Question # 1
Find A.M. between

(i) $3\sqrt{5}$ and $5\sqrt{5}$ (ii) $x-3$ and $x+5$

(iii) $1-x+x^2$ and $1+x+x^2$

Solution

(i) $3\sqrt{5}$ and $5\sqrt{5}$

Here $a=3\sqrt{5}$ and $b=5\sqrt{5}$, so

$$\text{A.M.} = \frac{a+b}{2} = \frac{3\sqrt{5}+5\sqrt{5}}{2} = \frac{8\sqrt{5}}{2} = 4\sqrt{5}$$

(ii) Same as (i) and (ii)

(iii) $1-x+x^2$ and $1+x+x^2$

Here $a=1-x+x^2$ and $b=1+x+x^2$

$$\text{A.M.} = \frac{a+b}{2} = \frac{1-x+x^2+1+x+x^2}{2} = \frac{2+2x^2}{2} = 1+x^2$$

Question # 2
If 5,8 are two A.Ms between a and b, find a and b.

Solution

Since 5, 8 are two A.Ms between a and b.
Therefor a, 5, 8, b are in A.P.

Here $a_1 = a$ and $d = 8 - 5 = 3$

Now $a_2 = a_1 + d \Rightarrow 5 = a + 3 \Rightarrow 5 - 3 = a \Rightarrow \boxed{a = 2}$

Also $a_4 = a_1 + 3d \Rightarrow b = 2 + 3(3) \Rightarrow \boxed{b = 11}$

Question # 3
Find 6 A.Ms between 2 and 5 .

solution

Let A_1, A_2, A_3, A_4, A_5 and A_6 are six A.Ms between 2 and 5.

Then $2, A_1, A_2, A_3, A_4, A_5, A_6, 5$ are in A.P.

Here $a_1 = 2$ and $a_8 = 5$

$\Rightarrow a_1 + 7d = 5 \Rightarrow 2 + 7d = 5$

$\Rightarrow 7d = 5 - 2 \Rightarrow 7d = 3 \Rightarrow d = \frac{3}{7}$

So $A_1 = a_2 = a_1 + d = 2 + \frac{3}{7} = \frac{17}{7}$

$A_2 = a_3 = a_1 + 2d = 2 + 2\left(\frac{3}{7}\right) = 2 + \frac{6}{7} = \frac{20}{7}$

$$A_3 = a_4 = a_1 + 3d = 2 + 3\left(\tfrac{3}{7}\right) = 2 + \tfrac{9}{7} = \tfrac{23}{7}$$

$$A_4 = a_5 = a_1 + 4d = 2 + 4\left(\tfrac{3}{7}\right) = 2 + \tfrac{12}{7} = \tfrac{26}{7}$$

$$A_5 = a_6 = a_1 + 5d = 2 + 5\left(\tfrac{3}{7}\right) = 2 + \tfrac{15}{7} = \tfrac{29}{7}$$

$$A_6 = a_7 = a_1 + 6d = 2 + 6\left(\tfrac{3}{7}\right) = 2 + \tfrac{18}{7} = \tfrac{32}{7}$$

Hence $\dfrac{17}{7}, \dfrac{20}{7}, \dfrac{23}{7}, \dfrac{26}{7}, \dfrac{29}{7}, \dfrac{32}{7}$ are six A.Ms between 2 and 5.

Question # 4

Find 4 A.Ms between $\sqrt{2}$ and $\dfrac{12}{\sqrt{2}}$.

Solution

Suppose A_1 , A_2 , A_3 and A_4 are four A.Ms between $\sqrt{2}$ and $\dfrac{12}{\sqrt{2}}$.

Then $\sqrt{2}$, A_1, A_2 , A_3, A_4 , $\dfrac{12}{\sqrt{2}}$ are in A.P.

Here $a_1 = \sqrt{2}$ and $a_6 = \dfrac{12}{\sqrt{2}}$

$$\Rightarrow a_1 + 5d = \frac{12}{\sqrt{2}} \quad \Rightarrow \sqrt{2} + 5d = \frac{12}{\sqrt{2}} \quad \Rightarrow 5d = \frac{12}{\sqrt{2}} - \sqrt{2}$$

$$\Rightarrow 5d = \frac{12 - 2}{\sqrt{2}} \qquad \Rightarrow 5d = \frac{10}{\sqrt{2}}$$

$$\Rightarrow d = \frac{2}{\sqrt{2}} = \frac{\left(\sqrt{2}\right)^2}{\sqrt{2}} \Rightarrow d = \sqrt{2}$$

Now $A_1 = a_2 = a_1 + d = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

$$A_2 = a_3 = a_1 + 2d = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$$

$$A_3 = a_4 = a_1 + 3d = \sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$$

$$A_4 = a_5 = a_1 + 4d = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$$

Hence $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$, $5\sqrt{2}$ are four A.Ms between $\sqrt{2}$ and $\dfrac{12}{\sqrt{2}}$.

Question # 5

Insert 7 A.Ms between 4 and 8.

Solution *Do yourself*

Question # 6

Find three A.Ms between 3 and 11

Solution *Do yourself*

Question # 7

Find n so that $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ may be the A.M. between a and b.

Solution

Since we know that A.M. (i)

But we have given A.M. $= \dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ ....(ii)

Comparing (i) and (ii)

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}$$

$$\Rightarrow \quad 2(a^n + b^n) = (a+b)(a^{n-1} + b^{n-1}) \qquad \textit{Cross} \times \textit{ing}$$

$$\Rightarrow \quad 2a^n + 2b^n = a^n + a^{n-1}b + ab^{n-1} + b^n$$

$$\Rightarrow \quad 2a^n + 2b^n - a^n - b^n = a^{n-1}b + ab^{n-1}$$

$$\Rightarrow \quad a^n + b^n = a^{n-1}b + ab^{n-1}$$

$$\Rightarrow \quad a^n - a^{n-1}b = ab^{n-1} - b^n$$

$$\Rightarrow \quad a^{n-1+1} - a^{n-1}b = ab^{n-1} - b^{n-1+1} \qquad \because n = n-1+1$$

$$\Rightarrow \quad a^{n-1}(a-b) = b^{n-1}(a-b)$$

$$\Rightarrow \quad a^{n-1} = b^{n-1}$$

$$\Rightarrow \quad \frac{a^{n-1}}{b^{n-1}} = 1$$

$$\Rightarrow \quad \left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^0 \qquad \because \left(\frac{a}{b}\right)^0 = 1$$

$$\Rightarrow \quad n-1 = 0 \quad \Rightarrow \quad \boxed{n = 1}$$

If you found any error, please report us at www.mathcity.org/error

Book: **Exercise 6.3**
Text Book of Algebra and Trigonometry Class XI
Punjab Textbook Board, Lahore.

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