

**1**

Determine the left hand limit and the right hand limit and then, find the limit of the following functions when  $x \rightarrow c$

$$f(x) = 2x^2 + x - 5, \quad c = 1$$

Chapter 1  
**FUNCTIONS AND  
LIMITS**

**Ex 1.4**

**U**  
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$$f(x) = 2x^2 + x - 5, \quad c = 1$$



L.H.L

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow 1^-} f(x)$$

$$= \lim_{x \rightarrow 1} (2x^2 + x - 5)$$

$$= 2(1)^2 + 1 - 5 = 2 + 1 - 5 = -2 \quad \checkmark$$

R.H.L

$$\lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow 1^+} f(x)$$

$$= \lim_{x \rightarrow 1} (2x^2 + x - 5)$$

$$= 2(1)^2 + 1 - 5 = 2 + 1 - 5 = -2 \quad \checkmark$$

$$\lim_{x \rightarrow 1} f(x) = -2.$$

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## Ex 1.4

1

Determine the left hand limit and the right hand limit and then, find the limit of the following functions when  $x \rightarrow c$

$$(ii) f(x) = \frac{x^2 - 9}{x - 3}, \quad c = -3$$

$$f(x) = \frac{x^2 - 9}{x - 3}, \quad c = -3$$

L.H.L

$$\begin{aligned} \lim_{x \rightarrow c^-} f(x) &= \lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3} \left( \frac{x^2 - 9}{x - 3} \right) \\ &= \frac{(-3)^2 - 9}{-3 - 3} = \frac{9 - 9}{-6} = 0 \quad \checkmark \end{aligned}$$

R.H.L

$$\begin{aligned} \lim_{x \rightarrow c^+} f(x) &= \lim_{x \rightarrow -3^+} f(x) = \lim_{x \rightarrow -3} \left( \frac{x^2 - 9}{x - 3} \right) \\ &= \frac{(-3)^2 - 9}{-3 - 3} = \frac{9 - 9}{-6} = 0 \quad \checkmark \end{aligned}$$

$$\lim_{x \rightarrow -3} f(x) = 0.$$

## Ex 1.4

1 Determine the left hand limit and the right hand limit and then, find the limit of the following functions when  $x \rightarrow c$

(iii)  $f(x) = |x - 5|$ ,  $c = 5$

$$f(x) = |x - 5|, \quad c = 5$$

$$f(x) = \begin{cases} x - 5 & \text{if } x - 5 \geq 0 \\ -(x - 5) & \text{if } x - 5 < 0 \end{cases}$$

$$f(x) = \begin{cases} x - 5 & \text{if } x \geq 5 \quad \checkmark \\ -x + 5 & \text{if } x < 5 \quad \checkmark \end{cases}$$

L.H.L  $\lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^-} (-x + 5) = -5 + 5 = 0 \quad \checkmark$

R.H.L  $\lim_{x \rightarrow 5^+} f(x) = \lim_{x \rightarrow 5^+} (x - 5) = 5 - 5 = 0 \quad \checkmark$

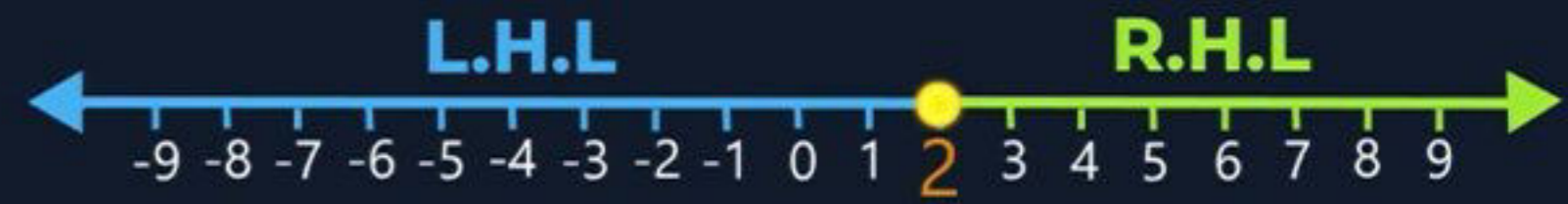
$$\lim_{x \rightarrow 5} f(x) = 0.$$



**Ex 1.4**

2 Discuss the continuity of  $f(x)$  at  $x = c$  (i)  $f(x) = \begin{cases} 2x + 5 & \text{if } x \leq 2 \\ 4x + 1 & \text{if } x > 2 \end{cases}$

$$f(x) = \begin{cases} 2x + 5 & \text{if } x \leq 2 \\ 4x + 1 & \text{if } x > 2 \end{cases}, \quad c = 2$$



L.H.L  $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2} (2x + 5) = 2(2) + 5 = 9$

R.H.L  $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} (4x + 1) = 4(2) + 1 = 9$

Value  $f(2) = 2(2) + 5 = 4 + 5 = 9$

Since  $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$

So  $f$  is continuous at  $c = 2$ .

**Continuous function**

A function  $f(x)$  is said to be continuous at  $c$  if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$$

2

Discuss the continuity of  $f(x)$  at  $x = c$

$$(ii) f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$$

**Ex 1.4**

$$f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \checkmark \\ 4 & \text{if } x = 1 \checkmark, \quad c = 1 \\ 2x & \text{if } x > 1 \checkmark \end{cases}$$



L.H.L.  $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (3x - 1) = 3(1) - 1 = 3 - 1 = 2$

R.H.L.  $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (2x) = 2(1) = 2$

Value  $f(1) = 4$

Since  $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) \neq f(1)$

So  $f$  is discontinuous at  $c = 1$ .

**Continuous function**

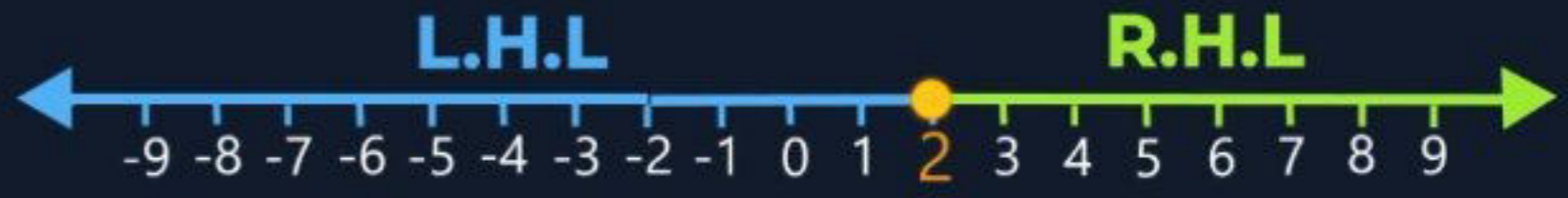
A function  $f(x)$  is said to be continuous at  $c$  if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$$

**Ex 1.4**

**3** If  $f(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$  Discuss continuity at  $x = 2$  and  $x = -2$

$$f(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$$



**Continuous function**

A function  $f(x)$  is said to be continuous at  $c$  if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$$

At  $x=2$

L.H.L  $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2} (x^2 - 1) = 4 - 1 = 3$

R.H.L  $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} (3) = 3$

Value  $f(2) = 3$

Since  $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$

So  $f$  is continuous at  $x=2$ .

Ex 1.4

3 If  $f(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$  Discuss continuity at  $x = 2$  and  $x = -2$

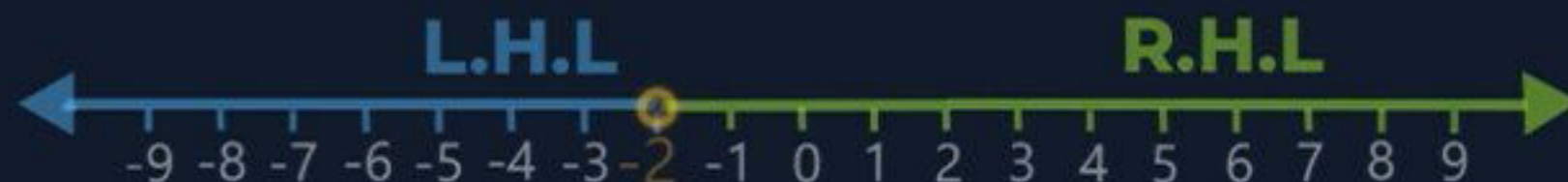
At  $x = -2$

L.H.L  $\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2} (3x) = 3(-2) = -6$

R.H.L  $\lim_{x \rightarrow -2^+} f(x) = \lim_{x \rightarrow -2} (x^2 - 1) = (-2)^2 - 1 = 3$

Since  $\lim_{x \rightarrow -2^-} f(x) \neq \lim_{x \rightarrow -2^+} f(x)$

∴  $f$  is discontinuous at  $x = -2$ .



Continuous function

A function  $f(x)$  is said to be continuous at  $c$  if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$$

4 If  $f(x) = \begin{cases} x+2 & , x \leq -1 \\ c+2 & , x > -1 \end{cases}$  find "c" so that  $\lim_{x \rightarrow -1} f(x)$  exists

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## Limit Exist

A function  $f(x)$  has limit at  $c$  if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x)$$

$$f(x) = \begin{cases} x+2 & , x \leq -1 \quad \checkmark \\ c+2 & , x > -1 \quad \checkmark \end{cases}$$

Give  $\lim_{x \rightarrow -1} f(x)$  exists.

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^+} f(x)$$

$$\lim_{x \rightarrow -1} (x+2) = \lim_{x \rightarrow -1} (c+2)$$

$$-1+2 = c+2$$

$$1 = c+2$$

$$1-2 = c$$

$$c = -1$$



5

Find the values  $m$  and  $n$ , so that given function  $f$  is continuous at  $x = 3$

$$(i) f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$$

**Ex 1.4**

**Continuous function**

A function  $f(x)$  is said to be continuous at  $c$  if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$$

$$f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$$

Given  $f(x)$  is continuous at  $x = 3$ .

$$\Rightarrow \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$$

$$\lim_{x \rightarrow 3} (mx) = \lim_{x \rightarrow 3} (-2x + 9) = n$$

$$3m = -2(3) + 9 = n$$

$$3m = 3 = n$$

$$3m = 3$$

$$m = 1$$

$$3 = n$$

$$n = 3.$$

**5**

Find the values  $m$  and  $n$ , so that given function  $f$  is continuous at  $x = 3$

$$(ii) f(x) = \begin{cases} mx & \text{if } x < 3 \\ x^2 & \text{if } x \geq 3 \end{cases}$$

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**Continuous function**

A function  $f(x)$  is said to be continuous at  $c$  if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$$

$$f(x) = \begin{cases} mx & \text{if } x < 3 \\ x^2 & \text{if } x \geq 3 \end{cases}$$

Given  $f(x)$  is continuous at  $x = 3$ .

$$\Rightarrow \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$$

$$\lim_{x \rightarrow 3} (mx) = \lim_{x \rightarrow 3} (x^2) = 3^2$$

$$3m = 3^2 = 9$$

$$3m = 9 = 9$$

$$3m = 9$$

$$m = 3.$$

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**6** If  $f(x) = \begin{cases} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} & , x \neq 2 \\ k & , x = 2 \end{cases}$  Find the values of  $k$  so that  $f$  is continuous at  $x = 2$

**Continuous function**

A function  $f(x)$  is said to be continuous at  $c$  if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$$

$$f(x) = \begin{cases} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} & , x \neq 2 \quad (x > 2, x < 2) \\ k & , x = 2 \quad \checkmark \end{cases}$$

Given  $f(x)$  is continuous at  $x = 2$

$$\Rightarrow f(2) = \lim_{x \rightarrow 2} f(x)$$

$$k = \lim_{x \rightarrow 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$$

$$k = \lim_{x \rightarrow 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} \times \frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}$$

$$k = \lim_{x \rightarrow 2} \frac{(\sqrt{2x+5})^2 - (\sqrt{x+7})^2}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$$

$$k = \lim_{x \rightarrow 2} \frac{2x+5 - x-7}{(x-2)(\sqrt{2x+5} + \sqrt{x+7})}$$

$$k = \lim_{x \rightarrow 2} \frac{\cancel{x-2}}{(\cancel{x-2})(\sqrt{2x+5} + \sqrt{x+7})}$$

$$k = \frac{1}{\sqrt{2(2)+5} + \sqrt{2+7}} = \frac{1}{3+3}$$

$$k = \frac{1}{6}$$

## Continuous function

A function  $f(x)$  is said to be continuous at  $c$  if

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$$

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