Chapter#1. 2nd year Functions and Limits

Function:
If A and B be two non-empty sets then f is said to be a function from set A to set B written as ;$f: A \to B$ and defined as

$i) D_f = A\ ii)\ for\ every\ a \in A\ thereexist\ only\ one\ b \in B\ s.that\ (a,b) \in f$

Domain:
The set of all possible inputs of a function is called domain.
*the domain of every function $f(x)$ is defined.
*the valves at which at $f(x)$ becomes undefined or complex valued will be excluded from real numbers.
*domain is also known as pre-images.

Range:
The set of all possible out puts of a function is called range.
*range is also known as images.

Types of functions:

i) Algebraic function:
Any function generated by algebraic operations is known as algebraic function. Algebraic functions are classified as below.

ii) **Polynomial function:**
A function P of the form

$$p(x) = a_n x_n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$$

$for\ all\ x, where\ the\ coefficients\ a_n, a_{n-1}, a_{n-2}. a_2, a_1, a_0\ are\ real\ numbers\ and\ exponents\ are\ non-negative\ integers, is\ called\ a\ polynomial\ function.$

iii) **Linear Function:**
If the degree of polynomial function is 1. Then it is called linear function.

iv) Quadratic Function:
If the degree of polynomial function is 2. Then it is called a quadratic function.

v) Identity function:
A function for which $f(x) = y\ or\ y = x$ is called identity function. It is denoted by I

vi) Constants Function:
A function for which $f(x) = b\ or\ y = b$ is called constant function.

vii) Rational function:
The quotient of two polynomials such as $f(x) = \frac{p(x)}{Q(x)}$ $where\ Q(x) \neq 0\ is\ called\ rational\ function$

viii) Exponential Function:
A function in which the variable appears as exponent (power) is called exponential function.
$e.g; y = e^{ax}, y = e^x\ e.t.c$

ix) Logarithmic Functions:
$if\ x = a^y\ then\ y = loga^x\ where\ a > o, a \neq 1\ is\ called\ logarithmic\ functions.$
$*\log_{10} x\ \ is\ knonw\ as\ common\ logarithm.$
$*\log_e x\ is\ known\ as\ natural\ logarithm.$

x) Hyperbolic Function:

$$sinhx = \frac{e^x - e^{-x}}{2},\quad coshx = \frac{e^x + e^{-x}}{2}$$
$$tanhx = \frac{e^x - e^{-x}}{e^x + e^{-x}},\quad sechx = \frac{2}{e^x + e^{-x}}$$
$$cschx = \frac{2}{e^x - e^{-x}},\quad cothx = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

xi) Inverse Hyperbolic function:

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right), \forall x$$
$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right), x \geq 1$$
$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), |x| < 1$$
$$\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right), x \neq 0$$
$$\operatorname{sech}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1-x^2}}{x}\right), 0 < x \leq 1$$
$$\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right), |x| < 1$$

xii) Explicit function:
If y is easily expressed in terms of x, then y is called explicit function.
$symbolically\ y = f(x)$

xiii) Implicit function:
If the two variables x and y are so mixed up such that y cannot be expressed in terms of x, then this type of function. Symbolically$f(x, y) = 0$

xiv) Parametric function:
If x and y are expressed in terms of third variable (say t) such as $x = f(t), y = g(t)\ then\ these\ equations\ are$ Called parametric equations.

xv) Even function:
A function f is said to be an even if $f(-x) = f(x)$ for every $x\ in\ domain\ of\ f.$

xvi) **Odd function**:
A function f is said to be odd if $f(-x) = -f(x)$ for every number x in the domain of f

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Exercise 1.1

Q1. Given that

$$\boldsymbol{a)f(x) = x^2 - x}$$

$\boldsymbol{b)f(x) = \sqrt{x+4}}$ $\quad$ $\boldsymbol{find\ i)f(-2)}$

$\boldsymbol{ii)\ f(a)}$ $\quad$ $\boldsymbol{iii)\ f(x-1)}$ $\quad$ $\boldsymbol{iv)\ f(x^2+4)}$

Solution:

$(a)f(x) = x^2 - x$

$(i)f(-2) = (-2)^2 - (-2) = 4 + 2 = 6$

$ii)\ f(0) = (0)^2 - 0 = 0$

$$iii)\ f(x-1) = (x-1)^2 - (x-1)$$
$$= x^2 + 1 - 2x - x + 1$$
$$= x^2 - 3x + 2$$

$iv)\ f(x^2+4) = ((x^2+4)^2 - (x^2+4)$
$$= x^4 + 16 + 8x^2 - x^2 - 4$$
$$= x^4 + 7x^2 + 12$$

(b) $f(x) = \sqrt{x+4}(i)f(-2) = \sqrt{-2+4} = \sqrt{2}$

$ii)f(0) = \sqrt{0+4} = \sqrt{4} = 2$

$iii)f(x-1) = \sqrt{x-1+4} = \sqrt{x+3}$

$xiv)\ f(x^2+4) = \sqrt{x^2+4-4} = \sqrt{x^2+8}$

Q2. Find $\boldsymbol{\frac{f(a+h)-f(a)}{h}}$ $\boldsymbol{and\ simlify\ where}$

$\boldsymbol{i)\ f(x) = 6x - 9}$ $\quad$ $\boldsymbol{ii)\ f(x) = sinx}$

$\boldsymbol{iii)f(x)x^3 + 2x^2 - 1}$ $\quad$ $\boldsymbol{(iv)f(x) = cosx}$

Solution:

$\boldsymbol{i)\ f(x) = 6x - 9}$

$$\frac{f(a+h)-f(a)}{h} = \frac{\{6(a+h)-9\}-(6a-9)}{h}$$
$$= \frac{(6a+6h-9-6a+9)}{h} = \frac{6h}{h} = 6$$

$\boldsymbol{ii)\ f(x) = sinx}$

$$\frac{f(a+h)-f(a)}{h} = \frac{sin(a+h) - sina}{h}$$
$$= \frac{1}{h}\left\{2cos\left(\frac{a+h+a}{2}\right) sin\left(\frac{a+h-a}{2}\right)\right\}$$
$$= \frac{1}{h}\left\{2cos\left(\frac{2a+h}{2}\right) sin\left(\frac{h}{2}\right)\right\}$$
$$= \frac{1}{h}\left\{2cos\left(a+\frac{h}{2}\right) sin\left(\frac{h}{2}\right)\right\}$$

$\boldsymbol{iii)\ (f(x) = x^3 + 2x^2 - 1}$

$$f(a+h) = (a+h)^3 + 2(a+h)^2 - 1$$
$$a^3 + b^3 + 3a^2h + 3ah^2 + 2a^2 + 2h^2 + 4ah - 1$$
$$f(a) = a^3 + 2a^2 - 1$$
$$\frac{f(a+h)-f(a)}{h}$$
$$= \frac{a^3 + h^3 + 3a^2 + 3ah^2 + 2a^2 + 2h^2 + 4ah - 1 - a^3 - 2a^2 + 1}{h}$$
$$= \frac{h(h^2 + 3a^2 + 3ah + 2h + 4a)}{h}$$
$$= h^2 + 3a^2 + 3ah + 2h + 4a$$

$\boldsymbol{iv)\ f(x) = cosx}$

$$\frac{f(a+h)-f(a)}{h} = \frac{cos(a+h) - cosa}{h}$$
$$= \frac{1}{h}\left(-2sin\left(\frac{a+h+a}{2}\right) sin\left(\frac{a+h-a}{2}\right)\right)$$
$$\frac{1}{h}\left(-2sin\left(\frac{2a+b}{2}\right) sin\left(\frac{b}{a}\right)\right)$$
$$-\frac{2}{h} sin\left(\frac{a+h}{2}\right) sin\left(\frac{h}{2}\right)$$

Q3. Express the following (a) the perimeter P of square as a function of its area A.

Solution:

Let each side of square be "x" then

Perimeter:

$$p = 4x \to (i)$$

Area:

$$a = x \times x = x^2 \Rightarrow x = \sqrt{A}$$
$$put\ valve\ of\ x\ in\ (i)$$

⇨ $P = 4\sqrt{A}$

b) The area A of a circle as a function of its circumference C.

Solution:

let r be the radius of circle then

Then

Area $= \pi r^2 \to (i)$

Circumference:

$C = 2\pi r \Rightarrow r = \frac{c}{2\pi}$ $put\ in(i)$

⇨ $A = \pi\left(\frac{c}{2\pi}\right)^2 = \pi.\left(\frac{c^2}{2\pi}\right) = \frac{c^2}{2\pi}$

⇨ $A = \frac{c^2}{2\pi}$

$\boldsymbol{(C) the\ volume\ V\ of\ a\ cube\ as\ a\ function\ of\ the\ area\ A\ of\ its\ base.}$

solution:

let each side of cube be x then

volume:

$V = x \times x \times x$

$V = x^3 \to (i)$

Area of base:

$A = x^2 \Rightarrow x = \sqrt{A}$ $put\ in\ (i)$

⇨ $V = \left(\sqrt{A}\right)^3 \Rightarrow V = A^{\frac{1}{3}}$

Q4. Find the domain and range of the functions g defined below.

$(i)g(x) = 2x - 5$

$D_y = (-\infty, +\infty), R_y = (-\infty, +\infty)$

$ii)g(x) = \sqrt{x^2 - 4}$

$g(x) becomes\ complex\ valued\ when\ x^2 - 4 < 0$

$or\ x^2 < 4\ or -2 < x < 2$

$D_y = R - (-2,2), R_y = [0, +\infty)$

$(iii)g(x) = \sqrt{x+1}$

$g(x) becomes\ complex\ valued\ when\ x + 1 < 0\ or$

$x < -1$ so $D_g = [-1, +\infty)$

$iv)\ g(x) = |x-3|$

$D_y = (-\infty, +\infty), R_y = [0, \infty)$

$v)\ g_x = \begin{cases} 6x+7 \text{ if } x \le -2 \\ 4x-3 \text{ if } x > -2 \end{cases}$

$D_y = (-\infty, -2] \cup (-2, +\infty)$

$R_y = (-\infty, -5] \cup (-11, +\infty)$

$vi)\ g(x) = \frac{x^2+3x+2}{x+1}, x \ne -1$

$D_y = R - \{-1\} \quad \because = \frac{x^2+3x+2}{x+1}$

$R_y = R - \{1\} \quad = \frac{(x+1)(x+2)}{x+1}$

$g(x) = x+2$

$viii)\ g(x) = \frac{x^2-16}{x-4}, x \ne 4 \quad g(-1) = -1+2 = 1$

$D_y = R - \{4\} \quad \because \frac{x^2-16}{x-4}$

$R_y = R - \{8\} \quad = \frac{(x-4)(x+4)}{x-4}$

$g(x) = x+4$

$g(x) = 4+4 = 8$

Q5. Given $f(x) = x^3 - ax^2 + bx + 1$ if $f(2) = -3$, $f(-1) = 0$ find the value of a and b

Solution:

$f(x) = x^3 - ax^2 + bx + 1$

⇨ $f(2) = (2)^3 - a(2)^2 + b(2) + 1$

⇨ $-3 = 8 - 4a + 2b + 1$

⇨ -4a+2b+12=0

⇨ $-2a + b + 6 = 0 \to (i)$

⇨ also $f(-1) = (-1)^3 + b(-1) + 1$

⇨ $0 = -1 - a - b + 1$

⇨ $-a - b = 0 \to (ii)$

⇨ $(i) + (ii) \quad -2a + b + 6 = 0$

$-a - b = 0$

$-3 + 6 = 0 \Rightarrow -3a = -6 \Rightarrow a = 2$

Put in $ii)\ -2 - b = 0 \Rightarrow b = -2$

Q6. A stone falls from a height of h after x second is approximately given by $h(x) = 40 - 10x^2$

i) when is the height of the stone when (a) $x = 1\ sec$?

$b)\ x = 1.5\ sec\ (c)\ x = 1.7\ sec$

$d)$ *when does the stone strike the ground.*

Solution:

$h(x) = 40 - 10x^2$

$(a)\quad h(1) = 40 - 10(1)^2 = 40 - 10 = 30$

$b)\ h(1.5) = 40 - 10(1.5)^2 = 40 - 22.5 = 17.5m$

$c)\ h(1.7) = 40 - 10(1.7)^2 = 40 - 28.9 = 11.1m$

$ii)$ *when does stone strikes the ground then* $h(x) = 0$

$h(x) = 40 - 10x^2$

⇨ $0 = 40 - 10x^2$

⇨ $10x^2 = 40$

⇨ $x^2 = 4$

⇨ $x = \pm 2$

⇨ $x = 2 \quad , (neglect\ -2)$

Q7. Show that the parametric equation:

$i)\ x = at^2, y = 2at$ *represented the equation: of parabola* $y^2 = 4ax$

Solution:

$$\mathbf{x = at^2 \to (1)}$$

$$\mathbf{y = 2at \Rightarrow t = \frac{y}{2a} \text{ put in } (i)}$$

⇨ $\mathbf{x = a\left(\frac{y}{2a}\right)^2 = a.\frac{y^2}{4a^2}}$

⇨ $\mathbf{x = \frac{y^2}{4a} \Rightarrow y^2 = 4ax}$

$(ii)\ x = acos\theta, y = bsin\theta$ *represent the equation of ellipse* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Solution:

$$\mathbf{x = acos\theta \to (1)}$$

$$\mathbf{y = bsin\theta \to (2)}$$

From (1)

$$\frac{x}{a} = cos\theta \to (3)$$

From (2)

$$\frac{y}{b} = sin\theta \to (4)$$

Squaring and adding (3) and (4)

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = (cos\theta)^2 + (sin\theta)^2$$

$$= \cos^2\theta + \sin^2\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$iii)\ x = asec\theta \to (1)$

$y = btan\theta \to (2)$

From 1)

$$\frac{x}{a} = sec\theta \Rightarrow \frac{x^2}{a^2} = \sec^2\theta \to (3)$$

From 2)

$$\frac{y}{b} = tan\theta \Rightarrow \frac{y^2}{b^2} = \tan^2\theta \to (4)$$

$(3) - (4)$

⇨ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta$

⇨ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Q8.prove the identities

$i)\ sinh2x = 2sinhxcoshx$

Solution:

$R.H.S = 2sinhxcoshx$

⇨ $2.\frac{e^x - e^{-x}}{2}.\frac{e^x + e^{-x}}{2} = \frac{e^{2x} - e^{-2x}}{2}$

⇨ $sinh2x = L.H.S$

Hence $\mathbf{sinh2x = 2sinhxcoshx}$

$iii)\ \operatorname{sech}^2 x = 1 - \tanh^2 x$

Solution:

$\boldsymbol{R.H.S = 1 - tanh^2 x}$

$$= 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2 = 1 - \frac{(e^x - e^{-x})^2}{(e^x + e^{-x})^2}$$
$$= \frac{(e^{2x} + e^{-2x} + 2) - (e^{2x} + e^{-2x} - 2)}{(e^x + e^{-x})^2}$$
$$= \frac{(e^{2x} + e^{-2x} + 2) - e^{2x} - e^{-2x} + 2)}{(e^x + e^{-x})^2}$$
$$= \frac{4}{(e^x + e^{-x})^2} = \left(\frac{2}{e^x + e^{-x}}\right)^2$$
$$= \frac{1}{\left(\frac{e^x + e^{-x}}{2}\right)^2} = \frac{1}{cosh^2} = sech^2 x = L.H.S$$

(iii) $\operatorname{csch}^2 x = \coth^2 x - 1$

Solution:

$R.H.S = \coth^2 x - 1$

$$= \left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right) - 1 = \frac{(e^x + e^{-x})}{(e^x - e^{-x})} - 1$$
$$= \frac{e^{2x} + e^{-2x} + 2 - e^{2x} - e^{-2x} + 2}{(e^x - e^{-x})^2}$$
$$= \frac{4}{(e^x - e^{-x})^2} = \left(\frac{2}{e^x - e^{-x}}\right)^2$$
$$= \left(\frac{1}{\frac{e^x - e^{-x}}{2}}\right)^2 = \frac{1}{\sinh^2 x} = cseh^2 x = L.H.S$$

$Hence\ csch^2\ x = \coth^2 x - 1$

Q9. Determine whether the given function f is even or odd.

Solution:

$i) f(x) = x^3 + x$

⇨ $f(-x) = (-x)^3 + (-x) = -x^3 - x$

⇨ $= -(x^3 + x) = -f(x)$

⇨ $thus\ f(x) is\ odd.$

⇨ $ii)\ f(x) = (x + 2)^2$

⇨ $f(-x) = (-x + 2)^2 \neq \pm f(x)$

$thus\ f(x) is\ neither\ even\ nor\ odd.$

$iii)\ \ f(x) = x\sqrt{x^2 + 5}$

⇨ $f(-x) = x\sqrt{(-x)^2 + 5}$

⇨ $= -x\sqrt{x^2 + 5} = -f(x)$

$thus\ f(x) is\ neither\ even\ nor\ odd.$

v) $f(x) = x^{\frac{1}{3}} + 6$

$\Rightarrow f(x) = x^{\frac{1}{3}} + 6$

$[(-x)^2] + 6$

$$= (x^2)^{\frac{1}{3}} + 6$$
$$= (x^2)^{\left(\frac{1}{3}\right)} + 6 = x^{\frac{2}{3}} + 6 = f(x)$$

$thus\ f(x) is\ even.$

$\boldsymbol{vi)\ f(x) = \frac{x^3 - x}{x^2 + 1}}$

⇨ $f(-x) = \frac{(-x)^2 - (-x)}{(-x)^2 + 1} = \frac{-x^3 + x}{x^2 + 1}$

⇨ $= -\frac{(x^3 - x)}{x^3 + 1} = -f(x)$

$thus\ f(x) is\ oddd.$

Composition of function:

If f is a function from set A to set B and g is a function from set B to set C then composition of f and g is denoted by

$$(fog)(x) = f(g(x)) \forall\ x \in A$$

Inverse of a function:

Let f be a bijective $(1 - 1\ and\ onto\ ) function\ from\ set\ A\ ro\ set\ b\ i.e\ f: A \to B\ then\ its\ inverse\ is\ f^{-1}\ which\ is\ surjective\ (onto) function\ from\ B\ to\ A\ i.e\ f^{-1}: B \to A\ in\ this\ case\ D_f: R_f\ on\ to\ R_f = D_{f^{-1}}$

Exercise 1.2

Q1. The real valued functions $f\ \ and\ g$ are defined below. find

$\boldsymbol{(a) fog(x)\ (b) gof(x)\ \ (c) fof(x)\ \ \ (d) gog(x)}$

i) $\quad f(x) = 2x + 1;\ \ g = \frac{3}{x-1}, x \neq 1$

S0lution:

$$(a) fog(x) = f(g(x)) = f\left(\frac{3}{x - 1}\right)$$
$$= 2\left(\frac{3}{x - 1}\right) + 1 = \frac{6}{x - 1} + 1 = \frac{6 + x - 1}{x - 1}$$
$$= \frac{5 + x}{x - 1}$$

b) $gof(x)\ \ = \ g(f(x))\ \ = \ \ g(2x + 1)$

$$= \frac{3}{2x + 1 - 1} = \frac{3}{2x}$$

c) $fof(x) = f(f(x)) = f(2x + 1)$

$$= 2(2x + 1) + 1 = 4x + 3$$

d) $gog(x)$=$g(g(x) = g\left(\frac{3}{x-1}\right) = \frac{3}{\frac{3}{x-1} - 1}$

$$= \frac{3}{\frac{3 - (x - 1)}{x - 1}} = \frac{3(x - 1)}{3 - x + 1} = \frac{3(x - 1)}{4 - x}$$

$\boldsymbol{ii) f(x) = \sqrt{x + 1}, g(x) = \frac{1}{x^2}}$

Solution:

$$a)\ fog(x) = f(g(x))$$
$$= f\left(\frac{1}{x^2}\right) = \sqrt{\frac{1}{x^2} + 1} = \sqrt{\frac{1 + x^2}{x^2}} = \frac{\sqrt{1 + x^2}}{x}$$

$$b) gof(x) = g(f(x)) = g(\sqrt{x+1}) = \frac{1}{(\sqrt{x+1})^2}$$

$$= \frac{1}{x+1}$$

$$c)\ fof(x) = f(f(x)) = f(\sqrt{x+1}) = \sqrt{\sqrt{x+1}+1}$$

$$d) gog(x) = g(g(x)) = g\left(\frac{1}{x^2}\right) = \frac{1}{\left(\frac{1}{x^2}\right)^2} = \frac{1}{\frac{1}{x^4}} = x^4$$

$(iii)\ f(x) = \frac{1}{\sqrt{x-1}}, g(x) = (x^2+1)^2$

Solution:

a) $fog(x) = f(g(x))$

$$f((x^2+1)^2) = \frac{1}{\sqrt{(x^2+1)^2 - 1}}$$

$$= \frac{1}{\sqrt{x^4+1+2x^2-1}} = \frac{1}{\sqrt{x^4+2x^2}} = \frac{1}{\sqrt{x^2(x^2+2)}}$$

$$= \frac{1}{x\sqrt{x^2+2}}$$

$$b)\ gof(x) = g(f(x)) = g\left(\frac{1}{\sqrt{x-1}}\right)$$

$$= \left[\left(\frac{1}{\sqrt{x-1}}\right)^2 + 1\right]^2 = \left(\frac{x}{x-1} + 1\right)^2$$

$$= \left(\frac{1+x-1}{x-1}\right)^2 = \left(\frac{x}{x-1}\right)^2$$

$c)$

$$fof(x) = f(f(x)) = f\left(\frac{1}{\sqrt{x-1}}\right)$$

$$= \frac{1}{\sqrt{\frac{1}{\sqrt{x-1}} - 1}} = \frac{1}{\left(\frac{1-\sqrt{x-1}}{\sqrt{x-1}}\right)^{\frac{1}{2}}}$$

$$= \left(\frac{1-\sqrt{x-1}}{\sqrt{x-1}}\right)^{-1\backslash 2}$$

$$= \left(\frac{\sqrt{x-1}}{1-\sqrt{x-1}}\right)^{\frac{1}{2}} = \sqrt{\frac{\sqrt{x-1}}{1-\sqrt{x-16}}}$$

d)

$$gog(x) = g(g(x))$$
$$= g(x^2+1)$$
$$= \left((x^2+1)^2 + 1\right)^2$$

(iv)

$f(x) = 3x^4 - 2x^2, g(x) = \frac{2}{\sqrt{x}}$

Solution:

$$a)\ fog(x) = f(g(x))$$

$$= f\left(\frac{2}{\sqrt{x}}\right) = 3\left(\frac{2}{\sqrt{x}}\right)^4 - 2\left(\frac{2}{\sqrt{x}}\right)^2$$

$$= 3(16/x^2) - \frac{8}{x} = \frac{48}{x^2} - \frac{8}{x} = \frac{48-8x}{x^2}$$

b)

$$gof(x) = g(f(x)) = g(3x^2 - 2x^2)$$
$$= g(3x^4 - 2x^2)$$

$$= \frac{2}{\sqrt{3x^4-2x^2}} = \frac{2}{\sqrt{x^2(3x^2-2)}} = \frac{2}{x\sqrt{3x^2-2}}$$

c)

$$fof(x) = f(f(x)) = f(3x^4 - 2x^2)$$
$$= 3(3x^4-2x^2)^4 - 2(3x^4-2x^2)^2$$

d)

$$gog(x) = g(g(x)) = g\left(\frac{2}{\sqrt{x}}\right)^{-\frac{1}{2}}$$

$$= \frac{2}{\sqrt{\frac{2}{\sqrt{x}}}} = \frac{2}{\left(\frac{2}{\sqrt{2}}\right)^{\frac{1}{2}}} = 2\left(\frac{2}{\sqrt{x}}\right)^{-\frac{1}{2}}$$

$$= 2\left(\frac{\sqrt{x}}{2}\right)^{\frac{1}{2}} = 2\sqrt{\frac{\sqrt{x}}{2}} = \sqrt{2}.\sqrt{2}\ \frac{\sqrt{\sqrt{2}}}{\sqrt{2}}$$

$$= \sqrt{2\sqrt{x}}$$

Q2.

For the real valued function f defined below, find

$(a) f^{-1}(x)\quad (b) f^{-1}(-1)$ ***and verify***

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

i) $f(x) = -2x + 8$

Solution:

$f(x) = -2x + 8$

$$let\ y = f(x) then$$

$$y = -2x + 8 \Rightarrow \frac{y-8}{-2} = x$$

⇨ $x = \frac{y-8}{-2}$

⇨ $f^{-1}(y) = \frac{y-8}{-2} \Rightarrow \because y = f(x)$

⇨ $f^{-1}(y) = x$

Replace y by x we have

⇨ $f^{-1}(x) = \frac{x-8}{-2}$

⇨ $put\ x = -1, f^{-1}(-1) = \frac{-1-8}{-2} = \frac{9}{2}$

ii) $f(x) = 3x^2 2 + 7$

Solution:

$$f(x) = 3x^2 + 7$$

$$let\ y = f(x) then\ y = 3x^2 + 7$$

⇨ $\frac{y-7}{3} = x^3$

⇨ $x = \left(\frac{y-7}{3}\right)^{\frac{1}{3}}$

⇨ $\because y = f(x) \Rightarrow f^{-1}(y) = x$

⇨ $f^{-1}(y) = \left(\frac{y-7}{3}\right)^{\frac{1}{3}}$

Replace y by x we have

⇨ $f^{-1}(x) = \left(\frac{x-7}{3}\right)^{\frac{1}{3}}$

Put $x = -1$ $f^{-1}(-1) = \left(\frac{-8}{3}\right)^{\frac{1}{3}}$

Verification:

$$f(f^{-1}(x)) = f\left[\left(\frac{x-7}{3}\right)^{\frac{1}{3}}\right] = 3\left[\left(\frac{x-7}{3}\right)^{\frac{1}{2}}\right]^2 + 7$$

$$= 3\left(\frac{x-7}{3}\right) + 7 = x - 7 + 7 = x$$

$$f^{-1}(f(x)) = f^{-1}(3x^3 + 7) = \left(\frac{3x^2 + 7 - 7}{3}\right)^{\frac{1}{3}}$$

$$= \left(\frac{3x^3}{3}\right)^{\frac{1}{3}} = x$$

hence $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

iii) $f(x) = (-x+9)^3$

⇨ $y = f(x) = (-x+9)^3$

let $y = f(x)$ *then* $y = (-x+9)^3$

$y^{\frac{1}{3}} = -x + 9$

⇨ $y^5 - 9 = -x$

⇨ $x = 9 - y^{\frac{1}{5}}$

⇨ $f^{-1}(y) = 9 - y^{\frac{1}{3}}$

$(\because y = f(x) \Rightarrow f^{-1}(y) = x$

replace y by x we have

$f^{-1}(x) = 9 - x^{\frac{1}{3}}$

Put $x = -1, f^{-1}(-1) = 9 - (-1)^{\frac{1}{3}} = 9 - (-1) = 0$

Verification:

$$f\left(f^{-1}(x)\right) = f\left(9 - x^{\frac{1}{3}}\right) = \left[-\left(9 - x^{\frac{1}{3}}\right) + 9\right]^3$$

$$= \left(-9 + x^{\frac{1}{3}} + 9\right)^3 = x$$

$$f^{-1}(f(x)) = f^{-1}((-x+9)^3)$$

$$= 9 - ((--x+9)^3)^{\frac{1}{3}} = 9 - (-x+9)$$

$$= 9 + x - 9 = x$$

Hence $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

iv) $f(x) = \frac{2x+1}{x-1}$

Let $y = f(x)$ ***then*** $y = \frac{2x+1}{x-1}$

⇨ $(x-1)y = 2x + 1$

⇨ $xy - y = 2x + 1$

⇨ $xy - 2x = y + 1$

⇨ $x(y-2) = 1 + y$

⇨ $x = \frac{1+y}{y-2}$

⇨ ***replace y by x we have***

⇨ $f^{-1}(x) = \frac{1+x}{x-2}$

⇨ ***put*** $x = -1, f^{-1}(-1) = \frac{1+(-1)}{-1-2} = 0$

Verification:

$$f\left(f^{-1}(x)\right) = f\left(\frac{1+x}{x-2}\right) = \frac{2\left(\frac{1+x}{x-2}\right) + 1}{\frac{1+x}{x-2} - 1}$$

$$\frac{\frac{2(1+x) + x + 2}{x-2}}{\frac{1 + x - (x-2)}{x-2}} = \frac{3x}{2x + 1 - 2x + 2} = \frac{3x}{3}$$

Hence $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Q3.

Without finding the inverse, state the domain and range of f^{-1}) ***i)*** $f(x) = \sqrt{x+2}$ ***ii)*** $f(x) = \frac{x-1}{x-4}, x \neq 4$

iii) $f(x) = \frac{1}{x+3}, x \neq -3$

iv) $f(x) = (x-5)^\wedge 2, x \geq 5$

Solution:

i) $f(x) = \sqrt{x+2}$

$\because f(x)$ ***becomes complex valued when*** $x + 2 < 0$ ***or*** $x < -2$

$D_f = [-2, +\infty), R_f = [0, +\infty)$

By definition of inverse function,

$D_{f^{-1}} = R_f = [0, +\infty)$

By definition of inverse function,

$D_{f^{-1}} = R_f = [0, +\infty), R_{f^{-1}} = D_f = [-2, +\infty)$

ii)

$f(x) = \frac{x-1}{x-4}, x \neq 4$

$D_f = R - \{4\}, \because f(x) = \frac{x-1}{x-4}, x \neq 4$

$R_f = R - \{1\}$ $y = \frac{x-1}{x-4}$

⇨ $yx - 4y = x - 1$

$xy - x = 4y - 1$

⇨ $x(y-1) = 4y - 1$

⇨ $x = \frac{4y-1}{y-1}$

$$f^{-1}(x) = \frac{4x-1}{x-1}, x \neq 1$$

By def. of inverse function.

$D_{f^{-1}} = R_f = R - \{1\}$

$R_{f^{-1}} = D_f = R - \{4\}$

iii)

$$f(x) = \frac{1}{x+3}, x \neq -3$$

$D_f = R - \{-3\}$ $\because$ $f(x) = \frac{1}{x+3}, x \neq -3$

$R_f = R - \{0\}$ $y = \frac{1}{x+3}$

By def. of inverse $x + 3 = \frac{1}{y}$

$D_{f^{-1}} = R_f = R - \{0\}$ $x = \frac{1}{y} - 3$

$R_{f^{-1}} = D_f = R - \{-3\}$ $f^{-1}(x) = \frac{1}{x} - 3, x \neq 0$

$R_{f^{-1}} = D_f = R - \{-3\}$

iv)

$$f(x) = (x-2)^2, \quad x \geq 5$$
$$D_f = [5,+\infty) \quad , R_f = [0,+\infty)$$

By definition of inverse function.

$$D_{f^{-1}} = R_f = [0,+\infty), \quad R_{f^{-1}} = D_f = [5,+\infty)$$

Limits of functions:

Let $f(x)$ be a function then a number L is said to be limit of $f(x)$ when x approaches to a from both left and right hand side of a , symbolically it is written as; $\lim_{x\to a} f(x) = L$

And read as "limit of f of x as approaches to a is equal to L"

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Theorems on limits of functions:

i) $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = L + M$

ii) $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = L - M$

iii) $\lim_{x\to a}[k, f(x)] = k\lim_{x\to a} f(x) = kL$

iv) $\lim_{x\to a} f(x)g(x) = k\lim_{x\to a} f(x).\lim_{x\to a} g(x) = LM$

v) $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}$

vi) $\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n = L^n$

Theorem:

Prove that $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$ *where n is an integer*

And $a > 0$

Proof:

Case 1:

Suppose n is a +ve integer.

$$L.H.S = \lim_{x\to a}\frac{x^n-a^n}{x-a}\left(\frac{0}{0}\right) form$$

$$= \lim_{x\to a}\frac{(x-a)(x^{n-1}+x^{n-2}a+x^{(n-3)}a^2+\cdots+xa^{n-2}+a^{n-1})}{x-a}$$

$$= \lim_{x\to a}(x^{n-1}+x^{n-2}a+x^{(n-3)}a^2+\cdots+xa^{n-2}+a^{n-1})$$
$$= a^{n-1}+a^{n-2}.a+a^{n-3}.a^2+\cdots+xa^{n-2}+a^{n-1}$$
$$= a^{n-1}+a^{n-1}+a^{n-1}+\cdots+a^{n-1}+a^{n-1}$$
$$= na^{n-1}$$
$$thus \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$$

Case 11:

Suppose n is +ve.

$$let\ n\ is -ve$$

$(where\ m\ is + ve\ integer)$

$$then \lim_{x\to a}\frac{x^n-a^n}{x-a} = \lim_{x\to a}\frac{x^{-m}-a^{-m}}{x-a}$$

$$= \lim_{x\to a}(x^{-m}-a^{-m}).\frac{1}{x-a} = \lim_{x\to a}\left(\frac{1}{x^m}.\frac{1}{a^n}\right) = \frac{1}{x-a}$$
$$= \lim_{x\to a}\left(\frac{a^m-x^m}{x^m a^m}\right).\frac{1}{x-a}$$
$$= \lim_{x\to a}\left(\frac{x^m-a^m}{x^m a^m}\right)\left(\frac{-1}{x-a}\right)$$
$$= \lim_{x\to a}\left(\frac{x^m-a^m}{x^m a^m}\right)\lim_{x\to a}\left(\frac{-1}{x-a}\right)$$
$$ma^{m-1}\left(\frac{-1}{a^{2m}}\right)$$
$$-ma^{m-1-2m} = -ma^{(-m-1)} = na^{(n-1)}$$

Thus $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1} \because n = -m$

Theorem:

Prove that $\lim_{x\to a}\frac{\sqrt{x+a}-\sqrt{a}}{a} = \frac{1}{2\sqrt{a}}$

$proof$:

$$L.H.S = \lim_{x\to 0}\frac{\sqrt{x+a}-\sqrt{a}}{x}\left(\frac{0}{0}\right) form$$
$$= \lim_{x\to 0}\left(\frac{\sqrt{x+a}-\sqrt{a}}{x}\times\frac{\sqrt{x+a}+\sqrt{a}}{\sqrt{x+a}+\sqrt{a}}\right)$$
$$= \lim_{x\to 0}\frac{x+a-a}{x(\sqrt{x+a}+\sqrt{a})}$$
$$= \lim_{x\to 0}\frac{1}{(\sqrt{x+a}+\sqrt{a})}$$
$$= \lim_{x\to 0}\frac{1}{(\sqrt{a}+\sqrt{a})}$$
$$= \lim_{x\to 0}\frac{1}{(2\sqrt{a})}$$
$$thus \lim_{x\to 0}\frac{\sqrt{x+a}-\sqrt{a}}{x} = 1/2\sqrt{a}$$

Theorem:

Prove that $\lim_{x\to+\infty}\left(1+\frac{1}{n}\right)^n = e$

Using Binomial theorem we have

$$\left(1+\frac{1}{n}\right)^n = 1+n\left(\frac{1}{n}\right)+\frac{n(n-1)}{2!}\left(\frac{1}{n}\right)^2+\frac{n(n-1)(n-2)}{3!}\left(\frac{1}{n}\right)^3+\cdots$$
$$= 1+1+\frac{1}{2!}\left(\frac{n-1}{n}\right)+\frac{1}{3!}\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)+\ldots.$$
$$2+\frac{1}{2!}\left(1-\frac{1}{n}\right)+\frac{1}{3!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)+\cdots$$
$$when\ n\to\infty, \frac{1}{n},\frac{2}{n},\frac{3}{n}..all\ tend\ to\ term$$

Thus,

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots$$
$$= 2+0.5+0.16667+..$$
$$= 2.718281$$

Thus,

$$\lim_{x\to+\infty}\left(1+\frac{1}{n}\right)^n = e$$

Deduction:

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$$

We know that

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \to (1)$$

$$put\ n = \frac{1}{x} \Rightarrow x = \frac{1}{n}\ in(i)$$

$$when\ n\to\infty, x\to 0$$

So (i)

$$\boldsymbol{\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e}$$

Theorem:

Prove that

$$\boldsymbol{\lim_{x\to 0}\frac{a^x-1}{x} = log_e a}$$

Proof:

$$L.H.S = \lim_{x\to a}\frac{a^x-1}{x}$$

$$\boldsymbol{put\ a^x - 1 = y \Rightarrow a^x = 1+y}$$

So $\boldsymbol{x = log_a(1+y)}$

As $\boldsymbol{x\to 0, y\to 0\ so}$

$\boldsymbol{L.H.S = \lim_{y\to 0}\frac{y}{log_a(1+y)}}$

$$\boldsymbol{= \lim_{y\to 0}\frac{y}{log_a(1+y)}}$$

$$\boldsymbol{= \lim_{y\to 0}\frac{1}{\frac{1}{y}log_a(1+y)} = \lim_{y\to 0}\frac{1}{log_a(1+y)^{\frac{1}{y}}}}$$

$$\boldsymbol{= \frac{1}{log_a e} = log_e a \qquad \because \lim_{y\to 0}(1+y)^{\frac{1}{y}} = e}$$

$$\boldsymbol{R.H.S}$$

Thus $\boldsymbol{\lim_{x\to 0}\frac{a^x-1}{x} = log_e a}$

Deduction:

$$\boldsymbol{\lim_{x\to 0}\left(\frac{e^x-1}{x}\right) = log_e e = 1}$$

Since we know that

$$\boldsymbol{\lim_{x\to 0}\left(\frac{a^x-1}{x}\right) = log_e a \to (i)}$$

$$\boldsymbol{put\ a = e\ in (1) we\ get}$$

$\boldsymbol{\lim_{x\to 0}\frac{e^x-1}{x} = log_e e = 1}$

Important results to remember:

$$\boldsymbol{i)\ \lim_{x\to +\infty}(e^x) = \infty}$$

$$\boldsymbol{ii)\ \lim_{x\to -\infty}(e^x) = \lim_{x\to -\infty}\left(\frac{1}{e^{-x}}\right) = 0}$$

$$\boldsymbol{iii)\ \lim_{x\to \pm\infty}\left(\frac{a}{x}\right) = 0\ ehere\ a\ is\ any\ real\ numbers.}$$

The Sandwich theorem:

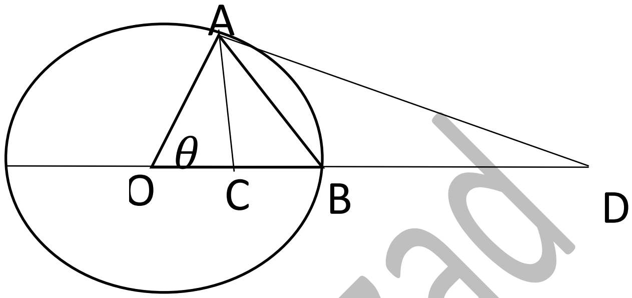
$let\ f, g\ and\ h\ be\ functiions\ s.that$ $f(x)\le g(x)\le h(x)$ For all numbers x in some open interval containing "c" $itself\ .if\ \lim_{x\to c} f(x) = L\ and$ $\lim_{x\to c} h(x) = L\ then\ g(x) is\ sandwhich\ b/w$ $f(x) and\ h(x) so\ that\ \lim_{x\to c} g(x) = L$

Theorem:

If θ is measured in radian, then $\lim_{\theta\to 0}\frac{sin\theta}{\theta} = 1$

Proof:

$draw\ a\ unit\ circle\ (radius\ 1) in\ which$



Area of $\triangle\ OAB = \frac{1}{2}(base)(perpendicual)$

$$= \frac{1}{2}|OB||AC|\ where \frac{|AC|}{|OA|} = sin\theta$$

$$= \frac{1}{2}(1)(sin\theta) \qquad |AC| = |OA| sin\theta$$

$$= \frac{1}{2}sin\theta \qquad |AC| = sin\theta$$

$$\because radius = |OA| = |OB| = 1$$

Area of sector $OAB = \frac{1}{2}r^2\theta$

$$= \frac{1}{2}(1)^2\theta = \frac{1}{2}\theta$$

Area of $\triangle\ OAD = \frac{1}{2}(base)(perpendicual)$

$$= \frac{1}{2}|OA||AD| \quad where\ \frac{|AD|}{|OA|} = tan\theta$$

$$= \frac{1}{2}tan\theta \qquad |AD| = tan\theta$$

Now by (1)

$$\frac{1}{2}sin\theta < \frac{1}{2}\theta < \frac{1}{2}tan\theta$$

$$or\quad sin\theta < \theta < tan\theta$$

$$or \quad \frac{sin\theta}{sin\theta} < \frac{\theta}{sin\theta} < \frac{sin\theta}{cos\theta}\times\frac{1}{sin\theta} \qquad (\div\ by\ sin\theta)$$

$$or \quad 1 < \frac{\theta}{sin\theta} < \frac{1}{cos\theta}$$

$$take\ reciprocal\ and\ limit\ \theta\to 0$$

$$\lim_{\theta\to 0}(1) > \lim_{\theta\to 0}\frac{sin\theta}{\theta} > \lim_{\theta\to 0}cos\theta$$

$$1 > \lim_{\theta\to 0}\frac{sin\theta}{\theta} > 1$$

$$applying\ sandwhich\ therom$$

$$\lim_{\theta\to 0}\frac{sin\theta}{\theta} = 1$$

Hence proved.

Exercise 1.3

Q1. Evaluate each limit by using theorems of limits.

$$i)\ \lim_{x\to 3}(2x+4)$$

Solution:

$$\lim_{x\to 3}(2x+4)$$

$$=\lim_{x\to 3}2x+\lim_{x\to 3}4=2(3)+4=10$$

$$ii)\ \ \lim_{x\to 1}(3x^2-2x+4)$$

Solution:

$$=3(1)^2-2(1)+4=3-2+4=5$$

$$iii)\ \lim_{x\to 3}\sqrt{x^2+x+4}$$

$$solution:\ \ \sqrt{(3)^2+3+4}=\sqrt{16}=4$$

$$iv)\ \lim_{x\to 2}x\sqrt{x^2-4}$$

$$solution: (2)\sqrt{(2)^2-4}=0$$

$$v)\ \ \lim_{x\to 2}\left(\sqrt{x^3+1}-\sqrt{x^2+5)}\right)$$

$$\sqrt{(2)^3+1}-\sqrt{(2)^3+5}=3-3=0$$

$$(vi)\ \lim_{x\to -2}\frac{2x^3+5x}{3x-2}$$

$$\frac{2(-2)^3+5(-2)}{3(-2)-2}=\frac{-16-10}{-8}=-\frac{26}{-8}=\frac{13}{4}$$

Q2. Evaluate each limit by using algebra techniques.

$$i)\ \lim_{x\to -1}\frac{x^3-x}{x+1}$$

Solution:

$$\lim_{x\to -1}\frac{x^3-x}{x+1}\qquad \left(\frac{0}{0}\right)form$$

$$=\lim_{x\to -1}\frac{x(x^2-1)}{x+1}=\lim_{x\to -1}\frac{x(x-1)(x+1)}{x+1}$$

$$=\lim_{x\to -1}x(x-1)=(-1)(-1-1)=2$$

$ii)$

$$\lim_{x\to 0}\left(\frac{3x^3+4x}{x^2+x}\right)$$

Solution:

$$\lim_{x\to 0}\left(\frac{3x^3+4x}{x^2+x}\right)\qquad \left(\frac{0}{0}\right)form$$

$$=\lim_{x\to 0}\frac{x(3x^2+4)}{x(x+1)}=\lim_{x\to 0}\frac{3x^2+4}{x+1}$$

$$=\frac{3(0)^2+4}{0+1}=\frac{4}{1}=4$$

$iii)$

$$\lim_{x\to 2}\frac{x^3-8}{x^2+x-6}\quad \left(\frac{0}{0}\right)form$$

$$=\lim_{x\to 2}\frac{(x)^3-(2)^3}{x^2+3x-2x-6}$$

$$=\lim_{x\to 2}\frac{(x-2)(x^2+4+2x)}{x(x+3)-2(x+3)}$$

$$=\lim_{x\to 2}\frac{(x-2)(x^2+4+2x)}{(x+3)(x-2)}$$

$$=\lim_{x\to 2}\frac{x^2+4+2x}{x+3}=\frac{(2)^2+4+2(2)}{2+3}=\frac{12}{5}$$

$iv)$

$$\lim_{x\to 1}\frac{x^3-3x^2+3x-1}{x^3-x}\qquad \left(\frac{0}{0}\right)form$$

$$=\lim_{x\to 1}\frac{(x-1)^3}{x(x^2-1)}\qquad \because (x-1)^3=x^3-3x^2+3x-1$$

$$\lim_{x\to 1}\frac{(x-1)^3}{x(x-1)(x+1)}=\lim_{x\to 1}\frac{(x-1)^2}{x(x+1)}$$

$$=\frac{(1-1)^2}{1(1+1)}=0$$

$v)$

$$\lim_{x\to -1}\left(\frac{x^3+x}{x^2-1}\right)$$

Solution:

$$\lim_{x\to -1}\left(\frac{x^3+x}{x^2-1}\right)\qquad \left(\frac{0}{0}\right)form$$

$$\lim_{x\to -1}\left(\frac{x^2(x+1)}{(x-1)(x+1)}\right)\lim_{x\to -1}\frac{x^2}{x-1}=\frac{(-1)^2}{-1-1}=\frac{1}{-2}$$

$vi)$

$$\lim_{x\to 4}\frac{2x^2-32}{x^3-4x^2}$$

Solution:

$$\lim_{x-4}\frac{2x^2-32}{x^3-4x^2}\qquad \left(\frac{0}{0}\right)form$$

$$=\lim_{x\to 4}\frac{2(x^2-16)}{x^2(x-4)}=\lim_{x\to 4}\frac{2(x-4)(x+4)}{x^2(x-4)}$$

$$\lim_{x\to 4}\frac{2(x+4)}{x^2}=\frac{2(4+4)}{4^2}=1$$

$vii)$

$$\lim_{x\to 2}\frac{\sqrt{x}-\sqrt{2}}{x-2}\times\frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}$$

$$=\lim_{x\to 2}\frac{\left(\sqrt{x}\right)^2-\left(\sqrt{2}\right)^2}{(x-2)\sqrt{x}+\sqrt{2}}$$

$$\lim_{x\to 2}\frac{x-2}{(x-2)(\sqrt{x}-\sqrt{2}}=\lim_{x\to 2}\frac{1}{\sqrt{x}+\sqrt{2}}=\frac{1}{\sqrt{2}+\sqrt{2}}$$

$$=\frac{1}{2\sqrt{2}}$$

$viii)$

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

Solution:

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\qquad \left(\frac{0}{0}\right)\ \ form$$

$$=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\times\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\lim_{h\to 0}\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x})}$$

$$=\frac{1}{\sqrt{x}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

$ix)$

$$\lim_{\theta\to a}\frac{x^n-a^n}{x^m-a^m}$$

Solution:

$\lim_{\theta \to a} \frac{x^n - a^n}{x^m - a^m}$ $\left(\frac{0}{0}\right)$ *form*

dividing up and down by $x - a$

$$= \lim_{x \to a} \left(\frac{\frac{x^n - a^n}{x - a}}{\frac{x^m - a^m}{x - a}} \right) = \frac{\lim_{x \to a} \frac{x^n - a^n}{x - a}}{\lim_{x \to a} \frac{x^m - a^m}{x - a}}$$

$$= \frac{na^{n-1}}{ma^{m-1}} \quad \left(\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1} \right)$$

$$= \frac{n}{m} a^{n-1-m+1} = \frac{n}{m} a^{n-m}$$

Q3. Evaluate the following limits.

$$i) \lim_{x \to 0} \frac{sin7x}{x}$$

Solution:

$$\lim_{x \to 0} \frac{sin7x}{x} \qquad \left(\frac{0}{0}\right) form$$

$$= 7 \left(\lim_{x \to 0} \frac{sin7x}{7x} \right) = 7(1) = 7$$

$$\because \lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1$$

ii)

$$\lim_{x \to 0} \frac{sinx^0}{x}$$

Solution:

$$\lim_{x \to 0} \frac{sinx^0}{x} \qquad \left(\frac{0}{0}\right) form$$

$$= \frac{\lim_{x \to 0} \frac{sin\pi x}{180}}{x}$$

$$\because 1^0 = \frac{\pi}{180} rad$$

$$so\ x^0 = \frac{\pi x}{180} rad$$

$$= \frac{\lim_{x \to 0} \frac{sin\pi x}{180}}{\frac{\pi x}{180}} \times \frac{\pi}{180}$$

$$1 \times \frac{\pi}{180} = \frac{\pi}{180}$$

Iii)

$$\lim_{\theta \to 0} \frac{1 - cos\theta}{sin\theta}$$

Solution:

$$\lim_{\theta \to 0} \frac{1 - cos\theta}{sin\theta} \qquad \left(\frac{0}{0}\right) form$$

$$= \lim_{\theta \to 0} \frac{1 - cos\theta}{sin\theta} \times \frac{1 + cos\theta}{1 + cos\theta}$$

$$= \lim_{0 \to 0} \frac{1 - \cos^2 \theta}{sin\theta(1 + cos\theta)}$$

$$= \lim_{\theta \to 0} \frac{1 - \cos^2 \theta}{sin\theta(1 + cos\theta)} = \lim_{\theta \to 0} \frac{\sin^2 \theta}{sin\theta(1 + cos\theta)}$$

$$= \lim_{\theta \to 0} \frac{sin\theta}{1 + cos\theta} = \frac{sin\theta}{1 + cos\theta} = \frac{0}{1 + 1} = 0$$

iv)

$$\lim_{x \to \pi} \frac{sinx}{\pi - x} \qquad \left(\frac{0}{0}\right) form$$

put $\pi - x = t$

⇨ $x = \pi - t$

when $x \to \pi$ *then* $t \to o$

So

$$\lim_{x \to \pi} \frac{sinx}{\pi - x} = \lim_{t \to 0} \frac{\sin(\pi - t)}{t}$$

$$= \lim_{t \to 0} \frac{sint}{t} \qquad \because \sin(\pi - \theta) = sin\theta$$

$$= 1$$

v)

$$\lim_{x \to 0} \frac{sinax}{sinbx}$$

Solution:

$$\lim_{x \to 0} \frac{sinax}{sinbx} \qquad \left(\frac{0}{0}\right) form$$

$$\lim_{x \to 0} \left(\frac{\frac{sinax}{ax} \times ax}{\frac{sinbx}{bx} \times bx} \right)$$

$$= \left(\frac{\lim_{x \to 0} \frac{sinax}{ax} \times ax}{\lim_{x \to 0} \frac{sinbx}{bx} \times bx} \right) = \frac{1 \times ax}{1 \times bx} = \frac{a}{b}$$

vi)

$$\lim_{x \to 0} \frac{x}{tanx}$$

Solution:

$$\lim_{x \to 0} \frac{x}{tanx} \qquad \left(\frac{0}{0}\right) form$$

$$= \lim_{x \to 0} x.cotx \qquad \because cotx = \frac{1}{tanx}$$

$$= \lim_{x \to 0} x.\frac{coxx}{sinx}$$

$$= \lim_{x \to 0} \frac{x}{sinx}.\lim_{x \to 0} cosx$$

$$= \left(\lim_{x \to 0} \frac{sinx}{x} \right)^{-1}.\lim_{x \to 0} cosx$$

$$= (1)^{-1}.cosx = 1.1 = 1$$

vii)

$$\lim_{x \to 0} \frac{1 - cos2x}{x^2}$$

Solution:

$$\lim_{x \to 0} \frac{1 - cos2x}{x^2} \qquad \frac{0}{0} form$$

$$= \lim_{x \to 0} \frac{2\sin^2 x}{x^2} \qquad \because 1 - cos\theta = 2\sin\left(\frac{\theta}{2}\right)$$

⇨ $1 - cos2\theta = 2\sin^2 \theta$

⇨ $= 2\left(\lim_{x \to 0} (\frac{sinx}{x} \right)^2 = 2(1)^2 = 2$

(viii)

$$\lim_{x \to 0} \left(\frac{1 - cosx}{\sin^2 \theta} \right)$$

Solution:

$$\lim_{x \to 0} \left(\frac{1 - cosx}{\sin^2 \theta} \right) \qquad \left(\frac{0}{0}\right) form$$

$$\lim_{x \to 0} \frac{1 - cosx}{1 - \cos^2 x} \qquad \because \sin^2\theta + \cos^2 \theta = 1$$

⇨ $\sin^2 \theta = 1 - \cos^2 \theta$

⇨ $\lim_{x \to 0} \frac{1 - cosx}{(1 - cosx)(1 + cosx)}$ $= (1 - cos\theta)(1 + cos\theta)$

$$\Rightarrow \quad = \lim_{\theta\to0}\frac{1}{1+cosx}=\frac{1}{1+\cos(0)}=\frac{1}{1+1}=\frac{1}{2}$$

ix)

$$\lim_{\theta\to0}\frac{\sin^2\theta}{\theta}$$

Solution:

$$\lim_{\theta\to0}\frac{\sin^2\theta}{\theta}\qquad\left(\frac{o}{o}\right) form$$

$$\lim_{\theta\to0}\frac{sin\theta}{\theta}\lim_{\theta\to0}sin\theta=1.sin\theta=1.0=0$$

$x)$

$$\lim_{x\to0}\frac{secx-cosx}{x}$$

Solution:

$$\lim_{x\to0}\frac{secx-cosc}{x}\qquad\left(\frac{0}{0}\right) form$$

$$=\lim_{x\to0}\frac{1}{x}\left(\frac{1}{cosx}-cosx\right)=\lim_{x\to0}\left(\frac{1-\cos^2x}{cosx}\right)$$

$$=\lim_{x\to0}\left(\frac{1}{x}\right)\left(\frac{\sin^2x}{cosx}\right)=\lim_{x\to0}\frac{sinx}{x}.\lim_{x\to0}\frac{sinx}{cosx}$$

$$=\lim_{x\to0}\frac{sinx}{x}.\lim_{x\to0}tanx=1.tan0=0$$

xi)

$$\lim_{\theta\to0}\frac{1-cosp\theta}{1+cosq\theta}$$

Solution:

$$\lim_{\theta\to0}\frac{1-cosp\theta}{1+cosq\theta}\qquad\left(\frac{o}{o}\right) form$$

$$=\lim_{\theta\to0}\frac{2\sin^2\left(\frac{p\theta}{2}\right)}{2\sin^2\frac{q\theta}{2}}=\frac{\left(\lim_{\theta\to0}sin\left(\frac{\theta}{2}\right)\right)^2}{\left(\lim_{\theta\to0}sin\left(\frac{q\theta}{2}\right)\right)^2}$$

$$=\frac{\left(\lim_{\theta\to0}\frac{sin\frac{p}{2}}{\frac{p\theta}{2}}\times\frac{p\theta}{2}\right)^2}{\left(\lim_{\theta\to0}\frac{sin\frac{q}{2}}{\frac{q\theta}{2}}\times\frac{q\theta}{2}\right)^2}=\frac{\left(1\times\frac{p\theta}{2}\right)^2}{\left(1\times\frac{q\theta}{2}\right)^2}$$

$$=\frac{\frac{p^2\theta^2}{4}}{\frac{q^2\theta^2}{4}}=p^2/q^2$$

xii)

$$\lim_{\theta\to0}\frac{tan\theta-sin\theta}{\sin^3\theta}$$

Solution:

$$\lim_{\theta\to0}\frac{tan\theta-sin\theta}{\sin^3\theta}\qquad\left(\frac{0}{0}\right) form$$

$$=\lim_{\theta\to0}\frac{1}{sin^3\theta}\left(\frac{sin\theta}{cos\theta}-sin\theta\right)$$

$$=\lim_{\theta\to0}\frac{1}{\sin^3\theta}(sin\theta-sin\theta cos\theta)$$

$$=\lim_{\theta\to0}\frac{sin\theta}{sin^3\theta}(1-cos\theta)=\lim_{\theta\to0}1/\sin^2\theta\ \ (1-cos\theta)$$

$$=\lim_{\theta\to0}\frac{1-cos\theta}{1-\cos^2\theta}=\lim_{\theta\to0}\frac{1-cos\theta}{(1-cos\theta)(1+cos\theta)}$$

$$=\frac{1}{1+cos\theta}=\frac{1}{1+1}=\frac{1}{2}$$

Q4.express each limit in terms of *e*

i) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$

Solution:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$$

$$=\left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\right]^2=e^2$$

ii) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\frac{n}{2}}$

Solution:

$$=\left[\lim_{n\to\infty}\left(1+\frac{}{n}\right)^n\right]^{\frac{1}{2}}=e^{\frac{1}{2}}$$

iii) $\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^n$

Solution:

$$\left[\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^{\frac{3n}{3}}\right]=\left[\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)\right]^{\frac{1}{3}}$$

$$=e^{\frac{1}{3}}$$

$iv)$

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n$$

Solution:

$$\lim_{n\to\infty}\left(1+\left(-\frac{1}{n}\right)\right)^n=\left[\lim_{n\to\infty}\left(1+\left(-\frac{1}{n}\right)\right)^{-n}\right]^{-1}$$

$$=e^{-1}$$

$v)$

$$\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^n$$

Solution:

$$\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^n$$

$$=\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^{\frac{4n}{4}}=\left[\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^{\frac{n}{4}}\right]^4=e^4$$

$vi)$

$$\lim_{x\to0}(1+3x)^{\frac{2}{x}}$$

Solution:

$$\lim_{x\to0}(1+3x)^{\frac{2}{x}}$$

$$=\lim_{x\to0}(1+3x)^{\frac{2}{x}\times\frac{3}{3}}=\lim_{x\to0}(1+3x)^{\frac{6}{3x}}$$

$$=\left[\lim_{x\to0}(1+3x)^{\frac{1}{3x}}\right]^6=e^6$$

vii)

$$\lim_{x\to 0}(1+2x^2)^{\frac{1}{x^2}}$$

Solution:

$$\lim_{x\to 0}(1+2x^2)^{\frac{1}{x^2}}$$

$$=\lim_{x\to 0}(1+2x^2)^{\frac{2}{2x^2}}=\left[\lim_{x\to 0}(1+2x^2)^{\frac{1}{2x^2}}\right]^2=e^2$$

viii)

$$\lim_{h\to 0}(1-2h)^{\frac{1}{h}}$$

Solution:

$$=\lim_{h\to 0}(1-2h)^{\frac{1}{h}}=\lim_{h\to 0}(1+(-2h))^{\frac{1}{h}}$$

$$=\lim_{h\to 0}(1+(-2h))^{\frac{-2}{-2h}}=\left[\lim_{h\to 0}(1+(-2h))^{\frac{1}{-2h}}\right]^{-2}$$

$$=e^{-2}$$

ix)

$$\lim_{x\to 0}\left(\frac{x}{1+x}\right)^x$$

Solution:

$$\lim_{x\to 0}\left(\frac{x}{1+x}\right)^x$$

$$=\lim_{x\to 0}\left(\frac{1+x}{x}\right)^{-x}=\lim_{x\to 0}\left(\frac{1}{x}+1\right)^{x(-1)}$$

$$=\left[\lim_{x\to 0}\left(\frac{1}{x}+1\right)^x\right]^{-1}=e^{-1}$$

(x)

$$\lim_{x\to 0}\left(\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}\right), x<0$$

Solution:

$$\lim_{x\to 0}\left(\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}\right)$$

Since $x<0$, *so let* $x=-t$ *where* $t>o$

as $x\to 0$, $t\to 0$

$$so\ \lim_{x\to 0}\left(\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}\right)=\lim_{t\to 0}\left(\frac{e^{\frac{1}{t}}-1}{e^{\frac{1}{t}}+1}\right)$$

$$=\lim_{t\to 0}\left(\frac{e^{\frac{1}{\frac{1}{t}}}-1}{e^{\frac{1}{\frac{1}{t}}}+1}\right)=\left(\frac{e^{\frac{1}{\frac{1}{0}}}-1}{e^{\frac{1}{\frac{1}{0}}}+1}\right)=\frac{e^{\left(\frac{1}{\infty}\right)}-1}{e^{\frac{1}{\infty}}+1}=\frac{\frac{1}{\infty}-1}{\frac{1}{\infty}+1}$$

$$=\frac{0-1}{0+1}=-\frac{1}{1}=-1$$

xi)

$$\lim_{x\to 0}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, x>0$$

Solution:

$$\lim_{x\to 0}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$$

$$=\lim_{x\to 0}\frac{e^{\frac{1}{x}}\left(1-\frac{1}{e^{\frac{1}{x}}}\right)}{e^{\frac{1}{x}}\left(1+\frac{1}{e^{\frac{1}{x}}}\right)}$$

$$=\frac{\left(1-\frac{1}{e^{\frac{1}{0}}}\right)}{\left(1+\frac{1}{e^{\frac{1}{0}}}\right)}=\frac{1-\frac{1}{e^{\infty}}}{1+\frac{1}{e^{\infty}}}=\frac{1-\frac{1}{\infty}}{1+\frac{1}{\infty}}=\frac{1-0}{1+0}=1$$

The left hand limit:

if $\lim_{x\to a^-} f(x)=L$ *it means* $f(x)$ *takes value* L *as* x *approaches to a from the left side of "a" (i.e from* $-\infty$ *to a) then* $\lim_{x\to a^-} f(x)=L$ *is called left hand limit.*

The Right hand limit:

if $\lim_{x\to a^+} f(x)=L$ *it means* $f(x)$ *takes value* L *as* x *approaches to a from the right side of* a*(i.e from a to* ∞*) then* $\lim_{x\to a^+} f(x)=L$ *is called right hand limit.*

Existence of Limit of function(criteria)

$$\lim_{x\to a} f(x)=L \text{ if and only if}$$

$$\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)=L$$

$$i.e\ L.H.S=R.H.S$$

Continuous Function:

A function f is said to be continuous at a number $x=o$ *if*

i) $f(a)$ *is defined ii)* $\lim_{x\to a} f(x)$ *exist. iii)* $\lim_{x\to a} f(x)=f(a)$

Discontinuous function:

A function $f(x)$is said to be discontinuous at $x = a$ if
$\lim_{x\to a} f(x) \neq f(a)$

- $if\ f(x) is\ not\ defined\ at\ x = a\ then\ f(x) is\ called\ discontinuous$
- Any function which does not satisfied at least one of three conditions of continuous is called discontinuous.

Exercise 1.4

Q1. Determine the left hand limit and the right hand limit and then find the limit of the following functions when $x \to c$

$i)\ f(x) = 2x^2 + x - 5,\ c = 1$

Solution:

$L.H.S$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2x^2 + x - 5) = 2(1)^2 + 1 + 5 = -2$

$R.H.S$

$\lim_{x\to +} f(x) = \lim_{x\to 1^+} (2x^2 + x - 5) = 2(1)^2 + 1 + 5 = -2$

$As\ L.H.S = R.H.S$

So,

$\lim_{x\to 1} f(x) = -2$

$ii)$

$f(x) = \frac{x^2-9}{x-3} \quad ,c = -3$

Solution:

$L.H.S$

$$\lim_{x\to -3^-} f(x) = \lim_{x\to -3^-} \frac{x^2 - 9}{x - 3}$$

$$\lim_{x\to -3^-} \frac{(x-3)(x+3)}{(x-3)} = \lim_{x\to -3^-} x + 3 = -3 + 3 = 0$$

$R.H.S$

$$\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+} \frac{x^2 - 9}{x - 3}$$

$$\lim_{x\to -3^+} \frac{(x-3)(x+3)}{(x-3)} = \lim_{x\to -3^+} x + 3 = -3 + 3 = 0$$

$As\ L.H.S = R.H.S$

So,

$$\lim_{x\to -3} f(x) = 0$$

$iii)$

$f(x) = |x - 5|, c = 5$

Solution:

L.H.S

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} |x - 5| = 5 - 5 = 0$$

R.H.S

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} |x - 5| = 5 - 5 = 0$$

As

$L.H.S = R.H.S$

So

$$\lim_{x\to 5} f(x) = 0$$

Q2. *Discuss the continuous of $f(x)$ at $x = c$*

i)

$$f(x) = \begin{cases} 2x + 5 & if\ x \leq 1 \\ 4x + 1 & if\ x > 2 \end{cases} \quad ,c = 2$$

Solution:

$L.H.S$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2x + 5) = 2(2) + 5 = 9$

R.H.S

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (4x + 1) = 4(2) + 1 = 9$

$$At\ x = 2$$

$$f(x) = 2x + 5$$

⇨ $f(2) = 2(2) + 5 = 9$

As $L.H.S = R.H.S\ so$

$$\lim_{x\to 2} f(x) = 9$$

⇨ $\lim_{x\to 2} f(x) = f(x) is\ continous\ at\ x = 2$

ii)

$$f(x) = \begin{cases} 3x - 1 if\ x < 1 \\ 4 \quad if\ x = 1, \quad c = 1 \\ 2x\ \ if\ x > 1 \end{cases}$$

Solution:

L.H.S

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} (3x - 1) = 3(1) - 1 = 2$

R.H.S

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2x) = 2(1) = 2$$

$$at\ x = 1, f(x) = 4 \Rightarrow f(1) = 4$$

$$as\ L.H.S = R.H.S\ so\ \lim_{x\to 1} f(x)\ exist.$$

But $\lim_{x\to 1} f(x) \neq f(1)$hence f(x) is discontinuous.

Q3.if $f(x) = \begin{cases} 3x\ \ if\ x \leq -2 \\ x^2 - 1\ if - 2 < x < 2 \\ 3\ if\ x \geq 2 \end{cases}$

Discuss continuity at $x = 2$ and $x = -2$

Solution:

i)

$x = 2$

$$L.H.S;.\ \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x^2 - 1)$$

$$= (2)^2 - 1 = 3$$

$$R.H.S\ \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 3 = 3$$

$$at\ x = 2, f(x) = 3 \Rightarrow f(2) = 3$$

$$\because L.H.S = R.H.S\ so\ \lim_{x\to 2} f(x)\ exist.$$

$$so\ \lim_{x\to 2} f(x) = f(2)$$

$$hence\ f\ is\ continous\ at\ x = 2$$

ii) $x = -2$

$$R.H.S;.\ \lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} (x^2 - 1)$$

$$= (-2)^2 - 1 = 3$$

$$L.H.S\ \lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} 3x = 3(-2) = -6$$

$$at\ x = -2, f(x) = 3x \Rightarrow f(-2) = 3(-2) = -6$$

$$\because L.H.S \neq R.H.S\ so\ .$$

$$hence\ f(x) is\ discontinous\ at\ x=-2$$

Q4. If $f(x)=\begin{cases} x+2, x\le -1 \\ c+2, x>-1 \end{cases}$ $if and\ "c"$ $so\ that\ \lim_{x\to -1} f(x)\ exist.$

solution:

L.H.S

$$\lim_{x\to -1^-} f(x)=\lim_{x\to -1^+}(x+2)=-1+2=1$$

$$R.H.S=\lim_{x\to -1^+} f(x)=\lim_{x\to -1^+}(c+2)=c+2$$

Given that $\lim_{x\to -1} f(x)\ exist\ .so$

$$L.H.S=R.H.S$$

- ⇨ $1+c+2$
- ⇨ $1-2=c$
- ⇨ $c=-1$

Q5. Find the value of m and n, so that given function f is continuous $at\ x=3$

$$\begin{cases} mx\ if\ x<3 \\ n\ if\ x=3 \\ -2x+9\ if\ x>3 \end{cases}$$

Solution:

$$L.H.S=\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}(mx)=3m$$

$$R.H.S=\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}(-2x+9)$$

$$=-2(3)+9=3$$

$$at\ x=3\ f(x)=n\ \Rightarrow f(3)=n$$

Given that $f(x) is\ continuous\ so\ L.H.S=R.H.S$

- ⇨ $3m=3$
- ⇨ $m=1$

We know that for a continuous function

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)=f(3)$$

$$3m=3=n$$

- ⇨ $n=3, m=1$

i)

$$f(x)=\begin{cases} mx\ if\ x<3 \\ x^2\ if\ x\ge 3 \end{cases}$$

Solution:

$$=\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}(mx)=3m$$

$$R.H.S=\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}(x^2)$$

$$=(3)^2=9$$

$$at\ x=3\ f(x)=x^2\Rightarrow f(3)=(3)^2=9$$

Given that $f(x) is\ continuous\ so\ L.H.S=R.H.S$

- ⇨ $3m=9$
- ⇨ $m=3$

Q6. If $f(x)=\begin{cases} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, x\ne 2 \\ k\ \ \ , x=2 \end{cases}$

Find value of $k\ so\ that\ f\ is\ continuos.$

Solution:

$$atx=2\ f(x)=k\quad \Rightarrow f(2)=k$$

Now $\lim_{x\to 2} f(x)=\lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$ $\left(\frac{0}{0}\right) form$

$$=\lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\times\frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}$$

$$=\lim_{x\to 2}\frac{2x+5-x-7}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$

$$=\lim_{x\to 2}\frac{x-2}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$

$$=\lim_{x\to 2}\frac{1}{\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$

$$\frac{1}{\left(\sqrt{2(2)+5}+\sqrt{2+7}\right)}=\frac{1}{6}$$

$\because given\ function\ is\ continuous\ at\ x=2$so

$$\lim_{x\to 2} f(x)=f(2)$$

⇨ $\frac{1}{6}=k\Rightarrow k=1/\mathbf{6}$

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Chapter#1. 2nd year Functions and Limits (MCQs)

- ***Each question has four possible answers. Select the correct answer and encircle it.***

1. The term function was introduced by:

(a) Euler (b) Newton
(c) Lagrange (d) Leibniz

2. The symbol y = f(x) i.e. y is equal to f of x, invented by Swiss mathematician ---------.

(a) Euler (b) Cauchy
(c) Leibniz (d) Newton

3. A function $P(x) = 6x^4 + 7x^3 + 5x + 1$ is called a polynomial function of degree ___________ with leading coefficient ------------.

(a) 4 , 6 (b) 2 , 7
(c) 2 , 3 (d) 2 , 5

4. If a variable y depends on a variable x in such a way that each value of x determines exactly one value of y, then y is a ------------ of x.

(a) Independent variable
(b) *not function*
(c) *function* (d) None of these

5. A function, in which the variables are ------------ numbers, then function is called a real valued function of real numbers.

(a) complex (b) rational
(c) *real* (d) None of these

6. $\boldsymbol{Let\ f(x) = x + \frac{1}{x},\ then\ f\left(\frac{1}{x}\right) =}$

(a) $f(x^2+1)$ (b) $f(x)$
(c) $\frac{1}{f(x)}$ (d) $f(x^2)$

7. $\boldsymbol{If\ f(x) = \frac{1-x}{1+x},\ then\ f(\cos x)\ equals}$:

(a) $2\tan^2\frac{x}{2}$ (b) $\tan^2\frac{x}{2}$
(c) $\tan^2 x$ (d) $\cot^2\frac{x}{2}$

8. Domain of the rational function $y = \frac{P(x)}{Q(x)}$ is:

(a) $Q(x) > 0$ (b) $Q(x) < 0$
(c) $Q(x) = 0$ (d) $Q(x) \neq 0$

9. For the function $f(x) = \frac{x^2-1}{x-1}$, f(1) is:

(a) $x + 1$ (b) undefined
(c) *indeterminate* (d) *zero*

10. If a function f is from a set X to a set Y, then set X is called the ------------ of f.

(a) domain (b) range
(c) co-domain (d) None of these

11. Let $f(x) = \frac{x}{x-2}$ then domain of f is the set of all real numbers except:

(a) 0 (b) 1

(c) 2 (d) 3

12. Let $f(x) = x^2$, real valued function then domain of f is the set of all:

(a) real numbers (b) integers
(c) complex numbers
(d) natural numbers

13. Let $f(x) = \frac{x}{x^2 - 4}$, then domain of f is the set of all real numbers except:

(a) 4, – 4 (b) 0
(c) 2, – 2 (d) 0, 4

14. If $f(x)=\sqrt{x+1}$, then domain of $f(x)$ is:

(a) $[0, \infty)$ (b) $[-1, \infty)$
(c) $[1, \infty]$ (d) $[1, \infty)$

15. If $f(x) = \sqrt{x+1}$, then range of $f(x)$ is:

(a) $(-\infty, \infty)$ (b) $[-\infty, \infty]$

(c) $[0, \infty)$ (d) $[-1, \infty)$

16. The domain of the function $f(x)=\frac{|x+2|}{x+2}$ is:

(a) R (b) R – { 2}
(c) R – {2 , –2} (d) R – {– 2}

17. The range of the function $f(x) = |x|$ is:

(a) $(-\infty, \infty)$ (b) $[0, \infty)$
(c) $(-\infty, 0]$ (d) $(0, \infty)$

18. Let $f(x) = x^2$, then range of f is the set of all:

(a) real numbers
(b) non-negative real numbers
(c) non-negative integers
(d) complex numbers

19. Let $f(x) = x^2 + 3$, then domain of f is the:

(a) Set of all integers
(b) Set of natural numbers
(c) Set of real numbers
(d) Set of rational numbers

20. Domain f^{-1} = Range f and Range f^{-1} = ------------.

(a) Domain f (b) Range f

(c) Domain f^{-1} (d) None of these

21. A function $P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$ is called a polynomial function of degree n, with leading coefficient a_n.

(a) $a_{n-1} = 0$ (b) $a_n = 0$
(c) $a_{n-1} \neq 0$ (d) $a_n \neq 0$

22. A function, in which the variable appears as exponent (power), is called a\an ------------ function.

(a) constant (b) explicit
(c) exponential (d) inverse

23. Let $f(x) = \sqrt{x^2 - 9}$, then range of f is the set:

(a) $]-\infty, \infty[$ (b) $[0, \infty)$
(c) $[3, \infty]$ (d) $[-3, 3]$

24. Which of the following functions is a polynomial function?

(a) $\frac{x^3 - 1}{\sqrt{x+2}}, x \neq -2$
(b) $x^5 + 6x^4 + 7x^3 + x^2 + \sqrt{x} + 4$
(c) $\frac{2x^2 + 7x + 4}{10}$
(d) $ax^2 + b\sqrt{x} + c$

25. If the degree of a polynomial function is ----------, then it is called a linear function.

(a) 0 (b) 1

(c) 2 (d) 3

26. Let X and Y be the set of real numbers, a function C : X → Y defined by C(x) = a ∀ x ∈ X , a ∈ Y and a is a constant number. Then C is called a\an ------------ function.

(a) constant (b) implicit
(c) identity (d) inverse

27. Which of the following is a rational function?

(a) $\frac{\sqrt{1 + x + x^2}}{2 + x}, x \neq -2$
(b) $\frac{2x^3 + 7x^2 + 8x + 1}{x^{3/2}}, x > 0$
(c) $\frac{1}{2}\sqrt{2x^2 + 3x + 7}$
(d) $\frac{2x^3 + 3x^2 + 7}{x - 5}, x \neq 5$

28. Which one is a constant function?

(a) $f(x) = x^2$ (b) $f(x) = x$
(c) $f(x) = x + 1$ (d) $f(x) = 14$

29. A function I : X → X for any set X, of the form I(x) = x ∀ x ∈ X is called a\an ------------ function.

(a) constant (b) implicit
(c) identity (d) inverse

30. If x and y are so mixed up and y cannot be expressed in terms of the independent variable x, then y is called a\an ------------ function of x.

(a) constant (b) explicit
(c) implicit (d) inverse

31. Which one is an identity function?

(a) $f(x) = \frac{1}{x}$ (b) $f(x) = g(x)$
(c) $f(x) = x$ (d) $f(x) = 1$

32. Which one is not an exponential function?

(a) 3^x (b) n^x
(c) $e^{x/2}$ (d) x^n

33. Which one is an exponential function?

(a) 2^x (b) x^2
(c) $\log_2 x$ (d) x^e

34. If $f(x) = ax + b$, where $a \neq 0$, a and b are real numbers, then $f(x)$ is a:

(a) constant function
(b) absolute linear function
(c) linear function
(d) quadratic function

35. $y = \log_a x$, where $a > 0$ and $a \neq 1$ is called a ------------- function of x.

(a) implicit (b) explicit
(c) exponential (d) logarithmic

36. $y = \log_{10} x$ is known as the --------- of

(a) common logarithmic
(b) natural logarithmic
(c) exponential (d) None of these

37. If $f(x) = |x|$, $f(x)$ is a

(a) constant function
(b) absolute function
(c) linear function
(d) quadratic function

38. If $x = e^y$, Then $y = \log_e x = \ln x$, is known as the ------------ of x.

(a) common logarithmic
(b) natural logarithmic
(c) exponential
(d) None of these

39. sinh x =

(a) $\frac{1}{2}\left(e^x - e^{-x}\right)$ (b) $\frac{1}{2}\left(e^x + e^{-x}\right)$
(c) $e^x - e^{-x}$ (d) $e^x + e^{-x}$

40. cosh x =

(a) $\frac{1}{2}\left(e^x - e^{-x}\right)$ (b) $\frac{1}{2}\left(e^x + e^{-x}\right)$
(c) $\frac{1}{4}\left(e^x - e^{-x}\right)$ (d) $\frac{1}{4}\left(e^x + e^{-x}\right)$

41. tanh x =

(a) $\frac{e^x + e^{-x}}{e^x - e^{-x}}$ (b) $\frac{1}{2}\left(e^x + e^{-x}\right)$
(c) $\frac{2}{e^x + e^{-x}}$ (d) $\frac{e^x - e^{-x}}{e^x + e^{-x}}$

42. sech x =

(a) $\frac{2}{e^x - e^{-x}}$ (b) $\frac{e^x + e^{-x}}{2}$
(c) $\frac{2}{e^x + e^{-x}}$ (d) $\frac{e^x + e^{-x}}{e^x - e^{-x}}$

43. csch x =

(a) $\frac{1}{2}\left(e^x - e^{-x}\right)$ (b) $\frac{2}{e^x + e^{-x}}$
(c) $\frac{e^x + e^{-x}}{e^x - e^{-x}}$ (d) $\frac{2}{e^x - e^{-x}}$

44. coth x =

(a) $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ (b) $\frac{e^x + e^{-x}}{e^x - e^{-x}}$
(c) $\frac{2}{e^x + e^{-x}}$ (d) $\frac{2}{e^x - e^{-x}}$

45. $\cosh^2 x - \sinh^2 x$ =

(a) 1 (b) -1
(c) 2 (d) -2

46. $\cosh^2 x + \sinh^2 x$ =

(a) $\cosh x^2$ (b) $\cosh 2x$
(c) $\sinh 2x$ (d) $\tanh 2x$

47. $\sinh^{-1} x$ =

(a) $\ell n\,(x + \sqrt{x^2 - 1}\,)$ $x \geq 1$
(b) $\ell n\,(x + \sqrt{x^2 + 1}\,)$ for all x
(c) $\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$, $|x|<1$
(d) $\ell n\left(\frac{1 + \sqrt{1 - x^2}}{x}\right)$ $0 < x \leq 1$

48. $\cosh^{-1} x$ =

(a) $\ell n\,(1 + \sqrt{x^2 - 1}\,)$ $x \geq 1$
(b) $\ell n\,(x + \sqrt{x^2 - 1}\,)$ $x \geq 1$
(c) $\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ $0 < x \leq 1$

(d) $\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$, $|x|<1$

49. $\tanh^{-1}x$ =

(a) $\frac{1}{2}\ell n\left(\frac{1+x}{1-x}\right)$ $|x| < 1$

(b) $\frac{1}{2}\ell n\left(\frac{x+1}{x-1}\right)$ $|x| < 1$

(c) $\ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ $0 < x \leq 1$

(d) $\ell n\left(\frac{1+\sqrt{1+x^2}}{x}\right)$ $x \neq 0$

50. $\text{sech}^{-1}x$ =

(a) $\frac{1}{2}\ell n\left(\frac{x+1}{x-1}\right)$ $|x| < 1$

(b) $\ell n\left(\frac{1+\sqrt{1+x^2}}{x}\right)$ $x \neq 0$

(c) $\ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ $0 < x \leq 1$

(d) $\ell n\left(\frac{1+x}{1-x}\right)$ $|x| < 1$

51. $\text{csch}^{-1}x$ =

(a) $\ell n\left(\frac{1+\sqrt{x^2+1}}{x}\right)$ $x \neq 0$

(b) $\ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ $0 < x \leq 1$

(c) $\ell n\left(\frac{1+\sqrt{1+x^2}}{x}\right)$ $x \neq 0$

(d) $\frac{1}{2}\ell n\left(\frac{x+1}{x-1}\right)$ $|x| < 1$

52. $\coth^{-1}x$ =

(a) $\ell n\left(\frac{x+2}{x-2}\right)$ $|x| < 2$

(b) $\frac{1}{2}\ell n\left(\frac{1+x}{1-x}\right)$ $|x| < 1$

(c) $\frac{1}{2}\ell n\left(\frac{x+1}{x}\right)$ $|x| \neq 0$

(d) $\frac{1}{2}\ell n\left(\frac{x+1}{x-1}\right)$ $x > 1$

53. Inverse hyperbolic functions are expressed in terms of natural:

(a) numbers (b) exponentials
(c) logarithms (d) sines

54. Which one is an explicit function?

(a) $x^2 + 2xy + y^3 + 7 = 0$

(b) $xy + x^3 y + xy^2 + 1 = 0$

(c) $y = x^3 + x^2 + \sqrt{x} + 10$

(d) $xy^2 + y^3 + x^2 y = 4$

55. y = $\sqrt{x-1}$ is a\an ------------ function of x.

(a) constant (b) implicit

(c) explicit (d) inverse

56. Which one is an implicit function?

(a) $y = f(x)$ (b) $f(x, y) = c$
(c) $x = f(u), y = g(u)$
(d) $y = f(u), u = g(x)$

57. Which one is an implicit function?

(a) $xy + xy^2 + x^2 + y = 2$
(b) $y = x^2 + 1$
(c) $x^3 + x^2 + x + 1 = y$
(d) $y = f(x)$

58. Which one is an explicit function?

(a) $y = f(x)$
(b) $f(x, y) = 0$
(c) $x = f(t), y = g(t)$
(d) none of these

59. Every relation, which can be represented by a linear equation in two variables, represents a:

(a) graph

(b) function

(c) cartesian product

(d) relation

60. A function from set X to set Y is denoted by:

(a) $f : X \to X$ (b) $f : Y \to Y$
(c) $f : X \to Y$ (d) $f : Y \to X$

61. If *y* is an image of *x* under the function f, we denote it by:

(a) $x = f(y)$ (b) $x = y$
(c) $y = f(x)$ (d) $f(x, y) = c$

62. The value of the parameter α, for which the function $f(x) = 1 + \alpha x, \alpha \neq 0$ is the inverse of itself is:

(a) 1 (b) – 1
(c) 2 (d) – 2

63. The curves $y = |x|^3 + 2|x|^2 + 1$ and $y = x^3 + 2x^2 + 1$ have the same graph for:

(a) $x > 0$ (b) $x \geq 0$
(c) $x \neq 0$ (d) all x

64. **Parametric equations x = a cos t, y = a sin t represent the equation of:**

(a) line (b) circle
(c) parabola (d) ellipse

65. **Parametric equations: x = a cos θ, y = b sin θ represent the equation of:**

(a) Parabola (b) hyperbola
(c) ellipse (d) circle

66. **Parametric equations x = a sec θ, y = b tan θ represent the equation of:**

(a) Line (b) parabola
(c) Ellipse (d) hyperbola

67. **If $f(x) = \frac{x}{x-1}$, $x \neq 1$ then $f^{-1}(x)$ equals**

(a) $\frac{x}{x-1}$ (b) $\frac{x-1}{x}$

(c) $\frac{x}{1-x}$ (d) $\frac{1-x}{x}$

68. **Inverse of $f(x) = \sqrt{x+1}$ is:**

(a) $f^{-1}(x) = x^2 - 1$

(b) $f^{-1}(x) = \frac{1}{\sqrt{x+1}}$

(c) $f^{-1}(x) = 1 - x^2$

(d) $f^{-1}(x) = x^2 + 1$

69. **Let $f(x) = 4 - x$, $g(x) = 2x + 1$, then $fog\,(x)$ is:**

(a) $5 + 2x$ (b) $3 - 2x$
(c) $2 + 3x$ (d) $2 - 3x$

70. **The perimeter P of square as a function of its area A is:**

(a) $\sqrt{A}$ (b) $2\sqrt{A}$

(c) $4\sqrt{A}$ (d) $\frac{1}{2}\sqrt{A}$

71. **The area A of a circle as a function of its circumference C is:**

(a) $\frac{C^2}{2\pi}$ (b) $\frac{C^2}{4\pi}$

(c) $\frac{C^2}{\pi}$ (d) $\frac{C}{4\pi}$

72. **The volume V of a cube as a function of the area A of its base is:**

(a) $\sqrt{A^2}$ (b) A^3

(c) $\sqrt[3]{A^3}$ (d) $\sqrt{A^3}$

73. **If $f(-x) = f(x)$ for all x in the domain of f, then f is :**

(a) constant function
(b) identity function
(c) even function
(d) odd function

74. **If $f(-x) = -f(x)$ for all x in the domain of f, then f is:**

(a) linear function
(b) identity function
(c) odd function
(d) even function

75. **If $f(x)$ is odd function. If and only if:**

(a) $f(-x) = -f(x)$ (b) $f(-x) = f(x)$

(c) $f(x) = 3f(-x)$ (d) $f(x) = -3f(-x)$

76. **$f(x)$ is even function. If and only if:**

(a) $f(-x) = -f(x)$

(b) $f(-x) = f(x)$

(c) $f(x) = 3f(-x)$

(d) $f(x) = -3f(-x)$

77. **If f is any function, then $\frac{f(x) + f(-x)}{2}$ is always:**

(a) even (b) odd

(c) one-one (d) zero

78. **f(x) = sin x + cos x is ------------ function.**

(a) even (b) odd

(c) composite

(d) neither even nor odd function

79. **Let $f(x)$ = cos x, then $f(x)$ is an: 12801079**

(a) even function (b) odd function
(c) power function (d) none of these

80. **Let $f(x) = x^3$ + sin x, then $f(x)$ is: 12801080**

(a) even function (b) odd function
(c) power function (d) none of these

81. **Let $f(x) = x^3$ + cos x, then $f(x)$ is: 12801081**

(a) an odd function (b) an even function
(c) neither even nor odd
(d) a constant function

82. **If $f(x) = x^2 + 1$, then the value of fof is equal to:**

(a) $x^4 + 2x^2 + 1$ (b) $x^4 - 2x^2 + 2$

(c) $x^4 + 2x^2 + 2$ (d) $x^4 + 2x^2 - 2$

83. **If a relation is given by:**

$R = \{(10,6),(10,4),(10,2)\}$ **then Dom of R is**

(a) { 2, 4, 6 ,10 } (b) { 2, 4, 6 }

(c) { } (d) { 10 }

84. $x^2 + y^2 = 4$ **is:**

(a) Function (b) Not a Function

(c) Ellipse (d) Line

85. $f(x) = x\sec x$, **then f(0)=**

(a) −1 (b) 0

(c) 1 (d) ∞

86. The linear function f(x) = ax+b is an identity function if:

(a) a = 0, b = 1 (b) a = 1, b = 0

(c) a = 1, b = 1 (d) a = 0, b = 1

87. Let f(x) = 4 – x, then $f^2(x) =$

(a) x (b) – x

(c) 4 + x (d) x – 4

88. Let $f(x) = \sqrt{x+4}$, **g(x) = 4 – x, then fog (x) =**

(a) $\sqrt{x}$ (b) $\sqrt{16-x}$

(c) $\sqrt{8-x}$ (d) $2\sqrt{x+4}+1$

89. Let f(x) = – 2, g(x) = 2x + 1, then fog(x) =

(a) 2x + 1 (b) – 2x

(c) 4x + 3 (d) – 2

90. Let f(x) = 4 – x, g(x) = – 2, then fog (x) =

(a) 2 (b) 6

(c) 8 (d) 5

91. The function y = $e^{x.\ln 2}$ **=** 2^x **is a\an ------------- function of x.**

(a) constant (b) explicit

(c) exponential (d) logarithmic

92. If y = f(x), then the variable x is called ------------ variable of a function f.

(a) dependent (b) independent

(c) image of y (d) None of these

93. $\lim_{x\to a}[f(x) - g(x)] =$ **------------.**

(a) $\lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

(b) $\lim_{x\to a} f(x) \times \lim_{x\to a} g(x)$

(c) $\lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

(d) $\lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

94. $\lim_{x\to a}[f(x)]^n =$ **------------.**

(a) $\lim_{x\to a} f(x)$ (b) $n \times [\lim_{x\to a} f(x)]$

(c) $[\lim_{x\to a} f(x)]^n$ (d) None of these

95. If k is any real number, then $\lim_{x\to a}[k.f(x)] =$ **------------.**

(a) $k \lim_{x\to a} f(x)$ (b) $k \times \lim_{x\to a} f(x)$

(c) $\lim_{x\to a} f(x)$ (d) None of these

96. $\lim_{x\to a} \dfrac{x^3 - a^3}{x - a} =$

(a) undefined (b) $3a^2$

(c) a^2 (d) 0

97. $\lim_{x\to 0} x \tan \dfrac{1}{x}$

(a) equals 0 (b) equals 1

(c) equals ∞ (d) does not exist.

98. If $\lim_{x\to 0} \dfrac{f(x)}{g(x)}$ **exists, then:**

(a) both $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ exist

(b) $\lim_{x\to 0} f(x)$ exist but $\lim_{x\to 0} g(x)$ need not exist

(c) $\lim_{x\to 0} f(x)$ need not exist but $\lim_{x\to 0} g(x)$ exist

(d) neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ may exist.

99. $\lim_{x\to a} f(x) = \ell$ **if and only if: 12801099**

(a) $\lim_{h\to 0} f(a+h) = \ell$

(b) $\lim_{h\to a} f(a+h) = \ell$

(c) $\lim_{x\to a} f(a+h) = 0$

(d) $\lim_{h\to 0} f(a+h) = 0$

100. $\lim_{x\to 1} \dfrac{x^2-1}{x-1}$ **is:**

(a) $x+1$ (b) 2

(c) indeterminate (d) 0

101. $\lim_{x\to 2} (2x^2 - 5x + 3) =$

(a) 1 (b) 2

(c) 3 (d) 4

102. $\lim_{x\to 3} \dfrac{x^2+1}{x+3} =$

(a) $\frac{3}{5}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{5}{3}$

103. $\lim_{x\to a} \dfrac{x^n - a^n}{x-a} =$ --------------

(a) $n\,a^n$ (b) $n\,a^{n-1}$

(c) 0 (d) does not exist

104. $\lim_{x\to 4} \dfrac{x^2-16}{x-4} =$

(a) 8 (b) 3

(c) 10 (d) 0

105. $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4} =$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{5}$

106. $\lim_{x\to 16} \dfrac{x-16}{\sqrt{x}-4} =$

(a) 2 (b) 5

(c) 8 (d) 7

107. $\lim_{x\to 2} \sqrt{\dfrac{x^3+2x+4}{x^2+5}} =$

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$

(c) $\frac{5}{3}$ (d) $\frac{3}{4}$

108. $\lim_{x\to 0} \dfrac{\sqrt{x+1}-1}{x} =$

(a) $\frac{1}{3}$ (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{5}$

109. $\lim_{x\to 0} \dfrac{(1+x)^3 - 1}{x} =$

(a) 1 (b) 5

(c) 3 (d) $\frac{1}{2}$

110. $\lim_{x\to 3} \dfrac{x^2-5x+6}{x-3} =$

(a) 1 (b) 3

(c) $\frac{1}{4}$ (d) None of these.

111. $\lim_{x\to\infty} \dfrac{x^2+1}{2x^3-x} =$

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) 0

112. $f(x) = \dfrac{x^2-9}{x-3}$; $x \neq 3$ **is discontinuous because:**

(a) $\lim_{x\to 3} f(x) = f(3)$

(b) $f(3)$ does not exist

(c) $\lim_{x\to 3} f(x)$ does not exist

(d) None of these

113. Let the function $f(x)$ be defined by $f(x) = \dfrac{1}{x}$, $x \neq 0$ and $f(0) = 0$. Then: 12801113

(a) $\lim_{x\to 0} f(x)$ exists and is equal to $f(0)$

(b) $\lim_{x\to 0} f(x)$ exists but is not equal to $f(0)$

(c) $f(x)$ is continuous at $x = 0$

(d) None of these.

114. Let $f(x)$ = sin x. Then

(a) $f(x)$ is continuous for all values of x

(b) $f(x)$ is continuous for all values

except $x = \frac{\pi}{2}$

(c) $f(x)$ is discontinuous at $x = 0$

(d) None of these.

115. The value of f(0) so that f(x) = $(x+1)^{1/x}$ is continuous at $x = 0$ is:

(a) 0 (b) $\frac{1}{e}$

(c) e (d) e^2

116. $x = 3\cos t$, $y = 3\sin t$ represent:

(a) Line (b) Circle

(c) Parabola (d) Hyperbola

117. $\lim_{\theta \to 0} \frac{1-\cos p\theta}{1-\cos q\theta}$ equals:

(a) 0 (b) $\frac{p}{q}$

(c) $\frac{p^2}{q^2}$ (d) $\frac{q^2}{p^2}$

118. If $f(x) = -2x + 8$ then $f^{-1}(x) =$:

(a) $\frac{8+x}{2}$ (b) $\frac{x-8}{2}$

(c) $\frac{8-x}{2}$ (d) $\frac{2}{8-x}$

119. If $f(x) = x^2 - x$ then $f(-2)$ is equal to:

(a) 2 (b) 6

(c) 0 (d) –6

120. $\lim_{x \to \infty}\left(1+\frac{x}{2}\right)^{\frac{1}{x}}$ equals :

(a) e (b) e^{-1}

(c) e^{-2} (d) $\sqrt{e}$

121. If $\lim_{x \to 0} \frac{f(x)-1}{x} = \ell n a; a > o,$ then:

(a) a^{-x} (b) a^{x}

(c) e^{-x} (d) e

1.	d	2.	a	3.	a	4.	c	5.	c
6.	b	7.	b	8.	d	9.	c	10	a
11.	c	12.	a	13	c	14.	b	15	c
16.	d	17	b	18.	b	19	c	20	a
21.	d	22.	c	23.	b	24.	c	25.	b
26.	a	27.	d	28.	d	29	c	30	c
31.	c	32	d	33.	a	34.	c	35	d
36.	a	37.	b	38.	b	39.	a	40	b
41.	d	42.	c	43	d	44.	b	45.	a
46.	b	47.	b	48.	b	49.	a	50	c
51.	a	52.	d	53.	c	54.	c	55.	c
56.	b	57.	a	58.	a	59.	b	60.	c
61.	c	62.	b	63.	b	64.	b	65.	c
66.	d	67.	a	68.	a	69.	b	70.	c
71.	b	72.	d	73.	c	74.	c	75.	a
76.	b	77.	c	78.	d	79.	a	80	b
81.	c	82.	a	83.	d	84	b	85	b
86.	b	87.	a	88.	c	89.	d	90	b
91.	c	92.	b	93.	a	94.	c	95	a
96.	b	97.	a	98.	a	99	a	100	b
101.	a	102.	d	103.	b	104.	a	105.	b
106.	c	107.	a	108.	c	109.	c	110.	a
111.	d	112.	b	113.	d	114.	a	115.	c
116.	b	117.	c	118.	c	119.	b	220	d
121.	b								

Answer Sheet