1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$ (b) $g \circ f(x)$ (c) $f \circ f(x)$ (d) $g \circ g(x)$

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UCADEMY

Ex 1.2

(i)

$$f(x) = 2x + 1$$

$$g(x) = \frac{3}{x-1}, \quad x \neq 1$$

$f(x) = 2x + 1$, $g(x) = \frac{3}{x-1}$, $x \neq 1$

$$f \circ g(x) = f(g(x)) = 2g(x) + 1$$
$$= 2\left(\frac{3}{x-1}\right) + 1$$
$$= \frac{6 + x - 1}{x - 1} = \frac{5 + x}{x - 1}.$$

$$g \circ f(x) = g(f(x)) = \frac{3}{f(x) - 1}$$
$$= \frac{3}{2x + \cancel{1} - \cancel{1}} = \frac{3}{2x}.$$

1 The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$ (b) $g \circ f(x)$ (c) $f \circ f(x)$ (d) $g \circ g(x)$

(i)

$$f(x) = 2x + 1$$

$$g(x) = \frac{3}{x-1}, \quad x \neq 1$$

$$f \circ f(x) = f(f(x)) = 2f(x) + 1$$
$$= 2(2x+1) + 1$$
$$= 4x + 2 + 1 = 4x + 3$$

$$g \circ g(x) = g(g(x)) = \frac{3}{g(x) - 1}$$
$$= \frac{3}{\frac{3}{x-1} - 1} = \frac{3}{\left(\frac{3-x+1}{x-1}\right)}$$
$$= \frac{3(x-1)}{4-x}.$$

1 The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$ (b) $g \circ f(x)$ (c) $f \circ f(x)$ (d) $g \circ g(x)$

Chapter 1
FUNCTIONS AND LIMITS

UCADEMY

Ex 1.2

(ii)

$$f(x) = \sqrt{x+1}$$

$$g(x) = \frac{1}{x^2}, \quad x \neq 0$$

$f(x) = \sqrt{x+1}$, $g(x) = \frac{1}{x^2}$, $x \neq 0$

$$f \circ g(x) = f(g(x)) = \sqrt{g(x) + 1}$$

$$= \sqrt{\frac{1}{x^2} + 1} = \sqrt{\frac{1+x^2}{x^2}}$$

$$= \frac{\sqrt{1+x^2}}{x}$$

$$g \circ f(x) = g(f(x)) = \frac{1}{f(x)^2} = \frac{1}{(\sqrt{x+1})^2}$$

$$= \frac{1}{x+1}.$$

1 The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$ (b) $g \circ f(x)$ (c) $f \circ f(x)$ (d) $g \circ g(x)$

Ex 1.2

(ii)

$$f(x) = \sqrt{x+1}$$

$$g(x) = \frac{1}{x^2}, \quad x \neq 0$$

$$f \circ f(x) = f(f(x)) = \sqrt{f(x)+1}$$

$$= \sqrt{\sqrt{x+1}+1}.$$

$$g \circ g(x) = g(g(x)) = \frac{1}{g(x)^2}$$

$$= \frac{1}{\left(\frac{1}{x^2}\right)^2} = \frac{1}{\left(\frac{1}{x^4}\right)}$$

$$= \frac{1 \cdot x^4}{1} = x^4.$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$ (b) $g \circ f(x)$ (c) $f \circ f(x)$ (d) $g \circ g(x)$

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Ex 1.2

(iii)

$$f(x) = \frac{1}{\sqrt{x-1}}, \quad x \neq 1$$

$$g(x) = (x^2+1)^2$$

$$f(x) = \frac{1}{\sqrt{x-1}}, \quad x \neq 1, \qquad g(x) = (x^2+1)^2$$

$$f \circ g(x) = f(g(x)) = \frac{1}{\sqrt{g(x)-1}} = \frac{1}{\sqrt{(x^2+1)^2-1}}$$

$$= \frac{1}{\sqrt{x^4+2x^2+\cancel{1}-\cancel{1}}} = \frac{1}{\sqrt{x^2(x^2+2)}}$$

$$= \frac{1}{x\sqrt{x^2+2}}.$$

1 The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$ (b) $g \circ f(x)$ (c) $f \circ f(x)$ (d) $g \circ g(x)$

Ex 1.2

(iii)

$$f(x) = \frac{1}{\sqrt{x-1}}, \quad x \neq 1$$

$$g(x) = (x^2+1)^2$$

$$g \circ f(x) = g(f(x)) = (f(x)^2 + 1)^2$$

$$= \left[\left(\frac{1}{\sqrt{x-1}}\right)^2 + 1\right]^2$$

$$= \left[\frac{1}{x-1} + 1\right]^2$$

$$= \left(\frac{\cancel{1} + x - \cancel{1}}{x-1}\right)^2$$

$$= \left(\frac{x}{x-1}\right)^2 = \frac{x^2}{(x-1)^2}$$

$$= \frac{x^2}{x^2-2x+1}.$$

1 The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$ (b) $g \circ f(x)$ (c) $f \circ f(x)$ (d) $g \circ g(x)$

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FUNCTIONS AND LIMITS

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Ex 1.2

(iii)

$$f(x) = \frac{1}{\sqrt{x-1}}, \quad x \neq 1$$

$$g(x) = (x^2+1)^2$$

$$f \circ f(x) = f(f(x)) = \frac{1}{\sqrt{f(x) - 1}}$$

$$= \frac{1}{\sqrt{\frac{1}{\sqrt{x-1}} - 1}}$$

$$= \sqrt{\frac{1}{\left(\frac{1-\sqrt{x-1}}{\sqrt{x-1}}\right)}} = \sqrt{\frac{\sqrt{x-1}}{1-\sqrt{x-1}}}$$

$$= \frac{\sqrt{\sqrt{x-1}}}{\sqrt{1-\sqrt{x-1}}}.$$

1 The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$ (b) $g \circ f(x)$ (c) $f \circ f(x)$ (d) $g \circ g(x)$

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Ex 1.2

(iii)

$$f(x) = \frac{1}{\sqrt{x-1}}, \quad x \neq 1$$

$$g(x) = (x^2+1)^2$$

$$g \circ g(x) = g(g(x))$$

$$= \left(g(x)^2 + 1\right)^2$$

$$= \left[\left\{(x^2+1)^2\right\}^2 + 1\right]^2$$

$$= \left[(x^2+1)^4 + 1\right]^2$$

1 The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$ (b) $g \circ f(x)$ (c) $f \circ f(x)$ (d) $g \circ g(x)$

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Ex 1.2

(iv)

$$f(x) = 3x^4 - 2x^2$$

$$g(x) = \frac{2}{\sqrt{x}}, \quad x \neq 0$$

$f(x) = 3x^4 - 2x^2$, $\quad g(x) = \frac{2}{\sqrt{x}}$, $\quad x \neq 0$

$$f \circ g(x) = f(g(x)) = 3(g(x))^4 - 2(g(x))^2$$

$$= 3\left(\frac{2}{\sqrt{x}}\right)^4 - 2\left(\frac{2}{\sqrt{x}}\right)^2$$

$$= 3\left(\frac{2^4}{(x^{\cancel{1/2}})^{\cancel{4}\,2}}\right) - 2\left(\frac{4}{x}\right)$$

$$= 3\left(\frac{16}{x^2}\right) - \frac{8}{x} = \frac{48}{x^2} - \frac{8}{x}$$

$$= \frac{48 - 8x}{x^2} = \frac{8(6-x)}{x^2}.$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$ (b) $g \circ f(x)$ (c) $f \circ f(x)$ (d) $g \circ g(x)$

Ex 1.2

(iv)

$$f(x) = 3x^4 - 2x^2$$

$$g(x) = \frac{2}{\sqrt{x}}, \quad x \neq 0$$

$$g \circ f(x) = g(f(x)) = \frac{2}{\sqrt{f(x)}}$$

$$= \frac{2}{\sqrt{3x^4 - 2x^2}}$$

$$= \frac{2}{\sqrt{x^2(3x^2-2)}}$$

$$= \frac{2}{x\sqrt{3x^2-2}}.$$

UCADEMY

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$ (b) $g \circ f(x)$ (c) $f \circ f(x)$ (d) $g \circ g(x)$

Ex 1.2

(iv)

$$f(x) = 3x^4 - 2x^2$$

$$g(x) = \frac{2}{\sqrt{x}}, \quad x \neq 0$$

$$f \circ f(x) = f(f(x))$$

$$= 3\,f(x)^4 - 2\,f(x)^2$$

$$= 3\left[3x^4 - 2x^2\right]^4 - 2\left[3x^4 - 2x^2\right]^2$$

$$g \circ g(x) = g(g(x))$$

$$= \frac{2}{\sqrt{g(x)}} = \frac{2}{\sqrt{\frac{2}{\sqrt{x}}}} = \frac{2}{\left(\frac{\sqrt{2}}{\sqrt{\sqrt{x}}}\right)}$$

$$= \frac{2\sqrt{\sqrt{x}}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}\sqrt{\sqrt{x}}}{\cancel{2}} = \sqrt{2\sqrt{x}}$$

UCADEMY

2

For the real valued function f defined below, find

(a) $f^{-1}(x)$ (b) $f^{-1}(-1)$ and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Ex 1.2

(i)

$$f(x) = -2x + 8$$

$$f(x) = -2x + 8$$

Let $y = f(x) \Rightarrow x = f^{-1}(y)$ ✓

$$y = -2x + 8$$

$$2x = 8 - y$$

$$x = \frac{8-y}{2}$$

$$f^{-1}(y) = \frac{8-y}{2}$$

Interchange y with x,

$$f^{-1}(x) = \frac{8-x}{2}.$$

2

For the real valued function f defined below, find

(a) $f^{-1}(x)$ (b) $f^{-1}(-1)$ and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

UCADEMY

Ex 1.2

(i) $f(x) = -2x + 8$

$$f^{-1}(-1) = \frac{8-(-1)}{2} = \frac{8+1}{2} = \frac{9}{2}$$

$$f(f^{-1}(x)) = -2f^{-1}(x) + 8 = -\cancel{2}\left(\frac{8-x}{\cancel{2}}\right) + 8$$

$$= -\cancel{8} + x + \cancel{8} = x$$

$$f^{-1}(f(x)) = \frac{8 - f(x)}{2} = \frac{8-(-2x+8)}{2}$$

$$= \frac{\cancel{8} + 2x - \cancel{8}}{2} = \frac{\cancel{2}x}{\cancel{2}} = x$$

Hence proved that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

2

For the real valued function f defined below, find

(a) $f^{-1}(x)$ (b) $f^{-1}(-1)$ and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

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UCADEMY

Ex 1.2

(ii) $f(x) = 3x^3 + 7$

$f(x) = 3x^3 + 7$

Let $y = f(x) \Rightarrow x = f^{-1}(y)$ ✓

$y = 3x^3 + 7$

$y - 7 = 3x^3$

$\left(\frac{y-7}{3}\right)^{1/3} = \left(x^{\cancel{3}}\right)^{1/\cancel{3}}$

$x = \left(\frac{y-7}{3}\right)^{1/3}$

$f^{-1}(y) = \left(\frac{y-7}{3}\right)^{1/3}$

Interchange y with x

$f^{-1}(x) = \left(\frac{x-7}{3}\right)^{1/3}.$

2

For the real valued function f defined below, find

(a) $f^{-1}(x)$ (b) $f^{-1}(-1)$ and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Ex 1.2

(ii) $f(x) = 3x^3 + 7$

$$f^{-1}(-1) = \left(\frac{-1-7}{3}\right)^{1/3} = \left(-\frac{8}{3}\right)^{1/3}$$

$$f(f^{-1}(x)) = 3\left[f^{-1}(x)\right]^3 + 7$$

$$= 3\left[\left(\frac{x-7}{3}\right)^{\cancel{1/3}}\right]^{\cancel{3}} + 7 = \cancel{3}\left(\frac{x-7}{\cancel{3}}\right) + 7$$

$$= x - \cancel{7} + \cancel{7} = x$$

$$f^{-1}(f(x)) = \left(\frac{f(x)-7}{3}\right)^{1/3} = \left(\frac{3x^3+\cancel{7}-\cancel{7}}{3}\right)^{1/3} = \left(\frac{\cancel{3}x^3}{\cancel{3}}\right)^{1/3}$$

$$= \left(x^{\cancel{3}}\right)^{1/\cancel{3}} = x$$

Hence $f(f^{-1}(x)) = f^{-1}(f(x)) = x.$

2

For the real valued function f defined below, find

(a) $f^{-1}(x)$ (b) $f^{-1}(-1)$ and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

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Ex 1.2

(iii)

$$f(x) = (-x+9)^3$$

$$f(x) = (-x+9)^3$$

Let $y = f(x) \Rightarrow x = f^{-1}(y)$ ✓

$$y = (-x+9)^3$$

$$y^{1/3} = -x+9$$

$$x = 9 - y^{1/3}$$

$$f^{-1}(y) = 9 - y^{1/3}$$

Replace y with x,

$$f^{-1}(x) = 9 - x^{1/3}.$$

2

For the real valued function f defined below, find

(a) $f^{-1}(x)$ (b) $f^{-1}(-1)$ and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Ex 1.2

(iii) $f(x) = (-x+9)^3$

$$f^{-1}(-1) = 9 - (-1)^{1/3}$$

$$f(f^{-1}(x)) = \left(-f^{-1}(x) + 9\right)^3$$

$$= \left[-(9 - x^{1/3}) + 9\right]^3 = \left[\cancel{-9} + x^{1/3} + \cancel{9}\right]^3$$

$$= \left(x^{\cancel{1/3}}\right)^{\cancel{3}} = x$$

$$f^{-1}(f(x)) = 9 - f(x)^{1/3} = 9 - \left[(-x+9)^{\cancel{3}}\right]^{\cancel{1/3}}$$

$$= \cancel{9} + x - \cancel{9} = x$$

Hence $f(f^{-1}(x)) = f^{-1}(f(x)) = x.$

2

For the real valued function f defined below, find

(a) $f^{-1}(x)$ (b) $f^{-1}(-1)$ and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

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Ex 1.2

(iv) $f(x) = \dfrac{2x+1}{x-1}, \quad x > 1$

$$f(x) = \frac{2x+1}{x-1}, \quad x > 1$$

Let $y = f(x) \Rightarrow x = f^{-1}(y)$ ✓

$$y = \frac{2x+1}{x-1}$$

$$xy - y = 2x + 1$$

$$xy - 2x = y + 1$$

$$x(y-2) = y + 1$$

$$x = \frac{y+1}{y-2}$$

$$f^{-1}(y) = \frac{y+1}{y-2}$$

Replace y with x,

$$f^{-1}(x) = \frac{x+1}{x-2}.$$

2 For the real valued function f defined below, find

(a) $f^{-1}(x)$ (b) $f^{-1}(-1)$ and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Ex 1.2

(iv) $f(x) = \dfrac{2x+1}{x-1}, \quad x > 1$

$$f^{-1}(-1) = \frac{-1+1}{-1-2} = \frac{0}{-3} = 0$$

$$f(f^{-1}(x)) = \frac{2f^{-1}(x)+1}{f^{-1}(x)-1}$$

$$= \frac{2\left(\frac{x+1}{x-2}\right)+1}{\frac{x+1}{x-2}-1} = \frac{\left(\frac{2(x+1)+x-2}{\cancel{x-2}}\right)}{\left(\frac{\cancel{x}+1-\cancel{x}+2}{\cancel{x-2}}\right)}$$

$$= \frac{2x+\cancel{2}+x-\cancel{2}}{3}$$

$$= \frac{\cancel{3}x}{\cancel{3}} = x$$

$$f^{-1}(f(x)) = \frac{f(x)+1}{f(x)-2}$$

$$= \frac{\frac{2x+1}{x-1}+1}{\frac{2x+1}{x-1}-2} = \frac{\left(\frac{2x+\cancel{1}+x-\cancel{1}}{\cancel{x-1}}\right)}{\left(\frac{\cancel{2x}+1-\cancel{2x}+2}{\cancel{x-1}}\right)}$$

$$= \frac{2x+x}{1+2} = \frac{\cancel{3}x}{\cancel{3}} = x$$

Hence

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x.$$

(iv)

$$f(x) = \frac{2x+1}{x-1}, \quad x > 1$$

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