$

$-2x^2 + 8 = 0$

$-2x^2 = -8$

$x^2 = 4$

$\Rightarrow x = \pm 2$

Thus X-Intercepts= $x = 2, \ x = -2$

Y-Intercept $x = 0$

$f(0) = -2(0)^2 + 8$

$f(0) = 8$

Thus Y-intercept $y = 8$

Axis of symmetry $x = h$

Thus Axis of symmetry $x = 0$

iv). $f(x) = -x^2 + 6x - 5$

Solution: we have $f(x) = -x^2 + 6x - 5$

$f(x) = -(x^2 - 6x) + 9$

$f(x) = -(x^2 - 6x + 9) + 9 - 5$

$f(x) = -(x-3)^2 + 4$

Compare with the general equation

$f\left(x\right)=a\left(x-h\right)^2+k$

$a=-1,\quad h=3,\quad k=4$

$a=-1<0$ Opens Down ward

Vertex $\left(h,k\right)=\left(3,4\right)$

X-Intercept Put $f\left(x\right)=0$

$$-x^2+6x-5=0$$
$$x^2-6x+5=0$$
$$x^2-5x-1x+5=0$$
$$x\left(x-5\right)-1\left(x-5\right)=0$$
$$\left(x-1\right)\left(x-5\right)=0$$

Either or

$x-1=0$ $\quad$ $x-5=0$

$x=1$ $\quad$ $x=5$

Thus x-intercepts $x=1,\quad x=5$

Y-Intercept $x=0$

$$f\left(0\right)=-\left(0\right)^2+6\left(0\right)-5$$
$$f\left(0\right)=-0+0-5$$
$$f\left(0\right)=-5$$

Thus y-intercept $y=-5$

Axis of symmetry $x=h$

Thus Axis of symmetry $x=3$

v). $f\left(x\right)=x^2+2x-3$

Solution: we have $f\left(x\right)=x^2+2x-3$

$f\left(x\right)=x^2+2x+1-1-3$

$f\left(x\right)=\left(x+1\right)^2-4$

$f\left(x\right)=\left(x-\left(-1\right)\right)^2-4$

Compare with the general equation

$f\left(x\right)=a\left(x-h\right)^2+k$

$a=1,\quad h=-1,\ k=-4$

$a=1>0$ Opens Up ward

Vertex $\left(h,k\right)=\left(-1,-4\right)$

X-Intercept Put $f\left(x\right)=0$

$$x^2+2x-3=0$$
$$x^2+3x-1x-3=0$$
$$x\left(x+3\right)-1\left(x-3\right)=0$$
$$\left(x-1\right)\left(x+3\right)=0$$

Either or

$x-1=0$ $\quad$ $x+3=0$

$x=1$ $\quad$ $x=-3$

Thus X-Intercepts $x=1,-3$

Y-Intercept $x=0$

$$f\left(0\right)=\left(0\right)^2+2\left(0\right)-3$$
$$f\left(0\right)=0+0-3$$
$$f\left(0\right)=-3$$

Thus Y-intercept $y=-3$

Axis of symmetry $x=h$

Thus Axis of symmetry $x=-1$

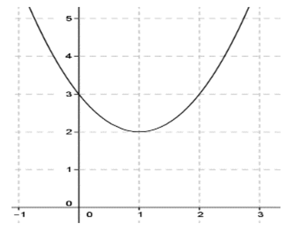
In Questions 4-6 guess the quadratic function for curve is given is ht figure.

Q4a) $f\left(x\right)=x^2+2x+3$

b) $f\left(x\right)=-x^2-2x+3$

c) $f\left(x\right)=x^2-2x+3$

d) $f\left(x\right)=-x^2+2x+3$



Sol: we have the graph upward

So possibilities are a and c which are upward

To find the vertex

Take $f\left(x\right)=x^2+2x+3$

$f\left(x\right)=x^2+2x+1+2$

$f\left(x\right)=\left(x+1\right)^2+2$

$f\left(x\right)=\left(x-\left(-1\right)\right)^2+2$

Compare with general eq $f\left(x\right)=a\left(x-h\right)^2+k$

$a=1,\quad h=-1,\quad k=2$

$a=1>0$ Opens Up ward

Vertex $\left(h,k\right)=\left(-1,2\right)$ Which is wrong/2nd quadrant

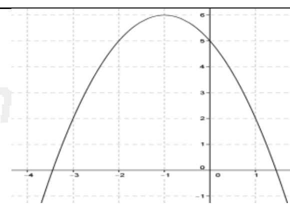
Take the last option, So the Option C is correct

Q5a) $g\left(x\right)=-x^2-2x-5$

b) $g\left(x\right)=-x^2+2x-5$

c) $g\left(x\right)=-x^2-2x+5$

d) $g\left(x\right)=-x^2+2x+5$



Solution: we have the graph downward

And y-intercept is positive so possible options are c & d

To find the vertex take Option C

Take $g\left(x\right)=-x^2-2x+5$

$g\left(x\right)=-\left(x^2+2x\right)+5$

$g\left(x\right)=-\left(x^2+2x+1\right)+5+1$

$g\left(x\right)=-\left(x+1\right)^2+6$

$g\left(x\right)=-\left(x-\left(-1\right)\right)^2+6$

Compare with general eq $f\left(x\right)=a\left(x-h\right)^2+k$

$a=-1,\quad h=-1,\quad k=6$

$a=-1<0$ Opens Up ward

Vertex $\left(h,k\right)=\left(-1,6\right)$ Which is Correct / 2nd quadrant

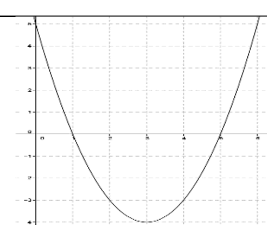
So the Option C is correct

Q6a) $h\left(x\right)=-x^2-6x+5$

b). $h\left(x\right)=x^2-6x+5$

c). $h\left(x\right)=x^2+6x+5$

d). $h\left(x\right)=-x^2-6x-5$



Sol: we have the graph upward

So possibilities are b and c which are upward

To find the vertex take option b

Take $h\left(x\right)=x^2-6x+5$

$h(x)=(x^2-6x)+5$

$h(x)=(x^2-6x+9)+5-9$

$h(x)=(x-3)^2-4$

Compare with general eq $f(x)=a(x-h)^2+k$

$a=1,\quad h=3,\quad k=-4$

$a=1>0$ Opens Up ward

Vertex $(h,k)=(3,-4)$ Which is correct/ 4[th]quadrant

So the Option b is correct

Exercise 8.3

Q1. Sketch graph of the following functions

i). $f(x)=(x-1)(x-3)$

Solution: we have $f(x)=(x-1)(x-3)$

X-Intercept Put $f(x)=0$

$(x-1)(x-3)=0$

Either $x-1=0$, $x=1$ or $x-3=0$, $x=3$

Thus X-Intercepts= 1 & 3

Y-Intercept $x=0$

$f(0)=(0-1)(0-3)$

$f(0)=(-1)(-3)$

$f(0)=3$

Thus Y-intercept $y=3$

Put $x=2$

$f(2)=(2-1)(2-3)$

$f(2)=(1)(-1)$

$f(2)=-1$

Put $x=4$

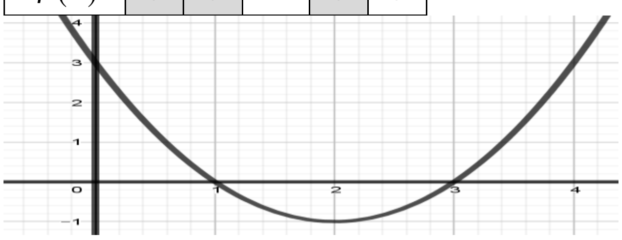
$f(4)=(4-1)(4-3)$

$f(4)=(3)(1)$

$f(4)=3$

So the table

x	0	1	2	3	4
$f(x)$	3	0	-1	0	3



ii). $f(x)=(x+4)(x+1)$

Solution: we have $f(x)=(x+4)(x+1)$

X-Intercept Put $f(x)=0$

$(x+4)(x+1)=0$

Either $x+4=0$, $x=-4$ or $x+1=0$, $x=-1$

Thus X-Intercepts= - 4 & - 1

Y-Intercept $x=0$

$f(0)=(0+4)(0+1)$

$f(0)=(4)(1)$

$f(0)=4$

Thus Y-intercept $y=4$

Put $x=-3$

$f(-3)=(-3+4)(-3+1)$

$f(-3)=(1)(-2)$

$f(-3)=-2$

Put $x=-2$

$f(-2)=(-2+4)(-2+1)$

$f(-2)=(2)(-1)$

$f(-2)=-2$

Put $x=-5$

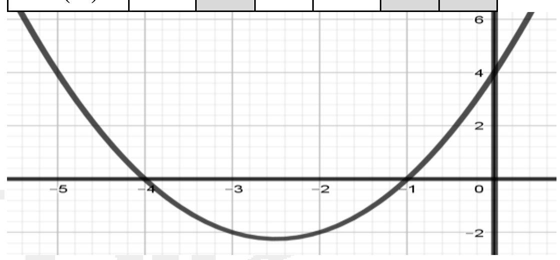
$f(-5)=(-5+4)(-5+1)$

$f(-5)=(-1)(-4)$

$f(-5)=4$

So the table

x	-5	-4	-3	-2	-1	0
$f(x)$	4	0	-2	-2	0	4



iii). $f(x)=2(x+1)(x-1)$

Solution: we have $f(x)=2(x+1)(x-1)$

X-Intercept Put $f(x)=0$

$2(x+1)(x-1)=0$

Either $x+1=0$, $x=-1$ or $x-1=0$, $x=1$

Thus X-Intercepts= - 1 & 1

Y-Intercept $x=0$

$f(0)=2(0+1)(0-1)$

$f(0)=2(1)(-1)$

$f(0)=-2$

Thus Y-intercept $y=-2$

Put $x=-2$

$f(-2)=2(-2+1)(-2-1)$

$f(-2)=2(-1)(-3)$

$f(-2)=6$

Put $x=2$

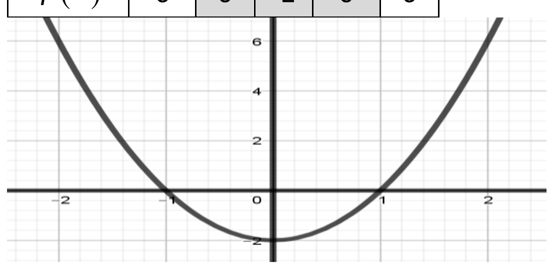
$f(2)=2(2+1)(2-1)$

$f(2)=2(3)(1)$

$f(2)=6$

So the table

x	-2	-1	0	1	2
$f(x)$	6	0	-2	0	6



iv). $f(x)=-2(x+1)(x-1)$

Solution: we have $f(x)=-2(x+1)(x-1)$

X-Intercept Put $f(x)=0$

$$-2(x+1)(x-1)=0$$

Either or

$x+1=0$ $x-1=0$

$x=-1$ $x=1$

Thus X-Intercepts= - 1 & 1

Y-Intercept $x=0$

$$f(0)=-2(0+1)(0-1)$$
$$f(0)=-2(1)(-1)$$
$$f(0)=2$$

Thus Y-intercept $y=2$

Put $x=-2$

$f(-2)=-2(-2+1)(-2-1)$

$f(-2)=-2(-1)(-3)$

$f(-2)=-6$

Put $x=2$

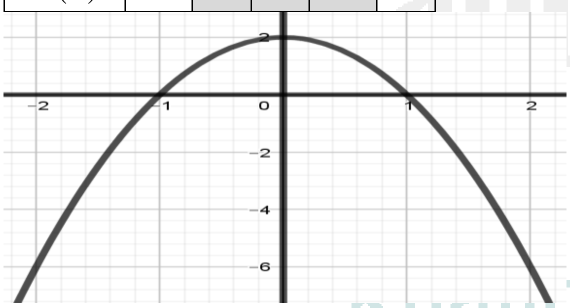
$f(2)=-2(2+1)(2-1)$

$f(2)=-2(3)(1)$

$f(2)=-6$

So the table

x	-2	-1	0	1	2
$f(x)$	-6	0	2	0	-6



Q2. Using factors to sketch graphs of following functions

i). $f(x)=x^2-2x-3$

Solution: we have $f(x)=x^2-2x-3$

$$f(x)=x^2-3x+1x-3$$
$$f(x)=x(x-3)+1(x-3)$$
$$f(x)=(x-3)(x+1)$$

X-Intercept Put $f(x)=0$

$$(x-3)(x+1)=0$$

Either or

$x-3=0$ $x+1=0$

$x=3$ $x=-1$

Thus X-Intercepts= 3 & - 1

Y-Intercept $x=0$

$$f(0)=(0-3)(0+1)$$
$$f(0)=(-3)(1)$$
$$f(0)=-3$$

Thus Y-intercept $y=-3$

Put $x=1$

$f(1)=(1-3)(1+1)$

$f(1)=(-2)(2)$

$f(1)=-4$

Put $x=2$

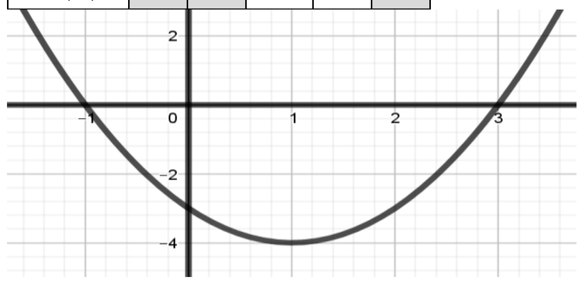
$f(2)=(2-3)(2+1)$

$f(2)=(-1)(3)$

$f(2)=-3$

So the table

x	-1	0	1	2	3
$f(x)$	0	-3	-4	-3	0



ii). $f(x)=-(x^2-x-2)$

Solution: we have $f(x)=-(x^2-x-2)$

$$f(x)=-(x^2-2x+1x-2)$$
$$f(x)=-\{x(x-2)+1(x-2)\}$$
$$f(x)=-(x-2)(x+1)$$

X-Intercept Put $f(x)=0$

$$-(x-2)(x+1)=0$$

Either or

$x-2=0$ $x+1=0$

$x=2$ $x=-1$

Thus X-Intercepts= 2 & - 1

Y-Intercept $x=0$

$$f(0)=-(0-2)(0+1)$$
$$f(0)=-(-2)(1)$$
$$f(0)=2$$

Thus Y-intercept $y=2$

Put $x=-2$

$$f(-2)=-(-2-2)(-2+1)$$
$$f(-2)=-(-4)(-1)$$
$$f(-2)=-4$$

Put $x=1$

$f(1)=-(1-2)(1+1)$

$f(1)=-(-1)(2)$

$f(1)=2$

Put $x=3$

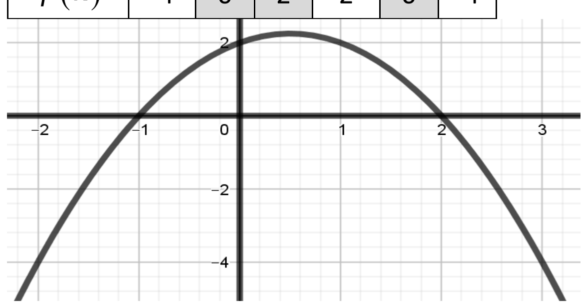
$f(3)=-(3-2)(3+1)$

$f(3)=-(1)(4)$

$f(3)=-4$

So the table

x	-2	-1	0	1	2	3
$f(x)$	-4	0	2	2	0	-4



iii). $f(x)=-x^2-4x-4$

Solution: we have $f(x)=-x^2-4x-4$

$f(x)=-\{x^2+4x+4\}$

$f(x)=-\{x^2+2(x)(2)+(2)^2\}$

$f(x)=-(x+2)^2$

X-Intercept Put $f(x)=0$

$-(x+2)^2=0$

Either or

$x+2=0$ $x+2=0$

$x=-2$ $x=-2$

Thus X-Intercept= 2

Y-Intercept $x=0$

$f(0)=-(0+2)^2$

$f(0)=-(4)$

$f(0)=-4$

Thus Y-intercept $y=-4$

Put $x=-4$

$f(-4)=-(-4+2)^2$

$f(-4)=-(-2)^2$

$f(-4)=-4$

Put $x=-3$

$f(-3)=-(-3+2)^2$

$f(-3)=-(-1)^2$

$f(-3)=-1$

Put $x=-1$

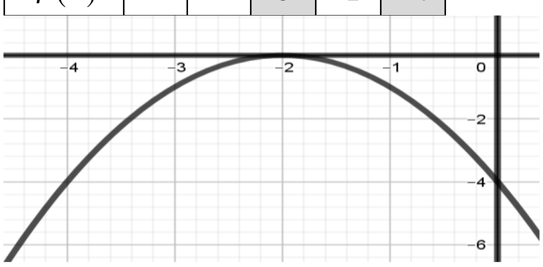
$f(-1)=-(-1+2)^2$

$f(-1)=-(1)^2$

$f(-1)=-1$

So the table

x	-4	-3	-2	-1	0
$f(x)$	-4	-1	0	-1	- 4



Q3. Find the equations of the graph of the function of the type $f(x)=x^2+bx+c$ which cross the x axis at the point (3,0) and (4,0)

Solution; we have $f(x)=x^2+bx+c$

At $(3,0)$ or $x=3$ and $y=f(x)=0$

$f(3)=(3)^2+b(3)+c=0$

$9+3b+c=0$

$c=-9-3b$..............................(1)

At $(4,0)$ or $x=4$ and $y=f(x)=0$

$f(4)=(4)^2+b(4)+c=0$

$16+4b+c=0$

Putting the value of c

$16+4b-9-3b=0$

$4b-3b=9-16$

$b=-7$

Putting the value of b in equation (1) we get

$c=-9-3(-7)$

$c=-9+21=12$

Putting the value of b and c in given equation

$f(x)=x^2-7x+12$

Q4. Find the equation of the graph of the function of the type $f(x)=ax^2+bx+c$ which

a). Cross the x-axis at the point (-5,0) and (3,0) and also passes through (-1,8)

Solution: we have $f(x)=ax^2+bx+c$

At $(3,0)$ or $x=3$ and $y=f(x)=0$

$f(3)=a(3)^2+b(3)+c=0$

$9a+3b+c=0$

$c=-9a-3b$..............................(1)

At $(-5,0)$ or $x=-5$ and $y=f(x)=0$

$f(-5)=a(-5)^2+b(-5)+c=0$

$25a-5b+c=0$

Putting the value of c

$25a-5b-9a-3b=0$

$25a-9a-5b-3b=$

$16a-8b=0$

$-8b=-16a$

$b=2a$..............................(2)

At $(-1,8)$ or $x=-1$ and $y=f(x)=8$

$f(-1)=a(-1)^2+b(-1)+c=8$

$a-b+c=8$

Putting the value of c

$a-b-9a-3b=8$

$a-9a-b-3b=8$

$-8a-4b=8$

Now putting the value of b

$-8a-4(2a)=8$

$-8a-8a=8$

$-16a=8$

$a=\frac{8}{-16}$

$a=\frac{-1}{2}$

Putting the value of a in equation (2) we get

$b=2\left(\frac{-1}{2}\right)$

$b=-1$

Putting the values of a & b in equation (1) we get

$c=-9a-3b$

$c=-9\left(\frac{-1}{2}\right)-3(-1)$

$c=\frac{9}{2}+3$

$c=\frac{15}{2}$

Putting the values of a, b and c in general equation

$$f(x)=-\frac{x^2}{2}-x+\frac{15}{2}$$

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b). Cross the x-axis at the point (-7,0) and (10,0) and also passes through (4,11)

Solution: we have $f(x)=ax^2+bx+c$

At $(10,0)$ or $x=10$ and $y=f(x)=0$

$f(10)=a(10)^2+b(10)+c=0$

$100a+10b+c=0$

$c=-100a-10b$..............................(1)

At $(-7,0)$ or $x=-7$ and $y=f(x)=0$

$f(-7)=a(-7)^2+b(-7)+c=0$

$49a-7b+c=0$

Putting the value of c

$49a-7b-100a-10b=0$

$-51a-17b=0$

$-17b=51a$

$b=-3a$..............................(2)

At $(4,11)$ or $x=4$ and $y=f(x)=11$

$f(4)=a(4)^2+b(4)+c=11$

$16a+4b+c=11$

Putting the value of c

$16a+4b-100a-10b=11$

$-84a-6b=11$

Now putting the value of b

$-84a-6(-3a)=11$

$-84a+18a=11$

$-66a=11$

$a=\frac{11}{-66}$

$a=\frac{-1}{6}$

Putting the value of a in equation (2) we get

$b=-3\left(\frac{-1}{6}\right)$

$b=\frac{1}{2}$

Putting the values of a & b in equation (1) we get

$c=-100a-10b$

$c=-100\left(\frac{-1}{6}\right)-10\left(\frac{1}{2}\right)$

$c=\frac{50}{3}-5$

$c=\frac{35}{3}$

Putting the values of a, b and c in general equation

$$f(x)=-\frac{x^2}{6}-\frac{x}{2}+\frac{35}{3}$$

Q5. Find the point of intersection graphically of the following linear functions with the coordinate axis

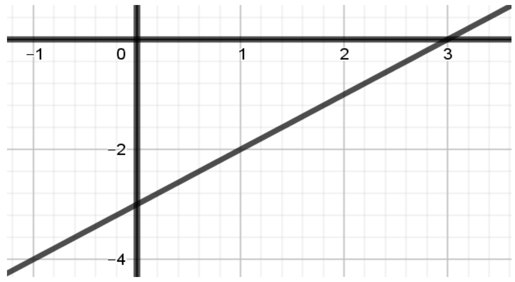
i). $f(x)=x-3$

Solution: we have $f(x)=x-3$

at x = 0 we get y = -3

at y = 0 we get x = 3

so (0,-3), (3,0)



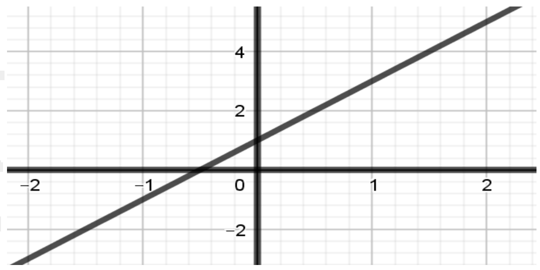
ii). $f(x)=2x+1$

Solution: we have $f(x)=2x+1$

at x = 0 we get y = 1

at y = 0 we get x = -1/2

so (0,1), (-1/2,0)



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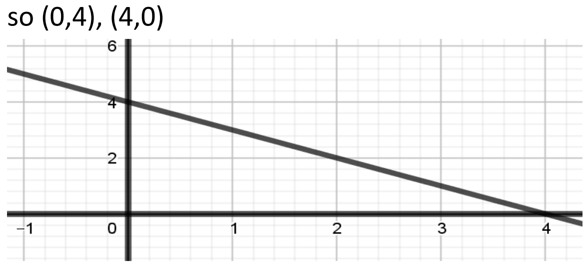
iii). $f(x)=-x+4$

Solution: we have $f(x)=-x+4$

at x = 0 we get y = 4

at y = 0 we get x = 4

so (0,4), (4,0)



Q6 Find point of intersection of following functions.

i). $f(x)=-x+2$ And $g(x)=2x+1$

Solution: we have $f(x)=-x+2$ And $g(x)=2x+1$

Take $f(x)=-x+2$

At $x=0$ we get $y=2$

At $y=f(x)=0$ we get $x=2$

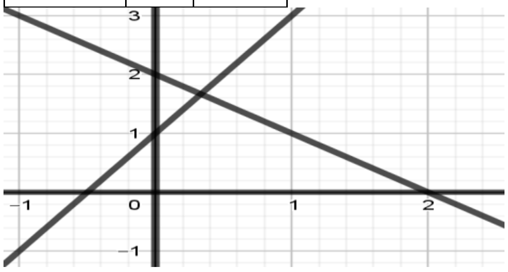
x	0	2
$f(x)$	2	0

Now take $g(x)=2x+1$

At $x=0$ we get $y=1$

At $y=g(x)=0$ we get $x=\frac{-1}{2}$

x	0	-1/2
$g(x)$	1	0



From the graph (1/3,5/3) is point of intersection

ii). $f(x)=3x-2$ And $g(x)=-x+6$

Solution: we have $f(x)=3x-2$ And

$g(x)=-x+6$

Take $f(x)=3x-2$

At $x=0$ we get $y=-2$

At $y=f(x)=0$ we get $x=\frac{2}{3}$

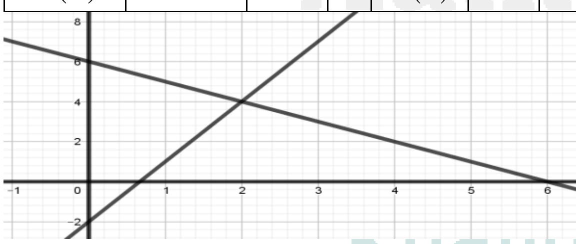
Now take $g(x)=-x+6$

At $x=0$ we get $y=6$

At $y=g(x)=0$ we get $x=6$

x	0	2/3
$f(x)$	-2	0

x	0	6
$g(x)$	6	0



From the graph (2,4) is point of intersection

iii). $f(x)=x+4$ And $g(x)=-2x+3$

Solution: we have $f(x)=x+4$ And

$g(x)=-2x+3$

Take $f(x)=x+4$

At $x=0$ we get $y=4$

At $y=f(x)=0$ we get $x=-4$

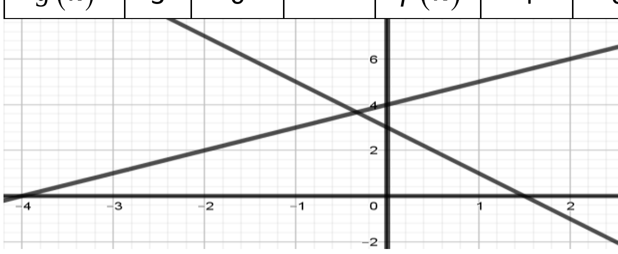
Now take $g(x)=-2x+3$

At $x=0$ we get $y=3$

At $y=g(x)=0$ we get $x=\frac{3}{2}$

x	0	3/2
$g(x)$	3	0

x	0	-4
$f(x)$	4	0



From the graph (-1/3,11/3) is point of intersection

Q7 Find the point of intersection graphically of the following functions.

i). $f(x)=-x^2+4$ and $g(x)=x+2$

Sol: Take $f(x)=-x^2+4$

At $x=0$ we get $y=4$

At $y=f(x)=0$ we get $-x^2+4=0$

$$-x^2=-4$$
$$x^2=4$$
$$x=\pm 2$$

At $x=-1$ we get $y=f(-1)=-(-1)^2+4$

$$y=-1+4$$
$$y=3$$

At $x=1$ we get $y=f(1)=-(1)^2+4$

$$y=-1+4$$
$$y=3$$

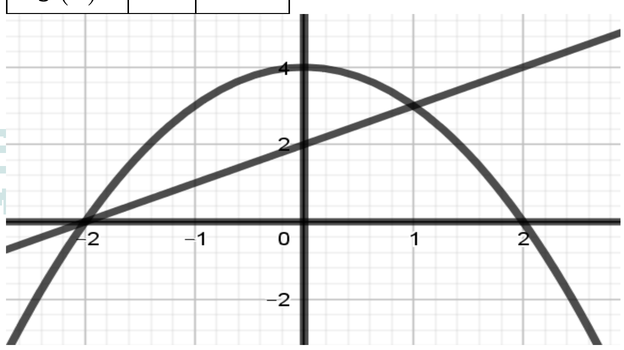
x	-2	-1	0	1	2
$f(x)$	0	3	4	3	0

Now take $g(x)=x+2$

At $x=0$ we get $y=2$

At $y=g(x)=0$ we get $x=-2$

x	0	-2
$g(x)$	2	0



From graph point of intersections are(-2,0) (1,3)

ii). $f(x)=x^2+x-3$ and $g(x)=-2x-5$

Sol: Given $f(x)=x^2+x-3$ & $g(x)=-2x-5$

Take $f(x)=x^2+x-3$

At $x=0$ we get $y=-3$

At $x=-2$

$$f(-2)=(-2)^2+(-2)-3$$
$$f(-2)=4-2-3$$
$$f(-2)=-1$$

At $x=-1$

$$f(-1)=(-1)^2+(-1)-3$$
$$f(-1)=1-1-3$$
$$f(-1)=-3$$

At $x=1$

$f(1)=(1)^2+(1)-3$

$f(1)=1+1-3$

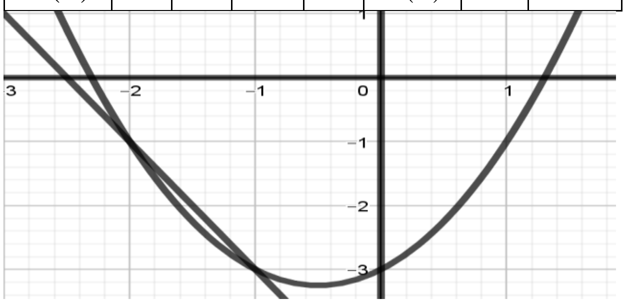
$f(1)=-1$

Now take $g(x)=-2x-5$

At $x=0$ we get $y=-5$

At $y=g(x)=0$ we get $x=\frac{-5}{2}$

x	-2	-1	0	1	x	0	-5/2
$f(x)$	-1	-3	-3	-1	$g(x)$	-5	0



From graph point of intersections are(-2,-1) (-1,-3)

iii). $f(x)=x^2-x-2$ and $g(x)=-3x-3$

Sol: Given $f(x)=x^2-x-2$ & $g(x)=-3x-3$

Take $f(x)=x^2-x-2$

At $x=0$ we get $y=-2$

At $y=f(x)=0$ we get $x^2-x-2=0$

$$x^2-2x+x-2=0$$

$$x(x-2)+1(x-2)=0$$

$$(x+1)(x-2)=0$$

Either or

$x+1=0$ $\quad$ $x-2=0$

$x=-1$ $\quad$ $x=2$

At $x=1$

$f(1)=(1)^2-(1)-2$

$f(1)=1-1-2$

$f(1)=-2$

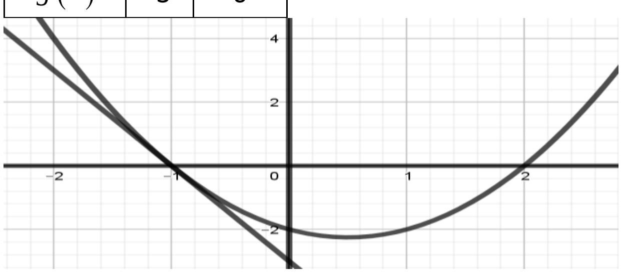
x	-1	0	1	2
$f(x)$	0	-2	-2	0

Now take $g(x)=-3x-3$

At $x=0$ we get $y=-3$

At $y=g(x)=0$ we get $x=-1$

x	0	-1
$g(x)$	-3	0



From the graph point of intersection is (-1,0)

Q8. The paths of two airplane A and B in the plane are determine by the straight lines
2x –y= 6 and 3x + y = 4 respectively. Find the point whose paths cross each other.

Sol: Given

2x-y =6 $\quad$ 3x+y = 4

Or y = 2x - 6 $\quad$ y = 4 – 3x

At $x=0$

$y=-6$ $\quad$ $y=4$

At $y=0$

$2x-6=0$ $\quad$ $4-3x=0$

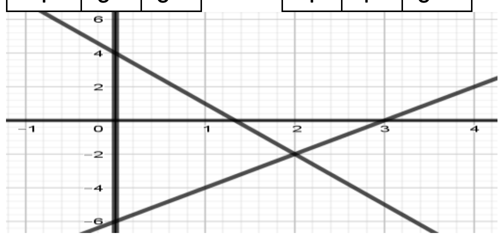
$2x=6$ $\quad$ $4=3x$

$x=3$ $\quad$ $x=\frac{4}{3}$

X	0	3
Y	6	0

and

X	0	4/3
Y	4	0



From the graph point of intersection is (2,-2)

Q9. A pilot makes a check flight in an air. Going directly into the wind. He covers a distance of 24km in 6 minutes. Going with the wind. He covers the distance in 4 minutes. Find his air speed and velocity of the wind in km/min

Solution; Let Speed of Airplane = x

Let Speed of Wind = y

Form the conditions

Flight into wind $\quad$ flight with wind

$x-y=\frac{24\ km}{6\ min}$ And $x+y=\frac{24\ km}{4\ min}$

$x-y=4\frac{km}{min}$.............(1)

$x+y=6\frac{km}{min}$..........(2)

$x=4+y$...................(3) Putting value of x in (2)

$$4+y+y=6$$

$$2y=6-4$$

$$2y=2$$

$$y=1$$

Putting the value of y in equation (3) we get

$$x=4+1$$

$$x=5$$

So Speed of Airplane = x = 5 km / min

Speed of Wind = y = 1 km / min