Chapter 6

Permutation combination and Probability

Exercise 6.1

Q1. Evaluate the following:

i). $\frac{9!0!}{5!4!}$

Solution: we have $\frac{9!0!}{5!4!}$

$\frac{9!0!}{5!4!}=\frac{9.8.7.6.5!.1}{5!.4.3.2.1}$

$\frac{9!0!}{5!4!}=\frac{9.8.7.6}{4.3.2.1}$

$\frac{9!0!}{5!4!}=9.2.7$

$\frac{9!0!}{5!4!}=126$

ii). $\frac{3!+4!}{5!-4!}$

Solution: we have $\frac{3!+4!}{5!-4!}$

$\frac{3!+4!}{5!-4!}=\frac{3.2.1+4.3.2.1}{5.4.3.2.1-4.3.2.1}$

$\frac{3!+4!}{5!-4!}=\frac{3.2.(1+4)}{4.3.2(5-1)}$

$\frac{3!+4!}{5!-4!}=\frac{5}{4.4}$

$\frac{3!+4!}{5!-4!}=\frac{5}{16}$

iii). $\frac{(n-1)!}{(n+1)!}$

Solution: we have $\frac{(n-1)!}{(n+1)!}$

$\frac{(n-1)!}{(n+1)!}=\frac{(n-1)!}{(n+1).n.(n-1)!}$

$\frac{(n-1)!}{(n+1)!}=\frac{1}{n(n+1)}$

iv). $\frac{10!}{(5!)^2}$

Solution: we have $\frac{10!}{(5!)^2}$

$\frac{10!}{(5!)^2}=\frac{10!}{(5)!(5)!}$

$\frac{10!}{(5!)^2}=\frac{10.9.8.7.6.5!}{5.4.3.2.1(5)!}$

$\frac{10!}{(5!)^2}=2.9.2.7$

$\frac{10!}{(5!)^2}=252$

Q2. Write the following in term of factorials.

i). 18.17.16.15.14

Solution: we have 18.17.16.15.14

$18.17.16.15.14=18.17.16.15.14.\frac{13!}{13!}$

$18.17.16.15.14=\frac{18.17.16.15.14.13!}{13!}$

$18.17.16.15.14=\frac{18!}{13!}$

ii). 2.4.6.8.10.12

Solution: we have 2.4.6.8.10.12

$2.4.6.8.10.12=2.2.2.2.3.2.4.2.5.2.6$

$2.4.6.8.10.12=2^6(1.2.3.4.5.6)$

$2.4.6.8.10.12=2^6 6!$

iii). $n(n^2-1)$

Solution: we have $n(n^2-1)$

$n(n^2-1)=n(n-1)(n+1)$

$n(n^2-1)=(n+1)n(n-1)$

$n(n^2-1)=\frac{(n+1)n(n-1)(n-2)!}{(n-2)!}$

$n(n^2-1)=\frac{(n+1)!}{(n-2)!}$

iv). $\frac{n(n+1)(n+2)}{3}$

Solution: we have $\frac{n(n+1)(n+2)}{3}$

$\frac{n(n+1)(n+2)}{3}=\frac{(n+2)(n+1)n}{3}.\frac{(n-1)!.2!}{(n-1)!.2!}$

$\frac{n(n+1)(n+2)}{3}=\frac{(n+2)!.2!}{(n-1)!.3!}$

Q3. Prove the following

i). $\frac{1}{6!}+\frac{2}{7!}+\frac{3}{8!}=\frac{75}{8!}$

Solution: take LHS $\frac{1}{6!}+\frac{2}{7!}+\frac{3}{8!}$

$\frac{1}{6!}+\frac{2}{7!}+\frac{3}{8!}=\frac{1}{6!}.\frac{7.8}{7.8}+\frac{2}{7!}.\frac{8}{8}+\frac{3}{8!}$

$\frac{1}{6!}+\frac{2}{7!}+\frac{3}{8!}=\frac{56+16+3}{8!}$

$\frac{1}{6!}+\frac{2}{7!}+\frac{3}{8!}=\frac{75}{8!}$

=RHS Hence proved.

ii). $\frac{(n+5)!}{(n+3)!}=n^2+9n+20$

Sol: Take LHS $\frac{(n+5)!}{(n+3)!}=\frac{(n+5)(n+4)(n+3)!}{(n+3)!}$

$\frac{(n+5)!}{(n+3)!}=(n+5)(n+4)$

$\frac{(n+5)!}{(n+3)!}=n^2+4n+5n+20$

$\frac{(n+5)!}{(n+3)!}=n^2+9n+20$

= RHS Hence proved

Q4. Find the value of n, when

i). $\dfrac{n(n!)}{(n-5)!}=\dfrac{12(n!)}{(n-4)!}$

Solution: we have $\dfrac{n(n!)}{(n-5)!}=\dfrac{12(n!)}{(n-4)!}$

$$\frac{n}{(n-5)!}=\frac{12}{(n-4)(n-5)!}$$

$$\frac{n}{1}=\frac{12}{(n-4)}$$

$$n(n-4)=12$$

$$n^2-4n-12=0$$

$$n^2-6n+2n-12=0$$

$$n(n-6)+2(n-6)=0$$

$$(n+2)(n-6)=0$$

$$\therefore n+2=0, \qquad n-6=0$$

$$n=-2 \qquad n=6$$

n should be positive so n = 6

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ii). $\dfrac{n!}{(n-4)!}:\dfrac{(n-1)!}{(n-4)!}=9:1$

Solution: we have $\dfrac{n!}{(n-4)!}:\dfrac{(n-1)!}{(n-4)!}=9:1$

$$\frac{n!}{(n-4)!}\div\frac{(n-1)!}{(n-4)!}=9\div 1$$

$$\frac{n!}{(n-4)!}\times\frac{(n-4)!}{(n-1)!}=9\times 1$$

$$\frac{n!}{(n-1)!}=9$$

$$\frac{n(n-1)!}{(n-1)!}=9$$

$$n=9$$

Q5. Show that $\dfrac{(2n)!}{n!}=2^n\left(1.3.5...(2n-1)\right)$

Solution: we take LHS $\dfrac{(2n)!}{n!}$

$$\frac{(2n)!}{n!}=\frac{\dfrac{2n(2n-2)(2n-4)(2n-6)...6.4.2}{(2n-1)(2n-3)(2n-5)...5.3.1}}{n(n-1)(n-2)...3.2.1}$$

$$\frac{(2n)!}{n!}=\frac{\dfrac{2n\times 2(n-1)\times 2(n-2)\times 2(n-3)...2\times 3.2\times 2.2\times 1}{(2n-1)(2n-3)(2n-5)...5.3.1}}{n(n-1)(n-2)...3.2.1}$$

$$\frac{(2n)!}{n!}=\frac{\dfrac{2^n n(n-1)(n-2)...3.2.1}{(2n-1)(2n-3)....5.3.1}}{n(n-1)(n-2)...3.2.1}$$

$$\frac{(2n)!}{n!}=2^n\left(1.3.5...(2n-5)(2n-3)(2n-1)\right)$$

Exercise 6.2

Q1. How many ways different batting orders are possible for a cricket team consisting of 11 players?

Sol: maximum Players in a cricket team $n=11$

All player allowed for batting $r=11$

Ways of batting = ${}^{11}P_{11}$

$${}^{11}P_{11}=\frac{11!}{(11-11)!}=\frac{11!}{0!}$$

$${}^{11}P_{11}=39,916,800$$

Q2. How many three digit numbers can be formed the digits 1,2,3,4 and 5 if repetitions

i). are allowed

Solution: when repetitions is allowed

digit are 1,2,3,4 and 5

and repetitions is allowed

Possible digits at Unit place = 5

Possible digits at Ten place = 5

Possible digits at Hundrad place = 5

Hundred	Ten	Unit
5	5	5

Total numbers of three digited number =

$5\times 5\times 5=125$

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ii). are not allowed

Solution: when repetitions is not allowed

we have to from three digited number

and the digits are 1,2,3,4 and 5 so

Possible digits at unit place = 5

Remaining Possible digits at Ten place = 5

Remaining Possible digits at Hundred place = 5

Hundred	Ten	unit
3	4	5

Then three digited number when repetitions are not allowed = $3\times 4\times 5=60$

Q3. A man has 4 coats, six shirts and three trousers. In how many ways can he dress himself with coat, shirt and trouser?

Solution: we have number of Coats = 4

Number of Shirts = 6

Number of Trousers = 3

The number of ways can he dress himself

$3\times 4\times 6=72$

Q4. In how many ways can four French books, two English books and three German books be arranged on a shelf so that all books in same language are together?

Solution: we have Ways for French books F= 4! = 24

Ways for English books E = 2! = 2

Ways for German books G = 3! = 6

Books arrangement in a Shelf

FEG FGE EGF

GEF GFE EFG

i.e. shelf arrangement $3!=6$

Arrangement of the books when same language book are together in the shelf $=3!(4!\times 2!\times 3!)$

$$=6(24\times 2\times 6)$$

$$=6(288)$$

$$=1728$$

Q5. How many different arrangements can be formed of the world “equation” if all the vowels are be kept together?

Solution: Arrangement for the vowels v = 5! = 120

Arrangement for the consonant c = 3! = 6

Arrangement can be possible

VCCC CVCC CCVC CCCV

Arrangement for the word= 4 (120 x 6) = 2880

Q6. A combination lock has five wheels, each labeled with the ten digits from 0 to 9. How many five number opening combinations are possible, assuming no digit is repeated?

i). When repetition is allowed

Solution: we have Numbers of digits = 10

When repetition is not allowed

Wheel at the unit place have possibilities = 10
Wheel at the ten place have 1 possibility less = 9
And so on

Ten Thousand	Thousand	Hundred	Ten	unit
6	7	8	9	10

So the five digited number= $10\times9\times8\times7\times6=30240$

ii). Assuming digits can be repeated.
Sol: Wheel at the unit place have possibilities = 10
Wheel at ten place have same possibilities = 10
And so on

Ten Thousand	Thousand	Hundred	Ten	Unit
10	10	10	10	10

So five digited number= $10\times10\times10\times10\times10=100000$

Q7 How many signals can be given by six flags of different colors when any number of them are use at a time?
Solution: we have total number of flags = 6

For one color flag= ${}^6P_1=6$ $\quad\because {}^nP_r=\dfrac{n!}{(n-r)!}$

For two color flags= ${}^6P_2=30$

For three color flags= ${}^6P_3=120$

For four color flags= ${}^6P_4=360$

For five color flags= ${}^6P_5=720$

For six color flags= ${}^6P_6=720$

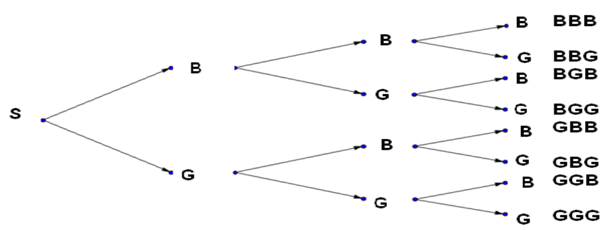
Any number of flags at a time=
${}^6P_1+{}^6P_2+{}^6P_3+{}^6P_4+{}^6P_5+{}^6P_6$
= 6 + 30 + 120 + 360 + 720 + 720 = 1956

Q8. A couple is planning to have three children. How many boy-girl combination are possible? Solve using
i) the multiplication principle

Sol: First baby	Second baby	Third baby
Boy or Girl	Boy or Girl	Boy or Girl
2	2	2

Combination for boy-girl = $2\times2\times2=8$

ii). A tree diagram



Exercise 6.3

Q1. Evaluate

i). ${}^6P_6$

Solution: we have ${}^6P_6=\dfrac{6!}{(6-6)!}$

${}^6P_6=\dfrac{6!}{0!}$

${}^6P_6=6!$

${}^6P_6=720$

ii). ${}^{20}P_2$

Solution: we have ${}^{20}P_2=\dfrac{20!}{(20-2)!}$

${}^{20}P_2=\dfrac{20!}{18!}$

${}^{20}P_2=\dfrac{20\times19\times18!}{18!}=20\times19$

${}^{20}P_2=380$

iii). ${}^7P_0$

Solution: we have ${}^7P_0$

${}^7P_0=\dfrac{7!}{(7-0)!}$

${}^7P_0=\dfrac{7!}{7!}$

${}^7P_0=1$

iv). ${}^5P_7$

Solution: we have $n<r$ which is not possible

Q2. Solve for n

i). ${}^nP_5=56\left({}^nP_3\right)$

Solution: we have ${}^nP_5=56\left({}^nP_3\right)$

$\dfrac{n!}{(n-5)!}=56\dfrac{n!}{(n-3)!}$

$\dfrac{1}{(n-5)!}=56\dfrac{1}{(n-3)(n-4)(n-5)!}$

$\dfrac{1}{1}=56\dfrac{1}{(n-3)(n-4)}$

$(n-3)(n-4)=56$

$(n-3)(n-4)=8\times7$

$\Rightarrow n-3=8$

$n=8+3$

$n=11$

ii). ${}^nP_5=9\left({}^{n-1}P_4\right)$

Solution: we have ${}^nP_5=9\left({}^{n-1}P_4\right)$

$\dfrac{n!}{(n-5)!}=\dfrac{9(n-1)!}{(n-1-4)!}$

$\dfrac{n(n-1)!}{(n-5)!}=\dfrac{9(n-1)!}{(n-5)!}$

$\Rightarrow n=9$

iii). ${}^{n^2}P_2=600$

Solution: we have ${}^{n^2}P_2=600$

$\dfrac{(n^2)!}{(n^2-2)!}=600$

$\dfrac{n^2(n^2-1)(n^2-2)!}{(n^2-2)!}=600$

$n^2(n^2-1)=25\times24$

$n^2(n^2-1)=25\times(25-1)$

$\Rightarrow n^2=25$

$n=5$

Q3 Prove following by Fundamental principle of counting

i). ${}^nP_r=n\left({}^{n-1}P_{r-1}\right)$

Sol; Taking RHS $n\left({}^{n-1}P_{r-1}\right)=n.\dfrac{(n-1)!}{(n-1-r+1)!}$

$=\dfrac{n(n-1)!}{(n-r)!}=\dfrac{n!}{(n-r)!}={}^nP_r$

= LHS
Hence proved

ii). ${}^nP_r={}^{n-1}P_r+r\left({}^{n-1}P_{r-1}\right)$

Solution; Taking RHS ${}^{n-1}P_r + r\left({}^{n-1}P_{r-1}\right)$

$$=\frac{(n-1)!}{(n-1-r)!}+r.\frac{(n-1)!}{(n-1-r+1)!}$$
$$=\frac{(n-1)!}{(n-r-1)!}+\frac{r.(n-1)!}{(n-r)!}$$
$$=\frac{(n-1)!}{(n-r-1)!}+\frac{r.(n-1)!}{(n-r)(n-r-1)!}$$
$$=\frac{(n-1)!}{(n-r-1)!}\left(1+\frac{r}{(n-r)}\right)$$
$$=\frac{(n-1)!}{(n-r-1)!}\left(\frac{n-r+r}{(n-r)}\right)$$
$$=\frac{n(n-1)!}{(n-r)(n-r-1)!}=\frac{n!}{(n-r)!}$$
$= {}^{n}P_r$ = LHS

Hence proved

Q4. In how many ways can a police department arrange eight suspects in a line up?

Solution; Total number of suspects $n=8$

Police department have to arrange $r=8$

there are eight suspects $= {}^{8}P_8$

$$=\frac{8!}{(8-8)!}=8!$$
$$=40320$$

Q5. How many different signals, each consisting of three flags hung one above the other, can be made from seven different flags?

Solution; Total number of flags n= 7

Signal consisting of flags r= 3

So ${}^{7}P_3=\frac{7!}{(7-3)!}$

$${}^{7}P_3=\frac{7!}{4!}$$
$${}^{7}P_3=\frac{7\times6\times5\times4!}{4!}$$
$${}^{7}P_3=7\times6\times5$$
$${}^{7}P_3=210$$

Q6. In how many ways can five students be seated in a row of eight seats if a certain two students?

i). insist on sitting next to each other

Solution; Total number of students n = 8

When two students insist next to each other

Then number of students to arrange r = 4

Let two students are A and B

Possible way to sit two students = 2

i.e. AB or BA

ways of sitting of 4 students $2\times{}^{8}P_4=\frac{2\times8!}{(8-4)!}$

$$2\times{}^{8}P_4=\frac{2\times8\times7\times6\times5\times4!}{4!}$$
$$2\times{}^{8}P=2\times8\times7\times6\times5$$
$$2\times{}^{8}P=3360$$

ii) refuse to sit next to each other

Solution; Total number of students n = 8

When two students refuse next to each other r = 4

Then ways for setting = ${}^{8}P_5-{}^{8}P_4$

$${}^{8}P_5-{}^{8}P_4=\frac{8!}{(8-5)!}-\frac{8!}{(8-4)!}$$
$${}^{8}P_5-{}^{8}P_4=\frac{8!}{3!}-\frac{8!}{4\times3!}$$
$${}^{8}P_5-{}^{8}P_4=\frac{8!}{3!}\left(\frac{1}{1}-\frac{1}{4}\right)$$
$${}^{8}P_5-{}^{8}P_4=\frac{8!}{3!}\left(\frac{4-1}{4}\right)$$
$${}^{8}P_5-{}^{8}P_4=\frac{8\times7\times6\times5\times4\times3!}{3!}.\frac{3}{4}$$
$${}^{8}P_5-{}^{8}P_4=8\times7\times6\times5\times3$$
$${}^{8}P_5-{}^{8}P_4=5040$$

Q7. How many numbers each lying between 10 & 1000 can be formed with digits 2,3,4,0,8,9 using only once?

Solution; Given digits are 2,3,4,0,8,9

Total number of digits = 6

Case 1 Number less than 1000

Possible digits at hundred place = 6

Possible digits at tan place = 5

Possible digits at unit place = 4

Than Possible numbers = 6 x 5 x 4 = 120

Case 2 Number greater than 10

Possible digit at unit place is 0 =1

Remaining Possible digits at ten place =5

Than possible numbers = 1 x 5 = 5

Case 3 Number less than 100

Possible digit at hundred place is 0

Possible digits at tan place = 5

Possible digits at unit place = 4

Than Possible numbers = 5 x 4 = 20

Possible number between 10 to 1000 are

Case 1 + case 2 + case 3 = 120 + 20 + 5

= 145

Q8. How many different words can be formed from the letters of the following words if the letters are taken all at a time?

i). BOOKWORM

Sol; Total number of words n = 8

O repeated $m_1=3$

Thus the required number of permutations

$$\binom{n}{m_1}=\binom{8}{3}=\frac{8!}{3!}=\frac{40320}{6}$$
$$\binom{n}{m_1}=6720$$

ii). BOOKKEEPER

Solution; Total number of words n = 10

O repeated $m_1=2$

K repeated $m_2=2$

E repeated $m_3=3$

Thus the required number of permutations

$$\binom{n}{m_1,m_2,m_3}=\binom{10}{2,2,3}=\frac{10!}{2!\times2!\times3!}$$
$$\binom{n}{m_1,m_2,m_3}=\frac{3628800}{24}=151200$$

iii). ABBOTTABAD

Solution; Total number of words n = 10

B repeated $m_1 = 3$ T repeated $m_2 = 2$

A repeated $m_3 = 3$

Thus the required number of permutations

$$\binom{n}{m_1, m_2, m_3} = \binom{10}{3,2,3} = \frac{10!}{3!\times 2!\times 3!}$$

$$\binom{n}{m_1, m_2, m_3} = \frac{3628800}{72} = 50400$$

iv). LETTER

Solution; Total number of words n = 6

E repeated $m_1 = 2$

T repeated $m_2 = 2$

Thus the required number of permutations

$$\binom{n}{m_1, m_2} = \binom{6}{2,2} = \frac{6!}{2!\times 2!}$$

$$\binom{n}{m_1, m_2} = \frac{720}{4} = 180$$

Q9. In how many distinct ways can $x^4y^3z^5$ be expressed without exponents?

Sol; Total number of variables without exponents n = 12

x repeated $m_1 = 4$ y repeated $m_2 = 3$

z repeated $m_3 = 5$

Thus the required number of permutations

$$\binom{n}{m_1, m_2, m_3} = \binom{12}{4,3,5} = \frac{12!}{4!\times 3!\times 5!} = \frac{479001600}{17280}$$

$$\binom{n}{m_1, m_2, m_3} = 27720$$

Q10. How many different ten-digit numerals can be formed from the digits: 3,3,3,3,1,1,1,7,7 and 5?

Sol; Total number of variables without exponents n = 10

3 repeated $m_1 = 4$

1 repeated $m_2 = 3$

7 repeated $m_3 = 2$

Thus the required number of permutations

$$\binom{n}{m_1, m_2, m_3} = \binom{10}{4,3,2} = \frac{10!}{4!\times 3!\times 2!}$$

$$\binom{n}{m_1, m_2, m_3} = \frac{3628800}{288} = 12600$$

Q11. In how many different ways can be six children seated at a round table if it certain two children

i). refuse to sit next to each other?

Solution; For table possible arrangement (n – 1)!

Here number of possible students = 6

Possible arrangement $= (6-1)! = 5!$

$= 5\times 4\times 3\times 2\times 1 = 120$

ii). Insist on sitting next to each other?

Sol; when two students insist next to each other

Or there should be two possibilities AB or BA

Then number of students will be = 5

Possible arrangement $= 2(5-1)! = 2(4)!$

$= 2\times 4!$

$= 2\times 4\times 3\times 2\times 1$

$= 48$

Q12. If five distinct keys are placed on a key ring. How many different orders are possible?

Solution; For key ring $\frac{(n-1)!}{2}$

Here numbers of keys $n = 5$

So possible order $= \frac{(5-1)!}{2} = \frac{4!}{2}$

$= \frac{4\times 3\times 2\times 1}{2} = 12$

Exercise 6.4

Q1. Solve the following for n

i). ${}^nC_2 = 36$

Solution; we have ${}^nC_2 = 36$

$$\frac{n!}{(n-2)!.2!} = 36$$

$$\frac{n(n-1)(n-2)!}{(n-2)!.} = 36\times 2 = 72$$

$$n(n-1) = 9\times 8$$

$$\Rightarrow n = 9$$

ii). ${}^{n+1}C_4 = 6.{}^{n-1}C_2$

Solution; we have ${}^{n+1}C_4 = 6.{}^{n-1}C_2$

$$\frac{(n+1)!}{(n+1-4)!.4!} = 6.\frac{(n-1)!}{(n-1-2)!.2!}$$

$$\frac{(n+1)n(n-1)!}{(n-3)!.4\times 3\times 2!} = 6.\frac{(n-1)!}{(n-3)!.2!}$$

$$\frac{(n+1)n}{4\times 3} = 6$$

$$n(n+1) = 6\times 12 = 72$$

$$n(n+1) = 8\times 9$$

$$\Rightarrow n = 8$$

iii). ${}^{n^2}C_2 = 30\ {}^nC_3$

Solution; we have ${}^{n^2}C_2 = 30\ {}^nC_3$

$$\frac{(n^2)!}{(n^2-2)!.2!} = 30.\frac{n!}{(n-3)!.3!}$$

$$\frac{(n^2)(n^2-1)(n^2-2)!}{(n^2-2)!.2!} = 30.\frac{n(n-1)(n-2)(n-3)!}{(n-3)!.3\times 2!}$$

$$(n^2)(n^2-1) = 10n(n-1)(n-2)$$

$$n.n(n-1)(n+1) = 10n(n-1)(n-2)$$

$$n(n+1) = 10(n-2)$$

$$n^2 + n = 10n - 20$$

$$n^2 + n - 10n + 20 = 0$$

$$n^2 - 9n + 20 = 0$$

$$n^2 - 5n - 4n + 20 = 0$$

$$n(n-5) - 4(n-5) = 0$$

$$(n-4)(n-5) = 0$$

$$\therefore n-4 = 0 \quad or \quad n-5 = 0$$

$$n = 4 \quad or \quad n = 5$$

Q2. Find n and r if ${}^nP_r = 840$ and ${}^nC_r = 35$

Solution; w have ${}^nP_r = 840$ and ${}^nC_r = 35$

$$\frac{n!}{(n-r)!.r!} = 35$$
$$\frac{n!}{(n-r)!} = 35r!$$
$${}^nP_r = 35r!$$
$$840 = 35r!$$
$$\Rightarrow r! = \frac{840}{35}$$
$$r! = 24$$
$$r! = 4!$$
$$\Rightarrow r = 4$$

When r = 4

$${}^nP_4 = 840$$
$$\frac{n!}{(n-4)!} = 840$$
$$n(n-1)(n-2)(n-3) = 7\times6\times5\times4$$
$$\Rightarrow n = 7$$

Q3. Find n when ${}^{2n}C_3 : {}^nC_2 = 36:3$

Solution; we have ${}^{2n}C_3 : {}^nC_2 = 36:3$

$${}^{2n}C_3 : {}^nC_2 = 36:3$$
$$\frac{(2n)!}{(2n-3)!.3!} \div \frac{(n)!}{(n-2)!.2!} = 36 \div 3$$
$$\frac{(2n)(2n-1)(2n-2)}{3} \times \frac{1}{(n)(n-1)} = 12$$
$$\frac{(2)(2n-1)\times 2}{3} = 12$$
$$2n-1 = \frac{12\times3}{4}$$
$$2n-1 = 9$$
$$2n = 10$$
$$n = 5$$

Q4. Prove that

i). ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$

Solution; LHS ${}^{n-1}C_r + {}^{n-1}C_{r-1}$

$$= \frac{(n-1)!}{(n-1-r)!r!} + \frac{(n-1)!}{(n-1-r+1)!(r-1)!}$$
$$= \frac{(n-1)!}{(n-r-1)!r!} + \frac{(n-1)!}{(n-r)!(r-1)!}$$
$$= \frac{(n-1)!}{(n-r-1)!r.(r-1)!} + \frac{(n-1)!}{(n-r)(n-r-1)!(r-1)!}$$
$$= \frac{(n-1)!}{(n-r-1)!(r-1)!}\left(\frac{1}{r} + \frac{1}{(n-r)}\right)$$
$$= \frac{(n-1)!}{(n-r-1)!(r-1)!}\left(\frac{n-r+r}{r(n-r)}\right)$$
$$= \frac{n(n-1)!}{(n-r)(n-r-1)!r.(r-1)!}$$
$$= \frac{n!}{(n-r)!r!} = {}^nC_r$$

=RHS

ii). $r.{}^nC_r = n.{}^{n-1}C_{r-1}$

Solution; Take RHS $n.{}^{n-1}C_{r-1}$

$$= n.\frac{(n-1)!}{(n-1-r+1)!(r-1)!}$$
$$= \frac{r}{r}.\frac{n(n-1)!}{(n-r)!(r-1)!}$$
$$= r.\frac{n!}{(n-r)!r.(r-1)!}$$
$$= r.\frac{n!}{(n-r)!r!} = r.{}^nC_r$$

=LHS

Q5. How many lines are determined by eight points if none of the three points are collinear? How many triangles are determined?

Solution; Maximum number of points n = 8

Line can pass through 2 points r = 2

So number of possible lines ${}^8C_2 = \frac{8!}{(8-2)!.2!}$

$$= \frac{8\times7\times6!}{6!.2}$$
$$= 4\times7 = 28$$

Triangle can be made through three non-collinear points, r= 3

So number of triangles = ${}^8C_3 = \frac{8!}{(8-3)!.3!}$

$$= \frac{8\times7\times6\times5!}{5!.3\times2}$$
$$= 8\times7 = 56$$

Q6. Three non-collinear points determine a circle. How many circles are determined by 5 such points?

Solution; Maximum number of points n = 5

Circle can pass through 3 points r = 3

So number of possible lines ${}^5C_3 = \frac{5!}{(5-3)!.3!}$

$$= \frac{5\times4\times3!}{2!.3!} = 10$$

Q7 A box contains 6 red balls and 4 green balls. In how many ways can 4 balls be chosen such that exactly 2 are green?

Solution; Red balls = 6 Green balls = 4

4 balls be chosen such that exactly 2 are green so other 2 will be red balls ${}^6C_2 \times {}^4C_2$

$$= \frac{6!}{(6-2)!.2!} \times \frac{4!}{(4-2)!.2!}$$
$$= \frac{6\times5\times4!}{4!.2} \times \frac{4\times3\times2!}{2!.2}$$
$$= 3\times5.2\times3 = 15\times6 = 90$$

Q8. From 12 books in how many ways can a selection of 5 be made

i). When one specified book is always included?

Solution; One specified book is included so n = 11

And number of book selected = 4

Possible ways of selection ${}^{11}C_4$

$$= \frac{11!}{(11-4)!.4!}$$
$$= \frac{11\times10\times9\times8\times7!}{7!4\times3\times2\times1}$$
$$= 11\times10\times3 = 330$$

ii). When one specified book is always exclude?

Solution; Total ways of selection – possible ways of selection of one book ${}^{12}C_5 - {}^{11}C_4$

$$= \frac{12!}{(12-5)!.5!} - \frac{11!}{(11-4)!.4!}$$
$$= \frac{12\times11\times10\times9\times8\times7!}{7!5\times4\times3\times2\times1} - \frac{11\times10\times9\times8\times7!}{7!4\times3\times2\times1}$$
$$= 11\times9\times8 - 11\times10\times3$$
$$= 792 - 330$$
$$= 462$$

Q9. How many diagonals can be drawn in a plane figure of 8 sides?

Solution; Number of vertices/points/corner n = 8

Line segment can pass through 2 points r= 2

Number of diagonals = possible numbers of lines – number of lines having shape/boundaries

Number of diagonals = ${}^{8}C_2 - 8$

$${}^{8}C_2 - 8 = \frac{8!}{(8-2)!.2!} - 8$$

$${}^{8}C_2 - 8 = \frac{8\times7\times6!}{6!.2} - 8$$

$$= 4\times7-8$$

$$= 28-8$$

$$= 20$$

Q10. A committee of seven persons is to be chosen from 10 men and 8 women. How many of these will have

i). exactly four men

Solution; Maximum number of men= 10

Maximum number of women= 8

Committee should have 7 members

For exactly four men then remaining for women

$${}^{10}C_4 \times {}^{8}C_3$$

$$= \frac{10!}{(10-4)!.4!} \times \frac{8!}{(8-3)!.3!}$$

$$= \frac{10\times9\times8\times7\times6!}{6!.4\times3\times2} \times \frac{8\times7\times6\times5!}{5!.3\times2}$$

$$= \frac{10\times9\times8\times7}{4\times3\times2} \times \frac{8\times7\times6}{3\times2}$$

$$= 10\times3\times7.8\times7$$

$$= 210\times56$$

$$= 11760$$

---

ii). At the most four men

Sol ${}^{10}C_0 \times {}^{8}C_7 + {}^{10}C_1 \times {}^{8}C_6 + {}^{10}C_2 \times {}^{8}C_5 + {}^{10}C_3 \times {}^{8}C_4 + {}^{10}C_4 \times {}^{8}C_3$

$$= 1\times8 + 10\times28 + 45\times56 + 120\times70 + 210\times56$$

$$= 8 + 280 + 2520 + 8400 + 11760$$

$$= 22968$$

Book answer = 32208

---

iii). At least four men.

Sol ${}^{10}C_4 \times {}^{8}C_3 + {}^{10}C_5 \times {}^{8}C_2 + {}^{10}C_6 \times {}^{8}C_1 + {}^{10}C_7 \times {}^{8}C_0$

$$= 210\times56 + 252\times28 + 210\times8 + 120\times1$$

$$= 11760 + 7056 + 1680 + 120$$

$$= 20616$$

Exercise 6.5

Q1: Let S={1,2,3,4,5,6} be the sample space of rolling a die. What is the probability of

a). rolling a 5?

Solution; S={1,2,3,4,5,6} so n(S)=6

E={5} so n(E)=1

Than the probability of rolling a 5

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

---

b). rolling a number less then one?

Solution; S={1,2,3,4,5,6} so n(S)=6

E={} so n(E)=0

Than the probability of rolling a 5 rolling a number less then one

$$P(E) = \frac{n(E)}{n(S)} = \frac{0}{6} = 0$$

---

c). rolling a number greater then 0?

Solution; S={1,2,3,4,5,6} so n(S)=6

E={1,2,3,4,5,6} so n(E)=6

Than probability of rolling a number greater then 0

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{6} = 1$$

---

d). rolling a multiple of 3?

Solution; S={1,2,3,4,5,6} so n(S)=6

E={3,6} so n(E)=2

Than probability of rolling a multiple of 3

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

---

e). rolling a number greater than or equal to 4?

Solution; S={1,2,3,4,5,6} so n(S)=6

E={4,5,6} so n(E)=3

Than probability of rolling a number greater then 4

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Q2. Give the sample space of rolling a pair of dice.

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$$
$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$$
$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$$
$$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$n(S) = 36$

a). what is the probability of

i). rolling a total of 7?

Sol; rolling a total of 7

$$E = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$$

$n(E) = 6$

Probability of rolling a total of 7

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

---

ii). Rolling a total of 11?

Solution; $E = \{(5,6),(6,5)\}$ $n(E) = 2$

Probability of rolling a total of 11

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

---

iii). Rolling a total of greater than or equal to 12?

Solution; $E = \{(6,6)\}$ $n(E) = 1$

Probability of rolling a total of 12

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{36}$$

---

b). which is more likely

i). rolling a total of 7 or a total of 9? Why?

Sol; For the total of 7 For the total of 9

$$E_1 = \begin{Bmatrix}(1,6),(2,5),(3,4),\\(4,3),(5,2),(6,1)\end{Bmatrix} \quad E_2 = \begin{Bmatrix}(4,5),(5,4),(6,3),\\(3,6)\end{Bmatrix}$$

$n(E_1) = 6$ $n(E_2) = 4$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6} \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

By comparing both the probabilities

We get $\frac{1}{6} > \frac{1}{9}$ or

$P(\text{total of }7) > P(\text{total of }9)$

ii). Rolling a total of 11 or a total of 3? Why?

Sol: For the total of 11

$E_1 = \{(5,6),(6,5)\}$

$n(E_1) = 2$

$P(E_1) = \frac{n(E_1)}{n(S)}$

$P(E_1) = \frac{2}{36}$

$P(E_1) = \frac{1}{18}$

For the total of 3

$E_2 = \{(1,2),(2,1)\}$

$n(E_2) = 2$

$P(E_2) = \frac{n(E_2)}{n(S)}$

$P(E_2) = \frac{2}{36}$

$P(E_2) = \frac{1}{18}$

By comparing both the probabilities

We get $\frac{1}{18} = \frac{1}{18}$ or

$P(\text{total of }11) = P(\text{total of }3)$

Q3. A true or false contains eight questions. If a student guesses the answer for each question, find the probability:

There are 8 questions having two options

Then $n(S) = 2^8 = 256$

a). 8 answers are correct

Solution; $n(E) = {}^8C_8 = 1$

$P(E) = \frac{n(E)}{n(S)} = \frac{1}{256}$

b). 7 answers are correct and 1 is incorrect

Solution; $n(E) = {}^8C_7 = 8$

$P(E) = \frac{n(E)}{n(S)} = \frac{8}{256} = \frac{1}{32}$

c). 6 answers are correct and 2 are incorrect

Solution; $n(E) = {}^8C_6 = 28$

$P(E) = \frac{n(E)}{n(S)} = \frac{28}{256} = \frac{7}{64}$

d). at least 6 answers are correct

Solution; $n(E) = {}^8C_6 + {}^8C_7 + {}^8C_8 = 28+8+1 = 37$

$P(E) = \frac{n(E)}{n(S)} = \frac{37}{256}$

Q4. A golf ball is selected at random from a container. If the container has 9 white, 8 green and 3 orange balls, find the probability that golf ball is

Sol; Total numbers of balls ={9 white +8 green+3 orange}

$S = \{20 \text{ balls}\}$ $n(S) = 20$

a). white

Solution; $E = \{9 \text{ white balls}\}$ $n(E) = 9$

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{9}{20}$

b). Green

Solution; $E = \{8 \text{ green balls}\}$

$n(E) = 8$

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{8}{20}$

$P(E) = \frac{2}{5}$

c). white or green

Solution; $E = \{9 \text{ white balls} + 8 \text{ green balls}\}$

$n(E) = 17$

$P(E) = \frac{n(E)}{n(S)} = \frac{17}{20}$

d). not white

Solution; $E = \{9 \text{ white balls}\}$ $n(E) = 9$

$P(E) = \frac{n(E)}{n(S)} = \frac{9}{20}$

$P(E') = 1 - P(E)$

$P(E') = 1 - \frac{9}{20}$

$P(E') = \frac{11}{20}$

Q5 A committee of 5 is to be selected at random from 6 men and 4 women. Find the probability that the committee will consist of

i). 3 men and 2 women

Solution; Total number of men or women= 6+4=10

Committee has member =5

Then number of sample space $n(S) = {}^{10}C_5 = 252$

$P(3 \text{ men and } 2 \text{ women}) = \frac{{}^6C_3 \times {}^4C_2}{{}^{10}C_5}$

$= \frac{20\times 6}{252} = \frac{120}{252} = \frac{10}{21}$

ii). 2 men and 3 women.

Sol; Total number of men or women= 6+4=10

Committee has member =5

Then number of sample space $n(S) = {}^{10}C_5 = 252$

$P(2 \text{ men and } 3 \text{ women}) = \frac{{}^6C_2 \times {}^4C_3}{{}^{10}C_5}$

$= \frac{15\times 4}{252}$

$= \frac{60}{252} = \frac{5}{21}$

Exercise 6.6

Q1. Suppose events A and B are such that $P(A) = \frac{2}{5}$, $P(B) = \frac{2}{5}$ and $P(A\cup B) = \frac{1}{2}$. Find $P(A\cap B)$

Sol; $P(A\cap B) = P(A) + P(B) - P(A\cup B)$

$P(A\cap B) = \frac{2}{5} + \frac{2}{5} - \frac{1}{2} = \frac{4+4-5}{10} = \frac{3}{10}$

Q2. If $P(A) = \frac{1}{3}$, $P(A\cup B) = \frac{1}{2}$ & $P(A\cap B) = \frac{1}{4}$. Find $P(B)$

Solution; $P(A\cap B) = P(A) + P(B) - P(A\cup B)$

$\frac{1}{4} = \frac{1}{3} + P(B) - \frac{1}{2}$

$\frac{1}{4} - \frac{1}{3} + \frac{1}{2} = P(B)$

$P(B) = \frac{3-4+6}{12} = \frac{5}{12}$

Q3. A sample space $S = P(A\cup B)$, $P(A) = 0.75$ and $P(B) = 0.65$. Find $P(A\cap B)$

Solution; Since we know that $P(\text{Sample space}) = 1$

So $P(A\cup B) = 1$

$P(A\cap B)=P(A)+P(B)-P(A\cup B)$
$P(A\cap B)=0.75+0.65-1$
$P(A\cap B)=0.40$

Q4. A bag contains 30 tickets numbered from 1 to 30. One ticket is selected at random. Find the probability that its number is either odd or the square of an integer?

Sol; sample space $S=\{1,2,3,4,...,30\}$ so $n(S)=30$

Event for odd integer
$A=\{1,3,5,7,9,11,13,15,17,19,21,23,25,27,29\}$ so
$n(A)=15$

Event for square of an integer $B=\{1,4,9,16,25\}$ so
$n(B)=5$

Event for square and odd integer $A\cap B=\{1,9,25\}$
so $n(A\cap B)=3$

Then $P(A)=\frac{15}{30}$, $P(B)=\frac{5}{30}$ and $P(A\cap B)=\frac{3}{30}$

we will find probability for odd or square of an integer
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$P(A\cup B)=\frac{15}{30}+\frac{5}{30}-\frac{3}{30}$
$=\frac{15+5-3}{30}=\frac{17}{30}$

Q5.Given $P(A)=0.5$ and $P(A\cup B)=0.6$. Find $P(B)$ if A and B are mutually exclusive.

Solution; Since any two events are mutually exclusive then $P(A\cap B)=0$
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$0.6=0.5+P(B)-0$
$P(B)=0.6-0.5$
$P(B)=0.1$

Q6. Suppose that each of the letters of the word MATHEMATICS are written on scrapes of paper of the same size, dropped into a bag, and mixed thoroughly. Find the probability of drawing an M or an A

Solution; The sample space
$S=\{M,A,T,H,E,M,A,T,I,C,S\}$ so $n(S)=11$

Event for drawing an M $E_1=\{M,M\}$ so $n(E_1)=2$

Event for drawing an A $E_2=\{A,A\}$ so $n(E_2)=2$

and $E_1\cap E_2=\{\ \}$ so $n(E_1\cap E_2)=0$

So the probability for drawing an M or an A
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$P(A\cup B)=\frac{2}{11}+\frac{2}{11}-\frac{0}{11}$
$P(A\cup B)=\frac{2+2}{11}$
$P(A\cup B)=\frac{4}{11}$

Q7. In a class of 100 students 50 have taken Physic and 80 have taken Mathematics. One student is selected at random. Show that the probability that he has taken both subjects (Hint: $P(P\cup M)\le 1$)

Sol; since the maximum numbers of students are 100

Number of elements in sample space so $n(S)=100$

Number students who has taken Physic $n(P)=50$

No students who has taken Mathematics $n(M)=80$

Then $P(Phy)=\frac{50}{100}$ $P(Math)=\frac{80}{100}$
$P(P\cap M)=P(P)+P(M)-P(P\cup M)$
$P(P\cap M)\ge\frac{50}{100}+\frac{80}{100}-1$
$P(P\cap M)\ge\frac{50+80-100}{100}$
$P(P\cap M)\ge\frac{30}{100}$
$P(P\cap M)\ge 0.30$

Q8. A student figures that the probability of passing an algebra test is $\frac{8}{9}$. what is the probability of failing the test?

Solution; Probability of passing an algebra test
i.e. $P(E)=\frac{8}{9}$

Probability of failing an algebra test
$P(E')=1-P(E)$
$P(E')=1-\frac{8}{9}$
$P(E')=\frac{9-8}{9}=\frac{1}{9}$

Q9. In the two dice experiment, given that the first die shows 4, what is the probability that the second die shows a number greater than 4?

Sol; For two dice when first die show 4
Then sample space
$S=\{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$
$n(S)=6$

Event for second die shows greater then 4
$E=\{(4,5),(4,6)\}$
$n(E)=2$

Then the probability
$P(E)=\frac{n(E)}{n(5)}$
$P(E)=\frac{2}{6}=\frac{1}{3}$