Chapter 1

Complex Numbers

Exercise 1.1

Q1. Simplify the following?

i). i^{14}

Sol: Given i^{14}

$i^{14} = \left(i^2\right)^7$

$i^{14} = \left(-1\right)^7 \quad \because i^2 = -1$

$i^{14} = -1$

ii). $(-i)^{23}$

Sol: Given $(-i)^{23}$

$(-i)^{23} = (-1)^{23} i^{23}$

$(-i)^{23} = -1.i^{2\times 11+1}$

$(-i)^{23} = -1.\left(i^2\right)^{11} i^1$

$(-i)^{23} = -1.(-1)^{11} i \quad \because i^2 = -1$

$(-i)^{23} = +1.i$

$(-i)^{23} = i$

iii). i^{-9}

Sol: Given $i^{-9} = \frac{1}{i^9}$

$i^{-9} = \frac{1}{i^{2\times 4+1}}$

$i^{-9} = \frac{1}{\left(i^2\right)^4 .i}$

$i^{-9} = \frac{1}{(-1)^4 .i} \quad \because i^2 = -1$

$i^{-9} = \frac{1}{i}$

Multiply and dividing by i

$i^{-9} = \frac{1}{i}.\frac{i}{i}$

$i^{-9} = \frac{i}{-1} \quad \because i^2 = -1$

$i^{-9} = -i$

iv). $(-i)^{-98}$

Sol: Given $(-i)^{-98}$

$(-i)^{-98} = \frac{1}{(-1)^{98} i^{98}}$

$(-i)^{-98} = \frac{1}{+1.\left(i^2\right)^{49}}$

$(-i)^{-98} = \frac{1}{(-1)^{49}} \quad \because i^2 = -1$

$(-i)^{-98} = -1$

Q2. Add the following complex numbers:

i). $3(1+2i), -2(1-3i)$

Sol: Given $3(1+2i), -2(1-3i)$

$3(1+2i) + \{-2(1-3i)\} = 3+6i-2+6i$

$3(1+2i) + \{-2(1-3i)\} = 1+12i$

ii). $\frac{1}{2}-\frac{2}{3}i, \frac{1}{4}-\frac{1}{3}i$

Sol: Given $\frac{1}{2}-\frac{2}{3}i, \frac{1}{4}-\frac{1}{3}i$

$\left(\frac{1}{2}-\frac{2}{3}i\right)+\left(\frac{1}{4}-\frac{1}{3}i\right) = \frac{1}{2}-\frac{2}{3}i+\frac{1}{4}-\frac{1}{3}i$

$\left(\frac{1}{2}-\frac{2}{3}i\right)+\left(\frac{1}{4}-\frac{1}{3}i\right) = \frac{2}{2}\times\frac{1}{2}+\frac{1}{4}-\frac{2}{3}i-\frac{1}{3}i$

$\left(\frac{1}{2}-\frac{2}{3}i\right)+\left(\frac{1}{4}-\frac{1}{3}i\right) = \frac{2+1}{4}-\frac{2+1}{3}i$

$\left(\frac{1}{2}-\frac{2}{3}i\right)+\left(\frac{1}{4}-\frac{1}{3}i\right) = \frac{3}{4}-\frac{3}{3}i$

$\left(\frac{1}{2}-\frac{2}{3}i\right)+\left(\frac{1}{4}-\frac{1}{3}i\right) = \frac{3}{4}-i$

iii). $(\sqrt{2},1),(1,\sqrt{2})$

Sol: Given $(\sqrt{2},1),(1,\sqrt{2})$

$(\sqrt{2},1)+(1,\sqrt{2}) = \sqrt{2}+1i+1+\sqrt{2}\,i$

$(\sqrt{2},1)+(1,\sqrt{2}) = 1+\sqrt{2}+1i+\sqrt{2}\,i$

$(\sqrt{2},1)+(1,\sqrt{2}) = 1(1+\sqrt{2})+i(1+\sqrt{2})$

$(\sqrt{2},1)+(1,\sqrt{2}) = (1+\sqrt{2})(1+i)$

Q3. Subtract the following complex numbers.

1). $3\sqrt{3}-5\sqrt{7}i, \sqrt{3}+2\sqrt{7}i$

Solution: we have $\left(3\sqrt{3}-5\sqrt{7}i\right),\left(\sqrt{3}+2\sqrt{7}i\right)$

$\left(3\sqrt{3}-5\sqrt{7}i\right)-\left(\sqrt{3}+2\sqrt{7}i\right) = 3\sqrt{3}-5\sqrt{7}i-\sqrt{3}-2\sqrt{7}i$

$\left(3\sqrt{3}-5\sqrt{7}i\right)-\left(\sqrt{3}+2\sqrt{7}i\right) = 3\sqrt{3}-\sqrt{3}-5\sqrt{7}i-2\sqrt{7}i$

$\left(3\sqrt{3}-5\sqrt{7}i\right)-\left(\sqrt{3}+2\sqrt{7}i\right) = 2\sqrt{3}-7\sqrt{7}i$

2). $\left(-3+\frac{1}{2}i\right),\left(3+\frac{1}{2}i\right)$

Sol: Given $\left(-3+\frac{1}{2}i\right),\left(3+\frac{1}{2}i\right)$

$\left(-3+\frac{1}{2}i\right)-\left(3+\frac{1}{2}i\right) = -3+0.5i-3-0.5i$

$\left(-3+\frac{1}{2}i\right)-\left(3+\frac{1}{2}i\right) = -3-3+0.5i-0.5i$

$\left(-3+\frac{1}{2}i\right)-\left(3+\frac{1}{2}i\right) = -6+0.i$

3). $(a,0),(2,-b)$

Sol: Given $(a,0),(2,-b)$

$(a,0)-(2,-b) = (a+0.i)-(2-bi)$

$(a,0)-(2,-b) = a+0.i-2+bi$

$(a,0)-(2,-b) = (a-2)+bi$

Q4. Multiply the following complex numbers

i). $2i, \quad 3i$

Solution: we have $2i, 3i$

$2i\times 3i = 6i^2$

$2i\times 3i = 6(-1) \quad \because i^2 = -1$

$2i\times 3i = -6$

ii). $3i, \quad 2(1-i)$

Sol: Given $3i, \quad 2(1-i)$

$3i\times 2(1-i)=6i(1-i)$

$3i\times 2(1-i)=6i-6i^2$

$3i\times 2(1-i)=6i-6(-1) \qquad \because i^2=-1$

$3i\times 2(1-i)=6+6i$

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iii). $\sqrt{2}+\sqrt{3}i, \quad 2\sqrt{2}-\sqrt{3}i$

Sol: Given $\left(\sqrt{2}+\sqrt{3}i\right),\left(2\sqrt{2}-\sqrt{3}i\right)$

$\left(\sqrt{2}+\sqrt{3}i\right)\left(2\sqrt{2}-\sqrt{3}i\right)=\sqrt{2}\left(2\sqrt{2}-\sqrt{3}i\right)+\sqrt{3}i\left(2\sqrt{2}-\sqrt{3}i\right)$

$\left(\sqrt{2}+\sqrt{3}i\right)\left(2\sqrt{2}-\sqrt{3}i\right)=2\left(\sqrt{2}\right)^2-\sqrt{6}i+2\sqrt{6}i-\left(\sqrt{3}i\right)^2$

$\left(\sqrt{2}+\sqrt{3}i\right)\left(2\sqrt{2}-\sqrt{3}i\right)=4-3(-1)+\sqrt{6}i \qquad \because i^2=-1$

$\left(\sqrt{2}+\sqrt{3}i\right)\left(2\sqrt{2}-\sqrt{3}i\right)=7+\sqrt{6}\,i$

Q5. Perform the indicated division and write the answer in the form of $a+ib$

i). $\dfrac{1+i}{i}$

Solution: we have $\dfrac{1+i}{i}$

$\dfrac{1+i}{i}=\dfrac{1+i}{i}\times\dfrac{i}{i}$

$\dfrac{1+i}{i}=\dfrac{i+(-1)}{-1} \qquad \because i^2=-1$

$\dfrac{1+i}{i}=-1(-1+i)$

$\dfrac{1+i}{i}=1-i$

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ii). $\dfrac{13}{5-12i}$

Sol: Given $\dfrac{13}{5-12i}$

$\dfrac{13}{5-12i}=\dfrac{13}{5-12i}\times\dfrac{5+12i}{5+12i}$

$\dfrac{13}{5-12i}=\dfrac{13(5+12i)}{5^2-12^2i^2}$

$\dfrac{13}{5-12i}=\dfrac{13(5+12i)}{25-144(-1)} \qquad \because i^2=-1$

$\dfrac{13}{5-12i}=\dfrac{13(5+12i)}{25+144}$

$\dfrac{13}{5-12i}=\dfrac{13(5+12i)}{169}$

$\dfrac{13}{5-12i}=\dfrac{(5+12i)}{13}$

$\dfrac{13}{5-12i}=\dfrac{5}{13}+\dfrac{12}{13}i$

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iii). $\dfrac{4-3i}{4+3i}$

Sol: Given $\dfrac{4-3i}{4+3i}$

$\dfrac{4-3i}{4+3i}=\dfrac{4-3i}{4+3i}\times\dfrac{4-3i}{4-3i}$

$\dfrac{4-3i}{4+3i}=\dfrac{(4-3i)^2}{4^2-3^2i^2}$

$\dfrac{4-3i}{4+3i}=\dfrac{4^2-3^2-24i}{16-9(-1)} \qquad \because i^2=-1$

$\dfrac{4-3i}{4+3i}=\dfrac{16-9-24i}{16+9}$

$\dfrac{4-3i}{4+3i}=\dfrac{7}{25}-\dfrac{24}{25}i$

Q6. Prove that the sum as well as Product of complex numbers and its conjugate is a real number.

Solution: First we have to Show that the sum of complex numbers and its conjugate is a real number. Let $z=a+ib, \quad \bar{z}=a-ib$

Then $z+\bar{z}=2a$ or $\bar{z}+z=2a$

Now we have to Show that the Product of complex numbers and its conjugate is a real number.

Let $z=a+ib, \quad \bar{z}=a-ib$

Then $z.\bar{z}=(a+ib)(a-ib)$

$z.\bar{z}=a^2-iab+iab+b^2$

$z.\bar{z}=a^2-b^2(-1) \qquad \because i^2=-1$

$z.\bar{z}=a^2-b^2$

Similarly $\bar{z}.z=(a-ib)(a+ib)$

$\bar{z}.z=a^2-iab+iab-b^2(-1) \qquad \because i^2=-1$

$\bar{z}.z=a^2+b^2$

Q7. If $z_1=1+2i, \quad z_2=2+3i$ evaluate

i). $|z_1+z_2|$

Sol: Given $|z_1+z_2|=|1+2i+2+3i|$

$|z_1+z_2|=|1+2+2i+3i|$

$|z_1+z_2|=|3+5i|$

$|z_1+z_2|=\sqrt{3^2+5^2}$

$|z_1+z_2|=\sqrt{34}$

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ii). $|z_1.z_2|$

Sol: Given $|z_1.z_2|=|(1+2i).(2+3i)|$

$|z_1.z_2|=|2+3i+4i+6i^2|$

$|z_1.z_2|=|2-6+7i|$

$|z_1.z_2|=|-4+7i|$

$|z_1.z_2|=\sqrt{4^2+7^2}$

$|z_1.z_2|=\sqrt{16+49}$

$|z_1.z_2|=\sqrt{65}$

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iii) $\left|\dfrac{z_1}{z_2}\right|$

Sol: Given $\left|\frac{z_1}{z_2}\right|=\left|\frac{1+2i}{2+3i}\right|$

$\left|\frac{z_1}{z_2}\right|=\left|\frac{1+2i}{2+3i}\times\frac{2-3i}{2-3i}\right|$

$\left|\frac{z_1}{z_2}\right|=\left|\frac{1+2i}{2+3i}\times\frac{2-3i}{2-3i}\right|$

$\left|\frac{z_1}{z_2}\right|=\left|\frac{2-3i+4i-6i^2}{2^2-3^2(-1)}\right| \quad \because i^2=-1$

$\left|\frac{z_1}{z_2}\right|=\left|\frac{2-6(-1)+i}{4+9}\right| \quad \because i^2=-1$

$\left|\frac{z_1}{z_2}\right|=\left|\frac{8}{13}+\frac{i}{13}\right|$

$\left|\frac{z_1}{z_2}\right|=\sqrt{\left(\frac{8}{13}\right)^2+\left(\frac{1}{13}\right)^2}$

$\left|\frac{z_1}{z_2}\right|=\sqrt{\frac{64}{169}+\frac{1}{169}}$

$\left|\frac{z_1}{z_2}\right|=\sqrt{\frac{65}{169}}$

$\left|\frac{z_1}{z_2}\right|=\frac{\sqrt{65}}{13}$

Q8. Separate into real and imaginary parts

i). $\frac{2+3i}{5-2i}$

Solution: we have $\frac{2+3i}{5-2i}$

$\frac{2+3i}{5-2i}=\frac{2+3i}{5-2i}\times\frac{5+2i}{5+2i}$

$\frac{2+3i}{5-2i}=\frac{10+4i+15i+6i^2}{5^2-2^2(-1)} \quad \because i^2=-1$

$\frac{2+3i}{5-2i}=\frac{10+6(-1)+19i}{25+4}$

$\frac{2+3i}{5-2i}=\frac{10-6+19i}{25+4}$

$\frac{2+3i}{5-2i}=\frac{4}{29}+\frac{19}{29}i$

Real part $x=\frac{4}{29}$ imaginary part $y=\frac{19}{29}$

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ii). $\frac{(1+2i)^2}{1-3i}$

Solution: we have $\frac{(1+2i)^2}{1-3i}$

$\frac{(1+2i)^2}{1-3i}=\frac{1+4(-1)+4i}{1-3i}\times\frac{1+3i}{1+3i}$

$\frac{(1+2i)^2}{1-3i}=\frac{-3+4i}{1-3i}\times\frac{1+3i}{1+3i}$

$\frac{(1+2i)^2}{1-3i}=\frac{-3-9i+4i+12i^2}{1^2+3^2}$

$\frac{(1+2i)^2}{1-3i}=\frac{-15}{10}-\frac{5}{10}i$

$\frac{(1+2i)^2}{1-3i}=\frac{-3}{2}-\frac{1}{2}i$

Real part $x=\frac{-3}{2}$ imaginary part $y=\frac{-1}{2}$

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iii). $\frac{1-i}{(1+i)^2}$

Sol: Given $\frac{1-i}{(1+i)^2}$

$\frac{1-i}{(1+i)^2}=\frac{1-i}{1+i^2+2i}$

$\frac{1-i}{(1+i)^2}=\frac{1-i}{2i}\times\frac{i}{i}$

$\frac{1-i}{(1+i)^2}=\frac{i-i^2}{2i^2}$

$\frac{1-i}{(1+i)^2}=\frac{-(-1)+i}{-2}$

$\frac{1-i}{(1+i)^2}=\frac{-1}{2}-\frac{i}{2}$

Real part $x=\frac{-1}{2}$ imaginary part $y=\frac{-1}{2}$

Exercise 1.2

Q1.i). If $z_1=1+i$, and $z_2=2+i$

Then verify that $z_1+z_2=z_2+z_1$

Solution; We have $z_1=1+i$, and $z_2=2+i$

take LHS

$z_1+z_2=1+i+2+i$

$z_1+z_2=1+2+i+i$

$z_1+z_2=3+2i$............................(1)

RHS

$z_2+z_1=2+i+1+i$

$z_2+z_1=2+1+i+i$

$z_2+z_1=3+2i$............................(2)

From (1) and (2) we get $z_1+z_2=z_2+z_1$

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ii). $z_1.z_2=z_2.z_1$

Solution; We have $z_1=1+i$, and $z_2=2+i$

take LHS $z_1z_2=(1+i).(2+i)$

$z_1z_2=2+i+2i-1$

$z_1z_2=1+3i$..................................(1)

Take RHS $z_2.z_1=(2+i).(1+i)$

$z_2z_1=2+2i+i-1$

$z_2z_1=2-1+2i+i$

$z_2z_1=1+3i$.............................(2)

From (1) and (2) we get $z_1.z_2=z_2.z_1$

Q2. Given that $z_1=-1-i, z_2=3+2i$ and $z_3=-2+3i$ then verify that

$z_1+(z_2+z_3)=(z_1+z_2)+z_3$

Sol: Given $z_1=-1-i, z_2=3+2i$ and $z_3=-2+3i$

i). LHS $z_1+(z_2+z_3)$

$z_1+(z_2+z_3)=-1-i+(3+2i-2+3i)$

$z_1+(z_2+z_3)=-1-i+(1+5i)$

$z_1+(z_2+z_3)=-1+1-i+5i$

$z_1+(z_2+z_3)=4i$............................(1)

RHS $\left(z_1+z_2\right)+z_3$

$\left(z_1+z_2\right)+z_3=\left(-1-i+3+2i\right)-2+3i$

$\left(z_1+z_2\right)+z_3=\left(2+i\right)+\left(-2+3i\right)$

$\left(z_1+z_2\right)+z_3=2-2+i+3i$

$\left(z_1+z_2\right)+z_3=4i$...............................(2)

From (1) and (2) we get LHS=RHS

$z_1+\left(z_2+z_3\right)=\left(z_1+z_2\right)+z_3$

Associative Law of Addition

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ii). $z_1.\left(z_2.z_3\right)=\left(z_1.z_2\right).z_3$

Solution: $z_1=-1-i, z_2=3+2i$ and $z_3=-2+3i$

Take LHS $z_1.\left(z_2.z_3\right)=\left(-1-i\right).\left\{\left(3+2i\right)\left(-2+3i\right)\right\}$

$z_1.\left(z_2.z_3\right)=\left(-1-i\right).\left\{-6+9i-4i+6\right\}$

$z_1.\left(z_2.z_3\right)=\left(-1-i\right).\left\{-6+9i-4i-6\right\}$

$z_1.\left(z_2.z_3\right)=\left(-1-i\right).\left(-12+5i\right)$

$z_1.\left(z_2.z_3\right)=12-5i+12i+5$

$z_1.\left(z_2.z_3\right)=17+7i$.....(1)

Now RHS $\left(z_1.z_2\right).z_3=\left\{\left(-1-i\right).\left(3+2i\right)\right\}\left(-2+3i\right)$

$\left(z_1.z_2\right).z_3=\left\{-3-2i-3i+2\right\}\left(-2+3i\right)$

$\left(z_1.z_2\right).z_3=\left(-1-5i\right)\left(-2+3i\right)$

$\left(z_1.z_2\right).z_3=2-3i+10i+15$

$\left(z_1.z_2\right).z_3=17+7i$......................(2)

From (1) and (3) we get $z_1.\left(z_2.z_3\right)=\left(z_1.z_2\right).z_3$

Associative Law of Multiplication

Q3 $z_1=\sqrt{3}+\sqrt{2}i, z_2=\sqrt{3}-\sqrt{2}i$ and $z_3=2-2i$

then verify that $z_1\left(z_2+z_3\right)=z_1.z_2+z_1.z_3$

Sol; $z_1=\sqrt{3}+\sqrt{2}i, z_2=\sqrt{3}-\sqrt{2}i$ and $z_3=2-2i$

Take RHS

$z_1.z_2+z_1.z_3=\left(\sqrt{3}+\sqrt{2}i\right)\left(\sqrt{3}-\sqrt{2}i\right)+\left(\sqrt{3}+\sqrt{2}i\right)\left(2-2i\right)$

$z_1.z_2+z_1.z_3=(3-\sqrt{6}i+\sqrt{6}i+2)+(2\sqrt{3}-2\sqrt{3}i+2\sqrt{2}i+2\sqrt{2})$

$z_1.z_2+z_1.z_3=5+2\sqrt{3}+2\sqrt{2}+2\sqrt{2}i-2\sqrt{3}i$.....(1)

Now LHS

$z_1\left(z_2+z_3\right)=\left(\sqrt{3}+\sqrt{2}i\right)\left(\sqrt{3}-\sqrt{2}i+2-2i\right)$

$z_1\left(z_2+z_3\right)=\left(\sqrt{3}+\sqrt{2}i\right)\left(2+\sqrt{3}-\sqrt{2}i-2i\right)$

$z_1\left(z_2+z_3\right)=2\sqrt{3}+3-\sqrt{6}i-2\sqrt{3}i+2\sqrt{2}i+\sqrt{6}i+2+2\sqrt{2}$

$z_1\left(z_2+z_3\right)=2\sqrt{3}+3+2+2\sqrt{2}-\sqrt{6}i-2\sqrt{3}i+2\sqrt{2}i+\sqrt{6}i$

$z_1\left(z_2+z_3\right)=5+2\sqrt{3}+2\sqrt{2}-2\sqrt{3}i+2\sqrt{2}i$.........(2)

From (1) and (2) we get the required result

Distributive Law of multiplication over addition

Q4 Find additive inverse of following complex numbers.

i). $2+3i$

Sol: Given $2+3i$

Let $a+ib$ is an additive inverse of $2+3i$

So by definition $a+ib+2+3i=0+0.i$

$a+2+i\left(b+3\right)=0+0.i$

Comparing real and imaginary parts we get

$a+2=0,\left(b+3\right)=0$

$a=-2, b=-3$ so $a+bi=-2-3i$

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ii). $2-3i$

Sol: Given $2-3i$

Additive inverse of $2-3i$ is $-2+3i$

Q5. Find the multiplicative inverse of the following complex numbers.

i). $\left(1+2i\right)^{-1}$

Sol: Given $\left(1+2i\right)^{-1}$

$\left(1+2i\right)^{-1}=\dfrac{1}{\left(1+2i\right)}$

$\left(1+2i\right)^{-1}=\dfrac{1}{1+2i}\times\dfrac{1-2i}{1-2i}$

$\left(1+2i\right)^{-1}=\dfrac{1-2i}{1-4\left(-1\right)}$

$\left(1+2i\right)^{-1}=\dfrac{1-2i}{1+4}$

$\left(1+2i\right)^{-1}=\dfrac{1}{5}-\dfrac{2}{5}i$

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ii). $\left(-1,2\right)^{-1}$

Sol: Given $\left(-1,2\right)^{-1}$

$\left(-1,2\right)^{-1}=\left(-1+2i\right)^{-1}$

$\left(-1,2\right)^{-1}=\dfrac{1}{-1+2i}\times\dfrac{-1-2i}{-1-2i}$

$\left(-1,2\right)^{-1}=\dfrac{-1-2i}{1-4\left(-1\right)}$

$\left(-1,2\right)^{-1}=\dfrac{-1-2i}{1+4}$

$\left(-1,2\right)^{-1}=\dfrac{-1}{5}-\dfrac{2}{5}i$

Q6. If $z_1=-3-\sqrt{3}i$ and $z_2=4+2i$ then verify that $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$

Solution; we have $z_1=-3-\sqrt{3}i$ and $z_2=4+2i$

For LHS

$z_1+z_2=-3-\sqrt{3}i+4+2i$

$z_1+z_2=-3+4-\sqrt{3}i+2i$

$z_1+z_2=1-\sqrt{3}i+2i$

now taking conjugate on both sides

$\overline{z_1+z_2}=\overline{1-\sqrt{3}i+2i}$

$\overline{z_1+z_2}=1+\sqrt{3}i-2i$....(1)

Now RHS

$\overline{z_1}+\overline{z_2}=\overline{-3-\sqrt{3}i}+\overline{4+2i}$

$\overline{z_1}+\overline{z_2}=-3+\sqrt{3}i+4-2i$

$\overline{z_1}+\overline{z_2}=1+\sqrt{3}i-2i$.....(2)

From (1) and (2) we get $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$

Q7. If $z_1 = -a - 2bi, z_2 = 2a + bi$ then verify that $\overline{z_1.z_2} = \overline{z_1}.\overline{z_2}$

Sol: Given $z_1 = -a - 2bi, z_2 = 2a + bi$

For LHS $z_1.z_2 = (-a-2bi)(2a+bi)$

$z_1.z_2 = -2a^2 - abi - 4abi + 2b^2$

$z_1.z_2 = 2b^2 - 2a^2 - abi - 4abi$

Now taking conjugate

$\overline{z_1.z_2} = \overline{2b^2 - 2a^2 - abi - 4abi}$

$\overline{z_1.z_2} = 2b^2 - 2a^2 + abi + 4abi$

$\overline{z_1.z_2} = 2b^2 - 2a^2 + 5abi$..............................(1)

$\overline{z_1}.\overline{z_2} = \overline{(-a-2bi)}.\overline{(2a+bi)}$

$\overline{z_1}.\overline{z_2} = (-a+2bi)(2a-bi)$

$\overline{z_1}.\overline{z_2} = -2a^2 + abi + 4abi - b^2(-1)$

$\overline{z_1}.\overline{z_2} = 2b^2 - 2a^2 + abi + 4abi$

$\overline{z_1}.\overline{z_2} = 2b^2 - 2a^2 + 5abi$..............................(2)

From (1) and (2) we get $\overline{z_1.z_2} = \overline{z_1}.\overline{z_2}$

Q8. If $\begin{matrix} z_1 = -a - 3bi, \\ z_2 = 2a - 3bi \end{matrix}$, then verify that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

Solution we have $z_1 = -a - 3bi, z_2 = 2a - 3bi$

Take LHS $\frac{z_1}{z_2} = \frac{-a-3bi}{2a-3bi}$

$\frac{z_1}{z_2} = \frac{-a-3bi}{2a-3bi} \times \frac{2a+3bi}{2a+3bi}$

$\frac{z_1}{z_2} = \frac{-2a^2 - 3abi - 6abi + 9b^2}{4a^2 + 9b^2}$

$\frac{z_1}{z_2} = \frac{9b^2 - 2a^2}{4a^2 + 9b^2} - \frac{3ab + 6ab}{4a^2 + 9b^2} i$

$\frac{z_1}{z_2} = \frac{9b^2 - 2a^2}{4a^2 + 9b^2} - \frac{9ab}{4a^2 + 9b^2} i$

$\overline{\left(\frac{z_1}{z_2}\right)} = \overline{\frac{9b^2 - 2a^2}{4a^2 + 9b^2} - \frac{9ab}{4a^2 + 9b^2} i}$

$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{9b^2 - 2a^2}{4a^2 + 9b^2} + \frac{9ab}{4a^2 + 9b^2} i$......(1)

RHS $\frac{\overline{z_1}}{\overline{z_2}} = \frac{-a+3bi}{2a+3bi}$

$\frac{\overline{z_1}}{\overline{z_2}} = \frac{-a+3bi}{2a+3bi} \times \frac{2a-3bi}{2a-3bi}$

$\frac{\overline{z_1}}{\overline{z_2}} = \frac{-2a^2 + 3abi + 6abi + 9b^2}{4a^2 + 9b^2}$

$\frac{\overline{z_1}}{\overline{z_2}} = \frac{9b^2 - 2a^2}{4a^2 + 9b^2} + \frac{3ab + 6ab}{4a^2 + 9b^2} i$

$\frac{\overline{z_1}}{\overline{z_2}} = \frac{9b^2 - 2a^2}{4a^2 + 9b^2} + \frac{9ab}{4a^2 + 9b^2} i$.........(2)

From (1) and (2) we get the required result

Q9. Show that for all complex number z_1 and z_2

i). $|z_1 z_2| = |z_1||z_2|$

Solution: Let $z_1 = a_1 + ib_1$, and $z_2 = a_2 + ib_2$

LHS $z_1.z_2 = (a_1 + ib_1)(a_2 + ib_2)$

$z_1.z_2 = a_1a_2 + ia_1b_2 + ia_2b_1 - b_1b_2$

$z_1.z_2 = (a_1a_2 - b_1b_2) + i(a_1b_2 + a_2b_1)$

Now $|z_1.z_2| = \sqrt{(a_1a_2 - b_1b_2)^2 + (a_1b_2 + a_2b_1)^2}$

$|z_1.z_2| = \sqrt{a_1^2a_2^2 + b_1^2b_2^2 - 2a_1a_2b_1b_2 + a_1^2b_2^2 + a_2^2b_1^2 + 2a_1a_2b_1b_2}$

$|z_1.z_2| = \sqrt{a_1^2(a_2^2 + b_2^2) + b_1^2(a_2^2 + b_2^2)}$

$|z_1.z_2| = \sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}$

$|z_1.z_2| = \sqrt{(a_1^2 + b_1^2)}\sqrt{(a_2^2 + b_2^2)}$

$|z_1.z_2| = |z_1||z_2|$

ii). $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

Solution: Let $z_1 = a_1 + ib_1$, and $z_2 = a_2 + ib_2$

LHS $\frac{z_1}{z_2} = \frac{a_1 + ib_1}{a_2 + ib_2}$

$\frac{z_1}{z_2} = \frac{a_1 + ib_1}{a_2 + ib_2} \times \frac{a_2 - ib_2}{a_2 - ib_2}$

$\frac{z_1}{z_2} = \frac{a_1a_2 - ia_1b_2 + ia_2b_1 + b_1b_2}{a_2^2 + b_2^2}$

$\frac{z_1}{z_2} = \frac{a_1a_2 + b_1b_2 + ia_2b_1 - ia_1b_2}{a_2^2 + b_2^2}$

$\frac{z_1}{z_2} = \frac{a_1a_2 + b_1b_2 + i(a_2b_1 - a_1b_2)}{a_2^2 + b_2^2}$

Now $\left|\frac{z_1}{z_2}\right| = \left|\frac{a_1a_2 + b_1b_2 + i(a_2b_1 - a_1b_2)}{a_2^2 + b_2^2}\right|$

$\left|\frac{z_1}{z_2}\right| = \frac{\sqrt{(a_1a_2 + b_1b_2)^2 + (a_2b_1 - a_1b_2)^2}}{a_2^2 + b_2^2}$

$\left|\frac{z_1}{z_2}\right| = \frac{\sqrt{a_1^2a_2^2 + b_1^2b_2^2 + 2a_1a_2b_1b_2 + a_1^2b_2^2 + a_2^2b_1^2 - 2a_1a_2b_1b_2}}{a_2^2 + b_2^2}$

$\left|\frac{z_1}{z_2}\right| = \frac{\sqrt{a_1^2a_2^2 + b_1^2b_2^2 + a_1^2b_2^2 + a_2^2b_1^2}}{a_2^2 + b_2^2}$

$\left|\frac{z_1}{z_2}\right| = \frac{\sqrt{a_1^2a_2^2 + a_1^2b_2^2 + b_1^2b_2^2 + a_2^2b_1^2}}{a_2^2 + b_2^2}$

$\left|\frac{z_1}{z_2}\right| = \frac{\sqrt{a_1^2(a_2^2 + b_2^2) + b_1^2(b_2^2 + a_2^2)}}{a_2^2 + b_2^2}$

$\left|\frac{z_1}{z_2}\right| = \frac{\sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}}{a_2^2 + b_2^2}$

$\left|\frac{z_1}{z_2}\right| = \sqrt{\frac{a_1^2 + b_1^2}{a_2^2 + b_2^2}}$

$\left|\frac{z_1}{z_2}\right| = \frac{\sqrt{a_1^2 + b_1^2}}{\sqrt{a_2^2 + b_2^2}} = \frac{|z_1|}{|z_2|}$

Q10. Separate into real and imaginary parts of the following complex numbers.

i). $2+3i$

Sol: Given $x+iy=2+3i$

$\Rightarrow x=2, y=3$

---

ii). $(3-2i)^2$

Sol: Given $x+iy=(3-2i)^2$

$x+iy=9-4-12i$

$x+iy=5-12i$

$\Rightarrow x=5, y=-12$

---

iii). $(3-4i)^{-1}$

Sol: Given $x+iy=(3-4i)^{-1}$

$x+iy=\frac{1}{3-4i}\times\frac{3+4i}{3+4i}$

$x+iy=\frac{3+4i}{9+16}$

$x+iy=\frac{3}{25}+\frac{4}{25}i$

$\Rightarrow x=\frac{3}{25}, \qquad y=\frac{4}{25}$

---

iv). $(2a-bi)^{-2}$

Sol: Given $x+iy=(2a-bi)^{-2}$

$x+iy=(2a-bi)^{-2}$

$x+iy=\frac{1}{(2a-bi)^2}$

$x+iy=\frac{1}{4a^2-b^2-4abi}$

$x+iy=\frac{1}{4a^2-b^2-4abi}\times\frac{4a^2-b^2+4abi}{4a^2-b^2+4abi}$

$x+iy=\frac{4a^2-b^2+4abi}{(4a^2-b^2)^2+(4ab)^2}$

$x+iy=\frac{4a^2-b^2+4abi}{16a^4+b^4-8a^2b^2+16a^2b^2}$

$x+iy=\frac{4a^2-b^2+4abi}{(4a^2+b^2)^2}$

$\Rightarrow x=\frac{4a^2-b^2}{(4a^2+b^2)^2}, y=\frac{4ab}{(4a^2+b^2)^2}$

V) $\frac{3-2i}{-1+i}$

Sol: Given $x+iy=\frac{3-2i}{-1+i}$

$x+iy=\frac{3-2i}{-1+i}\times\frac{-1-i}{-1-i}$

$x+iy=\frac{-3-3i+2i-2}{1+1}$

$x+iy=\frac{-5}{2}-\frac{1}{2}i$

$\Rightarrow x=\frac{-5}{2}, y=-\frac{1}{2}$

---

vi). $\left(\frac{5-2i}{2+3i}\right)^{-1}$

Sol: Given $\left(\frac{5-2i}{2+3i}\right)^{-1}$

$\left(\frac{5-2i}{2+3i}\right)^{-1}=\frac{2+3i}{5-2i}\times\frac{5+2i}{5+2i}$

$\left(\frac{5-2i}{2+3i}\right)^{-1}=\frac{10+4i+15i-6}{25+4}$

$\left(\frac{5-2i}{2+3i}\right)^{-1}=\frac{4}{29}+\frac{19}{29}i$

$\Rightarrow x=\frac{4}{29}, \qquad y=\frac{19}{29}$

---

vii). $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^2$

Sol: Given $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^2$

$\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^2=\frac{(\sqrt{3}-i)^2}{(\sqrt{3}+i)^2}$

$\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^2=\frac{3-1-2\sqrt{3}i}{3-1+2\sqrt{3}i}$

$\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^2=\frac{2-2\sqrt{3}i}{2+2\sqrt{3}i}$

$\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^2=\frac{1-\sqrt{3}i}{1+\sqrt{3}i}\times\frac{1-\sqrt{3}i}{1-\sqrt{3}i}$

$\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^2=\frac{1-3-2\sqrt{3}i}{1+3}$

$\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^2=\frac{-2-2\sqrt{3}i}{4}$

$\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^2=\frac{-1}{2}-\frac{\sqrt{3}}{2}i$

$\Rightarrow x=\frac{-1}{2}, y=-\frac{\sqrt{3}}{2}$

---

viii). $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{-2}$

Sol: Given $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{-2}$

$\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{-2}=\frac{(1-\sqrt{3}i)^2}{(1+\sqrt{3}i)^2}$

$\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{-2}=\frac{1-3-2\sqrt{3}i}{1-3+2\sqrt{3}i}$

$$\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{-2}=\frac{-2-2\sqrt{3}i}{-2+2\sqrt{3}i}$$

$$\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{-2}=\frac{-2\left(1+\sqrt{3}i\right)}{-2\left(1-\sqrt{3}i\right)}$$

$$\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{-2}=\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\times\frac{1+\sqrt{3}i}{1+\sqrt{3}i}$$

$$\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{-2}=\frac{1-3+2\sqrt{3}i}{1+3}$$

$$\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{-2}=\frac{-2+2\sqrt{3}i}{4}$$

$$\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{-2}=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$\Rightarrow x=\frac{-1}{2},\ y=\frac{\sqrt{3}}{2}$

Exercise 1.3

Solve the simultaneous linear equations with complex coefficients.

Q1. $z+w=3i,\qquad 2z+3w=2$

Sol: Given

$z+w=3i\cdots(1)$

$2z+3w=2\cdots(2)$

From equation 1 $\quad z=3i-w$................(3)

Put in equation 2

$2(3i-w)+3w=2$

$6i-2w+3w=2$

$w=2-6i$

Then put the value of z in equation 3 we get

$z=3i-2+6i$

$z=-2+9i$

Q2. $z-4w=3i,\qquad 2z+3w=11-5i$

Sol: Given

$z-4w=3i\cdots(1)$

$2z+3w=11-5i\cdots(2)$

Multiply eq 1 by 2 and subtract from eq 2

$2z+3w=11-5i$

$\pm2z\mp8w=\ 0\pm6i$

$11w=11-11i$

Or $w=1-i$ put in eq 1 we get

$z-4(1-i)=3i$

$z-4+4i=3i$

$z=4-4i+3i$

$z=4-i$

Q3. $3z+(2+i)w=11-i,\quad (2-i)z-w=-1+i$

Sol: Given $3z+(2+i)w=11-i\cdots(1)$

$(2-i)z-w=-1+i\cdots(2)$

Multiply eq 2 with $(2+i)$

$(2+i)(2-i)z-(2+i)w=(2+i)(-1+i)$

$(4+1)z-(2+i)w=-2+2i-i-1$

$5z-(2+i)w=-3+i$........(3)

Add (1) and (3)

$5z-(2+i)w=-3+i$

$3z+(2+i)w=11-i$

$8z\qquad =8$ or $z=1$

Putting the value of z in (2)

$1(2-i)-w=-1+i$

$2-i+1-i=w$

$w=3-2i$

Factorize polynomials $P(z)$ into linear factors.

Q4. $P(z)=z^2+4$

Sol: Given $P(z)=z^2+4$

$P(z)=z^2-(-4)$

$P(z)=z^2-(2i)^2$

$P(z)=(z-2i)(z+2i)$

Q5. $P(z)=3z^2+7$

Sol: Given $P(z)=3z^2+7$

$P(z)=3z^2-(-7)$

$P(z)=\left(\sqrt{3}z\right)^2-\left(\sqrt{7}i\right)^2$

$P(z)=\left(\sqrt{3}z-\sqrt{7}i\right)\left(\sqrt{3}z+\sqrt{7}i\right)$

Q6. $P(z)=z^3-2z^2+z-2$

Sol: Given $P(z)=z^3-2z^2+z-2$

$P(z)=z^2(z-2)+1(z-2)$

$P(z)=(z-2)(z^2+1)$

$P(z)=(z-2)(z^2-(-1))$

$P(z)=(z-2)(z^2-i^2)$

$P(z)=(z-2)(z+i)(z-i)$

Q7. $P(z)=z^3+6z+20$

Sol: Given $\quad P(z)=z^3+6z+20$

$P(1)=(1)^3+6(1)+20\neq0$

$P(-2)=(-2)^3+6(-2)+20=0$

Z+2 is a factor, so we arranging as a factor

$P(z)=z^3+2z^2-2z^2-4z+10z+20$

$P(z)=z^2(z-2)-2z(z-2)+10(z-2)$

$P(z)=(z-2)(z^2-2z+10)$

$P(z)=(z-2)(z^2-2z+1+9)$

$P(z)=(z-2)\left\{(z)^2-2(z)(1)+(1)^2+9\right\}$

$P(z)=(z-2)\{(z-1)^2-(3i)^2\}$

$P(z)=(z-2)(z-1+3i)(z-1-3i)$

2nd Method Q7. $P(z)=z^3+6z+20$

Sol: Given $P(z)=z^3+6z+20$

Using synthetic division

$$\begin{array}{r|rrrr} & 1 & 0 & 6 & 20 \\ -2 & & 2 & +4 & -20 \\ \hline & 1 & -2 & 10 & 0 \end{array}$$

Therefore $z=-2$ is a root & $z^2-2z+10$ is quotient

So $z+2$ is a factor and $z^2-2z+10$ is quotient

$P(z)=z^3+6z+20$

$P(z)=(z+2)(z^2-2z+10)$

$P(z)=(z-2)(z^2-2z+1+9)$

$P(z)=(z-2)\{(z)^2-2(z)(1)+(1)^2+9\}$

$P(z)=(z-2)\{(z-1)^2-(3i)^2\}$

$P(z)=(z-2)(z-1+3i)(z-1-3i)$

Solve the quadratic equations

Q8. $z^2+6z+13=0$

Sol: Given $z^2+6z+13=0$

Comparing with standard from of quadratic equation

$az^2+bz+c=0$ we get $a=1, b=6, c=13$ using

formula $z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ putting the values

$z=\frac{-(6)\pm\sqrt{(6)^2-4(1)(13)}}{2(1)}$

$z=\frac{-6\pm\sqrt{36-52}}{2}$

$z=\frac{-6\pm\sqrt{-16}}{2}$

$z=\frac{-6}{2}\pm\frac{4i}{2}=-3\pm 2i$

Q9. $z+\frac{2}{z}=2$

Sol: Given $z+\frac{2}{z}=2$

Multiply each term by z we get

$z.z+\frac{2}{z}.z=2.z$

$z^2+2=2z$

$z^2-2z+2=0$

Comparing with standard from of quadratic equation

$az^2+bz+c=0$ we get $a=1, b=-2, c=2$ using

formula $z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ putting the values

$z=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)}$

$z=\frac{2\pm\sqrt{4-8}}{2}$

$z=\frac{2\pm\sqrt{-4}}{2}$

$z=\frac{2}{2}\pm\frac{2i}{2}=1\pm i$

Q10. $2z^2+15=4z$

Sol: Given $2z^2+15=4z$ Rearranging

$2z^2-4z+15=0$

Comparing with standard from of quadratic equation

$az^2+bz+c=0$ we get $a=2, b=-4, c=15$ using

formula $z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ putting the values

$z=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(15)}}{2(2)}$

$z=\frac{4\pm\sqrt{16-120}}{4}$

$z=\frac{4\pm\sqrt{-104}}{4}$

$z=\frac{4\pm i\sqrt{4\times 26}}{4}$

$z=\frac{4}{4}\pm\frac{2i\sqrt{26}}{4}=1\pm i\frac{\sqrt{26}}{2}$

Q11. Show that each $z_1=-1+i$ And $z_2=-1-i$

Satisfies the equation $z^2+2z+2=0$

Solution If $z_1=-1+i$ is a solution of the given equation then it should satisfied the equation

i.e., $z^2+2z+2=0$

$(-1+i)^2+2(-1+i)+2=0$

$1-1-2i-2+2i+2=0$ Satisfied

Similarly, for $z_2=-1-i$

$z^2+2z+2=0$

$(-1-i)^2+2(-1-i)+2=0$

$1-1+2i-2-2i+2=0$ Satisfied

Q12 Determine where $1+2i$ is a solution of

$z^2-2z+5=0$

Solution If 1+2i is a solution of the given equation then it should satisfied the equation

$(1+2i)^2-2(1+2i)+5=0$

$(1)^2+(2i)^2+2(1)(2i)-2-4i+5=0$

$1+(-4)+4i-2-4i+5=0$

$-3-2+5+4i-4i=0$

$0=0$

So 1 + 2i is a solution of given equation