Chapter 2

Matrices And Determinant

Exercise 2.1

Q.1 Write the following product of matrices as a single matrix.

i). $\begin{bmatrix} 2 & -1 \\ 3 & -4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Sol: $\begin{bmatrix} 2 & -1 \\ 3 & -4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$=\begin{bmatrix} 2\times1-1\times0 & 2\times0-1\times1 \\ 3\times1-4\times0 & 3\times0-4\times1 \\ -1\times1+3\times0 & -1\times0+3\times1 \end{bmatrix}$

$=\begin{bmatrix} 2+0 & 0-1 \\ 3+0 & 0-4 \\ -1+0 & 0+3 \end{bmatrix}$

$=\begin{bmatrix} 2 & -1 \\ 3 & -4 \\ -1 & 3 \end{bmatrix}$

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ii) $\begin{bmatrix} 1 & 2 & -3 \\ -2 & -1 & 4 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 & -3 \\ -2 & -1 & 4 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$=\begin{bmatrix} 1\times0+2\times0-3\times1 & 1\times0+2\times1-3\times0 & 1\times1+2\times0-3\times0 \\ -2\times0-1\times0+4\times1 & -2\times0-1\times1+4\times0 & -2\times1-1\times0+4\times0 \end{bmatrix}$

$=\begin{bmatrix} 0+0-3 & 0+2-0 & 1+0+0 \\ -0-0+4 & 0-1+0 & -2+0+0 \end{bmatrix}$

$=\begin{bmatrix} -3 & 2 & 1 \\ 4 & -1 & -2 \end{bmatrix}$

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iii). $\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}\left(\begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}+\begin{bmatrix} -1 & -5 \\ 3 & 5 \end{bmatrix}\right)$

Solution: $\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}\left(\begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}+\begin{bmatrix} -1 & -5 \\ 3 & 5 \end{bmatrix}\right)$

$=\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2-1 & 3-5 \\ -4+3 & 1+5 \end{bmatrix}$

$=\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 6 \end{bmatrix}$

$=\begin{bmatrix} 1\times1-2\times-1 & 1\times-2-2\times6 \\ -1\times1+1\times-1 & -1\times-2+1\times6 \end{bmatrix}$

$=\begin{bmatrix} 1+2 & -2-12 \\ -1-1 & 2+6 \end{bmatrix}$

$=\begin{bmatrix} 3 & -14 \\ -2 & 8 \end{bmatrix}$

Q2.If possible, find matrix A

i). $A\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} -2 & 5 \\ 8 & -7 \end{bmatrix}$

Sol: $A\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} -2 & 5 \\ 8 & -7 \end{bmatrix}$

Let A B = C, where

$B=\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}, C=\begin{bmatrix} -2 & 5 \\ 8 & -7 \end{bmatrix}$

Now $|B|=\begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix}=2-0=2$

$|B|=2$ non singular

So $adj\ B=\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ then

$B^{-1}=\dfrac{1}{|B|}adj\ B$

$B^{-1}=\dfrac{1}{2}\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$

$\therefore AB=C$ Post multiply by B^{-1}

$A\,B\,B^{-1}=C\,B^{-1}$

$A\,I=C\,B^{-1}$

$A=C\,B^{-1}$

$A=\dfrac{1}{2}\begin{bmatrix} -2 & 5 \\ 8 & -7 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$

$A=\dfrac{1}{2}\begin{bmatrix} -2\times1+5\times0 & -2\times3+5\times2 \\ 8\times1-7\times0 & 8\times3-7\times2 \end{bmatrix}$

$A=\dfrac{1}{2}\begin{bmatrix} -2+0 & -6+10 \\ 8+0 & 24-14 \end{bmatrix}$

$A=\dfrac{1}{2}\begin{bmatrix} -2 & 4 \\ 8 & 10 \end{bmatrix}$

$A=\begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$

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ii) $A\begin{bmatrix} 2 & -3 \\ 4 & -6 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution: $A\begin{bmatrix} 2 & -3 \\ 4 & -6 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Let AB=C, where

$B=\begin{bmatrix} 2 & -3 \\ 4 & -6 \end{bmatrix}, C=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|B|=\begin{vmatrix} 2 & -3 \\ 4 & -6 \end{vmatrix}$

$=-12+12$

$=0$

B is singular so its inverse does not exists

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iii) $A\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Sol: $A\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Let AB = C, Where

$B=\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}, C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Now $|B|=\begin{vmatrix}1 & -2\\ 3 & 4\end{vmatrix}=4+6=10$

& $adj\ B=\begin{bmatrix}4 & 2\\ -3 & 1\end{bmatrix}$

$B^{-1}=\frac{1}{10}\begin{bmatrix}4 & 2\\ -3 & 1\end{bmatrix}$

A = C B^{-1}

$A=\frac{1}{10}\begin{bmatrix}4 & 2\\ -3 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$A=\frac{1}{10}\begin{bmatrix}4-0 & 2+0\\ 0-3 & 0+1\end{bmatrix}$

$A=\frac{1}{10}\begin{bmatrix}4 & 2\\ -3 & 1\end{bmatrix}=\begin{bmatrix}\frac{4}{10} & \frac{2}{10}\\ \frac{-3}{10} & \frac{1}{10}\end{bmatrix}$

$A=\begin{bmatrix}\frac{2}{5} & \frac{1}{5}\\ \frac{-3}{10} & \frac{1}{10}\end{bmatrix}$

Q3. Solve each of following matrix equation for x and y.

i). $\begin{bmatrix}x-2y\\ -x+2y\end{bmatrix}=\begin{bmatrix}2\\ 3\end{bmatrix}$

Sol: Given $\begin{bmatrix}x-2y\\ -x+2y\end{bmatrix}=\begin{bmatrix}2\\ 3\end{bmatrix}$

by definition of equal matrices their corresponding elements must be equal

x-2y=2

-x+2y=3 by adding

0=5

Which is impossible question is wrong

Or the lines are parallel

If we take the question with small change

$\begin{bmatrix}x-2y\\ -x+y\end{bmatrix}=\begin{bmatrix}2\\ 3\end{bmatrix}$

$x-2y=2 .....(1)$

$\underline{-x+y=3 ....(2)}$

$-y=5 \Rightarrow y=-5$

Put in (2)

-x + (-5) = 3

-x – 5 = 3

-x = 3 + 5

-x = 8 $\Rightarrow$ x = -8

ii) $\begin{bmatrix}-3 & x & 2\\ 4y & 0 & 3\end{bmatrix}=\begin{bmatrix}x & -3 & 2\\ 0 & y & 3\end{bmatrix}$

Solution: $\begin{bmatrix}-3 & x & 2\\ 4y & 0 & 3\end{bmatrix}=\begin{bmatrix}x & -3 & 2\\ 0 & y & 3\end{bmatrix}$

by definition of equal matrices their corresponding elements must be equal

-3 = x, 4y=0 $\Rightarrow$ y=0

Hence the solution x = -3, y = 0

iii) $\begin{bmatrix}4x+2 & 3 & 1\\ 5 & 3y+2 & -1\end{bmatrix}=\begin{bmatrix}2x & 3 & 1\\ 5 & 2y+4 & -1\end{bmatrix}$

Sol: $\begin{bmatrix}4x+2 & 3 & 1\\ 5 & 3y+2 & -1\end{bmatrix}=\begin{bmatrix}2x & 3 & 1\\ 5 & 2y+4 & -1\end{bmatrix}$

by definition of equal matrices their corresponding elements must be equal

$4x+2=2x$

$4x-2x=-2$ $3=3$ $1=1$

$2x=-2$

$x=-1$

And

$3y+2=2y+4$

$5=5$ $3y-2y=4-2$ $-1=-1$

$y=2$

Q.4 $A=\begin{bmatrix}1 & a\\ -1 & b\end{bmatrix}$, & $A^2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find values of a & b.

Sol: $A=\begin{bmatrix}1 & a\\ -1 & b\end{bmatrix}$, $A^2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$....(1)

$A^2=A.A=\begin{bmatrix}1 & a\\ -1 & b\end{bmatrix}\begin{bmatrix}1 & a\\ -1 & b\end{bmatrix}$

$A^2=\begin{bmatrix}1-a & a+ab\\ -1-b & -a+b^2\end{bmatrix}$............................(2)

By comparing equations (1) and (2)

$\begin{bmatrix}1-a & a+ab\\ -1-b & -a+b^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

by definition of equal matrices their corresponding elements must be equal

$1-a=1$

$1-1=a$ $-1-b=0$

$a=0$ $b=-1$

Q5. If A = $\begin{bmatrix}1 & 2\\ 0 & -1\end{bmatrix}$ and B= $\begin{bmatrix}2 & 3\\ -1 & 1\end{bmatrix}$ Show that

i). $(A+B)^2 = A^2 + 2AB + B^2$

ii). $(A-B)^2 = A^2 - 2AB + B^2$

iii). $(A+B)(A-B) = A^2 - B^2$

Sol: A = $\begin{bmatrix}1 & 2\\ 0 & -1\end{bmatrix}$ and B= $\begin{bmatrix}2 & 3\\ -1 & 1\end{bmatrix}$

$A^2 = A.A = \begin{bmatrix}1 & 2\\ 0 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & -1\end{bmatrix}$

$=\begin{bmatrix}1+0 & 2-2\\ 0-0 & 0+1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$B^2 = B.B = \begin{bmatrix}2 & 3\\ -1 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\ -1 & 1\end{bmatrix}$

$=\begin{bmatrix}4-3 & 6+3\\ -2-1 & -3+1\end{bmatrix}=\begin{bmatrix}1 & 9\\ -3 & -2\end{bmatrix}$

AB= $\begin{bmatrix}1 & 2\\ 0 & -1\end{bmatrix}\begin{bmatrix}2 & 3\\ -1 & 1\end{bmatrix}$

$$AB = \begin{bmatrix} 2-2 & 3+2 \\ 0+1 & 0-1 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 1 & -1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1+2 & 2+3 \\ 0-1 & -1+1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 3 & 5 \\ -1 & 0 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 1-2 & 2-3 \\ 0+1 & -1-1 \end{bmatrix}$$

$$A-B = \begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix}$$

Now (i) L.H.S = $(A+B)^2$

$$(A+B)^2 = \begin{bmatrix} 3 & 5 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ -1 & 0 \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} 9-5 & 15+0 \\ -3-0 & -5+0 \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} 4 & 15 \\ -3 & -5 \end{bmatrix} \text{.........................}(1)$$

R.H.S $= A^2 + 2AB + B^2$

$$A^2 + 2AB + B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2\begin{bmatrix} 0 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ -3 & -2 \end{bmatrix}$$

$$A^2 + 2AB + B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 10 \\ 2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ -3 & -2 \end{bmatrix}$$

$$A^2 + 2AB + B^2 = \begin{bmatrix} 1+0+1 & 0+10+9 \\ 0+2-3 & 1-2-2 \end{bmatrix}$$

$$A^2 + 2AB + B^2 = \begin{bmatrix} 2 & 19 \\ -1 & -3 \end{bmatrix} \text{......................}(2)$$

From equation (1) & (2), L.H.S $\neq$ R.H.S

i.e $(A+B)^2 \neq A^2 + 2AB + B^2$

(ii) L.H.S = $(A-B)^2$

$$\begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1-1 & 1+2 \\ -1-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -3 & 3 \end{bmatrix} \rightarrow (1)$$

$$A^2 - 2AB + B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 0 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ -3 & -2 \end{bmatrix}$$

$$A^2 - 2AB + B^2 = \begin{bmatrix} 1+0+1 & 0-10+9 \\ 0-2-3 & 1+2-2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 1 \end{bmatrix} \rightarrow (2)$$

From equation (1) and (2) L.H.S $\neq$ R.H.S

i.e $(A-B)^2 = A^2 - 2AB + B^2$

(iii) (A+B) (A-B)

$$(A+B)(A-B) = \begin{bmatrix} 3 & 5 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -3+5 & -3-10 \\ 1+0 & 1-0 \end{bmatrix} = \begin{bmatrix} 2 & -13 \\ 1 & 1 \end{bmatrix} \rightarrow (1)$$

R.H.S $A^2 - B^2$

$$A^2 - B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 9 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 1-1 & 0-9 \\ 0+3 & 1+2 \end{bmatrix}$$

$$A^2 - B^2 = \begin{bmatrix} 0 & -9 \\ 3 & 3 \end{bmatrix} \rightarrow (2)$$

From equation (1) & (2) L.H.S $\neq$ R.H.S

i.e (A+B) (A-B) $\neq$ $A^2 - B^2$

Q6. If $A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 1 & 2 & 5 \\ 0 & -2 & 1 & 6 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 3 & -1 & 4 \\ 3 & 1 & 2 & -1 \end{bmatrix}$

Then show that $(A+B)^t = A^t + B^t$

Solution: L.H.S = $(A+B)^t$

$$(A+B) = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 1 & 2 & 5 \\ 0 & -2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 3 & -1 & 4 \\ 3 & 1 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 0-1 & -1+3 & 2+1 \\ 3+1 & 1+3 & 2-1 & 5+4 \\ 0+3 & -2+1 & 1+2 & 6-1 \end{bmatrix}$$

$$(A+B) = \begin{bmatrix} 3 & -1 & 2 & 3 \\ 4 & 4 & 1 & 9 \\ 3 & -1 & 3 & 5 \end{bmatrix}$$ by taking the transpose

$$(A+B)^t = \begin{bmatrix} 3 & 4 & 3 \\ -1 & 4 & -1 \\ 2 & 1 & 3 \\ 3 & 9 & 5 \end{bmatrix} \text{.....}(1)$$

R.H.S = $A^t + B^t$

$$A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 1 & 2 & 5 \\ 0 & -2 & 1 & 6 \end{bmatrix} \Rightarrow A^t = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & -2 \\ -1 & 2 & 1 \\ 2 & 5 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 3 & -1 & 4 \\ 3 & 1 & 2 & -1 \end{bmatrix} \Rightarrow B^t = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 3 & 1 \\ 3 & -1 & 2 \\ 1 & 4 & -1 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & -2 \\ -1 & 2 & 1 \\ 2 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 3 \\ -1 & 3 & 1 \\ 3 & -1 & 2 \\ 1 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 3+1 & 0+3 \\ 0-1 & 1+3 & -2+1 \\ -1+3 & 2-1 & 1+2 \\ 2+1 & 5+4 & 6-1 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 3 & 4 & 3 \\ -1 & 4 & -1 \\ 2 & 1 & 3 \\ 3 & 9 & 5 \end{bmatrix} \text{.....}(2)$$

From equation (1) & (2) L.H.S = R.H.S

i.e., $(A+B)^t = A^t + B^t$

Q7. Find inverse of each of the following matrices.

i) $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

Solution: $|A| = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 4-(-3) = 7 \neq 0$

so A^{-1} exists adj A = $\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \dfrac{1}{|A|}$ adj A $\Rightarrow A^{-1} = \dfrac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$ Ans.

(ii) Let $A = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$

Solution: $|A| = \begin{vmatrix} 1 & x \\ 0 & 1 \end{vmatrix} = 1-0 = 1 \neq 0$,

So A^{-1} exists adj A = $\begin{bmatrix} 1 & -x \\ 0 & 1 \end{bmatrix}$

$A^{-1} = \dfrac{1}{|A|}$ adj A = $\Rightarrow A^{-1} = \dfrac{1}{1}\begin{bmatrix} 1 & -x \\ 0 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & -x \\ 0 & 1 \end{bmatrix}$ Ans.

(iii) Let $A = \begin{bmatrix} 1 & 0 \\ y & 1 \end{bmatrix}$

Solution: $|A| = \begin{vmatrix} 1 & 0 \\ y & 1 \end{vmatrix} = 1-0 = 1 \neq 0$,

So A^{-1} exists adj A = $\begin{bmatrix} 1 & 0 \\ -y & 1 \end{bmatrix}$

$A^{-1} = \dfrac{1}{|A|}$ adj A $\Rightarrow A^{-1} = \dfrac{1}{1}\begin{bmatrix} 1 & 0 \\ -y & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 0 \\ -y & 1 \end{bmatrix}$ Ans.

Q8. Solve the following system of linear equation.

(i) x - y = 2

2x + y = 3

Solution: system can be written in matrix form as.

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

A X = B

$\Rightarrow A^{-1} AX = A^{-1} B$

$IX = A^{-1} B$

$X = A^{-1} B \rightarrow (1)$

As $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$

$|A| = 1-(-2) = 1+2 = 3 \neq 0$, A^{-1} Exists

Adj A = $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$ Now $A^{-1} = \dfrac{1}{|A|}$. Adj A

$A^{-1} = \dfrac{1}{3}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 \\ -2/3 & 1/3 \end{bmatrix}$

Put this value in equation (1)

$$X = \begin{bmatrix} 1/3 & 1/3 \\ -2/3 & 1/3 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2/3+1 \\ -4/3+1 \end{bmatrix} = \begin{bmatrix} 5/3 \\ -1/3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5/3 \\ -1/3 \end{bmatrix} \Rightarrow \left.\begin{array}{l} x = 5/3 \\ y = -1/3 \end{array}\right\} \text{ Ans.}$$

(ii) x - y = 3

x + y = 5

Sol: following is the matrix form of above system

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Let A . X = B

$A^{-1} A . X = A^{-1} B$ applying A^{-1}

$I . X = A^{-1} B$

$X = A^{-1} B \rightarrow (1)$

$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1-(-1) = 1+1 = 2 \neq 0$,

So A^{-1} Exists Adj A = $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

So $A^{-1} = \dfrac{1}{|A|}$. Adj A

$A^{-1} = \dfrac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

Equation become

$X = A^{-1} B \Rightarrow \dfrac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$\Rightarrow X = \dfrac{1}{2}\begin{bmatrix} 3+5 \\ -3+5 \end{bmatrix}$

$\Rightarrow X = \dfrac{1}{2}\begin{bmatrix} 8 \\ 2 \end{bmatrix} = \begin{bmatrix} 8/2 \\ 2/2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \Rightarrow \left.\begin{array}{l} x = 4 \\ y = 1 \end{array}\right\} Ans$$

(iii) Given that 4x - 3y = -1

2x + 5y = 19

Sol: following is the matrix form of above system

$$\begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 19 \end{bmatrix}$$

Let A . X = B

$A^{-1} A . X = A^{-1} B$ applying A^{-1}

$I . X = A^{-1} B$

$X = A^{-1} B \rightarrow (1)$

$\Rightarrow |A| = \begin{vmatrix} 4 & -3 \\ 2 & 5 \end{vmatrix} = 20 + 6 = 26 \neq 0$,

So A^{-1} Exists

Adj A = $\begin{bmatrix} 5 & 3 \\ -2 & 4 \end{bmatrix}$ So $A^{-1} = \dfrac{1}{|A|}$. Adj A

$A^{-1} = \dfrac{1}{26}\begin{bmatrix} 5 & 3 \\ -2 & 4 \end{bmatrix}$

Equation become

$$X = \frac{1}{26}\begin{bmatrix} 5 & 3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -1 \\ 19 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{26}\begin{bmatrix} 5+57 \\ 2+76 \end{bmatrix} = \frac{1}{26}\begin{bmatrix} 52 \\ 78 \end{bmatrix} = \begin{bmatrix} 52/26 \\ 78/26 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \Rightarrow \left.\begin{matrix} x = 2 \\ y = 3 \end{matrix}\right\} Ans$$

Exercise 2.2

Q.1 Write the following sums as a single matrix

i. $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$

Solution: $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 4+3 \\ 5+(-2) \\ 6+(-1) \end{bmatrix}$

$$= \begin{bmatrix} 7 \\ 3 \\ 5 \end{bmatrix}$$

ii.

$$\begin{bmatrix} -2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -3 \end{bmatrix} = \begin{bmatrix} -2+1 & 3+0 & 4+(-3) \end{bmatrix}$$

Solution: $\begin{bmatrix} -2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -3 \end{bmatrix}$

$$= \begin{bmatrix} -2+1 & 3+0 & 4+(-3) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & 1 \end{bmatrix}$$

iii. $\begin{bmatrix} -2 & 3 & 2 \\ 1 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 1 \\ 0 & -1 & -2 \end{bmatrix}$

Solution: $\begin{bmatrix} -2 & 3 & 2 \\ 1 & 2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 1 \\ 0 & -1 & -2 \end{bmatrix}$

$$= \begin{bmatrix} -2+3 & 3+1 & 2+1 \\ 1+0 & 2-1 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 3 \\ 1 & 1 & -2 \end{bmatrix}$$

iv. $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 4 \\ 2 & 3 & 5 \\ 0 & 1 & 4 \end{bmatrix}$

Sol: Given $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 4 \\ 2 & 3 & 5 \\ 0 & 1 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 1+1 & 0+3 & -1+4 \\ 2+2 & 3+3 & 4+5 \\ 0+0 & 1+1 & 2+4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 3 \\ 4 & 6 & 9 \\ 0 & 2 & 6 \end{bmatrix}$$

Q2. Write the following product as a single matrix

i. $-2\begin{bmatrix} -1 & 2 & 4 \\ -3 & 5 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -4 & -8 \\ 6 & -10 & 4 \end{bmatrix}$

Solution: $-2\begin{bmatrix} -1 & 2 & 4 \\ -3 & 5 & -2 \end{bmatrix}$

$$= \begin{bmatrix} -2\times-1 & -2\times2 & -2\times4 \\ -2\times-3 & -2\times5 & -2\times-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -4 & -8 \\ 6 & -10 & 4 \end{bmatrix}$$

ii. $\begin{bmatrix} -1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

Sol: Given $\begin{bmatrix} -1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} (-1)(3)+(-2)(-2)+(-3)(1) \end{bmatrix}$$

$$= \begin{bmatrix} -3+4-3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \end{bmatrix}$$

iii. $\begin{bmatrix} 2 & -3 & 1 \\ 0 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix}$

Sol: Given $\begin{bmatrix} 2 & -3 & 1 \\ 0 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 6-6+0 & 0+3+1 \\ 0+4+0 & 0-2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 4 \\ 4 & -1 \end{bmatrix}$$

iv $\frac{1}{3}\begin{bmatrix} 6 & 3 & -9 \\ 12 & -15 & 3 \\ 0 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -3 \\ 4 & -5 & 1 \\ 0 & 2 & 3 \end{bmatrix}$

Sol: Given $\frac{1}{3}\begin{bmatrix} 6 & 3 & -9 \\ 12 & -15 & 3 \\ 0 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -3 \\ 4 & -5 & 1 \\ 0 & 2 & 3 \end{bmatrix}$ by taking common 3

v. $\begin{bmatrix} 2 & -3 & 1 \\ 0 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 3 \\ 4 & 2 & -1 \end{bmatrix}$

Sol: We have $\begin{bmatrix} 2 & -3 & 1 \\ 0 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 3 \\ 4 & 2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 0+3+4 & 2+6+2 & 0-9-1 \\ 0-2-4 & 0-4-2 & 0+6+1 \\ 0+0+4 & 1+0+2 & 0+0-1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 & -10 \\ -6 & -6 & 7 \\ 4 & 3 & -1 \end{bmatrix}$$

Q3. Let A = $\begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix}$

and C = $\begin{bmatrix} 0 & 1 & -2 \\ 0 & -1 & -1 \end{bmatrix}$ Find 2A + 3B -4C

Sol: Given A = $\begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix}$,

B = $\begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix}$ and C = $\begin{bmatrix} 0 & 1 & -2 \\ 0 & -1 & -1 \end{bmatrix}$

Now 2A + 3B -4C =

$$2\begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix}+3\begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix}-4\begin{bmatrix} 0 & 1 & -2 \\ 0 & -1 & -1 \end{bmatrix}$$

$$=2\begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix}+3\begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix}-4\begin{bmatrix} 0 & 1 & -2 \\ 0 & -1 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & -10 & 2 \\ 6 & 0 & -8 \end{bmatrix}+\begin{bmatrix} 3 & -6 & -9 \\ 0 & -3 & 15 \end{bmatrix}-\begin{bmatrix} 0 & 4 & -8 \\ 0 & -4 & -4 \end{bmatrix}$$

$$=\begin{bmatrix} 4+3-0 & -10-6-4 & 2-9+8 \\ 6+0-0 & 0-3+4 & -8+15+4 \end{bmatrix}$$

$$=\begin{bmatrix} 7 & -20 & 1 \\ 6 & 1 & 11 \end{bmatrix} \text{ Ans}$$

Q.4 Let A = $\begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 4 \\ 4 & 4 & 1 \end{bmatrix}$

Show that $\frac{1}{3}$ A^2-2A - 9I = 0

Sol: Given A = $\begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 4 \\ 4 & 4 & 1 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 4 \\ 4 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 4 \\ 4 & 4 & 1 \end{bmatrix}$$

$$A^2=\begin{bmatrix} 1+16+16 & 4+4+16 & 4+16+4 \\ 4+4+16 & 16+1+16 & 16+4+4 \\ 4+16+4 & 16+4+4 & 16+16+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 33 & 24 & 24 \\ 24 & 33 & 24 \\ 24 & 24 & 33 \end{bmatrix}$$

$$\text{Now } 2A = 2\begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 4 \\ 4 & 4 & 1 \end{bmatrix}=\begin{bmatrix} 2 & 8 & 8 \\ 8 & 2 & 8 \\ 8 & 8 & 2 \end{bmatrix}$$

$$\text{And } 9I = 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Now L.H.S $\frac{1}{3}$ A^2-2A - 9I

$$=\frac{1}{3}\begin{bmatrix} 33 & 24 & 24 \\ 24 & 33 & 24 \\ 24 & 24 & 33 \end{bmatrix}-\begin{bmatrix} 2 & 8 & 8 \\ 8 & 2 & 8 \\ 8 & 8 & 2 \end{bmatrix}-\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$=\begin{bmatrix} 11 & 8 & 8 \\ 8 & 11 & 8 \\ 8 & 8 & 11 \end{bmatrix}-\begin{bmatrix} 2 & 8 & 8 \\ 8 & 2 & 8 \\ 8 & 8 & 2 \end{bmatrix}-\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$=\begin{bmatrix} 11-2-9 & 8-8-0 & 8-8-0 \\ 8-8-0 & 11-2-9 & 8-8-0 \\ 8-8-0 & 8-8-0 & 11-2-9 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}= 0 \text{ Ans.}$$

Hence $\frac{1}{3}$ A^2-2A - 9I = 0

Q5 :Let A = $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, B = $\begin{bmatrix} 0 & i^2 \\ -i^2 & 0 \end{bmatrix}$, C= $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

Then verify the following

(i) $A^2 = B^2 = C^2 = -I$

(ii) AB = -BA = -C

(iii) BC = -CB = -A

(iv) CA = -AC = -B

To show

i). $A^2 = B^2 = C^2 = -I$

sol: A = $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, B = $\begin{bmatrix} 0 & i^2 \\ -i^2 & 0 \end{bmatrix}$ & C = $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$$

$$=\begin{bmatrix} i^2+0 & 0+0 \\ 0+0 & 0+i^2 \end{bmatrix}=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}=-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=-I \rightarrow (1)$$

$$B^2 = B.B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix}=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}=-I \rightarrow (2)$$

$$C^2 = C.C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 0+i^2 & 0+0 \\ 0+0 & i^2+0 \end{bmatrix}=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}=-I \rightarrow (3)$$

From (1), (2), (3)

$A^2 = B^2 = C^2 = -I$

(ii) AB = -BA = -C

sol: A = $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, B = $\begin{bmatrix} 0 & i^2 \\ -i^2 & 0 \end{bmatrix}$ & C = $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0+0 & -i+0 \\ 0-i & 0+0 \end{bmatrix}=\begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \text{ (-1) common}$$

$$AB = -\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}= -C \rightarrow (4)$$

-BA = - $\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}i&0\\0&-i\end{bmatrix}$

-BA = - $\begin{bmatrix}0+0&-i+0\\0-i&0+0\end{bmatrix}=-\begin{bmatrix}0&-i\\-i&0\end{bmatrix}$ = -C $\to$(5)

Combing (4) and (5) we get AB - BA = -C proved

(iii) BC = -CB = -A

Sol: A = $\begin{bmatrix}i&0\\0&-i\end{bmatrix}$, B = $\begin{bmatrix}0&i^2\\-i^2&0\end{bmatrix}$ & C = $\begin{bmatrix}0&i\\i&0\end{bmatrix}$

BC = $\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}0&i\\i&0\end{bmatrix}=\begin{bmatrix}0-i&0+0\\0+0&i+0\end{bmatrix}$

BC = $\begin{bmatrix}-i&0\\0&i\end{bmatrix}=-\begin{bmatrix}i&0\\0&-i\end{bmatrix}$ = -A $\to$(6)

-CB = - $\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}i&0\\0&-i\end{bmatrix}$ =- $\begin{bmatrix}0+i&0+0\\0+0&-i+0\end{bmatrix}$

-CB = - $\begin{bmatrix}i&0\\0&-i\end{bmatrix}$ =-A $\to$(7)

Combing (6) and (7)

AB = -BA = -C

(iv) CA = -AC = -B

Sol:A = $\begin{bmatrix}i&0\\0&-i\end{bmatrix}$, B = $\begin{bmatrix}0&i^2\\-i^2&0\end{bmatrix}$ & C = $\begin{bmatrix}0&i\\i&0\end{bmatrix}$

CA = $\begin{bmatrix}0&i\\i&0\end{bmatrix}\begin{bmatrix}i&0\\0&-i\end{bmatrix}=\begin{bmatrix}0+0&0-i^2\\i^2+0&0+0\end{bmatrix}$

CA = $\begin{bmatrix}0&-i^2\\i^2&0\end{bmatrix}=\begin{bmatrix}0&1\\-1&0\end{bmatrix}=-B\to$(8)

-AC = - $\begin{bmatrix}i&0\\0&-i\end{bmatrix}\begin{bmatrix}0&i\\i&0\end{bmatrix}=\begin{bmatrix}0+0&i^2+0\\0-i^2&0+0\end{bmatrix}$

-AC = - $\begin{bmatrix}0&i^2\\-i^2&0\end{bmatrix}$ = -B $\to$(9)

Combing (8) and (9)

CA = -AC = -B proved

Q6 $A=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix},B=\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix},C=\begin{bmatrix}3&2&6\\4&5&0\\1&3&4\end{bmatrix}$

And a,b are real numbers

i). $A+B=B+A$

Sol: we have $A=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix},B=\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix}$

Taking LHS $A+B=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}+\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix}$

$$=\begin{bmatrix}5+6&0+1&4+1\\-2+4&6+3&1+2\\3+0&2+1&1-5\end{bmatrix}=\begin{bmatrix}11&1&5\\2&9&3\\3&3&-4\end{bmatrix}....(1)$$

Now RHS

$$B+A=\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix}+\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}$$

$$B+A=\begin{bmatrix}6+5&1+0&1+4\\4-2&3+6&2+1\\0+3&1+2&-5+1\end{bmatrix}=\begin{bmatrix}11&1&5\\2&9&3\\3&3&-4\end{bmatrix}....(2)$$

From equations (1) and (2) we get

i.e., $A+B=B+A$

ii). $A+(B+C)=(A+B)+C$

Sol: $A=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix},B=\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix},C=\begin{bmatrix}3&2&6\\4&5&0\\1&3&4\end{bmatrix}$

Take LHS $A+(B+C)$

$$=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}+\left(\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix}+\begin{bmatrix}3&2&6\\4&5&0\\1&3&4\end{bmatrix}\right)$$

$$=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}+\begin{bmatrix}6+3&1+2&1+6\\4+4&3+5&2+0\\0+1&1+3&-5+4\end{bmatrix}$$

$$=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}+\begin{bmatrix}9&3&7\\8&8&2\\1&4&-1\end{bmatrix}$$

$$=\begin{bmatrix}5+9&0+3&4+7\\-2+8&6+8&1+2\\3+1&2+4&1-1\end{bmatrix}=\begin{bmatrix}14&3&11\\6&14&3\\4&6&0\end{bmatrix}...(1)$$

Now take RHS $(A+B)+C$

$$=\left(\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}+\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix}\right)+\begin{bmatrix}3&2&6\\4&5&0\\1&3&4\end{bmatrix}$$

$$=\begin{bmatrix}5+6&0+1&4+1\\-2+4&6+3&1+2\\3+0&2+1&1-5\end{bmatrix}+\begin{bmatrix}3&2&6\\4&5&0\\1&3&4\end{bmatrix}$$

$$=\begin{bmatrix}11&1&5\\2&9&3\\3&3&-4\end{bmatrix}+\begin{bmatrix}3&2&6\\4&5&0\\1&3&4\end{bmatrix}$$

$$=\begin{bmatrix}11+3&1+2&5+6\\2+4&9+5&3+0\\3+1&3+3&-4+4\end{bmatrix}=\begin{bmatrix}14&3&11\\6&14&3\\4&6&0\end{bmatrix}...(2)$$

From equations (1) and (2) we get

i.e., $A+(B+C)=(A+B)+C$

iii). $A+O=O+A=A$

Sol: we have $A=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}$ Take LHS $A+O$

$$A+O=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$=\begin{bmatrix}5+0&0+0&4+0\\-2+0&6+0&1+0\\3+0&2+0&1+0\end{bmatrix}=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}....(1)$$

Now take RHS $O+A$

$$O+A=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}+\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}$$

$$=\begin{bmatrix}0+5&0+0&0+4\\0-2&0+6&0+1\\0+3&0+2&0+1\end{bmatrix}=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}....(2)$$

From equations (1) and (2) we get

i.e., $A+O=O+A=A$

iv). $A+(-A)=(-A)+A=O$

Sol: Given $A=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}$ Take LHS $A+(-A)$

$$=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}+\begin{bmatrix}-5&0&-4\\2&-6&-1\\-3&-2&-1\end{bmatrix}$$

$$=\begin{bmatrix}5-5&0+0&4-4\\-2+2&6-6&1-1\\3-3&2-2&1-1\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}....(1)$$

Now RHS $(-A)+A$

$$=\begin{bmatrix}-5&0&-4\\2&-6&-1\\-3&-2&-1\end{bmatrix}+\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}$$

$$=\begin{bmatrix}-5+5&0+0&-4+4\\2-2&-6+6&-1+1\\-3+3&-2+2&-1+1\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}....(2)$$

From equations (1) and (2) we get

i.e., $A+(-A)=(-A)+A=O$

v). $(ab)A=a(bA)$

Sol: Given $A=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}$

Take LHS $(ab)A=(ab)\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}$

$$=\begin{bmatrix}5ab&0&4ab\\-2ab&6ab&ab\\3ab&2ab&ab\end{bmatrix}....(1)$$

Now take RHS $a(bA)$

$$=a\left(b\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}\right)=a\begin{bmatrix}5b&0&4b\\-2b&6b&b\\3b&2b&b\end{bmatrix}$$

$$=\begin{bmatrix}5ab&0&4ab\\-2ab&6ab&ab\\3ab&2ab&ab\end{bmatrix}....(2)$$

From equations (1) and (2) we get

i.e., $(ab)A=a(bA)$

vi). $a(A+B)=aA+aB$

Sol: $A=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}, B=\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix}$ Taking LHS

$$a(A+B)=a\left(\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}+\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix}\right)$$

$$=a\begin{bmatrix}5+6&0+1&4+1\\-2+4&6+3&1+2\\3+0&2+1&1-5\end{bmatrix}=a\begin{bmatrix}11&1&5\\2&9&3\\3&3&-4\end{bmatrix}$$

$$=\begin{bmatrix}11a&a&5a\\2a&9a&3a\\3a&3a&-4a\end{bmatrix}....(1)$$

Now RHS

$$aA+aB=a\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix}+a\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}$$

$$=\begin{bmatrix}6a&a&a\\4a&3a&2a\\0&a&-5a\end{bmatrix}+\begin{bmatrix}5a&0&4a\\-2a&6a&a\\3a&2a&a\end{bmatrix}$$

$$=\begin{bmatrix}6a+5a&a+0&a+4a\\4a-2a&3a+6a&2a+a\\0+3a&a+2a&-5a+a\end{bmatrix}=\begin{bmatrix}11a&a&5a\\2a&9a&3a\\3a&3a&-4a\end{bmatrix}....(2)$$

From equations (1) and (2) we get

i.e., $a(A+B)=aA+aB$

vii). $(a+b)A=aA+bA$

Sol: we have $A=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}$

Take LHS $(a+b)A$

$$=(a+b)\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}$$

$$=\begin{bmatrix}5(a+b)&0&4(a+b)\\-2(a+b)&6(a+b)&(a+b)\\3(a+b)&2(a+b)&(a+b)\end{bmatrix}....(1)$$

Now take RHS $aA+bA$

$$=a\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}+b\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}$$

$$=\begin{bmatrix}5a&0&4a\\-2a&6a&a\\3a&2a&a\end{bmatrix}+\begin{bmatrix}5b&0&4b\\-2b&6b&b\\3b&2b&b\end{bmatrix}$$

$$=\begin{bmatrix} 5a+5b & 0+0 & 4a+4b \\ -2a-2b & 6a+6b & a+b \\ 3a+3b & 2a+2b & a+b \end{bmatrix}$$

$$=\begin{bmatrix} 5(a+b) & 0 & 4(a+b) \\ -2(a+b) & 6(a+b) & (a+b) \\ 3(a+b) & 2(a+b) & (a+b) \end{bmatrix}\text{....}(2)$$

From equations (1) and (2) we get

i.e., $(a+b)A = aA + bA$

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viii). $A(BC) = (AB)C$

Sol: $A=\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}, B=\begin{bmatrix} 6 & 1 & 1 \\ 4 & 3 & 2 \\ 0 & 1 & -5 \end{bmatrix}, C=\begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}$

Take LHS $A(BC)$

$$=\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}\left(\begin{bmatrix} 6 & 1 & 1 \\ 4 & 3 & 2 \\ 0 & 1 & -5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}\right)$$

$$=\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 18+4+1 & 12+5+3 & 36+0+4 \\ 12+12+2 & 8+15+6 & 24+0+8 \\ 0+4-5 & 0+5-15 & 0+0-20 \end{bmatrix}$$

$$=\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 23 & 20 & 40 \\ 26 & 29 & 32 \\ -1 & -10 & -20 \end{bmatrix}$$

$$=\begin{bmatrix} 115+0-4 & 100+0-40 & 200+0-80 \\ -46+156-1 & -40+174-10 & -80+192-20 \\ 69+52-1 & 60+58-10 & 120+64-20 \end{bmatrix}$$

$$=\begin{bmatrix} 111 & 60 & 120 \\ 109 & 124 & 92 \\ 120 & 108 & 164 \end{bmatrix}\text{.....}(1)$$

Now take RHS $(AB)C$

$$=\left(\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 6 & 1 & 1 \\ 4 & 3 & 2 \\ 0 & 1 & -5 \end{bmatrix}\right)\begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}$$

$$=\begin{bmatrix} 30+0+0 & 5+0+4 & 5+0-20 \\ -12+24+0 & -2+18+1 & -2+12-5 \\ 18+8+0 & 3+6+1 & 3+4-5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}$$

$$=\begin{bmatrix} 30 & 9 & -15 \\ 12 & 17 & 5 \\ 26 & 10 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}$$

$$=\begin{bmatrix} 90+36-15 & 60+45-45 & 180+0-60 \\ 36+68+5 & 24+85+15 & 72+0+20 \\ 78+40+2 & 52+50+6 & 156+0+8 \end{bmatrix}$$

$$=\begin{bmatrix} 111 & 60 & 120 \\ 109 & 124 & 92 \\ 128 & 108 & 164 \end{bmatrix}\text{.....}(2)$$

From equations (1) and (2) we get

i.e., $A(BC) = (AB)C$

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ix). $A(B+C) = AB + AC$

Sol: $A=\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}, B=\begin{bmatrix} 6 & 1 & 1 \\ 4 & 3 & 2 \\ 0 & 1 & -5 \end{bmatrix}, C=\begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}$

Take LHS $A(B+C)$

$$=\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}\left(\begin{bmatrix} 6 & 1 & 1 \\ 4 & 3 & 2 \\ 0 & 1 & -5 \end{bmatrix}+\begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}\right)$$

$$=\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 6+3 & 1+2 & 1+6 \\ 4+4 & 3+5 & 2+0 \\ 0+1 & 1+3 & -5+4 \end{bmatrix}$$

$$=\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 9 & 3 & 7 \\ 8 & 8 & 2 \\ 1 & 4 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 45+0+4 & 15+0+16 & 35+0-4 \\ -18+48+1 & -6+48+4 & -14+12-1 \\ 27+16+1 & 9+16+4 & 21+4-1 \end{bmatrix}$$

$$=\begin{bmatrix} 49 & 31 & 31 \\ 31 & 46 & -3 \\ 44 & 29 & 24 \end{bmatrix}\text{......................}(1)$$

Now take RHS $AB + AC$

$$=\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 6 & 1 & 1 \\ 4 & 3 & 2 \\ 0 & 1 & -5 \end{bmatrix}+\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}$$

$$=\begin{bmatrix} 30+0+0 & 5+0+4 & 5+0-20 \\ -12+24+0 & -2+18+1 & -2+12-5 \\ 18+8+0 & 3+6+1 & 3+4-5 \end{bmatrix}$$

$$+\begin{bmatrix} 15+0+4 & 10+0+12 & 30+0+16 \\ -6+24+1 & -4+30+3 & -12+0+4 \\ 9+8+1 & 6+10+3 & 18+0+4 \end{bmatrix}$$

$$=\begin{bmatrix} 30 & 9 & -15 \\ 12 & 17 & 5 \\ 26 & 10 & 2 \end{bmatrix}+\begin{bmatrix} 19 & 22 & 46 \\ 19 & 29 & -8 \\ 18 & 19 & 22 \end{bmatrix}$$

$$=\begin{bmatrix} 30+19 & 9+22 & -15+46 \\ 12+19 & 17+29 & 5-8 \\ 26+18 & 10+19 & 2+22 \end{bmatrix}$$

$$=\begin{bmatrix} 49 & 31 & 31 \\ 31 & 46 & -3 \\ 44 & 29 & 24 \end{bmatrix}\text{.....}(2)$$

From equations (1) and (2) we get

i.e., $A(B+C) = AB + AC$

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x). $(A+B)C = AC + BC$

Sol: $A=\begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ 3 & 2 & 1 \end{bmatrix}, B=\begin{bmatrix} 6 & 1 & 1 \\ 4 & 3 & 2 \\ 0 & 1 & -5 \end{bmatrix}, C=\begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}$

LHS

$$(A+B)C=\left(\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}+\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix}\right)\begin{bmatrix}3&2&6\\4&5&0\\1&3&4\end{bmatrix}$$

$$=\begin{bmatrix}5+6&0+1&4+1\\-2+4&6+3&1+2\\3+0&2+1&1-5\end{bmatrix}\begin{bmatrix}3&2&6\\4&5&0\\1&3&4\end{bmatrix}$$

$$=\begin{bmatrix}11&1&5\\2&9&3\\3&3&-4\end{bmatrix}\begin{bmatrix}3&2&6\\4&5&0\\1&3&4\end{bmatrix}$$

$$=\begin{bmatrix}33+4+5&22+5+15&66+0+20\\6+36+3&4+45+9&12+0+12\\9+12-4&6+15-12&18+0-16\end{bmatrix}$$

$$(A+B)C=\begin{bmatrix}42&42&86\\45&58&24\\17&9&2\end{bmatrix}.....(1)$$

Now take RHS $AC+BC$

$$=\begin{bmatrix}5&0&4\\-2&6&1\\3&2&1\end{bmatrix}\begin{bmatrix}3&2&6\\4&5&0\\1&3&4\end{bmatrix}+\begin{bmatrix}6&1&1\\4&3&2\\0&1&-5\end{bmatrix}\begin{bmatrix}3&2&6\\4&5&0\\1&3&4\end{bmatrix}$$

$$=\begin{bmatrix}15+0+4&10+0+12&30+0+16\\-6+24+1&-4+30+3&-12+0+4\\9+8+1&6+10+3&18+0+4\end{bmatrix}$$

$$+\begin{bmatrix}18+4+1&12+5+3&36+0+4\\12+12+2&8+15+6&24+0+8\\0+4-5&0+5-15&0+0-20\end{bmatrix}$$

$$=\begin{bmatrix}19&22&46\\19&29&-8\\18&19&22\end{bmatrix}+\begin{bmatrix}23&20&40\\26&29&32\\-1&-10&-20\end{bmatrix}$$

$$=\begin{bmatrix}19+23&22+20&46+40\\19+26&29+29&-8+32\\18-1&19-10&22-20\end{bmatrix}$$

$$=\begin{bmatrix}42&42&86\\45&58&24\\17&9&2\end{bmatrix}.....(2)$$

From equations (1) and (2) we get

i.e., $(A+B)C=AC+BC$

Q7. Determine whether commutative property w.r.t multiplication holds in each of the following cases or not.

i). $A=\begin{bmatrix}p&q\\-q&p\end{bmatrix},B=\begin{bmatrix}s&t\\-t&s\end{bmatrix}$

Sol: we have to show that AB=BA

$$A.B=\begin{bmatrix}p&q\\-q&p\end{bmatrix}\begin{bmatrix}s&t\\-t&s\end{bmatrix}=\begin{bmatrix}ps-qt&pt+qs\\-qs-pt&-qt+ps\end{bmatrix}...(1)$$

$$B.A=\begin{bmatrix}s&t\\-t&s\end{bmatrix}\begin{bmatrix}p&q\\-q&p\end{bmatrix}=\begin{bmatrix}ps-qt&qs+pt\\-pt-qs&-qt+ps\end{bmatrix}...(2)$$

From equations (1) and (2) we get AB=BA

ii). $A=\begin{bmatrix}3&1&2\\2&1&3\end{bmatrix},B=\begin{bmatrix}1&2\\3&1\\2&3\end{bmatrix}$

Sol: we have to show that AB=BA

$$A.B=\begin{bmatrix}3&1&2\\2&1&3\end{bmatrix}\begin{bmatrix}1&2\\3&1\\2&3\end{bmatrix}=\begin{bmatrix}3+3+4&6+1+6\\2+3+6&4+1+9\end{bmatrix}$$

$$A.B=\begin{bmatrix}10&13\\11&14\end{bmatrix}...(1)$$

$$B.A=\begin{bmatrix}1&2\\3&1\\2&3\end{bmatrix}\begin{bmatrix}3&1&2\\2&1&3\end{bmatrix}$$

$$BA=\begin{bmatrix}3+4&1+2&2+6\\9+2&3+1&6+3\\6+6&2+3&4+9\end{bmatrix}$$

$$B.A=\begin{bmatrix}7&3&8\\11&4&9\\12&5&13\end{bmatrix}...(2)$$

From equations (1) and (2) we get $AB\neq BA$

Q8. Let $A=\begin{bmatrix}1&2&0\\3&-1&4\end{bmatrix}$ Show that

i). $\left(A^t\right)^t$

Sol: $A^t=\begin{bmatrix}1&3\\2&-1\\0&4\end{bmatrix}$ Taking transpose

$\left(A^t\right)^t=\begin{bmatrix}1&2&0\\3&-1&4\end{bmatrix}$ Again taking transpose

$\left(A^t\right)^t=A$

ii). $AA^t\neq A^tA$

Sol: Take LHS $AA^t=\begin{bmatrix}1&2&0\\3&-1&4\end{bmatrix}\begin{bmatrix}1&3\\2&-1\\0&4\end{bmatrix}$

$$=\begin{bmatrix}1+4+0&3-2+0\\3-2+0&9+1+16\end{bmatrix}$$

$$=\begin{bmatrix}5&1\\1&26\end{bmatrix}.....(1)$$

Now RHS

$$A^tA=\begin{bmatrix}1&3\\2&-1\\0&4\end{bmatrix}\begin{bmatrix}1&2&0\\3&-1&4\end{bmatrix}$$

$$=\begin{bmatrix}1+9&2-3&0+12\\2-3&4+1&0-4\\0+12&0-4&0+16\end{bmatrix}$$

$$=\begin{bmatrix}10&-1&12\\-1&5&-4\\12&-4&16\end{bmatrix}.....(2)$$

From equations (1) and (2) we get $AA^t\neq A^tA$

Q9. Solve the following matrix equations for X

i). $X-3A=2B$

if $A=\begin{bmatrix}1 & 0 & 3\\ -2 & 2 & 1\end{bmatrix}$ & $B=\begin{bmatrix}2 & 1 & 1\\ 3 & -1 & 4\end{bmatrix}$

Sol: $A=\begin{bmatrix}1 & 0 & 3\\ -2 & 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1 & 1\\ 3 & -1 & 4\end{bmatrix}$

take $X-3A=2B$

$X=3A+2B$

$$X=3\begin{bmatrix}1 & 0 & 3\\ -2 & 2 & 1\end{bmatrix}+2\begin{bmatrix}2 & 1 & 1\\ 3 & -1 & 4\end{bmatrix}$$

$$X=\begin{bmatrix}3 & 0 & 9\\ -6 & 6 & 3\end{bmatrix}+\begin{bmatrix}4 & 2 & 2\\ 6 & -2 & 8\end{bmatrix}$$

$$X=\begin{bmatrix}3+4 & 0+2 & 9+2\\ -6+6 & 6-2 & 3+8\end{bmatrix}$$

$$X=\begin{bmatrix}7 & 2 & 11\\ 0 & 4 & 11\end{bmatrix}$$

---

ii). $2(X-A)=B$ if $A=\begin{bmatrix}1 & 2 & 2\\ 3 & -1 & 2\end{bmatrix}$ &

$B=\begin{bmatrix}4 & 6 & 2\\ 0 & -4 & 2\end{bmatrix}$

Sol: Since $A=\begin{bmatrix}1 & 2 & 2\\ 3 & -1 & 2\end{bmatrix}$ & $B=\begin{bmatrix}4 & 6 & 2\\ 0 & -4 & 2\end{bmatrix}$

take $2(X-A)=B$

$X-A=\frac{1}{2}B$

Putting the values of A and B

$X=A+\frac{1}{2}B$

$$X=\begin{bmatrix}1 & 2 & 2\\ 3 & -1 & 2\end{bmatrix}+\frac{1}{2}\begin{bmatrix}4 & 6 & 2\\ 0 & -4 & 2\end{bmatrix}$$

$$X=\begin{bmatrix}1 & 2 & 2\\ 3 & -1 & 2\end{bmatrix}+\begin{bmatrix}2 & 3 & 1\\ 0 & -2 & 1\end{bmatrix}$$

$$X=\begin{bmatrix}1+2 & 2+3 & 2+1\\ 3+0 & -1-2 & 2+1\end{bmatrix}$$

$$X=\begin{bmatrix}3 & 5 & 3\\ 3 & -3 & 3\end{bmatrix}$$

Exercise 2.3

Q1. If $A=\begin{bmatrix}1 & 3 & 1\\ -1 & 2 & 0\\ 2 & 0 & -2\end{bmatrix}$, then find

$A_{11}, A_{21}, A_{23}, A_{31}, A_{32}, A_{33}$. Also find $|A|$

Sol: we have $A=\begin{bmatrix}1 & 3 & 1\\ -1 & 2 & 0\\ 2 & 0 & -2\end{bmatrix}$

$$A_{11}=(-1)^{1+1}\begin{vmatrix}2 & 0\\ 0 & -2\end{vmatrix}=(+1)(-4-0)=-4$$

$$A_{21}=(-1)^{2+1}\begin{vmatrix}3 & 1\\ 0 & -2\end{vmatrix}=(-1)(-6-0)=6$$

$$A_{23}=(-1)^{2+3}\begin{vmatrix}1 & 3\\ 2 & 0\end{vmatrix}=(-1)(0-6)=6$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix}3 & 1\\ 2 & 0\end{vmatrix}=(+1)(0-2)=-2$$

$$A_{32}=(-1)^{3+2}\begin{vmatrix}1 & 1\\ -1 & 0\end{vmatrix}=(-1)(0+1)=-1$$

$$A_{33}=(-1)^{3+3}\begin{vmatrix}1 & 3\\ -1 & 2\end{vmatrix}=(+1)(2+3)=5$$

Now using formula

$|A|=a_{31}A_{31}+a_{32}A_{32}+a_{33}A_{33}$

$|A|=(2)(-2)+(0)(-1)+(-2)(5)$

$|A|=-4+0-10$

$|A|=-14$

Q2. Without evaluating state the reason for the following equalities.

i). $\begin{vmatrix}1 & 2 & 0\\ 3 & 1 & 0\\ -1 & 2 & 0\end{vmatrix}=0$

Sol: Reason $R_3=0$ in LHS

ii). $\begin{vmatrix}1 & 2 & 3\\ -8 & 4 & -12\\ 2 & -1 & 3\end{vmatrix}=0$

Sol: Reason R_2 is a multiple of C_3 in LHS

iii). $\begin{vmatrix}1 & 3 & -2\\ 3 & -1 & 1\\ 2 & 1 & 4\end{vmatrix}=\begin{vmatrix}1 & 3 & 2\\ 3 & 1 & 1\\ -2 & 1 & 4\end{vmatrix}$

Sol: Reason $|A|=|A^t|$

iv). $\begin{vmatrix}3 & 2 & 0\\ 1 & 1 & -3\\ 2 & 4 & -6\end{vmatrix}=-3\begin{vmatrix}3 & 2 & 0\\ 1 & 1 & 1\\ 2 & 4 & 2\end{vmatrix}$

Sol: Reason Taking -3 as common from C_3 of LHS

v). $\begin{vmatrix}1 & 0 & -1\\ 3 & 2 & 1\\ 1 & -1 & 0\end{vmatrix}=-\begin{vmatrix}1 & 0 & -1\\ 1 & -1 & 0\\ 3 & 2 & 1\end{vmatrix}$

Sol: Reason; interchanging R_2 and R_3 of LHS

vi). $\begin{vmatrix}2 & 0 & 1\\ 3 & 1 & 2\\ 1 & 2 & 2\end{vmatrix}=\begin{vmatrix}2 & 0 & 1\\ 5 & 5 & 6\\ 1 & 2 & 2\end{vmatrix}$

Sol: Reason; Appling R_2+2R_3 on LHS to get RHS

Q3. Let A be a square matrix of order 3. Verify property (2) of determinants that is $|A^t|=|A|$

Sol: $A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$ then $|A|=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$

Expanding by R_1

$$|A|=a_{11}\begin{vmatrix}a_{22} & a_{23}\\ a_{32} & a_{33}\end{vmatrix}-a_{12}\begin{vmatrix}a_{21} & a_{23}\\ a_{31} & a_{33}\end{vmatrix}+a_{13}\begin{vmatrix}a_{21} & a_{22}\\ a_{31} & a_{32}\end{vmatrix}$$

$$|A|=a_{11}(a_{22}a_{33}-a_{23}a_{32})-a_{12}(a_{21}a_{33}-a_{23}a_{31})$$
$$+a_{13}(a_{21}a_{32}-a_{22}a_{31})$$

$|A| = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33} - a_{12}a_{23}a_{31}$
$+a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}$.......(1)

$$|A^t| = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

Expanding by R_1

$$|A^t| = a_{11}\begin{vmatrix} a_{22} & a_{32} \\ a_{23} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{32} \\ a_{13} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{22} \\ a_{13} & a_{23} \end{vmatrix}$$

$|A^t| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{21}(a_{12}a_{33} - a_{32}a_{13})$
$+a_{31}(a_{12}a_{23} - a_{22}a_{13})$

$|A^t| = a_{11}a_{22}a_{33} - a_{11}a_{32}a_{23} - a_{21}a_{12}a_{33} + a_{21}a_{32}a_{13}$
$+a_{31}a_{12}a_{23} - a_{31}a_{22}a_{13}$.......(2)

From equations (1) & (2) we get $|A^t| = |A|$

Q4. Evaluate the following determinants;

i). $\begin{vmatrix} 0 & 1 & 3 \\ -1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix}$

Sol: $\begin{vmatrix} 0 & 1 & 3 \\ -1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix}$

Expanding by R_1

$$= 0\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} + 3\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix}$$

$= 0(2-1) - 1(-1-2) + 3(-1-4)$

$= 0(1) - 1(-3) + 3(-5) = 0 + 3 - 15 = -12$

ii). $\begin{vmatrix} 3 & 4 & -2 \\ 2 & 4 & -6 \\ -4 & 2 & 0 \end{vmatrix}$

Sol: $\begin{vmatrix} 3 & 4 & -2 \\ 2 & 4 & -6 \\ -4 & 2 & 0 \end{vmatrix}$

Expanding by R_1

$$= 3\begin{vmatrix} 4 & -6 \\ 2 & 0 \end{vmatrix} - 4\begin{vmatrix} 2 & -6 \\ -4 & 0 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 4 \\ -4 & 2 \end{vmatrix}$$

$= 3(0+12) - 4(0-24) - 2(4+16)$

$= 3(12) - 4(-24) - 2(20) = 36 + 96 - 40$

$= 92$

iii). $\begin{vmatrix} 3 & 1 & 2 \\ 6 & -5 & 4 \\ -9 & 8 & -7 \end{vmatrix}$

Sol: $\begin{vmatrix} 3 & 1 & 2 \\ 6 & -5 & 4 \\ -9 & 8 & -7 \end{vmatrix}$

Expanding by R_1

$$= 3\begin{vmatrix} -5 & 4 \\ 8 & -7 \end{vmatrix} - 1\begin{vmatrix} 6 & 4 \\ -9 & -7 \end{vmatrix} + 2\begin{vmatrix} 6 & -5 \\ -9 & 8 \end{vmatrix}$$

$= 3(35-32) - 1(-42+36) + 2(48-45)$

$= 3(3) - 1(-6) + 2(3) = 9 + 6 + 6 = 21$

iv). $\begin{vmatrix} 2 & 1 & -3 \\ 1 & 1 & 0 \\ -2 & 3 & 4 \end{vmatrix}$

Sol: $\begin{vmatrix} 2 & 1 & -3 \\ 1 & 1 & 0 \\ -2 & 3 & 4 \end{vmatrix}$

Expanding by R_1

$$= 2\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 0 \\ -2 & 4 \end{vmatrix} + (-3)\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix}$$

$= 2(4-0) - 1(4-0) - 3(3+2)$

$= 2(4) - (4) - 3(5) = 8 - 4 - 15 = -11$

Q5. Show that

i). $\begin{vmatrix} a & b & c \\ l & m & n \\ x & y & z \end{vmatrix} = \begin{vmatrix} a & l & x \\ b & m & y \\ c & n & z \end{vmatrix}$

Solution: Take LHS $\begin{vmatrix} a & b & c \\ l & m & n \\ x & y & z \end{vmatrix}$

Take a transpose

$= \begin{vmatrix} a & l & x \\ b & m & y \\ c & n & z \end{vmatrix}$ =RHS, Hence proved

ii). $\begin{vmatrix} a & b & c \\ 1-3a & 2-3b & 3-3c \\ 4 & 5 & 6 \end{vmatrix} = \begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$

Solution: Take LHS $\begin{vmatrix} a & b & c \\ 1-3a & 2-3b & 3-3c \\ 4 & 5 & 6 \end{vmatrix}$

Apply the property P6 i.e., if any row is a sum of two terms, then its determinants can be written as the sum of two determinants.

$$= \begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} + \begin{vmatrix} a & b & c \\ -3a & -3b & -3c \\ 4 & 5 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} - 3\begin{vmatrix} a & b & c \\ a & b & c \\ 4 & 5 & 6 \end{vmatrix}$$

In RHS R_1 and R_2 are identical so it will be zero

$= \begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} - 3(0) = \begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$ =RHS,

Hence proved

iii). $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix} = 0$

Solution: Take LHS $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a+b+c & a+b+c & a+b+c \end{vmatrix} R_3 + R_2$

$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

In second determinant R_1 and R_3 are identical so it will be zero

$= (a+b+c).0 = 0$ =RHS, Hence proved

iv). $\begin{vmatrix} bc & ca & ab \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Solution: Take LHS $\begin{vmatrix} bc & ca & ab \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

$= \frac{abc}{abc}\begin{vmatrix} bc & ca & ab \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

Multiplying aC_1, bC_2, cC_3

$= \frac{1}{abc}\begin{vmatrix} abc & abc & abc \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Taking common abc from R_1

$= \frac{abc}{abc}\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

=RHS, Hence proved

v). $\begin{vmatrix} bc & a^3 & 1/a \\ ca & b^3 & 1/b \\ ab & c^3 & 1/c \end{vmatrix} = 0$

Solution: Take LHS $\begin{vmatrix} bc & a^3 & 1/a \\ ca & b^3 & 1/b \\ ab & c^3 & 1/c \end{vmatrix}$

$= \frac{abc}{abc}\begin{vmatrix} bc & a^3 & 1/a \\ ca & b^3 & 1/b \\ ab & c^3 & 1/c \end{vmatrix}$

Multiplying aR_1, bR_2, cR_3

$= \frac{1}{abc}\begin{vmatrix} abc & a^4 & 1 \\ abc & b^4 & 1 \\ abc & c^4 & 1 \end{vmatrix}$

Taking common abc from C_1

$= \frac{abc}{abc}\begin{vmatrix} 1 & a^4 & 1 \\ 1 & b^4 & 1 \\ 1 & c^4 & 1 \end{vmatrix} = \frac{abc}{abc}(0) = 0 = RHS$

$C_1 \& C_3$ are identical, Hence proved

vi). $\begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

Solution: Take LHS $\begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix}$

$\begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1-1 & \beta-\alpha & \beta^2-\alpha^2 \\ 1-1 & \gamma-\alpha & \gamma^2-\alpha^2 \end{vmatrix} \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}$

$= \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 0 & \beta-\alpha & \beta^2-\alpha^2 \\ 0 & \gamma-\alpha & \gamma^2-\alpha^2 \end{vmatrix}$

$= \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 0 & \beta-\alpha & (\beta-\alpha)(\beta+\alpha) \\ 0 & \gamma-\alpha & (\gamma-\alpha)(\gamma+\alpha) \end{vmatrix}$

Taking common $\gamma-\alpha$ from R_3 and $\beta-\alpha$ from R_2

$= (\beta-\alpha)(\gamma-\alpha)\begin{vmatrix} 1 & \alpha & \alpha^2 \\ 0 & 1 & \beta+\alpha \\ 0 & 1 & \gamma+\alpha \end{vmatrix}$

Expanding by C_1

$= (\beta-\alpha)(\gamma-\alpha)\left\{1\begin{vmatrix} 1 & \beta+\alpha \\ 1 & \gamma+\alpha \end{vmatrix} - 0 + 0\right\}$

$= (\beta-\alpha)(\gamma-\alpha)(\gamma+\alpha-\beta-\alpha)$

$= (\beta-\alpha)(\gamma-\alpha)(\gamma-\beta)$

$= -(\alpha-\beta)(\gamma-\alpha)(\gamma-\beta)$

$= +(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha) = RHS$

Hence proved

vii). $\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^3 & \beta^3 & \gamma^3 \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$

Solution: Take LHS $\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^3 & \beta^3 & \gamma^3 \end{vmatrix}$

$\begin{vmatrix} 1 & 1-1 & 1-1 \\ \alpha & \beta-\alpha & \gamma-\alpha \\ \alpha^3 & \beta^3-\alpha^3 & \gamma^3-\alpha^3 \end{vmatrix} \begin{matrix} C_2 - C_1 \\ C_3 - C_1 \end{matrix}$

$= \begin{vmatrix} 1 & 0 & 0 \\ \alpha & \beta-\alpha & \gamma-\alpha \\ \alpha^3 & \beta^3-\alpha^3 & \gamma^3-\alpha^3 \end{vmatrix}$

$$=\begin{vmatrix} 1 & 0 & 0 \\ \alpha & \beta-\alpha & \gamma-\alpha \\ \alpha^3 & (\beta-\alpha)(\beta^2+\beta\alpha+\alpha^2) & (\gamma-\alpha)(\gamma^2+\gamma\alpha+\alpha^2) \end{vmatrix}$$

Taking common $\gamma-\alpha$ from C_3 and $\beta-\alpha$ from C_2

$$=(\beta-\alpha)(\gamma-\alpha)\begin{vmatrix} 1 & 0 & 0 \\ \alpha & 1 & 1 \\ \alpha^3 & \beta^2+\beta\alpha+\alpha^2 & \gamma^2+\gamma\alpha+\alpha^2 \end{vmatrix}$$

Expanding by R_1

$$=(\beta-\alpha)(\gamma-\alpha)$$

$$\left\{1\begin{vmatrix} 1 & 1 \\ \beta^2+\beta\alpha+\alpha^2 & \gamma^2+\gamma\alpha+\alpha^2 \end{vmatrix}-0+0\right\}$$

$$=-(\alpha-\beta)(\gamma-\alpha)(\gamma^2+\gamma\alpha+\alpha^2-\beta^2-\beta\alpha-\alpha^2)$$

$$=-(\alpha-\beta)(\gamma-\alpha)(\gamma^2-\beta^2+\gamma\alpha-\beta\alpha)$$

$$=-(\alpha-\beta)(\gamma-\alpha)\{(\gamma-\beta)(\gamma+\beta)+\alpha(\gamma-\beta)\}$$

$$=-(\alpha-\beta)(\gamma-\alpha)(\gamma-\beta)\{(\gamma+\beta)+\alpha\}$$

$$=+(\alpha-\beta)(\gamma-\alpha)(\beta-\gamma)(\alpha+\gamma+\beta)=RHS$$

Hence proved

viii). $\begin{vmatrix} \gamma\cos\theta & 0 & \gamma\sin\theta \\ 1 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{vmatrix}=\gamma$

Solution: Take LHS $\begin{vmatrix} \gamma\cos\theta & 0 & \gamma\sin\theta \\ 1 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{vmatrix}$

Expanding by C_2

$$=-0+1\begin{vmatrix} \gamma\cos\theta & \gamma\sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}-0$$

$$=\gamma\cos^2\theta+\gamma\sin^2\theta$$

$$=\gamma(\cos^2\theta+\sin^2\theta)=\gamma \qquad \because \cos^2\theta+\sin^2\theta=1$$

Hence proved

Q6. Identify singular and non singular matrices.

i). $\begin{vmatrix} 7 & 1 & 3 \\ 6 & 2 & -2 \\ 5 & 1 & 1 \end{vmatrix}$

Sol: $\begin{vmatrix} 7 & 1 & 3 \\ 6 & 2 & -2 \\ 5 & 1 & 1 \end{vmatrix}$

Expanding by R_1

$$=7\begin{vmatrix} 2 & -2 \\ 1 & 1 \end{vmatrix}-1\begin{vmatrix} 6 & -2 \\ 5 & 1 \end{vmatrix}+3\begin{vmatrix} 6 & 2 \\ 5 & 1 \end{vmatrix}$$

$$=7(2+2)-1(6+10)+3(6-10)$$

$$=7(4)-1(16)+3(-4)$$

$$=28-16-12=0$$

∴Singular matrix

ii). $\begin{vmatrix} 1 & -1 & 1 \\ 3 & -2 & 1 \\ -2 & -3 & 2 \end{vmatrix}$

Sol: $\begin{vmatrix} 1 & -1 & 1 \\ 3 & -2 & 1 \\ -2 & -3 & 2 \end{vmatrix}$

Expanding by R_1

$$=1\begin{vmatrix} -2 & 1 \\ -3 & 2 \end{vmatrix}-(-1)\begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix}+1\begin{vmatrix} 3 & -2 \\ -2 & -3 \end{vmatrix}$$

$$=(-4+3)+(6+2)+(-9-4)$$

$$=(-1)+(8)+(-13)=-6\neq 0$$

∴Non singular matrix

iii). $\begin{vmatrix} 3 & 2 & -3 \\ 3 & 6 & -3 \\ -1 & 0 & 1 \end{vmatrix}$

Sol: we have $\begin{vmatrix} 3 & 2 & -3 \\ 3 & 6 & -3 \\ -1 & 0 & 1 \end{vmatrix}$

Expanding by R_1

$$=3\begin{vmatrix} 6 & -3 \\ 0 & 1 \end{vmatrix}-2\begin{vmatrix} 3 & -3 \\ -1 & 1 \end{vmatrix}+(-3)\begin{vmatrix} 3 & 6 \\ -1 & 0 \end{vmatrix}$$

$$=3(6-0)-2(3-3)-3(0+6)$$

$$=18-0-18=0$$

∴Singular matrix

Q7. Find the value of λ if A is singular.

$$A=\begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix}$$

Sol: $\begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix}$

Expanding by R_1

$$(-\lambda)\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix}-1\begin{vmatrix} 1 & 1 \\ 0 & -\lambda \end{vmatrix}+0=0$$

$$-\lambda(\lambda^2-1)-(-\lambda-0)=0$$

$$-\lambda^3+\lambda+\lambda=0$$

$$-\lambda^3+2\lambda=0$$

$$-\lambda(\lambda^2-2)=0$$

$$\therefore \lambda=0 \qquad or \quad \lambda^2-2=0$$

$$\lambda^2=2$$

$$\lambda=\pm\sqrt{2}$$

Hence $S.S=\{0,\pm\sqrt{2}\}$

Q8. Solve for x

i). $\begin{vmatrix} x & 2 & 3 \\ 0 & -1 & 1 \\ 0 & 4 & 5 \end{vmatrix}=9$

Sol: $\begin{vmatrix} x & 2 & 3 \\ 0 & -1 & 1 \\ 0 & 4 & 5 \end{vmatrix}=9$

Expanding by C_1

$$x\begin{vmatrix}-1 & 1\\ 4 & 5\end{vmatrix}-0+0=9$$

$$x(-5-4)=9$$

$$-9x=9$$

$$\Rightarrow x=-1$$

ii). $\begin{vmatrix}-1 & 0 & 1\\ x^2 & 1 & x\\ 2 & 3 & 4\end{vmatrix}=-6$

Sol: $\begin{vmatrix}-1 & 0 & 1\\ x^2 & 1 & x\\ 2 & 3 & 4\end{vmatrix}=-6$

Expanding by R_1

$$-1\begin{vmatrix}1 & x\\ 3 & 4\end{vmatrix}-0+1\begin{vmatrix}x^2 & 1\\ 2 & 3\end{vmatrix}=-6$$

$$-(4-3x)+(3x^2-2)=-6$$

$$-4+3x+3x^2-2=-6$$

$$3x^2+3x-6+6=0$$

$$3x^2+3x=0$$

$$3x(x+1)=0$$

$\therefore x+1=0$ *or* $3x=0$

$x=-1$ $\quad x=0$

$$S.S=\{0,-1\}$$

Q9. Show that the inverse of a square matrix exist, then it is unique.

Sol: Let A is a non-singular matrix, then by definition there exists a Square matrix such that

$$AA^{-1}=A^{-1}A=I$$

Uniqueness;

Let A be a square matrix suppose B and C are two inverses of A, So we have

$$AB=BA=I.....(1)$$

$$AC=CA=I......(2)$$

Now CAB=C(AB)=C(I) from equation (1)

$\Rightarrow$ CAB=C.....(3)

Again CAB=(CA)B=(I)B from equation (2)

$\Rightarrow$ CAB=B.....(4)

By comparing equations (3) and (4)

B=C

i.e., inverse of a square matrix is unique

Q10. Let $A=\begin{bmatrix}0 & 2 & 2\\ -1 & 3 & 2\\ 1 & 0 & 5\end{bmatrix}$. *Find* A^{-1}

Sol: Given $A=\begin{bmatrix}0 & 2 & 2\\ -1 & 3 & 2\\ 1 & 0 & 5\end{bmatrix}$.

Its determinant $|A|=\begin{vmatrix}0 & 2 & 2\\ -1 & 3 & 2\\ 1 & 0 & 5\end{vmatrix}$

Expanding by R_1

$$|A|=0-2\begin{vmatrix}-1 & 2\\ 1 & 5\end{vmatrix}+2\begin{vmatrix}-1 & 3\\ 1 & 0\end{vmatrix}$$

$$|A|=0-2(-5-2)+2(0-3)$$

$$|A|=14-6=8\neq 0$$

So A^{-1} exists, now cofactors

$$A_{11}=(-1)^{1+1}\begin{vmatrix}3 & 2\\ 0 & 5\end{vmatrix}=+(15-0)=15$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}-1 & 2\\ 1 & 5\end{vmatrix}=-(-5-2)=7$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}-1 & 3\\ 1 & 0\end{vmatrix}=+(0-3)=-3$$

$$A_{21}=(-1)^{2+1}\begin{vmatrix}2 & 2\\ 0 & 5\end{vmatrix}=-(10-0)=-10$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}0 & 2\\ 1 & 5\end{vmatrix}=+(0-2)=-2$$

$$A_{23}=(-1)^{2+3}\begin{vmatrix}0 & 2\\ 1 & 0\end{vmatrix}=-(0-2)=2$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix}2 & 2\\ 3 & 2\end{vmatrix}=+(4-6)=-2$$

$$A_{32}=(-1)^{3+2}\begin{vmatrix}0 & 2\\ -1 & 2\end{vmatrix}=-(0+2)=-2$$

$$A_{33}=(-1)^{3+3}\begin{vmatrix}0 & 2\\ -1 & 3\end{vmatrix}=+(0+2)=2$$

$$\therefore adj\ A=\begin{vmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{vmatrix}$$

$$adj\ A=\begin{vmatrix}15 & -10 & -2\\ 7 & -2 & -2\\ -3 & 2 & 2\end{vmatrix}$$

We have

$$A^{-1}=\frac{1}{|A|}adj\ A=\frac{1}{8}\begin{vmatrix}15 & -10 & -2\\ 7 & -2 & -2\\ -3 & 2 & 2\end{vmatrix}$$

$$A^{-1}=\begin{vmatrix}\frac{15}{8} & \frac{-10}{8} & \frac{-2}{8}\\ \frac{7}{8} & \frac{-2}{8} & \frac{-2}{8}\\ \frac{-3}{8} & \frac{2}{8} & \frac{2}{8}\end{vmatrix}=\begin{vmatrix}\frac{15}{8} & \frac{-5}{4} & \frac{-1}{4}\\ \frac{7}{8} & \frac{-1}{4} & \frac{-1}{4}\\ \frac{-3}{8} & \frac{1}{4} & \frac{1}{4}\end{vmatrix}$$

Q11. Let $A=\begin{bmatrix}3 & -1\\ 4 & 2\end{bmatrix}$ show that $|A^{-1}|=\dfrac{1}{|A|}$

Sol: we have $A=\begin{bmatrix}3 & -1\\ 4 & 2\end{bmatrix}$

$$|A|=\begin{vmatrix}3 & -1\\ 4 & 2\end{vmatrix}=6+4=10$$

$$\text{adj } A=\begin{bmatrix}2 & 1\\ -4 & 3\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{|A|}\text{adj } A=\frac{1}{10}\begin{bmatrix}2 & 1\\ -4 & 3\end{bmatrix}=\begin{bmatrix}\frac{2}{10} & \frac{1}{10}\\ \frac{-4}{10} & \frac{3}{10}\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}\frac{1}{5} & \frac{1}{10}\\ \frac{-2}{5} & \frac{3}{10}\end{bmatrix}$$

now $\left|A^{-1}\right|=\begin{vmatrix}\frac{1}{5} & \frac{1}{10}\\ \frac{-2}{5} & \frac{3}{10}\end{vmatrix}=\frac{1}{5}.\frac{3}{10}-\frac{1}{10}.\frac{-2}{5}=\frac{3+5}{50}$

$$\left|A^{-1}\right|=\frac{5}{50}=\frac{1}{10}$$

$$\left|A^{-1}\right|=\frac{1}{|A|}$$

Q12. Verify that $(AB)^{-1}=B^{-1}A^{-1}$,

$A=\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}, B=\begin{bmatrix}-1 & 1\\ 2 & 3\end{bmatrix}$

Sol: $A=\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}, B=\begin{bmatrix}-1 & 1\\ 2 & 3\end{bmatrix}$

$|A|=\begin{vmatrix}2 & 3\\ 1 & 0\end{vmatrix}$ $\quad |B|=\begin{vmatrix}-1 & 1\\ 2 & 3\end{vmatrix}$

$|A|=0-3$ $\quad |B|=-3-2$

$|A|=-3$ $\quad |B|=-5$

$\text{adj } A=\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}$ $\quad \text{adj } B=\begin{bmatrix}3 & -1\\ -2 & -1\end{bmatrix}$

$A^{-1}=\frac{1}{|A|}\text{adj } A$ $\quad B^{-1}=\frac{1}{|B|}\text{adj } B$

$A^{-1}=\frac{1}{-3}\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}$ $\quad B^{-1}=\frac{1}{-5}\begin{bmatrix}3 & -1\\ -2 & -1\end{bmatrix}$

Now

$$AB=\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1\\ 2 & 3\end{bmatrix}=\begin{bmatrix}-2+6 & 2+9\\ -1+0 & 1+0\end{bmatrix}$$

$$AB=\begin{bmatrix}4 & 11\\ -1 & 1\end{bmatrix}$$

$$|AB|=\begin{vmatrix}4 & 11\\ -1 & 1\end{vmatrix}=4+11=15$$

$$\text{adj }(AB)=\begin{bmatrix}1 & -11\\ 1 & 4\end{bmatrix}$$

$$(AB)^{-1}=\frac{1}{|AB|}\text{adj }(AB)$$

$$(AB)^{-1}=\frac{1}{15}\begin{bmatrix}1 & -11\\ 1 & 4\end{bmatrix}\text{.......(1)}$$

Now

$$B^{-1}.A^{-1}=\frac{1}{-5}.\frac{1}{-3}\begin{bmatrix}3 & -1\\ -2 & -1\end{bmatrix}\begin{bmatrix}0 & -3\\ -1 & 2\end{bmatrix}$$

$$B^{-1}.A^{-1}=\frac{1}{15}\begin{bmatrix}0+1 & -9-2\\ 0+1 & 6-2\end{bmatrix}$$

$$B^{-1}.A^{-1}=\frac{1}{15}\begin{bmatrix}1 & -11\\ 1 & 4\end{bmatrix}\text{.......}(2)$$

From equations (1) & (2) we get $(AB)^{-1}=B^{-1}A^{-1}$

Q13 If A and B are non singular matrices, then show that

Sol; Let A and B are non singular matrices then by definition

$$AA^{-1}=A^{-1}A=I\text{......}(1)$$

$$BB^{-1}=B^{-1}B=I\text{......}(2)$$

i). To show that $\left(A^{-1}\right)^{-1}=A$

$AA^{-1}=I$ from (1)

$A^{-1}A=I$ from (1)

Thus A is the inverse of A^{-1}

i.e., $\left(A^{-1}\right)^{-1}=A$

---

ii). To show that $(AB)^{-1}=B^{-1}.A^{-1}$

Sol; Let A and B are non singular matrices then by definition

$$AA^{-1}=A^{-1}A=I\text{......}(1)$$

$$BB^{-1}=B^{-1}B=I\text{......}(2)$$

Associative law holds in Matrices

$$\begin{aligned}(AB)\left(B^{-1}.A^{-1}\right)&=A\left(BB^{-1}\right)A^{-1}\\ &=A(I)A^{-1}\\ &=AA^{-1}\\ &=I\text{....}(3)\end{aligned}$$

Similarly

$$\begin{aligned}\left(B^{-1}.A^{-1}\right)(AB)&=B^{-1}\left(A^{-1}A\right)B\\ &=B^{-1}(I)B\\ &=B^{-1}B\\ &=I\text{....}(4)\end{aligned}$$

From equations (3) & (4)

$B^{-1}.A^{-1}$ is the inverse of AB

i.e., $(AB)^{-1}=B^{-1}.A^{-1}$

Q14. Verify that $(AB)^{t}=B^{t}.A^{t}$ if

i). $A=\begin{bmatrix}2 & -1 & 3\\ 1 & 0 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 2\\ 2 & 2\\ 3 & 0\end{bmatrix}$

Sol: $A^{t}=\begin{bmatrix}2 & 1\\ -1 & 0\\ 3 & 1\end{bmatrix}, B^{t}=\begin{bmatrix}1 & 2 & 3\\ 2 & 2 & 0\end{bmatrix}$

$$AB=\begin{bmatrix}2 & -1 & 3\\ 1 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 2 & 2\\ 3 & 0\end{bmatrix}$$

$$AB=\begin{bmatrix}2-2+9 & 4-2+0\\ 1+0+3 & 2+0+0\end{bmatrix}$$ now

$$AB=\begin{bmatrix}9 & 2\\ 4 & 2\end{bmatrix}$$

$$\Rightarrow (AB)^{t}=\begin{bmatrix}9 & 4\\ 2 & 2\end{bmatrix}\text{........(1)}$$

$$B^tA^t=\begin{bmatrix}1&2&3\\2&2&0\end{bmatrix}\begin{bmatrix}2&1\\-1&0\\3&1\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}2-2+9&1+0+3\\4-2+0&2+0+0\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}9&4\\2&2\end{bmatrix}\text{........}(2)$$

From equations (1) & (2) we get $(AB)^t=B^t.A^t$

ii). $A=\begin{bmatrix}1&2&0\\-1&1&4\end{bmatrix}, B=\begin{bmatrix}1&1\\2&3\\1&-2\end{bmatrix}$

Sol: we have $A=\begin{bmatrix}1&2&0\\-1&1&4\end{bmatrix}, B=\begin{bmatrix}1&1\\2&3\\1&-2\end{bmatrix}$

$$AB=\begin{bmatrix}1&2&0\\-1&1&4\end{bmatrix}\begin{bmatrix}1&1\\2&3\\1&-2\end{bmatrix}$$

$$AB=\begin{bmatrix}1+4+0&1+6+0\\-1+2+4&-1+3-8\end{bmatrix}$$

$$AB=\begin{bmatrix}5&7\\5&-6\end{bmatrix}$$

$$\Rightarrow (AB)^t=\begin{bmatrix}5&5\\7&-6\end{bmatrix}\text{..............................}(1)$$

Now $A^t=\begin{bmatrix}1&-1\\2&1\\0&4\end{bmatrix}, B^t=\begin{bmatrix}1&2&1\\1&3&-2\end{bmatrix}$

$$B^tA^t=\begin{bmatrix}1&2&1\\1&3&-2\end{bmatrix}\begin{bmatrix}1&-1\\2&1\\0&4\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}1+4+0&-1+2+4\\1+6+0&-1+3-8\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}5&5\\7&-6\end{bmatrix}\text{........}(2)$$

From equations (1) & (2) we get $(AB)^t=B^t.A^t$

Q15. Let $A=\begin{bmatrix}2&3\\-1&1\end{bmatrix}$ verify that $(A^{-1})^t=(A^t)^{-1}$

Sol; $A=\begin{bmatrix}2&3\\-1&1\end{bmatrix}$ $\qquad A^t=\begin{bmatrix}2&-1\\3&1\end{bmatrix}$

$|A|=\begin{vmatrix}2&3\\-1&1\end{vmatrix}$ $\qquad |A^t|=\begin{vmatrix}2&-1\\3&1\end{vmatrix}$

$|A|=2+3$ $\qquad |A^t|=2+3$

$|A|=5$ $\qquad |A^t|=5$

$\text{adj } A=\begin{bmatrix}1&-3\\1&2\end{bmatrix}$ $\qquad \text{adj } A^t=\begin{bmatrix}1&1\\-3&2\end{bmatrix}$

$A^{-1}=\dfrac{1}{|A|}\text{adj } A$ $\qquad (A^t)^{-1}=\dfrac{1}{|A^t|}\text{adj } A^t$

$A^{-1}=\dfrac{1}{5}\begin{bmatrix}1&-3\\1&2\end{bmatrix}$ $\qquad (A^t)^{-1}=\begin{bmatrix}1&1\\-3&2\end{bmatrix}\text{...}(1)$

$$(A^{-1})^t=\frac{1}{5}\begin{bmatrix}1&1\\-3&2\end{bmatrix}\text{....}(2)$$

From equations (1) & (2) we get $(A^{-1})^t=(A^t)^{-1}$

Exercise 2.4

Q1. Let $A=\begin{bmatrix}1&-3&4\\-3&2&-5\\4&-5&0\end{bmatrix}$ & $B=\begin{bmatrix}5&6&7\\6&-8&3\\7&3&1\end{bmatrix}$

Show that A & B are symmetric. Also show that A+B is symmetric

Sol: Given that

$$A=\begin{bmatrix}1&-3&4\\-3&2&-5\\4&-5&0\end{bmatrix}\Rightarrow A^t=\begin{bmatrix}1&-3&4\\-3&2&-5\\4&-5&0\end{bmatrix}$$

$\Rightarrow A^t=A \qquad \Rightarrow A$ is symmetric

$$B=\begin{bmatrix}5&6&7\\6&-8&3\\7&3&1\end{bmatrix}\Rightarrow B^t=\begin{bmatrix}5&6&7\\6&-8&3\\7&3&1\end{bmatrix}$$

$B^t=B \qquad \Rightarrow B$ is symmetric

Now $A+B=\begin{bmatrix}1&-3&4\\-3&2&-5\\4&-5&0\end{bmatrix}+\begin{bmatrix}5&6&7\\6&-8&3\\7&3&1\end{bmatrix}$

$$A+B=\begin{bmatrix}1+5&-3+6&4+7\\-3+6&2-8&-5+3\\4+7&-5+3&0+1\end{bmatrix}$$

$$A+B=\begin{bmatrix}6&3&11\\3&-6&-2\\11&-2&1\end{bmatrix}$$

$A+B$ is symmetric

$$\Rightarrow (A+B)^t=\begin{bmatrix}6&3&11\\3&-6&-2\\11&-2&1\end{bmatrix}$$

$$\Rightarrow (A+B)^t=A+B$$

Q2. $A=\begin{bmatrix}0&1&-2\\-1&0&3\\2&-3&0\end{bmatrix}$ & $B=\begin{bmatrix}0&-6&11\\6&0&-7\\-11&7&0\end{bmatrix}$

Show that A+B is skew symmetric

Sol: $A=\begin{bmatrix}0&1&-2\\-1&0&3\\2&-3&0\end{bmatrix}$ & $B=\begin{bmatrix}0&-6&11\\6&0&-7\\-11&7&0\end{bmatrix}$

$$A+B=\begin{bmatrix}0&1&-2\\-1&0&3\\2&-3&0\end{bmatrix}+\begin{bmatrix}0&-6&11\\6&0&-7\\-11&7&0\end{bmatrix}$$

$$A+B=\begin{bmatrix}0+0 & 1-6 & -2+11\\ -1+6 & 0+0 & 3-7\\ 2-11 & -3+7 & 0+0\end{bmatrix}$$

$$A+B=\begin{bmatrix}0 & -5 & 9\\ 5 & 0 & -4\\ -9 & 4 & 0\end{bmatrix}(A+B)^t=\begin{bmatrix}0 & 5 & -9\\ -5 & 0 & 4\\ 9 & -4 & 0\end{bmatrix}$$

$$(A+B)^t=-\begin{bmatrix}0 & -5 & 9\\ 5 & 0 & -4\\ -9 & 4 & 0\end{bmatrix}$$

$(A+B)^t=-(A+B)$, $A+B$ is skew symmetric

Q3. If $A=\begin{bmatrix}3 & 2 & 1\\ 4 & 5 & 6\\ -2 & 3 & 4\end{bmatrix}$, then show that

a). $A+A^t$ is symmetric

Sol: $A=\begin{bmatrix}3 & 2 & 1\\ 4 & 5 & 6\\ -2 & 3 & 4\end{bmatrix}$ & $A^t=\begin{bmatrix}3 & 4 & -2\\ 2 & 5 & 3\\ 1 & 6 & 4\end{bmatrix}$

$$A+A^t=\begin{bmatrix}3 & 2 & 1\\ 4 & 5 & 6\\ -2 & 3 & 4\end{bmatrix}+\begin{bmatrix}3 & 4 & -2\\ 2 & 5 & 3\\ 1 & 6 & 4\end{bmatrix}$$

$$A+A^t=\begin{bmatrix}3+3 & 2+4 & 1-2\\ 4+2 & 5+5 & 6+3\\ -2+1 & 3+6 & 4+4\end{bmatrix}$$

$$A+A^t=\begin{bmatrix}6 & 6 & -1\\ 6 & 10 & 9\\ -1 & 9 & 8\end{bmatrix}$$

Now $(A+A^t)^t=\begin{bmatrix}6 & 6 & -1\\ 6 & 10 & 9\\ -1 & 9 & 8\end{bmatrix}$

$(A+A^t)^t=A+A^t$ $\quad$ $A+A^t$ is symmetric

---

b). $A-A^t$ is skew symmetric

Sol: $A=\begin{bmatrix}3 & 2 & 1\\ 4 & 5 & 6\\ -2 & 3 & 4\end{bmatrix}$ & $A^t=\begin{bmatrix}3 & 4 & -2\\ 2 & 5 & 3\\ 1 & 6 & 4\end{bmatrix}$

$$A-A^t=\begin{bmatrix}3 & 2 & 1\\ 4 & 5 & 6\\ -2 & 3 & 4\end{bmatrix}-\begin{bmatrix}3 & 4 & -2\\ 2 & 5 & 3\\ 1 & 6 & 4\end{bmatrix}$$

$$A-A^t=\begin{bmatrix}3-3 & 2-4 & 1+2\\ 4-2 & 5-5 & 6-3\\ -2-1 & 3-6 & 4-4\end{bmatrix}$$

$$A-A^t=\begin{bmatrix}0 & -2 & 3\\ 2 & 0 & 3\\ -3 & -3 & 0\end{bmatrix}$$

Now $(A-A^t)^t=\begin{bmatrix}0 & 2 & -3\\ -2 & 0 & -3\\ 3 & 3 & 0\end{bmatrix}$

$$(A-A^t)^t=-\begin{bmatrix}0 & -2 & 3\\ 2 & 0 & 3\\ -3 & -3 & 0\end{bmatrix}$$

$$(A-A^t)^t=-(A-A^t)$$

$A-A^t$ is skew symmetric

Q4 If A is a square matrix of order 3, then show that

a). $A+A^t$ is symmetric

Sol: Let A is a square matrix of order 3

$$(A+A^t)^t=A^t+(A^t)^t$$
$$=A^t+A$$
$$(A+A^t)^t=A+A^t$$

$\therefore$ $A+A^t$ is symmetric

---

b). $A-A^t$ is skew symmetric

Sol: Let A is a square matrix of order 3

$$(A-A^t)^t=A^t-(A^t)^t$$
$$=A^t-A$$
$$=-A+A^t$$

$(A+A^t)^t=-(A-A^t)$ $\therefore$ $A-A^t$ is skew symmetric

Q5. Reduce each of the following matrices to the indicated form

i). $\begin{bmatrix}1 & 3 & -1\\ 2 & 1 & 4\\ 3 & 4 & -5\end{bmatrix}$ Echelon form

Solution: $\begin{bmatrix}1 & 3 & -1\\ 2 & 1 & 4\\ 3 & 4 & -5\end{bmatrix}$

$$\begin{bmatrix}1 & 3 & -1\\ 2-2 & 1-6 & 4+2\\ 3-3 & 4-9 & -5+3\end{bmatrix}\begin{matrix}R_2-2R_1\\ R_2-3R_1\end{matrix}$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 3 & -1\\ 0 & -5 & 6\\ 0 & -5 & -2\end{bmatrix}$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 3 & -1\\ 0 & -5 & 6\\ 0 & -5+5 & -2-6\end{bmatrix}R_3-R_2$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 3 & -1\\ 0 & -5 & 6\\ 0 & 0 & -8\end{bmatrix}$$

Required echelon form

---

ii). $\begin{bmatrix}1 & 0 & -2\\ 2 & 1 & 1\\ 3 & 2 & 3\end{bmatrix}$ Echelon form

Sol: $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -2 \\ 2-2 & 1-0 & 1+4 \\ 3-3 & 2-0 & 3+6 \end{bmatrix} \begin{matrix} R_2-2R_1 \\ R_3-3R_1 \end{matrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 5 \\ 0 & 2 & 9 \end{bmatrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 5 \\ 0 & 2-2 & 9-10 \end{bmatrix} R_3-2R_2$

$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 5 \\ 0 & 0 & -1 \end{bmatrix}$

Required echelon form

iii). $\begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & 2 \\ 4 & 1 & 7 \end{bmatrix}$ Reduce echelon form

Sol: $\begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & 2 \\ 4 & 1 & 7 \end{bmatrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & 2 \\ 2 & -3 & 1 \\ 4 & 1 & 7 \end{bmatrix} \quad R_2 \leftrightarrow R_1$

$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & 2 \\ 2-2 & -3-2 & 1-4 \\ 4-4 & 1-4 & 7-8 \end{bmatrix} \begin{matrix} R_2-2R_1 \\ R_3-4R_1 \end{matrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & 2 \\ 0 & -5 & -3 \\ 0 & -3 & -1 \end{bmatrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & 2 \\ 0 & -5+6 & -3+2 \\ 0 & -3 & -1 \end{bmatrix} R_2-2R_3$

$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & -3 & -1 \end{bmatrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 1-1 & 2+1 \\ 0 & 1 & -1 \\ 0 & -3+3 & -1-3 \end{bmatrix} \begin{matrix} R_3+3R_2 \\ R_1-R_2 \end{matrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -4 \end{bmatrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \frac{-1}{4}R_3$

$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 3-3 \\ 0 & 1 & -1+1 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} R_1-3R_3 \\ R_2+R_3 \end{matrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Required reduce echelon form

iv). $\begin{bmatrix} 0 & 2 & 3 \\ 3 & -4 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ Reduce echelon form

Sol: $\begin{bmatrix} 0 & 2 & 3 \\ 3 & -4 & 1 \\ 1 & -1 & 2 \end{bmatrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & -1 & 2 \\ 3 & -4 & 1 \\ 0 & 2 & 3 \end{bmatrix} \quad R_3 \leftrightarrow R_1$

$\underset{\sim}{R}\begin{bmatrix} 1 & -1 & 2 \\ 3-3 & -4+3 & 1-6 \\ 0 & 2 & 3 \end{bmatrix} R_2-3R_1$

$\underset{\sim}{R}\begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & -5 \\ 0 & 2 & 3 \end{bmatrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 5 \\ 0 & 2 & 3 \end{bmatrix} \quad -R_2$

$\underset{\sim}{R}\begin{bmatrix} 1 & -1+1 & 2+5 \\ 0 & 1 & 5 \\ 0 & 2-2 & 3-10 \end{bmatrix} \begin{matrix} R_1+R_2 \\ R_3-2R_2 \end{matrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 5 \\ 0 & 0 & -7 \end{bmatrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \quad \frac{-1}{7}R_3$

$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 7-7 \\ 0 & 1 & 5-5 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} R_1-7R_3 \\ R_2-5R_3 \end{matrix}$

$\underset{\sim}{R}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Required reduce echelon form

Q6. Find the inverse of the following matrices by using elementary row and column operations.

i). $\begin{bmatrix} 4 & -2 & 5 \\ 2 & 1 & 0 \\ -1 & 2 & 3 \end{bmatrix}$

Sol: Let $A=\begin{bmatrix} 4 & -2 & 5 \\ 2 & 1 & 0 \\ -1 & 2 & 3 \end{bmatrix}$

$$|A|=\begin{vmatrix} 4 & -2 & 5 \\ 2 & 1 & 0 \\ -1 & 2 & 3 \end{vmatrix}$$

$$|A|=4\begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix}-(-2)\begin{vmatrix} 2 & 0 \\ -1 & 3 \end{vmatrix}+5\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}$$

$$|A|=4(3-0)+2(6+0)+5(4+1)$$

$$|A|=12+12+25=49\neq 0$$

So A is non singular so A^{-1} exists

$$Now[A/I]=\left[\begin{array}{ccc|ccc} 4 & -2 & 5 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ -1 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} -1 & 2 & 3 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 4 & -2 & 5 & 1 & 0 & 0 \end{array}\right] R_1\leftrightarrow R_3$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & -2 & -3 & 0 & 0 & -1 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 4 & -2 & 5 & 1 & 0 & 0 \end{array}\right] -R_1$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & -2 & -3 & 0 & 0 & -1 \\ 2-2 & 1+4 & 0+6 & 0 & 1 & 0+2 \\ 4-4 & -2+8 & 5+12 & 1 & 0 & 0+4 \end{array}\right] \begin{matrix} R_2-2R_1 \\ R_3-4R_1 \end{matrix}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & -2 & -3 & 0 & 0 & -1 \\ 0 & 5 & 6 & 0 & 1 & 2 \\ 0 & 6 & 17 & 1 & 0 & 4 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & -2 & -3 & 0 & 0 & -1 \\ 0 & 5 & 6 & 0 & 1 & 2 \\ 0 & 6-5 & 17-6 & 1 & 0-1 & 4-2 \end{array}\right] R_3-R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & -2 & -3 & 0 & 0 & -1 \\ 0 & 5 & 6 & 0 & 1 & 2 \\ 0 & 1 & 11 & 1 & -1 & 2 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & -2 & -3 & 0 & 0 & -1 \\ 0 & 1 & 11 & 1 & -1 & 2 \\ 0 & 5 & 6 & 0 & 1 & 2 \end{array}\right] R_3\leftrightarrow R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & -2+2 & -3+22 & 0+2 & 0-2 & -1+4 \\ 0 & 1 & 11 & 1 & -1 & 2 \\ 0 & 5-5 & 6-55 & 0-5 & 1+5 & 2-10 \end{array}\right] \begin{matrix} R_1+2R_2 \\ R_3-5R_2 \end{matrix}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 0 & 19 & 2 & -2 & 3 \\ 0 & 1 & 11 & 1 & -1 & 2 \\ 0 & 0 & -49 & -5 & 6 & -8 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 0 & 19 & 2 & -2 & 3 \\ 0 & 1 & 11 & 1 & -1 & 2 \\ 0 & 0 & 1 & 5/49 & -6/49 & 8/49 \end{array}\right] \frac{-1}{49}R_3$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 0 & 19-19 & 2-19\left(\frac{5}{49}\right) & -2-19\left(\frac{-6}{49}\right) & 3-19\left(\frac{8}{49}\right) \\ 0 & 1 & 11-11 & 1-11\left(\frac{5}{49}\right) & -1-11\left(\frac{-6}{49}\right) & 2-11\left(\frac{8}{49}\right) \\ 0 & 0 & 1 & 5/49 & -6/49 & 8/49 \end{array}\right] \begin{matrix} R_1-19R_3 \\ R_2-11R_3 \end{matrix}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/49 & 16/49 & -5/49 \\ 0 & 1 & 0 & -6/49 & 17/49 & 10/49 \\ 0 & 0 & 1 & 5/49 & -6/49 & 8/49 \end{array}\right]$$

$$\left[I/A^{-1}\right]\Rightarrow A^{-1}=\frac{1}{49}\begin{bmatrix} 3 & 16 & -5 \\ -6 & 17 & 10 \\ 5 & -6 & 8 \end{bmatrix}$$

---

ii). $\begin{bmatrix} 3 & -1 & 6 \\ 1 & 3 & 4 \\ -1 & 5 & 1 \end{bmatrix}$

Sol: Let $A=\begin{bmatrix} 3 & -1 & 6 \\ 1 & 3 & 4 \\ -1 & 5 & 1 \end{bmatrix}$

$$|A|=\begin{vmatrix} 3 & -1 & 6 \\ 1 & 3 & 4 \\ -1 & 5 & 1 \end{vmatrix}$$

$$|A|=3\begin{vmatrix} 3 & 4 \\ 5 & 1 \end{vmatrix}-(-1)\begin{vmatrix} 1 & 4 \\ -1 & 1 \end{vmatrix}+6\begin{vmatrix} 1 & 3 \\ -1 & 5 \end{vmatrix}$$

$$|A|=3(3-20)+(1+4)+6(5+3)$$

$$|A|=3(-17)+(5)+6(8)=-51+5+48=2\neq 0$$

So A is nonsingular so A^{-1} exists

$$Now[A/I]=\left[\begin{array}{ccc|ccc} 3 & -1 & 6 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 & 1 & 0 \\ -1 & 5 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 0 & 1 & 0 \\ 3 & -1 & 6 & 1 & 0 & 0 \\ -1 & 5 & 1 & 0 & 0 & 1 \end{array}\right] R_1\leftrightarrow R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 0 & 1 & 0 \\ 3-3 & -1-9 & 6-12 & 1 & 0-3 & 0 \\ -1+1 & 5+3 & 1+4 & 0 & 0+1 & 1 \end{array}\right] \begin{matrix} R_2-3R_1 \\ R_3+R_1 \end{matrix}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 0 & 1 & 0 \\ 0 & -10 & -6 & 1 & -3 & 0 \\ 0 & 8 & 5 & 0 & 1 & 1 \end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 0 & 1 & 0 \\ 0 & -10+8 & -6+5 & 1+0 & -3+1 & 0+1 \\ 0 & 8 & 5 & 0 & 1 & 1 \end{array}\right] R_2+R_3$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc}1 & 3 & 4 & 0 & 1 & 0\\ 0 & -2 & -1 & 1 & -2 & 1\\ 0 & 8 & 5 & 0 & 1 & 1\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc}1 & 3 & 4 & 0 & 1 & 0\\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 1 & -\frac{1}{2}\\ 0 & 8 & 5 & 0 & 1 & 1\end{array}\right]\frac{-1}{2}R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc}1 & 3-3 & 4-3\left(\frac{1}{2}\right) & 0-3\left(\frac{-1}{2}\right) & 1-3 & 0-3\left(\frac{-1}{2}\right)\\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 1 & -\frac{1}{2}\\ 0 & 8-8 & 5-8\left(\frac{1}{2}\right) & 0-8\left(\frac{-1}{2}\right) & 1-8 & 1-8\left(\frac{-1}{2}\right)\end{array}\right]\begin{matrix}R_1-3R_2\\ R_3-8R_2\end{matrix}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc}1 & 0 & 5/2 & 3/2 & -2 & 3/2\\ 0 & 1 & 1/2 & -1/2 & 1 & -1/2\\ 0 & 0 & 1 & 4 & -7 & 5\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc}1 & 0 & \frac{5}{2}-\frac{5}{2} & \frac{3}{2}-\frac{5}{2}(4) & -2-\frac{5}{2}(-7) & \frac{3}{2}-\frac{5}{2}(5)\\ 0 & 1 & \frac{1}{2}-\frac{1}{2} & \frac{-1}{2}-\frac{1}{2}(4) & 1-\frac{1}{2}(-7) & \frac{-1}{2}-\frac{1}{2}(5)\\ 0 & 0 & 1 & 4 & -7 & 5\end{array}\right]\begin{matrix}R_1-\frac{5}{2}R_3\\ R_2-\frac{1}{2}R_3\end{matrix}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|ccc}1 & 0 & 0 & -17/2 & 31/2 & -11\\ 0 & 1 & 0 & -5/2 & 9/2 & -3\\ 0 & 0 & 1 & 4 & -7 & 5\end{array}\right]$$

$$\left[I/A^{-1}\right]\Rightarrow A^{-1}=\frac{1}{2}\begin{bmatrix}-17 & 31 & -22\\ -5 & 9 & -6\\ 8 & -14 & 10\end{bmatrix}$$

Q7. Find ranks of each of the following matrices

i). $\begin{bmatrix}1 & 0 & -2\\ 2 & 2 & 1\\ -1 & 2 & 3\end{bmatrix}$

Sol: $\begin{bmatrix}1 & 0 & -2\\ 2 & 2 & 1\\ -1 & 2 & 3\end{bmatrix}$

$$\begin{bmatrix}1 & 0 & -2\\ 2-2 & 2-0 & 1+2\\ -1+1 & 2+0 & 3-2\end{bmatrix}\begin{matrix}R_2-2R_1\\ R_3+R_1\end{matrix}$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 0 & -2\\ 0 & 2 & 5\\ 0 & 2 & 1\end{bmatrix}$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 0 & -2\\ 0 & 2 & 5\\ 0 & 2-2 & 1-5\end{bmatrix}R_3-R_2$$

$$\underset{\sim}{R}\begin{bmatrix}1 & 0 & -2\\ 0 & 2 & 5\\ 0 & 0 & -4\end{bmatrix}$$

Number of nonzero rows are 3

∴Rank of the matrix = 3

ii). $\begin{bmatrix}3 & 1 & -4\\ 0 & 2 & 1\\ 1 & -1 & -2\end{bmatrix}$

Sol: $\begin{bmatrix}3 & 1 & -4\\ 0 & 2 & 1\\ 1 & -1 & -2\end{bmatrix}$

$$\underset{\sim}{R}\begin{bmatrix}1 & -1 & -2\\ 0 & 2 & 1\\ 3 & 1 & -4\end{bmatrix}R_1\leftrightarrow R_3$$

$$\underset{\sim}{R}\begin{bmatrix}1 & -1 & -2\\ 0 & 2 & 1\\ 3-3 & 1+3 & -4+6\end{bmatrix}R_3-3R_1$$

$$\underset{\sim}{R}\begin{bmatrix}1 & -1 & -2\\ 0 & 2 & 1\\ 0 & 4 & 2\end{bmatrix}$$

$$\underset{\sim}{R}\begin{bmatrix}1 & -1 & -2\\ 0 & 2 & 1\\ 0 & 4-4 & 2-2\end{bmatrix}R_3-2R_2$$

$$\underset{\sim}{R}\begin{bmatrix}1 & -1 & -2\\ 0 & 2 & 1\\ 0 & 0 & 0\end{bmatrix}$$

Number of nonzero rows are 2

∴Rank of the matrix = 2

Exercise 2.5

Q1 Solve following system of eqs by matrix method.

$$4x-3y+z=11$$

i). $2x+y-4z=-1$

$$x+2y-2z=1$$

Sol: System of eqs can be written as in matrix form

$$\begin{bmatrix}4 & -3 & 1\\ 2 & 1 & -4\\ 1 & 2 & -2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}11\\ -1\\ 1\end{bmatrix}$$

Let AX=B, $\Rightarrow X=A^{-1}B....(1)$ where

$$A=\begin{bmatrix}4 & -3 & 1\\ 2 & 1 & -4\\ 1 & 2 & -2\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}11\\ -1\\ 1\end{bmatrix}$$

Now $|A|=\begin{vmatrix}4 & -3 & 1\\ 2 & 1 & -4\\ 1 & 2 & -2\end{vmatrix}$

$$|A|=4\begin{vmatrix}1 & -4\\ 2 & -2\end{vmatrix}-(-3)\begin{vmatrix}2 & -4\\ 1 & -2\end{vmatrix}+1\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix}$$

$$|A|=4(-2+8)+3(-4+4)+(4-1)$$

$$|A|=4(6)+3(0)+(3)=27\neq 0$$

So A^{-1} exists

$$A_{11}=(-1)^{1+1}\begin{vmatrix}1 & -4\\ 2 & -2\end{vmatrix}=+(-2+8)=6$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}2 & -4\\ 1 & -2\end{vmatrix}=-(-4+4)=0$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix}=+(4-1)=3$$

$$A_{21}=(-1)^{2+1}\begin{vmatrix}-3 & 1\\ 2 & -2\end{vmatrix}=-(6-2)=-4$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}4 & 1\\ 1 & -2\end{vmatrix}=+(-8-1)=-9$$

$$A_{23}=(-1)^{2+3}\begin{vmatrix}4 & -3\\ 1 & 2\end{vmatrix}=-(8+3)=-11$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix}-3 & 1\\ 1 & -4\end{vmatrix}=+(12-1)=11$$

$$A_{32}=(-1)^{3+2}\begin{vmatrix}4 & 1\\ 2 & -4\end{vmatrix}=-(-16-2)=18$$

$$A_{33}=(-1)^{3+3}\begin{vmatrix}4 & -3\\ 2 & 1\end{vmatrix}=+(4+6)=10$$

$$\text{adj } A=\begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix}$$

$$\text{adj } A=\begin{bmatrix}6 & -4 & 11\\ 0 & -9 & 18\\ 3 & -11 & 10\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\text{adj } A\Rightarrow A^{-1}=\frac{1}{27}\begin{bmatrix}6 & -4 & 11\\ 0 & -9 & 18\\ 3 & -11 & 10\end{bmatrix}$$

Putting in equation (1)

$$X=A^{-1}B=\frac{1}{27}\begin{bmatrix}6 & -4 & 11\\ 0 & -9 & 18\\ 3 & -11 & 10\end{bmatrix}\begin{bmatrix}11\\ -1\\ 1\end{bmatrix}$$

$$X=\frac{1}{27}\begin{bmatrix}66+4+11\\ 0+9+18\\ 33+11+10\end{bmatrix}=\frac{1}{27}\begin{bmatrix}81\\ 27\\ 54\end{bmatrix}=\begin{bmatrix}3\\ 1\\ 2\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ 1\\ 2\end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal

$\Rightarrow x=3, y=1, z=2$

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ii). $x+y+z=1$

$x+y-2z=3$

$2x+y+z=2$

Sol: System of eqs can be written as in matrix form

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & -2\\ 2 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 3\\ 2\end{bmatrix}$$

Let AX=B, $\Rightarrow X=A^{-1}B....(1)$ where

$$A=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & -2\\ 2 & 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}1\\ 3\\ 2\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & 1 & 1\\ 1 & 1 & -2\\ 2 & 1 & 1\end{vmatrix}$$

$$|A|=1\begin{vmatrix}1 & -2\\ 1 & 1\end{vmatrix}-1\begin{vmatrix}1 & -2\\ 2 & 1\end{vmatrix}+1\begin{vmatrix}1 & 1\\ 2 & 1\end{vmatrix}$$

$$|A|=(1+2)-(1+4)+(1-2)$$

$$|A|=(3)-(5)+(-1)=-3\neq 0$$

So A^{-1} exists

$$A_{11}=(-1)^{1+1}\begin{vmatrix}1 & -2\\ 1 & 1\end{vmatrix}=+(1+2)=3$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}1 & -2\\ 2 & 1\end{vmatrix}=-(1+4)=-5$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}1 & 1\\ 2 & 1\end{vmatrix}=+(1-2)=-1$$

$$A_{21}=(-1)^{2+1}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix}=-(1-1)=0$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}1 & 1\\ 2 & 1\end{vmatrix}=+(1-2)=-1$$

$$A_{23}=(-1)^{2+3}\begin{vmatrix}1 & 1\\ 2 & 1\end{vmatrix}=-(1-2)=1$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix}1 & 1\\ 1 & -2\end{vmatrix}=+(-2-1)=-3$$

$$A_{32}=(-1)^{3+2}\begin{vmatrix}1 & 1\\ 1 & -2\end{vmatrix}=-(-2-1)=3$$

$$A_{33}=(-1)^{3+3}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix}=+(1-1)=0$$

$$\text{adj } A=\begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix}$$ putting the values

$$\text{adj } A=\begin{bmatrix}3 & 0 & -3\\ -5 & -1 & 3\\ -1 & 1 & 0\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\text{adj } A\Rightarrow A^{-1}=\frac{-1}{3}\begin{bmatrix}3 & 0 & -3\\ -5 & -1 & 3\\ -1 & 1 & 0\end{bmatrix}$$

Putting in equation (1)

$$X=A^{-1}B=\frac{-1}{3}\begin{bmatrix}3 & 0 & -3\\ -5 & -1 & 3\\ -1 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\ 3\\ 2\end{bmatrix}$$

$$X=\frac{-1}{3}\begin{bmatrix}3+0-6\\ -5-3+6\\ -1+3+0\end{bmatrix}=\frac{-1}{3}\begin{bmatrix}-3\\ -2\\ 2\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ \frac{2}{3}\\ \frac{-2}{3}\end{bmatrix}$$

By definition of equal matrices there corresponding elements are equal

$\Rightarrow x=1, y=\frac{2}{3}, z=\frac{-2}{3}$

Q2. Solve following system of equations by Gauss elimination method and Gauss Jordan method.

$x-y+4z=4$

i). $2x+2y-z=2$

$3x-2y+3z=-3$

Sol: The augmented matrix of the given system is

$$[A/B]=\left[\begin{array}{ccc|c}1&-1&4&4\\2&2&-1&2\\3&-2&3&-3\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-1&4&4\\2-2&2+2&-1-8&2-8\\3-3&-2+3&3-12&-3-12\end{array}\right]\begin{array}{l}R_2-2R_1\\R_3-3R_1\end{array}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-1&4&4\\0&4&-9&-6\\0&1&-9&-15\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-1&4&4\\0&1&-9&-15\\0&4&-9&-6\end{array}\right]R_3\leftrightarrow R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-1&4&4\\0&1&-9&-15\\0&4-4&-9+36&-6+60\end{array}\right]R_3-4R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-1&4&4\\0&1&-9&-15\\0&0&27&54\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-1&4&4\\0&1&-9&-15\\0&0&1&2\end{array}\right]\frac{1}{27}R_3$$

The required system in row echelon form is

$$\begin{array}{lll}x-y+4z=4 & & x-y+4z=4....(1)\\0x+y-9z=-15 & or & y-9z=-15....(2)\\0x+0y+z=2 & & z=2....(3)\end{array}$$

Put z=2 in equation (2)

$y-9(2)=-15\Rightarrow y=-15+18=3$

Put z=2,y=3 in equation (1)

$x-3+4(2)=4$

$x-3+8=4$

$x+5=4$

$x=4-5=-1$

Thus $x=-1, y=3, z=2$

Now for Reduce echelon form or Gauss Jardan method

The augmented matrix of the given system is

$$[A/B]=\left[\begin{array}{ccc|c}1&-1&4&4\\2&2&-1&2\\3&-2&3&-3\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-1&4&4\\2-2&2+2&-1-8&2-8\\3-3&-2+3&3-12&-3-12\end{array}\right]\begin{array}{l}R_2-2R_1\\R_3-3R_1\end{array}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-1&4&4\\0&4&-9&-6\\0&1&-9&-15\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-1&4&4\\0&1&-9&-15\\0&4&-9&-6\end{array}\right]R_3\leftrightarrow R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1+0&-1+1&4-9&4-15\\0&1&-9&-15\\0&4-4&-9+36&-6+60\end{array}\right]\begin{array}{l}R_1+R_2\\R_3-4R_2\end{array}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&0&-5&-11\\0&1&-9&-15\\0&0&27&54\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&0&-5&-11\\0&1&-9&-15\\0&0&1&2\end{array}\right]\frac{1}{27}R_3$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&0&-5+5&-11+10\\0&1&-9+9&-15+18\\0&0&1&2\end{array}\right]\begin{array}{l}R_2+9R_3\\R_1+5R_3\end{array}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&0&0&-1\\0&1&0&3\\0&0&1&2\end{array}\right]$$

The required system in row reduce echelon form is,

$$\begin{array}{lll}x+0y+0z=-1 & & x=-1\\0x+y+0z=3 & or & y=3\\0x+0y+z=2 & & z=2\end{array}$$

Thus $x=-1, y=3, z=2$

$$\begin{array}{ll} & 2x+4y-z=0\\ \text{ii).} & x-2y-2z=2\\ & -5x-8y+3z=-2\end{array}$$

Sol: The augmented matrix of the given system is

$$[A/B]=\left[\begin{array}{ccc|c}2&4&-1&0\\1&-2&-2&2\\-5&-8&3&-2\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-2&-2&2\\2&4&-1&0\\-5&-8&3&-2\end{array}\right]R_1\leftrightarrow R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-2&-2&2\\2-2&4+4&-1+4&0-4\\-5+5&-8-10&3-10&-2+10\end{array}\right]\begin{array}{l}R_2-2R_1\\R_3+5R_1\end{array}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-2&-2&2\\0&8&3&-4\\0&-18&-7&8\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-2&-2&2\\0&1&3/8&-1/2\\0&-18&-7&8\end{array}\right]\frac{1}{8}R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1&-2&-2&2\\0&1&3/8&-1/2\\0&-18+18&-7+18\left(\frac{3}{8}\right)&8+18\left(\frac{-1}{2}\right)\end{array}\right]R_3+18R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1 & -2 & -2 & 2\\ 0 & 1 & 3/8 & -1/2\\ 0 & 0 & -1/4 & -1\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1 & -2 & -2 & 2\\ 0 & 1 & 3/8 & -1/2\\ 0 & 0 & 1 & 4\end{array}\right] -4R_3$$

The required system in row echelon form is

$$x-2y-2z=2 \qquad x-2y-2z=2....(1)$$

$$0x+y+\frac{3}{8}z=-\frac{1}{2} \quad or \quad y+\frac{3}{8}z=-\frac{1}{2}....(2)$$

$$0x+0y+z=4 \qquad z=4....(3)$$

Put z=4 in equation (2)

$$y+\frac{3}{8}(4)=-\frac{1}{2}\Rightarrow y=-\frac{1}{2}-\frac{3}{2}=-2$$

Put z=4,y=-2 in equation (1)

$$x-2(-2)-2(4)=2$$

$$x+4-8=2$$

$$x-4=2$$

$$x=2+4=6$$

Thus $x=6, y=-2, z=4$

Now for Reduce echelon form or Gauss Jardan method

The augmented matrix of the given system is

$$[A/B]=\left[\begin{array}{ccc|c}2 & 4 & -1 & 0\\ 1 & -2 & -2 & 2\\ -5 & -8 & 3 & -2\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1 & -2 & -2 & 2\\ 2 & 4 & -1 & 0\\ -5 & -8 & 3 & -2\end{array}\right] R_1\leftrightarrow R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1 & -2 & -2 & 2\\ 2-2 & 4+4 & -1+4 & 0-4\\ -5+5 & -8-10 & 3-10 & -2+10\end{array}\right] \begin{array}{l}R_2-2R_1\\ R_3+5R_1\end{array}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1 & -2 & -2 & 2\\ 0 & 8 & 3 & -4\\ 0 & -18 & -7 & 8\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1 & -2 & -2 & 2\\ 0 & 1 & 3/8 & -1/2\\ 0 & -18 & -7 & 8\end{array}\right] \frac{1}{8}R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1 & -2+2 & -2+2\left(\frac{3}{8}\right) & 2+2\left(\frac{-1}{2}\right)\\ 0 & 1 & 3/8 & -1/2\\ 0 & -18+18 & -7+18\left(\frac{3}{8}\right) & 8+18\left(\frac{-1}{2}\right)\end{array}\right]$$

R_1+2R_2

R_3+18R_2

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1 & 0 & -5/4 & 1\\ 0 & 1 & 3/8 & -1/2\\ 0 & 0 & -1/4 & -1\end{array}\right]$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1 & 0 & -5/4 & 1\\ 0 & 1 & 3/8 & -1/2\\ 0 & 0 & 1 & 4\end{array}\right] -4R_3$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1 & 0 & \frac{-5}{4}+\frac{5}{4} & 1+\frac{5}{4}(4)\\ 0 & 1 & \frac{3}{8}-\frac{3}{8} & \frac{-1}{2}-\frac{3}{8}(4)\\ 0 & 0 & 1 & 4\end{array}\right] \begin{array}{l}R_1+\frac{5}{4}R_3\\ R_2-\frac{3}{8}R_3\end{array}$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c}1 & 0 & 0 & 6\\ 0 & 1 & 0 & -2\\ 0 & 0 & 1 & 4\end{array}\right]$$

The required system in row reduce echelon form is,

$$x+0y+0z=6 \qquad x=6$$

$$0x+y+0z=-2 \quad or \quad y=-2$$

$$0x+0y+z=4 \qquad z=4$$

Thus $x=6, y=-2, z=4$

Q3. Use Cramer's rule to solve the following system of equations

$$x-2y=-4$$

i). $3x-y=-5$

$$2x+z=-1$$

Sol: System of eqs can be written as in matrix form

$$\begin{bmatrix}1 & -2 & 0\\ 3 & 1 & 0\\ 2 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-4\\ -5\\ -1\end{bmatrix}$$

Let AX=B

$$A=\begin{bmatrix}1 & -2 & 0\\ 3 & 1 & 0\\ 2 & 0 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}-4\\ -5\\ -1\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & -2 & 0\\ 3 & 1 & 0\\ 2 & 0 & 1\end{vmatrix}$$

Expanding by C_3

$$|A|=0-0+1\begin{vmatrix}1 & -2\\ 3 & 1\end{vmatrix}$$

$$|A|=0+1(1+6)$$

$$|A|=7\neq 0$$

∴Solution exists

$$|A_X|=\begin{vmatrix}-4 & -2 & 0\\ -5 & 1 & 0\\ -1 & 0 & 1\end{vmatrix}$$

Expanding by C_3

$$|A_X|=0-0+1\begin{vmatrix}-4 & -2\\ -5 & 1\end{vmatrix}$$

$$|A_X|=0+1(-4-10)$$

$$|A_X|=-14$$

$$|A_Y|=\begin{vmatrix}1 & -4 & 0\\ 3 & -5 & 0\\ 2 & -1 & 1\end{vmatrix}$$

Expanding by C_3

$$|A_Y| = 0-0+1\begin{vmatrix}1 & -4\\3 & -5\end{vmatrix}$$

$$|A_Y| = 0+1(-5+12)$$

$$|A_Y| = 7$$

$$|A_Z| = \begin{vmatrix}1 & -2 & -4\\3 & 1 & -5\\2 & 0 & -1\end{vmatrix}$$

Expanding by R_3

$$|A_Z| = 2\begin{vmatrix}-2 & -4\\1 & -5\end{vmatrix} - 0 + (-1)\begin{vmatrix}1 & -2\\3 & 1\end{vmatrix}$$

$$|A_Z| = 2(10+4)-1(1+6)$$

$$|A_Z| = 2(14)-(7) = 28-7 = 21$$

$$\therefore x = \frac{|A_X|}{|A|} \qquad \therefore y = \frac{|A_Y|}{|A|} \qquad \therefore z = \frac{|A_Z|}{|A|}$$

$$x = \frac{-14}{7} \qquad y = \frac{7}{7} \qquad z = \frac{21}{7}$$

$$\Rightarrow x = -2 \qquad y = 1 \qquad z = 3$$

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ii).
$$x-y+2z=10$$
$$2x+y-2z=-4$$
$$3x+y+z=7$$

Sol: System of equations can be written as in matrix form

$$\begin{bmatrix}1 & -1 & 2\\2 & 1 & -2\\3 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}10\\-4\\7\end{bmatrix}$$

Let AX=B

$$A = \begin{bmatrix}1 & -1 & 2\\2 & 1 & -2\\3 & 1 & 1\end{bmatrix}, X = \begin{bmatrix}x\\y\\z\end{bmatrix}, B = \begin{bmatrix}10\\-4\\7\end{bmatrix}$$

$$|A| = \begin{vmatrix}1 & -1 & 2\\2 & 1 & -2\\3 & 1 & 1\end{vmatrix}$$

Expanding by R_1

$$|A| = 1\begin{vmatrix}1 & -2\\1 & 1\end{vmatrix} - (-1)\begin{vmatrix}2 & -2\\3 & 1\end{vmatrix} + 2\begin{vmatrix}2 & 1\\3 & 1\end{vmatrix}$$

$$|A| = 1(1+2)+(2+6)+2(2-3)$$

$$|A| = 3+8-2 = 9 \neq 0$$

∴Solution exists

$$|A_X| = \begin{vmatrix}10 & -1 & 2\\-4 & 1 & -2\\7 & 1 & 1\end{vmatrix}$$

Expanding by R_1

$$|A_X| = 10\begin{vmatrix}1 & -2\\1 & 1\end{vmatrix} - (-1)\begin{vmatrix}-4 & -2\\7 & 1\end{vmatrix} + 2\begin{vmatrix}-4 & 1\\7 & 1\end{vmatrix}$$

$$|A_X| = 10(1+2)+1(-4+14)+2(-4-7)$$

$$|A_X| = 30+10-22 = 18$$

$$|A_Y| = \begin{vmatrix}1 & 10 & 2\\2 & -4 & -2\\3 & 7 & 1\end{vmatrix}$$

Expanding by R_1

$$|A_Y| = 1\begin{vmatrix}-4 & -2\\7 & 1\end{vmatrix} - 10\begin{vmatrix}2 & -2\\3 & 1\end{vmatrix} + 2\begin{vmatrix}2 & -4\\3 & 7\end{vmatrix}$$

$$|A_Y| = 1(-4+14)-10(2+6)+2(14+12)$$

$$|A_Y| = 10-80+52 = -18$$

$$|A_Z| = \begin{vmatrix}1 & -1 & 10\\2 & 1 & -4\\3 & 1 & 7\end{vmatrix}$$

Expanding by R_1

$$|A_Z| = 1\begin{vmatrix}1 & -4\\1 & 7\end{vmatrix} - (-1)\begin{vmatrix}2 & -4\\3 & 7\end{vmatrix} + 10\begin{vmatrix}2 & 1\\3 & 1\end{vmatrix}$$

$$|A_Z| = 1(7+4)+1(14+12)+10(2-3)$$

$$|A_Z| = 11+26-10 = 27$$

$$\therefore x = \frac{|A_X|}{|A|} \qquad \therefore y = \frac{|A_Y|}{|A|} \qquad \therefore z = \frac{|A_Z|}{|A|}$$

$$x = \frac{18}{9} \qquad y = \frac{-18}{9} \qquad z = \frac{27}{9}$$

$$\Rightarrow x = 2 \qquad y = -2 \qquad z = 3$$

Q4. Solve the following homogeneous equations

i).
$$x_1 - x_2 + x_3 = 0..........(1)$$
$$x_1 + 2x_2 - x_3 = 0..........(2)$$
$$2x_1 + x_2 + 3x_3 = 0..........(3)$$

Sol: System of equations can be written as in matrix form

$$\begin{bmatrix}1 & -1 & 1\\1 & 2 & -1\\2 & 1 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

Let AX=0

$$|A| = \begin{vmatrix}1 & -1 & 1\\1 & 2 & -1\\2 & 1 & 3\end{vmatrix}$$

$$|A| = 1\begin{vmatrix}2 & -1\\1 & 3\end{vmatrix} - (-1)\begin{vmatrix}1 & -1\\2 & 3\end{vmatrix} + 1\begin{vmatrix}1 & 2\\2 & 1\end{vmatrix}$$

$$|A| = 1(6+1)+1(3+2)+1(1-4)$$

$$|A| = 7+5-3 = 9 \neq 0$$

So, the system has trivial solution

$$eq(1) \Rightarrow \quad x_1 - x_2 + x_3 = 0$$

$$eq(2) \Rightarrow \quad \underline{x_1 + 2x_2 - x_3 = 0} \qquad adding$$

$$2x_1 + x_2 = 0.......(4)$$

$$\times eq(2)\, by 3 \Rightarrow 3x_1 + 6x_2 - 3x_3 = 0$$

$$eq(3) \Rightarrow \quad \underline{2x_1 + x_2 + 3x_3 = 0} \qquad \text{adding}$$

$$5x_1 + 7x_2 = 0.......(5)$$

$$\times eq(4)\, by 7 \Rightarrow 14x_1 + 7x_2 = 0$$

$$eq(3) \Rightarrow \quad \underline{\pm 5x_1 \pm 7x_2 = 0} \quad \text{subtracting}$$

$$9x_1 = 0 \Rightarrow x_1 = 0$$

Put $x_1=0$ in eq (4) we get $x_2=0$

Put $x_1=0$ and $x_2=0$ in eq (1) we get $x_3=0$

Thus $x_1=0$, $x_2=0$, $x_3=0$

$$\begin{aligned} &x_1+x_2+2x_3=0 \\ \text{ii).}\ &-2x_1+x_2-x_3=0 \\ &-x_1+5x_2+4x_3=0 \end{aligned}$$

Sol: System of equations can be written as in matrix form

$$\begin{bmatrix} 1 & 1 & 2 \\ -2 & 1 & -1 \\ -1 & 5 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Let AX=0

$$|A|=\begin{vmatrix} 1 & 1 & 2 \\ -2 & 1 & -1 \\ -1 & 5 & 4 \end{vmatrix}$$

$$|A|=1\begin{vmatrix} 1 & -1 \\ 5 & 4 \end{vmatrix}-1\begin{vmatrix} -2 & -1 \\ -1 & 4 \end{vmatrix}+2\begin{vmatrix} -2 & 1 \\ -1 & 5 \end{vmatrix}$$

$$|A|=1(4+5)-1(-8-1)+2(-10+1)$$

$$|A|=9+9-18=0$$

So, the system has non trivial solution

$$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & 2 \\ -2 & 1 & -1 \\ -1 & 5 & 4 \end{bmatrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 3 \\ 0 & 6 & 6 \end{bmatrix}\begin{matrix} R_2+2R_1 \\ R_3+R_1 \end{matrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{matrix} \frac{1}{2}R_2 \\ \frac{1}{2}R_3 \end{matrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}R_3-R_2$$

The required system

$$x_1+x_2+2x_3=0.....(1)$$

$$0x_1+x_2+x_3=0.......(2)$$

$$0x_1+0x_2+0x_3=0....(3)$$

$$x_1+x_2+2x_3=0.....(1)$$

Or $x_2+x_3=0.......(2)$

$0=0 \quad true$

Put an ordinary value $x_3=t$ then

$x_2=-t$

And putting both $x_3=t$, $x_2=-t$ in eq (1)

We get $x_1=-t$

Thus $x_1=-t$, $x_2=-t$, $x_3=t$

Q5. For what value of λ the following system of homogeneous equations has a non trivial solution. Solve the system.

$$x_1+5x_2+3x_3=0..........(1)$$

$$5x_1+x_2-\lambda x_3=0..........(2)$$

$$x_1+2x_2+\lambda x_3=0..........(3)$$

Sol: System of equations can be written as in matrix form

$$\begin{bmatrix} 1 & 5 & 3 \\ 5 & 1 & -\lambda \\ 1 & 2 & \lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}........(a)$$

Let AX=0

$$|A|=\begin{vmatrix} 1 & 5 & 3 \\ 5 & 1 & -\lambda \\ 1 & 2 & \lambda \end{vmatrix}$$

$$|A|=1\begin{vmatrix} 1 & -\lambda \\ 2 & \lambda \end{vmatrix}-5\begin{vmatrix} 5 & -\lambda \\ 1 & \lambda \end{vmatrix}+3\begin{vmatrix} 5 & 1 \\ 1 & 2 \end{vmatrix}$$

$$|A|=1(\lambda+2\lambda)-5(5\lambda+\lambda)+3(10-1)$$

$$|A|=3\lambda-30\lambda+27=0$$

$$|A|=-27\lambda+27=0 \Rightarrow \lambda=1$$

Put $\lambda=1$ in the given system (a)

$$\begin{bmatrix} 1 & 5 & 3 \\ 5 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}\underset{\sim}{R}\begin{bmatrix} 1 & 5 & 3 \\ 0 & -24 & -16 \\ 0 & -3 & -2 \end{bmatrix}\begin{matrix} R_2-5R_1 \\ R_3-R_1 \end{matrix}$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 5 & 3 \\ 0 & 0 & 0 \\ 0 & -3 & -2 \end{bmatrix}R_2-8R_3$$

$$\underset{\sim}{R}\begin{bmatrix} 1 & 5 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{bmatrix}R_2\leftrightarrow R_3$$

The required system

$$x_1+5x_2+3x_3=0..........(1)$$

$$0-3x_2-2x_3=0..........(2)$$

Here x_3 is the free variable $\therefore x_3=t$

Put in eq (2) $-3x_2-2t=0$ then $x_2=\dfrac{-2t}{3}$

Both values put in eq (1)

$$x_1+5\left(\frac{-2t}{3}\right)+3t=0$$

$$x_1=\frac{10t}{3}-3t=\frac{t}{3}$$

Thus $x_1=\dfrac{t}{3}$, $x_2=\dfrac{-2t}{3}$, $x_3=t$