

Q No 1 L.H.S =  $\tan \theta + \cot \theta$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$$

$$= \frac{1}{\cos \theta \cdot \sin \theta} \quad \because \sin^2 \theta + \cos^2 \theta = 1$$

$$= \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} = \sec \theta \cdot \operatorname{cosec} \theta$$

$$= \operatorname{cosec} \theta \cdot \sec \theta = \text{R.H.S proved}$$

Domain:  $\{ \theta \mid \theta \in \mathbb{R} \text{ but } \theta \neq n\frac{\pi}{2}, n \in \mathbb{Z} \}$

Q No 2 L.H.S =  $\sec \theta \operatorname{cosec} \theta \cdot \sin \theta \cos \theta$

$$= \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} \cdot \sin \theta \cos \theta$$

$$= 1 = \text{R.H.S}$$

Domain =  $\mathbb{R} - \{ n\frac{\pi}{2} \}; n \in \mathbb{Z}$

Q No. 3 L.H.S =  $\cos \theta + \tan \theta \cdot \sin \theta$

$$= \cos \theta + \frac{\sin \theta}{\cos \theta} \cdot \sin \theta$$

$$= \cos \theta + \frac{\sin^2 \theta}{\cos \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta} = \sec \theta = \text{R.H.S proved}$$

Domain =  $\mathbb{R} - \{ (2n+1)\frac{\pi}{2} \}; n \in \mathbb{Z}$

Q No. 4 L.H.S =  $\operatorname{cosec} \theta + \tan \theta \sec \theta$

$$= \frac{1}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta}$$

$$= \frac{1}{\sin \theta} + \frac{\sin \theta}{\cos^2 \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos^2 \theta}$$

$$= \frac{1}{\sin \theta \cdot \cos^2 \theta} = \frac{1}{\sin \theta} \cdot \frac{1}{\cos^2 \theta}$$

$$= \operatorname{cosec} \theta \cdot \sec^2 \theta = \text{R.H.S proved}$$

Domain =  $\{ \theta \mid \theta \in \mathbb{R} \text{ but } \theta \neq n\frac{\pi}{2}, n \in \mathbb{Z} \}$

Kohman  
18-1-2003  
Saturday

$$\begin{aligned}
 \text{Qno 5 L.H.S} &= \sec^2\theta - \operatorname{cosec}^2\theta && \because 1 + \cot^2\theta = \operatorname{cosec}^2\theta \\
 &= (1 + \tan^2\theta) - (1 + \cot^2\theta) && 1 + \tan^2\theta = \sec^2\theta \\
 &= 1 + \tan^2\theta - 1 - \cot^2\theta \\
 &= \tan^2\theta - \cot^2\theta = \text{R.H.S proved}
 \end{aligned}$$

$$\begin{aligned}
 \text{Qno 6 L.H.S} &= \cot^2\theta - \cos^2\theta \\
 &= \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta = \cos^2\theta \left( \frac{1}{\sin^2\theta} - 1 \right) \\
 &= \cos^2\theta \left( \frac{1 - \sin^2\theta}{\sin^2\theta} \right) \\
 &= \cos^2\theta \left( \frac{\cos^2\theta}{\sin^2\theta} \right) && \because 1 - \sin^2\theta = \cos^2\theta \\
 &= \cos^2\theta \cdot \cot^2\theta = \text{R.H.S}
 \end{aligned}$$

$$\begin{aligned}
 \text{Qno 7 L.H.S} &= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) \\
 &= \sec^2\theta - \tan^2\theta \\
 &= (1 + \tan^2\theta) - \tan^2\theta && \because 1 + \tan^2\theta = \sec^2\theta \\
 &= 1 + \tan^2\theta - \tan^2\theta \\
 &= 1 = \text{R.H.S}
 \end{aligned}$$

$$\begin{aligned}
 \text{Qno 8 L.H.S} &= 2\cos^2\theta - 1 && \because \sin^2\theta + \cos^2\theta = 1 \\
 &= 2(1 - \sin^2\theta) - 1 && \because \cos^2\theta = 1 - \sin^2\theta \\
 &= 2 - 2\sin^2\theta - 1 \\
 &= 1 - 2\sin^2\theta = \text{R.H.S}
 \end{aligned}$$

$$\begin{aligned}
 \text{Qno 9 R.H.S} &= \frac{1 - \tan^2\theta}{1 + \tan^2\theta} \\
 &= \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}} = \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}} \\
 &= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{1} \\
 &= \cos^2\theta - \sin^2\theta = \text{R.H.S}
 \end{aligned}$$

$$\begin{aligned}
 \text{Q.No. 10} \quad \text{R.H.S} &= \frac{\cot\theta - 1}{\cot\theta + 1} \\
 &= \frac{\frac{\cos\theta}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} + 1} = \frac{\frac{\cos\theta - \sin\theta}{\sin\theta}}{\frac{\cos\theta + \sin\theta}{\sin\theta}} \\
 &= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Q.No. 11} \quad \text{L.H.S} &= \frac{\sin\theta}{1 + \cos\theta} + \text{~~cos\theta~~ } \cot\theta. \quad \text{<sup>1+2=3</sup> * correction} \\
 &= \frac{\sin\theta + \cos\theta(1 + \cos\theta)}{1 + \cos\theta} + \frac{\cos\theta}{\sin\theta} \\
 &= \frac{\sin^2\theta + \cos\theta(1 + \cos\theta)}{(1 + \cos\theta)\sin\theta} \\
 &= \frac{\sin^2\theta + \cos\theta + \cos^2\theta}{(1 + \cos\theta)\sin\theta} \\
 &= \frac{(\sin^2\theta + \cos^2\theta) + \cos\theta}{(1 + \cos\theta)\sin\theta} \\
 &= \frac{1 + \cos\theta}{(1 + \cos\theta)\sin\theta} \quad \because \sin^2\theta + \cos^2\theta = 1 \\
 &= \frac{1}{\sin\theta} = \operatorname{cosec}\theta = \text{R.H.S}
 \end{aligned}$$

$$\begin{aligned}
 \text{Q.No. 12} \quad \text{L.H.S} &= \frac{\cot^2\theta - 1}{1 + \cot^2\theta} \\
 &= \frac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{1 + \frac{\cos^2\theta}{\sin^2\theta}} = \frac{\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}} \\
 &= \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta + \cos^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{1} \\
 &= \cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta) \\
 &= \cos^2\theta - 1 + \cos^2\theta \\
 &= 2\cos^2\theta - 1 = \text{R.H.S proved}
 \end{aligned}$$

QNO.13 R.H.S =  $(\operatorname{cosec}\theta + \cot\theta)^2$

$$= \left( \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} \right)^2$$

$$= \left( \frac{1 + \cos\theta}{\sin\theta} \right)^2$$

$$= \frac{(1 + \cos\theta)^2}{\sin^2\theta} = \frac{(1 + \cos\theta)^2}{1 - \cos^2\theta}$$

$$= \frac{(1 + \cos\theta)(1 + \cos\theta)}{(1 - \cos\theta)(1 + \cos\theta)}$$

$$= \frac{1 + \cos\theta}{1 - \cos\theta} = \text{R.H.S proved}$$

QNO.14 L.H.S =  $(\sec\theta - \tan\theta)^2$

$$= \left( \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \right)^2 = \left( \frac{1 - \sin\theta}{\cos\theta} \right)^2$$

$$= \frac{(1 - \sin\theta)^2}{\cos^2\theta} = \frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}$$

$$= \frac{(1 - \sin\theta)(1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}$$

$$= \frac{1 - \sin\theta}{1 + \sin\theta} = \text{R.H.S proved}$$

QNO.15

$$\text{L.H.S} = \frac{2\tan\theta}{1 + \tan^2\theta} = \frac{2\tan\theta}{\sec^2\theta}$$

$$= 2\tan\theta \cdot \cos^2\theta$$

$$= 2 \frac{\sin\theta}{\cos\theta} \cdot \cos^2\theta = 2\sin\theta \cos\theta = \text{R.H.S}$$

QNO.16

$$\text{L.H.S} = \frac{1 - \sin\theta}{\cos\theta}$$

$$= \frac{1 - \sin\theta}{\cos\theta} \cdot \frac{1 + \sin\theta}{1 + \sin\theta} \quad \text{Rationalizing}$$

$$= \frac{1 - \sin^2\theta}{\cos\theta(1 + \sin\theta)} = \frac{\cos^2\theta}{\cos\theta(1 + \sin\theta)}$$

$$= \frac{\cos\theta}{1 + \sin\theta} = \text{R.H.S proved}$$

Q.No. 17 L.H.S =  $(\tan\theta + \cot\theta)^2$   
 $= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$   
 $= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)^2$   
 $= \left(\frac{1}{\cos\theta\sin\theta}\right)^2 \quad \because \sin^2\theta + \cos^2\theta = 1$   
 $= (\sec\theta \operatorname{cosec}\theta)^2$   
 $= \sec^2\theta \operatorname{cosec}^2\theta = \text{R.H.S. proved}$

Q.No. 18 L.H.S =  $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

See Alternative Solution at the end.

$$= \frac{\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta} + 1} = \frac{\frac{\sin\theta + 1 - \cos\theta}{\cos\theta}}{\frac{\sin\theta - 1 + \cos\theta}{\cos\theta}}$$

$$= \frac{\sin\theta + 1 - \cos\theta}{\sin\theta - 1 + \cos\theta}$$

$$= \frac{\sin\theta + 1 - \cos\theta}{\sin\theta - 1 + \cos\theta} \cdot \frac{\sin\theta + 1 + \cos\theta}{\sin\theta + 1 + \cos\theta}$$

$$= \frac{(\sin\theta + 1) - \cos\theta}{(\sin\theta + \cos\theta) - 1} \cdot \frac{(\sin\theta + 1) + \cos\theta}{(\sin\theta + \cos\theta) + 1}$$

$$= \frac{(\sin\theta + 1)^2 - (\cos\theta)^2}{(\sin\theta + \cos\theta)^2 - (1)^2}$$

$$= \frac{\sin^2\theta + 2\sin\theta + 1 - \cos^2\theta}{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}$$

$$= \frac{\cancel{\sin^2\theta} + 2\sin\theta + \cancel{\sin^2\theta}}{1 + 2\sin\theta\cos\theta - 1} = \frac{2\sin\theta}{2\sin\theta\cos\theta}$$

$$= \frac{\cancel{2}\sin\theta}{\cancel{2}\sin\theta\cos\theta} + \frac{\cancel{2}\sin\theta}{\cancel{2}\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$$

$$= \tan\theta + \sec\theta = \text{R.H.S. proved}$$

$$\begin{aligned}
 \text{Q.No 19 L.H.S} &= \frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} \\
 &= \frac{1}{\operatorname{cosec}\theta - \cot\theta} \cdot \frac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta + \cot\theta} - \frac{1}{\sin\theta} \\
 &= \frac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta} - \frac{1}{\sin\theta} && \because 1 + \cot^2\theta = \operatorname{cosec}^2\theta \\
 &= \frac{\operatorname{cosec}\theta + \cot\theta}{1} - \frac{1}{\sin\theta} && \because 1 = \operatorname{cosec}^2\theta - \cot^2\theta \\
 &= \operatorname{cosec}\theta + \cot\theta - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} \\
 &= \frac{\cos\theta}{\sin\theta} = \cot\theta \quad \text{--- (i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{R.H.S} &= \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta} \\
 &= \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta} \cdot \frac{\operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}\theta - \cot\theta} \\
 &= \frac{1}{\sin\theta} \cdot \frac{\operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta} \\
 &= \frac{1}{\sin\theta} \cdot \frac{(\operatorname{cosec}\theta - \cot\theta)}{1} && \because 1 + \cot^2\theta = \operatorname{cosec}^2\theta \\
 &&& \because 1 = \operatorname{cosec}^2\theta - \cot^2\theta \\
 &= \frac{1}{\sin\theta} - (\operatorname{cosec}\theta - \cot\theta) = \frac{1}{\sin\theta} - \operatorname{cosec}\theta + \cot\theta \\
 &= \frac{1}{\sin\theta} - \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta \quad \text{--- (ii)}
 \end{aligned}$$

From (i) & (ii) L.H.S = R.H.S proved

$$\begin{aligned}
 \text{Q.No 20 L.H.S} &= \sin^3\theta - \cos^3\theta \\
 &= (\sin\theta)^3 - (\cos\theta)^3 \\
 &= (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta) \\
 &= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta) \\
 &= \text{R.H.S} \quad \text{proved}
 \end{aligned}$$

$$\begin{aligned}
 \text{QNO. 21 L.H.S} &= \sin^6 \theta - \cos^6 \theta \\
 &= (\sin^2 \theta)^3 - (\cos^2 \theta)^3 \\
 &= (\sin^2 \theta - \cos^2 \theta) ((\sin^2 \theta)^2 + (\cos^2 \theta)^2 + \sin^2 \theta \cos^2 \theta) \\
 &= (\sin^2 \theta - \cos^2 \theta) ((\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta) \\
 &= (\sin^2 \theta - \cos^2 \theta) ((\sin^2 \theta + \cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta) \\
 &= (\sin^2 \theta - \cos^2 \theta) (1 - \sin^2 \theta \cos^2 \theta) \\
 &= \text{R.H.S proved}
 \end{aligned}$$

$$\begin{aligned}
 \text{QNO. 22 L.H.S} &= \sin^6 \theta + \cos^6 \theta \\
 &= (\sin^2 \theta)^3 + (\cos^2 \theta)^3 \\
 &= (\sin^2 \theta + \cos^2 \theta) ((\sin^2 \theta)^2 + (\cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta) \\
 &= (1) ((\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 3 \sin^2 \theta \cos^2 \theta) \\
 &= ((\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta) \\
 &= (1)^2 - 3 \sin^2 \theta \cos^2 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta = \text{R.H.S}
 \end{aligned}$$

$$\begin{aligned}
 \text{QNO. 23 L.H.S} &= \frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} \\
 &= \frac{1 - \sin \theta + 1 + \sin \theta}{(1 + \sin \theta)(1 - \sin \theta)} = \frac{2}{1 - \sin^2 \theta} \\
 &= \frac{2}{\cos^2 \theta} = 2 \sec^2 \theta = \text{R.H.S proved}
 \end{aligned}$$

$$\begin{aligned}
 \text{QNO 24 L.H.S} &= \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} + \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \\
 &= \frac{(\cos \theta + \sin \theta)^2 + (\cos \theta - \sin \theta)^2}{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)} \\
 &= \frac{\cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta + \cos^2 \theta + \sin^2 \theta - 2 \cos \theta \sin \theta}{\cos^2 \theta - \sin^2 \theta} \\
 &= \frac{2 \cos^2 \theta + 2 \sin^2 \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{2(\cos^2 \theta + \sin^2 \theta)}{(1 - \sin^2 \theta) - \sin^2 \theta} \\
 &= \frac{2(1)}{1 - \sin^2 \theta - \sin^2 \theta} = \frac{2}{1 - 2 \sin^2 \theta} = \text{R.H.S proved}
 \end{aligned}$$

**Question # 18** [by **Hafiz Syed Rizwan** (FSc, Session: 2007-2009) Punjab College of Science, Lahore.]

$$\text{L.H.S} = \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1}$$

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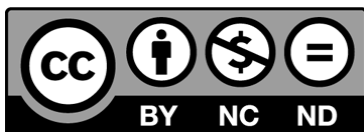
$$\begin{aligned} &= \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} \times \frac{\tan \theta + \sec \theta}{\tan \theta + \sec \theta} \\ &= \frac{(\tan \theta + \sec \theta - 1)(\tan \theta + \sec \theta)}{\tan^2 \theta - \tan \theta \sec \theta + \tan \theta + \tan \theta \sec \theta - \sec^2 \theta + \sec \theta} \\ &= \frac{(\tan \theta + \sec \theta - 1)(\tan \theta + \sec \theta)}{\tan^2 \theta - \sec^2 \theta + \tan \theta + \sec \theta} && \because 1 + \tan^2 \theta = \sec^2 \theta \\ &= \frac{(\tan \theta + \sec \theta - 1)(\tan \theta + \sec \theta)}{-1 + \tan \theta + \sec \theta} && \because \tan^2 \theta - \sec^2 \theta = -1 \\ &= \tan \theta + \sec \theta = \text{R.H.S} \end{aligned}$$

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If you have any alternative solution then you can send us at [Admin@MathCity.org](mailto:Admin@MathCity.org)

Book: **Exercise 9.4 (Page 311)**  
*Text Book of Algebra and Trigonometry Class XI*  
*Punjab Textbook Board, Lahore.*

Available online at <http://www.MathCity.org> in PDF Format  
 (Picture format to view online).  
 Page setup: A4 (8.27 in × 11.02 in).  
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