

Question # 1 & 2*Do yourself***Question # 3**

In which quadrant are the terminal arms of the angle lie when

(i) $\sin \theta < 0$ and $\cos \theta > 0$

(ii) $\cot \theta > 0$ and $\operatorname{cosec} \theta > 0$

Solutions(i) Since $\sin \theta < 0$ so θ lies in *IIIrd* or *IVth* quadrant.Also $\cos \theta > 0$ so θ lies in *Ist* or *IVth* quadrant. $\Rightarrow \theta$ lies in *IVth* quadrant(ii) Since $\cot \theta > 0$ so θ lies in *Ist* or *IIIrd* quadrant.Also $\csc \theta > 0$ so θ lies in *Ist* or *IInd* quadrant $\Rightarrow \theta$ lies in *Ist* quadrant.**Question # 3 (iii), (iv) and***Do yourself as above***Question # 4**

Find the values of the remaining trigonometric functions:

(i) $\sin \theta = \frac{12}{13}$ and the terminal arm of the angle is in quad. I.

(ii) $\cos \theta = \frac{9}{41}$ and the terminal arm of the angle is in quad. IV.

(iv) $\tan \theta = -\frac{1}{3}$ and the terminal arm of the angle is in quad. II.

Solutions

(i) Since $\sin^2 \theta + \cos^2 \theta = 1$

$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$

$\Rightarrow \cos \theta = \pm \sqrt{1 - \sin^2 \theta}$

As terminal ray lies in *Ist* quadrant so $\cos \theta$ is +ive.

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\Rightarrow \cos \theta = \sqrt{1 - \left(\frac{12}{13}\right)^2} \quad \because \sin \theta = \frac{12}{13}$$

$$= \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} \Rightarrow \boxed{\cos \theta = \frac{5}{13}}$$

Now

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{13} \cdot \frac{13}{5} \Rightarrow \boxed{\tan \theta = \frac{12}{5}}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{1}{12/13} = \frac{13}{12} \quad \Rightarrow \quad \boxed{\csc \theta = \frac{13}{12}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{5/13} = \frac{13}{5} \quad \Rightarrow \quad \boxed{\sec \theta = \frac{13}{5}}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{12/5} = \frac{5}{12} \quad \Rightarrow \quad \boxed{\cot \theta = \frac{5}{12}}$$

(ii) Since $\sin^2 \theta + \cos^2 \theta = 1$
 $\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$
 $\Rightarrow \sin \theta = \pm \sqrt{1 - \cos^2 \theta}$

As terminal ray lies in *IVth* quadrant so $\sin \theta$ is -ive .

$$\begin{aligned} \sin \theta &= -\sqrt{1 - \cos^2 \theta} \\ \Rightarrow \sin \theta &= -\sqrt{1 - \left(\frac{9}{41}\right)^2} \\ &= -\sqrt{1 - \frac{81}{1681}} = -\sqrt{\frac{1600}{1681}} = -\frac{40}{41} \quad \Rightarrow \quad \boxed{\sin \theta = -\frac{40}{41}} \end{aligned}$$

Now

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-40/41}{9/41} = -\frac{40}{41} \cdot \frac{41}{9} = -\frac{40}{9} \quad \Rightarrow \quad \boxed{\tan \theta = -\frac{40}{9}}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{1}{-40/41} = -\frac{41}{40} \quad \Rightarrow \quad \boxed{\csc \theta = -\frac{41}{40}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{9/11} = \frac{11}{9} \quad \Rightarrow \quad \boxed{\sec \theta = \frac{11}{9}}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{-40/9} = -\frac{9}{40} \quad \Rightarrow \quad \boxed{\cot \theta = -\frac{9}{40}}$$

(iv) Since $\sec^2 \theta = 1 + \tan^2 \theta$
 $\Rightarrow \sec \theta = \pm \sqrt{1 + \tan^2 \theta}$

As terminal ray is in *IInd* quadrant so $\sec \theta$ is -ive.

$$\begin{aligned} \Rightarrow \sec \theta &= -\sqrt{1 + \tan^2 \theta} \\ \Rightarrow \sec \theta &= -\sqrt{1 + \left(-\frac{1}{3}\right)^2} = -\sqrt{1 + \frac{1}{9}} = -\sqrt{\frac{10}{9}} \\ \Rightarrow \boxed{\sec \theta} &= -\frac{\sqrt{10}}{3} \end{aligned}$$

$$\text{Now } \cos \theta = \frac{1}{\sec \theta} = \frac{1}{-\sqrt{10}/3} = -\frac{3}{\sqrt{10}} \Rightarrow \boxed{\cos \theta = -\frac{3}{\sqrt{10}}}$$

$$\text{Also } \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\Rightarrow \sin \theta = (\tan \theta)(\cos \theta) = \left(-\frac{1}{3}\right)\left(-\frac{3}{\sqrt{10}}\right) \Rightarrow \boxed{\sin \theta = \frac{1}{\sqrt{10}}}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{1}{1/\sqrt{10}} \Rightarrow \boxed{\csc \theta = \sqrt{10}}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{-1/3} \Rightarrow \boxed{\cot \theta = -3}$$

Question # 4 (iii) and (v)

Do yourself as above.

Question # 5

If $\cot \theta = \frac{15}{8}$ and terminal arm of the angle is not in quad. I, find the values of $\cos \theta$ and $\operatorname{cosec} \theta$.

Solution

As $\cot \theta$ is +ive and it is not in *Ist* quadrant so it is in *IIIrd* quadrant
($\cot \theta$ +ive in *Ist* and *IIIrd* quadrant)

$$\text{Now } \csc^2 \theta = 1 + \cot^2 \theta \Rightarrow \csc \theta = \pm \sqrt{1 + \cot^2 \theta}$$

As terminal ray is in *IIIrd* quadrant so $\csc \theta$ is -ive.

$$\begin{aligned} \csc \theta &= -\sqrt{1 + \cot^2 \theta} \\ \Rightarrow \csc \theta &= -\sqrt{1 + \left(\frac{15}{8}\right)^2} = -\sqrt{1 + \frac{225}{64}} & \because \cot \theta = \frac{15}{8} \\ &= -\sqrt{\frac{289}{64}} \Rightarrow \boxed{\csc \theta = -\frac{17}{8}} \end{aligned}$$

$$\sin \theta = \frac{1}{\csc \theta} = \frac{1}{-17/8} \Rightarrow \boxed{\sin \theta = -\frac{8}{17}}$$

$$\text{Now } \frac{\cos \theta}{\sin \theta} = \cot \theta$$

$$\Rightarrow \cos \theta = \cot \theta \sin \theta = \left(\frac{15}{8}\right)\left(-\frac{8}{17}\right) \Rightarrow \boxed{\cos \theta = -\frac{15}{17}}$$

Question # 6

If $\operatorname{cosec} \theta = \frac{m^2 + 1}{2m}$ and $\left(0 < \theta < \frac{\pi}{2}\right)$, find the values of the remaining trigonometric function.

Solution

Since $0 < \theta < \frac{\pi}{2}$ therefore terminal ray lies in *Ist* quadrant.

$$\begin{aligned}\text{Now } 1 + \cot^2 \theta &= \csc^2 \theta \\ \Rightarrow \cot^2 \theta &= \csc^2 \theta - 1 \\ \Rightarrow \cot \theta &= \pm \sqrt{\csc^2 \theta - 1}\end{aligned}$$

As terminal ray of θ is in *Ist* quadrant so $\cot \theta$ is +ive.

$$\begin{aligned}\cot \theta &= \sqrt{\csc^2 \theta - 1} \\ \Rightarrow \cot \theta &= \sqrt{\left(\frac{m^2 + 1}{2m}\right)^2 - 1} = \sqrt{\frac{(m^2 + 1)^2}{(2m)^2} - 1} \quad \because \csc \theta = \frac{m^2 + 1}{2m} \\ &= \sqrt{\frac{m^4 + 2m^2 + 1}{4m^2} - 1} = \sqrt{\frac{m^4 + 2m^2 + 1 - 4m^2}{4m^2}} = \sqrt{\frac{m^4 - 2m^2 + 1}{4m^2}} \\ &= \sqrt{\frac{(m^2 - 1)^2}{(2m)^2}} = \frac{m^2 - 1}{2m} \quad \Rightarrow \boxed{\cot \theta = \frac{m^2 - 1}{2m}} \\ \sin \theta &= \frac{1}{\csc \theta} = \frac{1}{\left(\frac{m^2 + 1}{2m}\right)} = \frac{2m}{m^2 + 1} \quad \Rightarrow \boxed{\sin \theta = \frac{2m}{m^2 + 1}}\end{aligned}$$

$$\text{Now } \frac{\cos \theta}{\sin \theta} = \cot \theta \quad \Rightarrow \cos \theta = (\cot \theta)(\sin \theta)$$

$$\Rightarrow \cos \theta = \left(\frac{m^2 - 1}{2m}\right)\left(\frac{2m}{m^2 + 1}\right) \quad \Rightarrow \boxed{\cos \theta = \left(\frac{m^2 - 1}{m^2 + 1}\right)}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{m^2 - 1}{m^2 + 1}} \quad \Rightarrow \boxed{\sec \theta = \left(\frac{m^2 + 1}{m^2 - 1}\right)}$$

$$\tan \theta = \frac{1}{\cot \theta} = \frac{1}{\frac{m^2 - 1}{2m}} \quad \Rightarrow \boxed{\tan \theta = \left(\frac{2m}{m^2 - 1}\right)}$$

Question # 7

If $\tan \theta = \frac{1}{\sqrt{7}}$ and the terminal arm of the angle is not in the II quad. Find the value of

$$\frac{\csc^2 \theta - \sec^2 \theta}{\csc^2 \theta + \sec^2 \theta}$$

Solution

$\tan \theta$ is +ive and terminal arm is not in the *IIIrd* quadrant, therefore terminal arm lies in *Ist* quadrant.

$$\begin{aligned}\text{Now } \sec^2 \theta &= 1 + \tan^2 \theta \\ \Rightarrow \sec \theta &= \pm \sqrt{1 + \tan^2 \theta}\end{aligned}$$

as terminal arm is in the first quadrant so $\sec \theta$ is +ive.

$$\sec \theta = \sqrt{1 + \tan^2 \theta}$$

$$\sec \theta = \sqrt{1 + \left(\frac{1}{\sqrt{7}}\right)^2} = \sqrt{1 + \frac{1}{7}} = \sqrt{\frac{8}{7}} \Rightarrow \boxed{\sec \theta = \frac{2\sqrt{2}}{\sqrt{7}}}$$

Now $\cos \theta = \frac{1}{\sec \theta} = \frac{1}{2\sqrt{2}/\sqrt{7}} \Rightarrow \boxed{\cos \theta = \frac{\sqrt{7}}{2\sqrt{2}}}$

Now $\frac{\sin \theta}{\cos \theta} = \tan \theta \Rightarrow \sin \theta = (\tan \theta)(\cos \theta)$

$$\Rightarrow \sin \theta = \left(\frac{1}{\sqrt{7}}\right)\left(\frac{\sqrt{7}}{2\sqrt{2}}\right) \Rightarrow \boxed{\sin \theta = \frac{1}{2\sqrt{2}}}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{1}{1/2\sqrt{2}} \Rightarrow \boxed{\csc \theta = 2\sqrt{2}}$$

Now
$$\frac{\csc^2 \theta - \sec^2 \theta}{\csc^2 \theta + \sec^2 \theta} = \frac{(2\sqrt{2})^2 - \left(2\sqrt{2}/\sqrt{7}\right)^2}{(2\sqrt{2})^2 + \left(2\sqrt{2}/\sqrt{7}\right)^2} = \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$= \frac{48/7}{64/7} = \frac{48}{7} \cdot \frac{7}{64} = \frac{4}{3} \quad \text{Answer}$$

Question # 8

If $\cot \theta = \frac{5}{2}$ and the terminal arm of the angle is in the I quad., find the value of

$$\frac{3\sin \theta + 4\cos \theta}{\cos \theta - \sin \theta}$$

Solution

Since $\csc^2 \theta = 1 + \cot^2 \theta$
 $\Rightarrow \csc \theta = \pm \sqrt{1 + \cot^2 \theta}$

As terminal ray is in Ird quadrant so $\csc \theta$ is +ive.

$$\csc \theta = \sqrt{1 + \cot^2 \theta} = \sqrt{1 + \left(\frac{5}{2}\right)^2} = \sqrt{1 + \frac{25}{4}} = \sqrt{\frac{29}{4}} = \frac{\sqrt{29}}{2}$$

Now $\sin \theta = \frac{1}{\csc \theta} = \frac{1}{\sqrt{29}/2} \Rightarrow \boxed{\sin \theta = \frac{2}{\sqrt{29}}}$

Now $\frac{\cos \theta}{\sin \theta} = \cot \theta \Rightarrow \cos \theta = (\cot \theta)(\sin \theta)$

$$\Rightarrow \cos \theta = \left(\frac{5}{2}\right)\left(\frac{2}{\sqrt{29}}\right) \Rightarrow \boxed{\cos \theta = \frac{5}{\sqrt{29}}}$$

$$\begin{aligned}
 \text{Now } \frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta} &= \frac{3\left(\frac{2}{\sqrt{29}}\right) + 4\left(\frac{5}{\sqrt{29}}\right)}{\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}} = \frac{\frac{6}{\sqrt{29}} + \frac{20}{\sqrt{29}}}{\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}} \\
 &= \frac{\frac{6+20}{\sqrt{29}}}{\frac{5-2}{\sqrt{29}}} = \frac{\frac{26}{\sqrt{29}}}{\frac{3}{\sqrt{29}}} = \frac{26}{\sqrt{29}} \cdot \frac{\sqrt{29}}{3} = \frac{26}{3} \quad \text{Answer}
 \end{aligned}$$

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Book: **Exercise 9.2 (Page #301)**
Text Book of Algebra and Trigonometry Class XI
Punjab Textbook Board, Lahore.

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