

Resolving it into partial fraction:

**Question # 1**

$$\frac{1}{x^2 - 1}$$

$$\text{Solution} \quad \frac{1}{x^2 - 1} = \frac{1}{(x-1)(x+1)}$$

$$\frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

Multiplying both sides by  $(x-1)(x+1)$  we get

$$1 = A(x+1) + B(x-1) \dots \dots \dots \text{(i)}$$

Put  $x-1=0 \Rightarrow x=1$  in equation (i)

$$1 = A(1+1) + B(0) \Rightarrow 1 = 2A + 0 \Rightarrow A = \frac{1}{2}$$

Now put  $x+1=0 \Rightarrow x=-1$  in equation (i)

$$1 = A(0) + B(-1-1) \Rightarrow 1 = 0 - 2B \Rightarrow B = -\frac{1}{2}$$

Hence

$$\begin{aligned} \frac{1}{(x-1)(x+1)} &= \frac{A}{x-1} + \frac{B}{x+1} \\ &= \frac{1}{2(x-1)} - \frac{1}{2(x+1)} \quad \text{Answer} \end{aligned}$$

**Question # 2**

$$\begin{array}{rcl} \frac{x^2(x^2+1)}{(x+1)(x-1)} & & x^2+2 \\ \text{Solution} \quad \frac{x^2(x^2+1)}{(x+1)(x-1)} & = & \frac{x^4+x^2}{(x^2-1)} \\ & & \begin{array}{c} x^2-1 | x^4+x^2 \\ \underline{-} x^4-x^2 \\ \hline 2x^2 \\ \begin{array}{c} + \\ - \end{array} 2x^2-2 \\ \hline 2 \end{array} \\ & = & x^2+2+\frac{2}{(x^2-1)} \\ & = & x^2+2+\frac{2}{(x+1)(x-1)} \end{array}$$

Now consider

$$\frac{2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

Multiplying both sides by  $(x+1)(x-1)$

$$2 = A(x-1) + B(x+1) \dots \dots \dots \text{(i)}$$

Put  $x+1=0 \Rightarrow x=-1$  in equation (i)

$$2 = A(-1-1) + B(0) \Rightarrow 2 = -2A + 0 \Rightarrow A = -1$$

Now put  $x-1=0 \Rightarrow x=1$  in equation (i)

$$2 = A(0) + B(1+1) \Rightarrow 2 = 0 + 2B \Rightarrow B = 1$$

$$\text{So } \frac{2}{(x+1)(x-1)} = \frac{-1}{x+1} + \frac{1}{x-1}$$

Hence

$$\begin{aligned}\frac{x^2(x^2+1)}{(x+1)(x-1)} &= x^2 + 2 + \frac{-1}{(x+1)} + \frac{1}{(x-1)} \\ &= x^2 + 2 - \frac{1}{(x+1)} + \frac{1}{(x-1)}\end{aligned}\quad \text{Answer}$$


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### Question # 3

$$\begin{aligned}&\frac{2x+1}{(x-1)(x+2)(x+3)} \\ \text{Solution } &\frac{2x+1}{(x-1)(x+2)(x+3)} \\ &\frac{2x+1}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3}\end{aligned}$$

Multiplying both side by  $(x-1)(x+2)(x+3)$

$$2x+1 = A(x+2)(x+3) + B(x-1)(x+3) + C(x-1)(x+2) \dots \dots \dots \text{(i)}$$

Put  $x-1=0 \Rightarrow x=1$  in equation (i)

$$2(1)+1 = A(1+2)(1+3) + B(0) + C(0)$$

$$3 = A(3)(4) + 0 + 0 \Rightarrow 3 = 12A \Rightarrow \frac{3}{12} = A \Rightarrow \boxed{A = \frac{1}{4}}$$

Now put  $x+2=0 \Rightarrow x=-2$  in equation (i)

$$2(-2)+1 = A(0) + B(-2-1)(-2+3) + C(0)$$

$$-4+1 = 0 + B(-3)(1) + 0 \Rightarrow -3 = -3B \Rightarrow \boxed{B = 1}$$

Now put  $x+3=0 \Rightarrow x=-3$  in equation (i)

$$2(-3)+1 = A(0) + B(0) + C(-3-1)(-3+2)$$

$$-6+1 = 0 + 0 + C(-4)(-1) \Rightarrow -5 = 4C \Rightarrow \boxed{C = -\frac{5}{4}}$$

So

$$\begin{aligned}\frac{2x+1}{(x-1)(x+2)(x+3)} &= \frac{\cancel{1/4}}{x-1} + \frac{1}{x+2} + \frac{-5\cancel{1/4}}{x+3} \\ &= \frac{1}{4(x-1)} + \frac{1}{x+2} - \frac{5}{4(x+3)}\end{aligned}\quad \text{Answer}$$


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### Question # 4

$$\begin{aligned}&\frac{3x^2-4x-5}{(x-2)(x^2+7x+10)} \\ \text{Solution } &\frac{3x^2-4x-5}{(x-2)(x^2+7x+10)} \\ &= \frac{3x^2-4x-5}{(x-2)(x+5)(x+2)}\end{aligned}$$

Now resolving into partial fraction.

$$\begin{aligned}\frac{3x^2-4x-5}{(x-2)(x+5)(x+2)} &= \frac{A}{x-2} + \frac{B}{x+5} + \frac{C}{x+2} \\ &\therefore x^2 + 7x + 10 = x^2 + 5x + 2x + 10 \\ &= x(x+5) + 2(x+5) \\ &= (x+5)(x+2)\end{aligned}$$

$$\left[ \begin{array}{l} \text{Do yourself. You will get} \\ A = -\frac{1}{28}, B = \frac{30}{7}, C = -\frac{5}{4} \end{array} \right]$$

## Question # 5

$$\frac{1}{(x-1)(2x-1)(3x-1)}$$

$$\textbf{\textit{Solution}} \quad \frac{1}{(x-1)(2x-1)(3x-1)}$$

$$\frac{1}{(x-1)(2x-1)(3x-1)} = \frac{A}{x-1} + \frac{B}{2x-1} + \frac{C}{3x-1}$$

Multiplying both side by  $(x-1)(2x-1)(3x-1)$ .

Put  $x - 1 = 0 \Rightarrow x = 1$  in equation (i)

$$1 = A(2(1)-1)(3(1)-1) + B(0) + C(0) \Rightarrow 1 = A(1)(2) + 0 + 0$$

$$\Rightarrow 1 = 2A \quad \Rightarrow \boxed{A = \frac{1}{2}}$$

Put  $2x - 1 = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$  in equation (i)

$$1 = A(0) + B\left(\frac{1}{2}-1\right)\left(3\left(\frac{1}{2}\right)-1\right) + C(0) \quad \Rightarrow 1 = 0 + B\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) + 0$$

$$\Rightarrow 1 = -\frac{1}{4}B \quad \Rightarrow \boxed{B = -4}$$

Put  $3x - 1 = 0 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$  in equation (i)

$$1 = A(0) + B(0) + C\left(\frac{1}{3}-1\right)\left(2\left(\frac{1}{3}\right)-1\right) \Rightarrow 1 = 0 + 0 + C\left(-\frac{2}{3}\right)\left(-\frac{1}{3}\right)$$

$$\Rightarrow 1 = \frac{2}{9}C \Rightarrow \boxed{C = \frac{9}{2}}$$

Hence

$$\begin{aligned}\frac{1}{(x-1)(2x-1)(3x-1)} &= \frac{\frac{1}{2}}{x-1} + \frac{-4}{2x-1} + \frac{\frac{9}{2}}{3x-1} \\ &= \frac{1}{2(x-1)} - \frac{4}{2x-1} + \frac{9}{2(3x-1)} \quad \text{Answer}\end{aligned}$$

## **Question # 6**

$$\frac{x}{(x-a)(x-b)(x-c)}$$

### *Solution*

$$\frac{x}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

Multiplying both sides by  $(x-a)(x-b)(x-c)$ .

Put  $x-a=0 \Rightarrow x=a$  in equation (i)

$$a = A(a-b)(a-c) + B(0) + C(0)$$

$$\Rightarrow a = A(a-b)(a-c) + 0 + 0 \quad \Rightarrow \boxed{A = \frac{a}{(a-b)(a-c)}}$$

Now put  $x - b = 0 \Rightarrow x = b$  in equation (i)

$$a = A(0) + B(b-a)(b-c) + C(0)$$

$$\Rightarrow a = 0 + B(b-a)(b-c) + 0 \quad \Rightarrow \quad B = \frac{b}{(b-a)(b-c)} \quad \text{Now put}$$

$x - c = 0 \Rightarrow x = c$  in equation (i)

$$c = A(0) + B(0) + C(c-a)(c-b)$$

$$\Rightarrow c = 0 + 0 + C(c-a)(c-b) \quad \Rightarrow \boxed{B = \frac{c}{(c-a)(c-b)}}$$

S<sub>0</sub>

$$\begin{aligned} \frac{x}{(x-a)(x-b)(x-c)} &= \frac{\cancel{a}/(a-b)(a-c)}{x-a} + \frac{\cancel{b}/(b-a)(b-c)}{x-b} + \frac{\cancel{c}/(c-a)(c-b)}{x-c} \\ &= \frac{a}{(a-b)(a-c)(x-a)} + \frac{b}{(b-a)(b-c)(x-b)} + \frac{c}{(c-a)(c-b)(x-c)} \end{aligned}$$

## **Question # 7**

$$\frac{6x^3 + 5x^2 - 7}{2x^2 - x - 1}$$

### *Solution*

$$\begin{array}{r}
 & & 3x + 4 \\
 & & \hline
 2x^2 - x - 1 & \overline{)6x^3 + 5x^2 - 7} \\
 & - & 6x^3 - 3x^2 - 3x \\
 & & + & + \\
 & & 8x^2 + 3x - 7 \\
 & & - & 8x^2 - 4x - 4 \\
 & & & + & + \\
 & & & 7x - 3
 \end{array}$$

$$= 3x + 4 + \frac{7x - 3}{2x^2 - x - 1}$$

$$= 3x + 4 + \frac{7x - 3}{2x^2 - 2x + x - 1}$$

$$= 3x + 4 + \frac{7x - 3}{2x(x-1) + 1(x-1)} = 3x + 4 + \frac{7x - 3}{(x-1)(2x+1)}$$

## Now Consider

$$\frac{7x-3}{(x-1)(2x+1)} = \frac{A}{x-1} + \frac{B}{2x+1}$$

*Find value of A & B yourself*

*You will get A =  $\frac{4}{3}$  and B =  $\frac{13}{3}$*

so

$$\frac{7x-3}{(x-1)(2x+1)} = \frac{\cancel{4}/_3}{x-1} + \frac{\cancel{13}/_3}{2x+1} = \frac{4}{3(x-1)} + \frac{13}{3(2x+1)}$$

Hence

$$\frac{6x^3 + 5x^2 - 7}{2x^2 - x - 1} = 3x + 4 + \frac{4}{3(x-1)} + \frac{13}{3(2x+1)}$$

## Question # 8

$$\begin{array}{r}
 \begin{array}{c}
 2x^3 + x^2 - 5x + 3 \\
 \hline
 2x^3 + x^2 - 3x \\
 \hline
 2x^3 + x^2 - 5x + 3 \\
 \hline
 2x^3 + x^2 - 3x \\
 \hline
 -2x + 3
 \end{array}
 &
 \begin{array}{c}
 1 \\
 2x^3 + x^2 - 3x \\
 \hline
 2x^3 + x^2 - 3x \\
 \hline
 - \\
 - \\
 + \\
 \hline
 -2x + 3
 \end{array}
 \end{array}$$

**Solution**

$$\begin{aligned}
 &= 1 + \frac{-2x + 3}{2x^3 + x^2 - 3x} \\
 &= 1 + \frac{-2x + 3}{x(2x^2 + x - 3)} \\
 &= 1 + \frac{-2x + 3}{x(x(2x + 3) - 1(2x + 3))} \\
 &= 1 + \frac{-2x + 3}{x(2x + 3)(x - 1)}
 \end{aligned}$$

Now consider

Put  $x=0$  in equation (i)

$$3 - 2(0) = A(2(0) + 3)((0) - 1) + B(0) + C(0) \quad \Rightarrow \quad 3 - 0 = A(0 + 3)(-1) + 0 + 0$$

$$\Rightarrow 3 = -3A \quad \Rightarrow \quad \boxed{A = -1}$$

Now put  $2x + 3 = 0 \Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2}$  in equation (i)

$$3 - 2\left(-\frac{3}{2}\right) = A(0) + B\left(-\frac{3}{2}\right)\left(-\frac{3}{2} - 1\right) + C(0) \Rightarrow 3 + 3 = 0 + B\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right) + 0$$

$$\Rightarrow 6 = \frac{15}{4}B \Rightarrow B = (6)\left(\frac{4}{15}\right) \Rightarrow \boxed{B = \frac{8}{5}}$$

Now put  $x - 1 = 0 \Rightarrow x = 1$  in equation (i)

$$3 - 2(1) = A(0) + B(0) + C(1)(2(1) + 3) \Rightarrow 1 = 0 + 0 + 5C \quad \Rightarrow \boxed{C = \frac{1}{5}}$$

$$\text{So } \frac{3-2x}{x(2x+3)(x-1)} = \frac{-1}{x} + \frac{8/5}{2x+3} + \frac{1/5}{x-1} = -\frac{1}{x} + \frac{8}{5(2x+3)} + \frac{1}{5(x-1)}$$

Hence  $\frac{2x^3 + x^2 - 5x + 3}{2x^3 + x^2 - 3x} = 1 - \frac{1}{x} + \frac{8}{5(2x+3)} + \frac{1}{5(x-1)}$     Answer

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## Question # 9

$$\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}$$

$$\text{Solution} \quad \frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}$$

$$= \frac{(x-1)(x^2 - 3x - 5x + 15)}{(x-2)(x^2 - 4x - 6x + 24)}$$

$$= \frac{(x-1)(x^2 - 8x + 15)}{(x-2)(x^2 - 10x + 24)}$$

$$= \frac{x^3 - 8x^2 + 15x - x^2}{x^3 - 10x^2 + 24x - 2x^2}$$

$$= \frac{x^3 - 9x^2 + 23x - 15}{x^3 - 12x^2 + 44x - 48}$$

$$x^2 - 12x + 44x - 48$$

$$\begin{array}{r} & & 1 \\ & & \hline x^3 - 12x^2 + 44x - 48 & )x^3 - 9x^2 + 23x - 15 \\ & x^3 - 12x^2 + 44x - 48 \\ \hline & & 3x^2 - 21x + 33 \end{array}$$

$$= 1 + \frac{3x^2 - 21x + 33}{x^3 - 12x^2 + 44x - 48} = 1 + \frac{3x^2 - 21x + 33}{(x-2)(x-4)(x-6)}$$

Now Suppose

$$\frac{3x^2 - 21x + 33}{(x-2)(x-4)(x-6)} = \frac{A}{x-2} + \frac{B}{x-4} + \frac{C}{x-6}$$

*Find value of A, B and C yourself*  
*You will get A = 3/8, B = 3/4, C = 15/8*

So

$$\begin{aligned} \frac{3x^2 - 21x + 33}{(x-2)(x-4)(x-6)} &= \frac{3/8}{x-2} + \frac{3/4}{x-4} + \frac{15/8}{x-6} \\ &= \frac{3}{8(x-2)} + \frac{3}{4(x-4)} + \frac{15}{8(x-6)} \end{aligned}$$

Hence

$$\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)} = 1 + \frac{3}{8(x-2)} + \frac{3}{4(x-4)} + \frac{15}{8(x-6)} \quad \text{Answer}$$


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### Question # 10

*Solution*

$$\frac{1}{(1-ax)(1-bx)(1-cx)}$$

$$\frac{1}{(1-ax)(1-bx)(1-cx)} = \frac{A}{1-ax} + \frac{B}{1-bx} + \frac{C}{1-cx}$$

Multiplying both sides by  $(1-ax)(1-bx)(1-cx)$ .

$$1 = A(1-bx)(1-cx) + B(1-ax)(1-cx) + C(1-ax)(1-bx) \dots \dots \dots \text{(i)}$$

Put  $1-ax=0 \Rightarrow ax=1 \Rightarrow x=\frac{1}{a}$  in equation (i).

$$1 = A\left(1-b\cdot\frac{1}{a}\right)\left(1-c\cdot\frac{1}{a}\right) + B(0) + C(0) \Rightarrow 1 = A\left(1-\frac{b}{a}\right)\left(1-\frac{c}{a}\right) + 0 + 0$$

$$\Rightarrow 1 = A\left(\frac{a-b}{a}\right)\left(\frac{a-c}{a}\right) \Rightarrow 1 = A\frac{(a-b)(a-c)}{a^2} \Rightarrow A = \frac{a^2}{(a-b)(a-c)}$$

*Find value of B & C yourself as A.*  
*You will get B =  $\frac{b^2}{(b-a)(b-c)}$ , C =  $\frac{c^2}{(c-a)(c-b)}$*

Hence

$$\begin{aligned} \frac{1}{(1-ax)(1-bx)(1-cx)} &= \frac{(a-b)(a-c)}{1-ax} + \frac{(b-a)(b-c)}{1-bx} + \frac{(c-a)(c-b)}{1-cx} \\ &= \frac{a^2}{(a-b)(a-c)(1-ax)} + \frac{b^2}{(b-a)(b-c)(1-bx)} + \frac{c^2}{(c-a)(c-b)(1-cx)} \end{aligned}$$

Answer

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### Question # 11

*Solution*

$$\frac{x^2 + a^2}{(x^2 + b^2)(x^2 + c^2)(x^2 + d^2)}$$

$$\frac{x^2 + a^2}{(x^2 + b^2)(x^2 + c^2)(x^2 + d^2)}$$

Put  $y = x^2$  in above.

$$\frac{y+a^2}{(y+b^2)(y+c^2)(y+d^2)}$$

Now consider

$$\frac{y+a^2}{(y+b^2)(y+c^2)(y+d^2)} = \frac{A}{y+b^2} + \frac{B}{y+c^2} + \frac{C}{y+d^2}$$

Put  $y + b^2 = 0 \Rightarrow y = -b^2$  in equation (i)

$$-b^2 + a^2 = A(-b^2 + c^2)(-b^2 + d^2) + B(0) + C(0)$$

$$\Rightarrow a^2 - b^2 = A(c^2 - b^2)(d^2 - b^2) + 0 + 0 \quad \Rightarrow \quad A = \frac{a^2 - b^2}{(c^2 - b^2)(d^2 - b^2)}$$

Now put  $y + c^2 = 0 \Rightarrow y = -c^2$  in equation (i)

$$-c^2 + a^2 = A(0) + B(-c^2 + b^2)(-b^2 + d^2) + C(0)$$

$$\Rightarrow a^2 - c^2 = 0 + B(b^2 - c^2)(d^2 - c^2) + 0 \quad \Rightarrow \boxed{B = \frac{a^2 - c^2}{(b^2 - c^2)(d^2 - c^2)}}$$

Now put  $y + d^2 = 0 \Rightarrow y = -d^2$  in equation (i)

$$-d^2 + a^2 = A(0) + B(0) + C(-d^2 + b^2)(-d^2 + c^2)$$

$$\Rightarrow a^2 - d^2 = 0 + 0 + C(b^2 - d^2)(c^2 - d^2) \quad \Rightarrow \quad C = \frac{a^2 - d^2}{(b^2 - d^2)(c^2 - d^2)}$$

Hence

$$\begin{aligned} \frac{y+a^2}{(y+b^2)(y+c^2)(y+d^2)} &= \frac{\frac{a^2-b^2}{(c^2-b^2)(d^2-b^2)}}{y+b^2} + \frac{\frac{a^2-c^2}{(b^2-c^2)(d^2-c^2)}}{y+c^2} + \frac{\frac{a^2-d^2}{(b^2-d^2)(c^2-d^2)}}{y+d^2} \\ &= \frac{a^2-b^2}{(c^2-b^2)(d^2-b^2)(y+b^2)} + \frac{a^2-c^2}{(b^2-c^2)(d^2-c^2)(y+c^2)} + \frac{a^2-d^2}{(b^2-d^2)(c^2-d^2)(y+d^2)} \end{aligned}$$

Since  $y = x^2$

$$= \frac{a^2 - b^2}{(c^2 - b^2)(d^2 - b^2)(x^2 + b^2)} + \frac{a^2 - c^2}{(b^2 - c^2)(d^2 - c^2)(x^2 + c^2)} + \frac{a^2 - d^2}{(b^2 - d^2)(c^2 - d^2)(x^2 + d^2)}$$

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**Book:** **Exercise 5.1**  
*Text Book of Algebra and Trigonometry Class XI*

Punjab Textbook Board, Lahore.

*Available online at <http://www.MathCity.org> in PDF Format*

(Picture format to view online).

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