1

Evaluate each limit by using theorems of limits:

(i) $\lim_{x \to 3} (2x + 4)$

Ex 1.3

$$\lim_{x \to 3} (2x + 4)$$

$$= \lim_{x \to 3} (2x) + \lim_{x \to 3} (4)$$

$$= 2 \cdot \lim_{x \to 3} (x) + 4$$

$$= 2 \cdot (3) + 4$$

$$= 6 + 4$$

$$= 10$$

1

Evaluate each limit by using theorems of limits:

(ii) $\lim_{x\to 1}(3x^2 - 2x + 4)$

$$\lim_{x\to 1}(3x^2-2x+4)$$

$$= \lim_{x\to 1}(3x^2) - \lim_{x\to 1}(2x) + \lim_{x\to 1}(4)$$

$$= 3\lim_{x\to 1}(x^2) - 2\lim_{x\to 1}(x) + 4$$

$$= 3\cdot(1^2) - 2(1) + 4$$

$$= 3 - 2 + 4$$

$$= 5.$$

1

Evaluate each limit by using theorems of limits:

(iii) $\lim_{x \to 3} \sqrt{x^2 + x + 4}$

$$\lim_{x \to 3} \sqrt{x^2 + x + 4}$$

$$= \sqrt{\lim_{x \to 3} (x^2 + x + 4)}$$

$$= \sqrt{\lim_{x \to 3} (x^2) + \lim_{x \to 3} (x) + \lim_{x \to 3} (4)}$$

$$= \sqrt{3^2 + 3 + 4}$$

$$= \sqrt{9 + 3 + 4} = \sqrt{16} = 4.$$

1 Evaluate each limit by using theorems of limits:

(iv) $\lim_{x \to 2} x\sqrt{x^2 - 4}$

$$\lim_{x \to 2} \underline{x} \, \underline{\sqrt{x^2 - 4}}$$

$$= \left(\lim_{x \to 2} x\right) \cdot \left(\lim_{x \to 2} \sqrt{x^2 - 4}\right)$$

$$= 2 \cdot \sqrt{\lim_{x \to 2} (x^2 - 4)}$$

$$= 2\sqrt{\lim_{x \to 2} (x^2) - \lim_{x \to 2} (4)}$$

$$= 2\sqrt{2^2 - 4}$$

$$= 2\sqrt{4 - 4} = 2\sqrt{0} = 0.$$

1

Evaluate each limit by using theorems of limits:

(v) $\lim_{x\to 2}\left(\sqrt{x^3+1}-\sqrt{x^2+5}\right)$

UCADEMY

Ex 1.3

$$\lim_{x\to 2}\left(\sqrt{x^3+1}-\sqrt{x^2+5}\right)$$

$$=\lim_{x\to 2}\sqrt{x^3+1}-\lim_{x\to 2}\sqrt{x^2+5}$$

$$=\sqrt{\lim_{x\to 2}(x^3+1)}-\sqrt{\lim_{x\to 2}(x^2+5)}$$

$$=\sqrt{\lim_{x\to 2}(x^3)+\lim_{x\to 2}(1)}-\sqrt{\lim_{x\to 2}(x^2)+\lim_{x\to 2}(5)}$$

$$=\sqrt{2^3+1}-\sqrt{2^2+5}$$

$$=\sqrt{9}-\sqrt{9}=0.$$

1 Evaluate each limit by using theorems of limits:

(vi) $\lim_{x\to -2} \frac{2x^3+5x}{3x-2}$

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FUNCTIONS AND LIMITS

UCADEMY

Ex 1.3

$$\lim_{x\to -2} \frac{2x^3+5x}{3x-2}$$

$$= \frac{\lim_{x\to -2}(2x^3+5x)}{\lim_{x\to -2}(3x-2)}$$

$$= \frac{\lim_{x\to -2}(2x^3)+\lim_{x\to -2}(5x)}{\lim_{x\to -2}(3x)-\lim_{x\to -2}(2)}$$

$$= \frac{2\lim_{x\to -2}(x^3)+5\lim_{x\to -2}(x)}{3\lim_{x\to -2}(x)-2} = \frac{2(-2)^3+5(-2)}{3(-2)-2}$$

$$= \frac{-16-10}{-6-2} = \frac{\cancel{-}26}{\cancel{-}8} = \frac{13}{4} \quad \text{Ans}$$

2 Evaluate each limit by using algebraic techniques.

(i) $\lim_{x \to -1} \frac{x^3 - x}{x + 1}$

Chapter 1

FUNCTIONS AND LIMITS

UCADEMY

Ex 1.3

$$\lim_{x \to -1} \frac{x^3 - x}{x + 1}$$

$$= \lim_{x \to -1} \frac{x(x^2 - 1)}{x + 1}$$

$$= \lim_{x \to -1} \frac{x\cancel{(x+1)}(x-1)}{\cancel{x+1}}$$

$$= \lim_{x \to -1} x(x-1)$$

$$= (-1)(-1-1)$$

$$= (-1)(-2) = 2.$$

2 Evaluate each limit by using algebraic techniques.

(ii) $\lim_{x\to 0}\left(\frac{3x^3+4x}{x^2+x}\right)$

$$\lim_{x\to 0}\left(\frac{3x^3+4x}{x^2+x}\right)$$

$$= \lim_{x\to 0}\frac{\cancel{x}(3x^2+4)}{\cancel{x}(x+1)}$$

$$= \lim_{x\to 0}\frac{3x^2+4}{x+1}$$

$$= \frac{3(0)^2+4}{0+1} = \frac{0+4}{0+1}$$

$$= \frac{4}{1} = 4.$$

2 Evaluate each limit by using algebraic techniques.

(iii) $\lim_{x\to 2} \frac{x^3-8}{x^2+x-6}$

UCADEMY

Ex 1.3

$$\lim_{x\to 2} \frac{x^3-8}{x^2+x-6}$$

$$= \lim_{x\to 2} \frac{x^3-2^3}{x^2+3x-2x-6}$$

$$= \lim_{x\to 2} \frac{(x-2)(x^2+2x+2^2)}{x(x+3)-2(x+3)}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{(x+3)\cancel{(x-2)}}$$

$$= \frac{2^2+2(2)+4}{2+3} = \frac{4+4+4}{5} = \frac{12}{5}.$$

2 Evaluate each limit by using algebraic techniques.

(iv) $\lim_{x \to 1} \frac{x^3 - 3x^2 + 3x - 1}{x^3 - x}$

Ex 1.3

$$\lim_{x \to 1} \frac{x^3 - 3x^2 + 3x - 1}{x^3 - x}$$

$$= \lim_{x \to 1} \frac{(x-1)^3}{x(x^2-1)}$$

$$= \lim_{x \to 1} \frac{(x-1)^{\cancel{3}\,2}}{x(x+1)\cancel{(x-1)}}$$

$$= \lim_{x \to 1} \frac{(x-1)^2}{x(x+1)} = \frac{(1-1)^2}{1(1+1)}$$

$$= \frac{0}{2} = 0$$

2 Evaluate each limit by using algebraic techniques.

(v) $\lim_{x \to -1} \left(\dfrac{x^3 + x^2}{x^2 - 1} \right)$

$$\lim_{x \to -1} \left(\frac{x^3 + x^2}{x^2 - 1} \right)$$

$$= \lim_{x \to -1} \frac{x^2 \cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$$

$$= \lim_{x \to -1} \frac{x^2}{x - 1}$$

$$= \frac{(-1)^2}{-1-1} = \frac{1}{-2} = -\frac{1}{2}.$$

2 Evaluate each limit by using algebraic techniques.

(vi) $\lim_{x \to 4} \frac{2x^2 - 32}{x^3 - 4x^2}$

UCADEMY

Ex 1.3

$$\lim_{x \to 4} \frac{2x^2 - 32}{x^3 - 4x^2}$$

$$= \lim_{x \to 4} \frac{2(x^2 - 16)}{x^2(x-4)}$$

$$= \lim_{x \to 4} \frac{2(x+4)\cancel{(x-4)}}{x^2\cancel{(x-4)}}$$

$$= \lim_{x \to 4} \frac{2(x+4)}{x^2}$$

$$= \frac{2(4+4)}{4^2} = \frac{2(8)}{16} = \frac{16}{16} = 1.$$

2 Evaluate each limit by using algebraic techniques.

(vii) $\lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$

UCADEMY

Ex 1.3

$$\lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$$

$$= \lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2} \times \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}$$

$$= \lim_{x\to 2} \frac{(\sqrt{x})^2-(\sqrt{2})^2}{(x-2)(\sqrt{x}+\sqrt{2})}$$

$$= \lim_{x\to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}(\sqrt{x}+\sqrt{2})}$$

$$= \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}.$$

2 Evaluate each limit by using algebraic techniques.

(viii) $\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

Ex 1.3

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + h - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.$$

2 Evaluate each limit by using algebraic techniques.

(ix) $\lim\limits_{x\to a} \dfrac{x^n - a^n}{x^m - a^m}$

Ex 1.3

Theorem

$$\lim_{x\to a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

$$\lim_{x\to a} \frac{x^n - a^n}{x^m - a^m}$$

$$= \lim_{x\to a} \frac{\left(\frac{x^n - a^n}{x-a}\right)}{\left(\frac{x^m - a^m}{x-a}\right)}$$

$$= \frac{\lim\limits_{x\to a} \frac{x^n - a^n}{x-a}}{\lim\limits_{x\to a} \frac{x^m - a^m}{x-a}} = \frac{na^{n-1}}{ma^{m-1}}$$

$$= \frac{n}{m} a^{n-\cancel{1}-m+\cancel{1}} = \frac{n}{m} a^{n-m}.$$

3 Evaluate the following limits.

(i) $\lim_{x\to 0} \frac{\sin 7x}{x}$

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UCADEMY

Ex 1.3

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{x\to 0} \frac{\sin 7x}{x}$$

$$= \lim_{x\to 0} \frac{\sin 7x}{7x} \times 7$$

$$= 7 \cdot \lim_{x\to 0} \frac{\sin 7x}{7x}$$

$$= 7 \cdot 1 = 7$$

Ans.

(ii) $\lim_{x\to 0} \frac{\sin x^\circ}{x}$

Ex 1.3

$$\lim_{x\to 0} \frac{\sin x^\circ}{x}$$

$$= \lim_{x\to 0} \frac{\sin \frac{\pi}{180}x}{\frac{\pi}{180}x} \times \frac{\pi}{180}$$

$$= \frac{\pi}{180} \cdot \lim_{x\to 0} \frac{\sin\left(\frac{\pi}{180}x\right)}{\frac{\pi}{180}x}$$

$$= \frac{\pi}{180} \cdot 1 \quad = \frac{\pi}{180}.$$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

3 Evaluate the following limits.

(iii) $\lim_{\theta \to 0} \frac{1 - \cos\theta}{\sin\theta}$

Ex 1.3

$$\lim_{\theta \to 0} \frac{1 - \cos\theta}{\sin\theta}$$

$$= \lim_{\theta \to 0} \frac{1 - \cos\theta}{\sin\theta} \times \frac{1 + \cos\theta}{1 + \cos\theta}$$

$$= \lim_{\theta \to 0} \frac{1 - \cos^2\theta}{\sin\theta(1 + \cos\theta)}$$

$$= \lim_{\theta \to 0} \frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}(1 + \cos\theta)}$$

$$= \frac{\sin 0}{1 + \cos 0} = \frac{0}{1 + 1} = \frac{0}{2} = 0.$$

3 **Evaluate the following limits.**

(iii) $\lim_{\theta \to 0} \frac{1 - \cos\theta}{\sin\theta}$

$$\lim_{\theta \to 0} \frac{1 - \cos\theta}{\sin\theta}$$

$$= \lim_{\theta \to 0} \frac{1 - \cos\theta}{\sin\theta} \times \frac{1 + \cos\theta}{1 + \cos\theta}$$

$$= \lim_{\theta \to 0} \frac{1 - \cos^2\theta}{\sin\theta(1 + \cos\theta)}$$

$$= \lim_{\theta \to 0} \frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}(1 + \cos\theta)}$$

$$= \frac{\sin 0}{1 + \cos 0} = \frac{0}{1 + 1} = \frac{0}{2} = 0.$$

3 Evaluate the following limits.

(v) $\lim_{x\to0} \frac{\sin ax}{\sin bx}$

Ex 1.3

$$\lim_{\theta\to0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{x\to0} \frac{\sin ax}{\sin bx}$$

$$= \lim_{x\to0} \frac{\frac{\sin ax}{ax} \times a\cancel{x}}{\frac{\sin bx}{bx} \times b\cancel{x}}$$

$$= \frac{a}{b} \cdot \frac{\lim_{x\to0} \frac{\sin ax}{ax}}{\lim_{x\to0} \frac{\sin bx}{bx}}$$

$$= \frac{a}{b} \cdot \frac{1}{1} = \frac{a}{b}.$$

3 Evaluate the following limits.

(vi) $\lim_{x\to 0} \dfrac{x}{\tan x}$

Ex 1.3

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{x\to 0} \frac{x}{\tan x}$$

$$= \lim_{x\to 0} x \div \tan x$$

$$= \lim_{x\to 0} x \div \frac{\sin x}{\cos x} = \lim_{x\to 0} x \cdot \frac{\cos x}{\sin x}$$

$$= \lim_{x\to 0} \frac{x\cos x}{\sin x} = \lim_{x\to 0} \frac{\left(\frac{\cancel{x}\cos x}{\cancel{x}}\right)}{\left(\frac{\sin x}{x}\right)}$$

$$= \frac{\lim_{x\to 0} \cos x}{\lim_{x\to 0} \frac{\sin x}{x}} = \frac{\cos 0}{1} = \frac{1}{1} = 1.$$

3 Evaluate the following limits.

(vii) $\lim\limits_{x\to 0} \dfrac{1-\cos 2x}{x^2}$

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Ex 1.3

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$$

$$= \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$$

$$= 2\lim_{x\to 0} \frac{\sin x \cdot \sin x}{x \cdot x}$$

$$= 2\lim_{x\to 0} \left(\frac{\sin x}{x} \cdot \frac{\sin x}{x}\right)$$

$$= 2\left(\lim_{x\to 0} \frac{\sin x}{x}\right)\cdot\left(\lim_{x\to 0} \frac{\sin x}{x}\right) = 2\cdot 1\cdot 1 = 2.$$

3 Evaluate the following limits.

(viii) $\lim_{x\to 0} \frac{1-\cos x}{\sin^2 x}$

$$\lim_{x\to 0} \frac{1-\cos x}{\sin^2 x}$$

$$= \lim_{x\to 0} \frac{1-\cos x}{1-\cos^2 x}$$

$$= \lim_{x\to 0} \frac{1\cancel{-\cos x}}{(1+\cos x)\cancel{(1-\cos x)}}$$

$$= \frac{1}{1+\cos 0} = \frac{1}{1+1}$$

$$= \frac{1}{2}$$

(ix) $\lim_{\theta \to 0} \frac{\sin^2\theta}{\theta}$

Ex 1.3

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\sin^2\theta}{\theta}$$

$$= \lim_{\theta \to 0} \frac{\sin\theta \cdot \sin\theta}{\theta}$$

$$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \cdot \lim_{\theta \to 0} \sin\theta$$

$$= 1 \cdot \sin 0$$

$$= 1 \cdot 0 = 0.$$

3 Evaluate the following limits.

(x) $\lim_{x \to 0} \frac{\sec x - \cos x}{x}$

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$

$$\lim_{x \to 0} \frac{\sec x - \cos x}{x}$$

$$= \lim_{x \to 0} \frac{1}{x}\left[\sec x - \cos x\right]$$

$$= \lim_{x \to 0} \frac{1}{x}\left[\frac{1}{\cos x} - \cos x\right]$$

$$= \lim_{x \to 0} \frac{1}{x}\left(\frac{1 - \cos^2 x}{\cos x}\right) = \lim_{x \to 0} \frac{1}{x} \cdot \frac{\sin^2 x}{\cos x}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{\cos x} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{\sin x}{\cos x}$$

$$= 1 \cdot \frac{\sin 0}{\cos 0} = 1 \cdot \frac{0}{1} = 1 \cdot 0 = 0.$$

3 Evaluate the following limits.

(xi) $\lim_{\theta\to 0} \frac{1-\cos p\theta}{1-\cos q\theta}$

UCADEMY

Ex 1.3

$$\lim_{\theta\to 0} \frac{1-\cos p\theta}{1-\cos q\theta}$$

$$= \lim_{\theta\to 0} \frac{\cancel{2}\sin^2 \frac{p\theta}{2}}{\cancel{2}\sin^2 \frac{q\theta}{2}}$$

$$= \lim_{\theta\to 0} \left(\frac{\sin \frac{p\theta}{2}}{\sin \frac{q\theta}{2}}\right)^2 = \left(\lim_{\theta\to 0} \frac{\sin \frac{p\theta}{2}}{\sin \frac{q\theta}{2}}\right)^2$$

$$= \left(\lim_{\theta\to 0} \frac{\frac{\sin \frac{p\theta}{2}}{p\theta/2} \times \frac{p\cancel{\theta}}{\cancel{2}}}{\frac{\sin \frac{q\theta}{2}}{\frac{q\theta}{2}} \times \frac{q\cancel{\theta}}{\cancel{2}}}\right)^2 = \left(\frac{1\times p}{1\times q}\right)^2 = \left(\frac{p}{q}\right)^2$$

$$= \frac{p^2}{q^2}.$$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

$$1-\cos p\theta = 2\sin^2 \frac{p\theta}{2}$$

3 Evaluate the following limits.

(xii) $\lim_{\theta\to 0} \frac{\tan\theta - \sin\theta}{\sin^3\theta}$

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Ex 1.3

$$\lim_{\theta\to 0} \frac{\tan\theta - \sin\theta}{\sin^3\theta}$$

$$= \lim_{\theta\to 0} \frac{1}{\sin^3\theta}\left(\frac{\sin\theta}{\cos\theta} - \sin\theta\right) = \lim_{\theta\to 0} \frac{1}{\sin^3\theta}\left(\frac{\sin\theta - \sin\theta\cos\theta}{\cos\theta}\right)$$

$$= \lim_{\theta\to 0} \frac{\cancel{\sin\theta}}{\sin^{\cancel{3}\,2}\theta}\left(\frac{1-\cos\theta}{\cos\theta}\right) = \lim_{\theta\to 0} \frac{1}{1-\cos^2\theta} \cdot \frac{1-\cos\theta}{\cos\theta}$$

$$= \lim_{\theta\to 0} \frac{\cancel{1-\cos\theta}}{\cos\theta(1+\cos\theta)\cancel{(1-\cos\theta)}}$$

$$= \frac{1}{\cos 0(1+\cos 0)} = \frac{1}{1(1+1)} = \frac{1}{2}.$$

4 Express each limit in terms of e: (i) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{2}$$

$$=\left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right]^{2}$$

$$=[e]^{2}=e^{2}.$$

Theorem

$$\lim_{n\to\infty}(1+\frac{1}{n})^{n}=e$$

4 Express each limit in terms of e: (ii) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\frac{n}{2}}$

Ex 1.3

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\frac{n}{2}}$$

$$= \lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{1}{2}}$$

$$= \left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{1}{2}}$$

$$= e^{1/2}.$$

$$= \sqrt{e}.$$

Theorem

$$\lim_{n\to\infty}(1+\frac{1}{n})^{n} = e$$

4 Express each limit in terms of e:

(iii) $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n$

Ex 1.3

Theorem

$$\lim_{n\to\infty}(1+\frac{1}{n})^n = e$$

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n$$

$$= \lim_{n\to\infty}\left(1+\frac{1}{-n}\right)^n$$

$$= \lim_{n\to\infty}\left[\left(1+\frac{1}{-n}\right)^{-n}\right]^{-1}$$

$$= \left[\lim_{n\to\infty}\left(1+\frac{1}{-n}\right)^{-n}\right]^{-1}$$

$$= e^{-1} = \frac{1}{e}.$$

4 Express each limit in terms of e:

(iv) $\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^{n}$

Ex 1.3

$$\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^{n}$$

$$= \lim_{n\to\infty}\left[\left(1+\frac{1}{3n}\right)^{3n}\right]^{\frac{1}{3}}$$

$$= \left[\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^{3n}\right]^{\frac{1}{3}}$$

$$= e^{1/3}$$

Theorem

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$$

4 Express each limit in terms of e:

(v) $\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^n$

Ex 1.3

$$\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^n$$

$$= \lim_{n\to\infty}\left[\left(1+\frac{4}{n}\right)^{\frac{n}{4}}\right]^4$$

$$= \left[\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^{\frac{n}{4}}\right]^4$$

$$= e^4$$

Theorem

$$\lim_{n\to\infty}(1+\frac{1}{n})^n = e$$

4 Express each limit in terms of e: (vi) $\lim_{x\to 0}(1+3x)^{\frac{2}{x}}$

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Ex 1.3

Theorem

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$$

$$\lim_{x\to 0}(1+3x)^{\frac{2}{x}}$$

$$= \lim_{x\to 0}(1+3x)^{\frac{3\times 2}{3\times x}}$$

$$= \lim_{x\to 0}(1+3x)^{\frac{6}{3x}}$$

$$= \lim_{x\to 0}\left[(1+3x)^{\frac{1}{3x}}\right]^{6}$$

$$= \left[\lim_{x\to 0}(1+3x)^{\frac{1}{3x}}\right]^{6} = e^{6}.$$

4 Express each limit in terms of e:

(vii) $\lim_{x\to 0}(1+2x^2)^{\frac{1}{x^2}}$

$$\lim_{x\to 0}(1+2x^2)^{\frac{1}{x^2}}$$

$$= \lim_{x\to 0}\left[(1+2x^2)^{\frac{1}{2x^2}}\right]^2$$

$$= \left[\lim_{x\to 0}(1+2x^2)^{\frac{1}{2x^2}}\right]^2$$

$$= e^2.$$

Theorem

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$$

4 Express each limit in terms of e : (viii) $\lim_{h \to 0} (1-2h)^{\frac{1}{h}}$

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FUNCTIONS AND LIMITS

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Ex 1.3

$$\lim_{h \to 0} (1-2h)^{\frac{1}{h}}$$

$$= \lim_{h \to 0} \left(1+(-2h)\right)^{\frac{1}{h}}$$

$$= \lim_{h \to 0} \left[\left(1+(-2h)\right)^{\frac{1}{-2h}}\right]^{-2}$$

$$= \left[\lim_{h \to 0} \left(1+(-2h)\right)^{\frac{1}{-2h}}\right]^{-2}$$

$$= e^{-2} = \frac{1}{e^2}.$$

Theorem

$$\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e$$

4 Express each limit in terms of e:

(ix) $\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^{x}$

UCADEMY

Ex 1.3

Theorem

$$\lim_{n\to\infty}(1+\frac{1}{n})^{n}=e$$

$$\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^{x}$$

$$=\lim_{x\to\infty}\left(\frac{1+x}{x}\right)^{-x}$$

$$=\lim_{x\to\infty}\left(\frac{1}{x}+\frac{\cancel{x}}{\cancel{x}}\right)^{-x}$$

$$=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{-x}$$

$$=\lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{-1}=\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right]^{-1}=e^{-1}$$

$$=\frac{1}{e}.$$

4

Express each limit in terms of e: (x) $\lim_{x\to 0} \frac{e^{1/x}-1}{e^{1/x}+1}, \quad x<0$

$$\lim_{x\to 0} \frac{e^{1/x}-1}{e^{1/x}+1}, \quad x<0 \checkmark$$

Let $x=-t, \quad t>0$

$$\frac{1}{x} = -\frac{1}{t} \checkmark$$

As $x\to 0, \quad t\to 0$

$$\lim_{x\to 0} \frac{e^{1/x}-1}{e^{1/x}+1} = \lim_{t\to 0} \frac{e^{-1/t}-1}{e^{-1/t}+1} = \frac{e^{-\infty}-1}{e^{-\infty}+1}$$

$$= \frac{0-1}{0+1} = \frac{-1}{1} = -1$$

Ans.

(xi) $\lim_{x\to 0} \dfrac{e^{1/x}-1}{e^{1/x}+1}, \quad x>0$

Ex 1.3

$$\lim_{x\to 0} \frac{e^{1/x}-1}{e^{1/x}+1}, \quad x>0 \quad \checkmark$$

$$= \lim_{x\to 0} \frac{e^{1/x} - e^{1/x}\cdot e^{-1/x}}{e^{1/x} + e^{1/x}\cdot e^{-1/x}}$$

$$= \lim_{x\to 0} \frac{\cancel{e^{1/x}}\left(1 - e^{-1/x}\right)}{\cancel{e^{1/x}}\left(1 + e^{-1/x}\right)}$$

$$= \frac{1 - e^{-\infty}}{1 + e^{-\infty}} = \frac{1-0}{1+0} = 1$$

LECTURES BY

AKHTAR ABBAS

UNIVERSITY OF JHANG

03326297570