Question # 1 & 2

Do yourself

Question # 3
In which quadrant are the terminal arms of the angle lie when

(i) \( \sin \theta < 0 \) and \( \cos \theta > 0 \)
(ii) \( \cot \theta > 0 \) and \( \csc \theta > 0 \)

Solutions

(i) Since \( \sin \theta < 0 \) so \( \theta \) lies in IIIrd or IVth quadrant.
   Also \( \cos \theta > 0 \) so \( \theta \) lies in Ist or IVth quadrant.
   \( \implies \theta \) lies in IVth quadrant

(ii) Since \( \cot \theta > 0 \) so \( \theta \) lies in Ist or IIIrd quadrant.
   Also \( \csc \theta > 0 \) so \( \theta \) lies in Ist or IIInd quadrant
   \( \implies \theta \) lies in Ist quadrant.

Question # 3 (iii), (iv) and …

Do yourself as above

Question # 4
Find the values of the remaining trigonometric functions:

(i) \( \sin \theta = \frac{12}{13} \) and the terminal arm of the angle is in quad. I.

(ii) \( \cos \theta = \frac{9}{41} \) and the terminal arm of the angle is in quad. IV.

(iv) \( \tan \theta = -\frac{1}{3} \) and the terminal arm of the angle is in quad. II.

Solutions

(i) Since \( \sin^2 \theta + \cos^2 \theta = 1 \)
   \( \implies \cos^2 \theta = 1 - \sin^2 \theta \)
   \( \implies \cos \theta = \pm \sqrt{1 - \sin^2 \theta} \)
   As terminal ray lies in Ist quadrant so \( \cos \theta \) is +ive.
   \( \cos \theta = \sqrt{1 - \sin^2 \theta} \)
   \( \implies \cos \theta = \sqrt{1 - \left(\frac{12}{13}\right)^2} \)
   \( = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} \)
   \( \implies \cos \theta = \frac{5}{13} \)

Now

\( \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{12/13}{5/13} = \frac{12}{5} \)
\( \implies \tan \theta = \frac{12}{5} \)
\[
\csc \theta = \frac{1}{\sin \theta} = \frac{1}{12/13} = \frac{13}{12} \quad \Rightarrow \quad \csc \theta = \frac{13}{12}
\]

\[
\sec \theta = \frac{1}{\cos \theta} = \frac{1}{5/13} = \frac{13}{5} \quad \Rightarrow \quad \sec \theta = \frac{13}{5}
\]

\[
\cot \theta = \frac{1}{\tan \theta} = \frac{1}{12/5} = \frac{5}{12} \quad \Rightarrow \quad \cot \theta = \frac{5}{12}
\]

(ii) Since \(\sin^2 \theta + \cos^2 \theta = 1\)

\[
\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta
\]

\[
\Rightarrow \sin \theta = \pm \sqrt{1 - \cos^2 \theta}
\]

As terminal ray lies in IVth quadrant so \(\sin \theta\) is –ive.

\[
\sin \theta = -\sqrt{1 - \cos^2 \theta}
\]

\[
\Rightarrow \sin \theta = -\sqrt{1 - \left(\frac{9}{41}\right)^2}
\]

\[
= -\sqrt{1 - \frac{81}{1681}} = -\sqrt{\frac{1600}{1681}} = -\frac{40}{41} \quad \Rightarrow \quad \sin \theta = -\frac{40}{41}
\]

Now

\[
\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-40/41}{9/41} = -\frac{40}{41} \cdot \frac{41}{9} = -\frac{40}{9} \quad \Rightarrow \quad \tan \theta = -\frac{40}{9}
\]

\[
\csc \theta = \frac{1}{\sin \theta} = \frac{1}{-40/41} = -\frac{41}{40} \quad \Rightarrow \quad \csc \theta = -\frac{41}{40}
\]

\[
\sec \theta = \frac{1}{\cos \theta} = \frac{1}{9/11} = \frac{41}{9} \quad \Rightarrow \quad \sec \theta = \frac{41}{9}
\]

\[
\cot \theta = \frac{1}{\tan \theta} = \frac{1}{-40/9} = -\frac{9}{40} \quad \Rightarrow \quad \cot \theta = -\frac{9}{40}
\]

(iv) Since \(\sec^2 \theta = 1 + \tan^2 \theta\)

\[
\Rightarrow \sec \theta = \pm \sqrt{1 + \tan^2 \theta}
\]

As terminal ray is in IIInd quadrant so \(\sec \theta\) is –ive.

\[
\Rightarrow \sec \theta = -\sqrt{1 + \tan^2 \theta}
\]

\[
\Rightarrow \sec \theta = -\sqrt{1 + \left(-\frac{1}{3}\right)^2} = -\sqrt{1 + \frac{1}{9}} = -\sqrt{\frac{10}{9}}
\]
\[ \sec \theta = -\frac{\sqrt{10}}{3} \]

Now \[ \cos \theta = \frac{1}{\sec \theta} = \frac{1}{-\frac{\sqrt{10}}{3}} = -\frac{3}{\sqrt{10}} \]

\[ \sin \theta = \tan \theta \]

\[ \Rightarrow \sin \theta = \left( \tan \theta \right) \left( \cos \theta \right) = \left( -\frac{1}{3} \right) \left( -\frac{3}{\sqrt{10}} \right) \]

\[ \csc \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{\sqrt{10}}{3}} \]

\[ \cot \theta = \frac{1}{\tan \theta} = \frac{1}{-\frac{1}{3}} \]

\[ \Rightarrow \cos \theta = \cot \theta \sin \theta = \left( \frac{15}{8} \right) \left( -\frac{8}{17} \right) \]

**Question # 4 (iii) and (v)**

Do yourself as above.

**Question # 5**

If \( \cot \theta = \frac{15}{8} \) and terminal arm of the angle is not in quad. I, find the values of \( \cos \theta \) and \( \csc \theta \).

**Solution**

As \( \cot \theta \) is +ive and it is not in \( I \)st quadrant, so it is in \( III \)rd quadrant

\[ \text{(cot \( \theta \) +ive in \( I \)st and \( III \)rd quadrant)} \]

Now

\[ \csc^2 \theta = 1 + \cot^2 \theta \]

\[ \Rightarrow \csc \theta = \pm \sqrt{1 + \cot^2 \theta} \]

As terminal ray is in \( III \)rd quadrant so \( \csc \theta \) is –ive.

\[ \csc \theta = -\sqrt{1 + \cot^2 \theta} \]

\[ \Rightarrow \csc \theta = -\sqrt{1 + \left( \frac{15}{8} \right)^2} = -\sqrt{1 + \frac{225}{64}} \]

\[ = -\sqrt{1 + \frac{289}{64}} \]

\[ = -\sqrt{\frac{289}{64}} \]

\[ \Rightarrow \csc \theta = -\frac{17}{8} \]

\[ \sin \theta = \frac{1}{\csc \theta} = \frac{1}{-\frac{17}{8}} \]

\[ \Rightarrow \sin \theta = -\frac{8}{17} \]

Now

\[ \frac{\cos \theta}{\sin \theta} = \cot \theta \]

\[ \Rightarrow \cos \theta = \cot \theta \sin \theta = \left( \frac{15}{8} \right) \left( -\frac{8}{17} \right) \]

\[ \Rightarrow \cos \theta = -\frac{15}{17} \]
Question # 6
If \( \csc \theta = \frac{m^2 + 1}{2m} \) and \( 0 < \theta < \frac{\pi}{2} \), find the values of the remaining trigonometric function.

**Solution**

Since \( 0 < \theta < \frac{\pi}{2} \), therefore terminal ray lies in 1st quadrant.

Now \( 1 + \cot^2 \theta = \csc^2 \theta \)

\( \Rightarrow \cot^2 \theta = \csc^2 \theta - 1 \)

\( \Rightarrow \cot \theta = \pm \sqrt{\csc^2 \theta - 1} \)

As terminal ray of \( \theta \) is in 1st quadrant so \( \cot \theta \) is +ive.

\( \cot \theta = \sqrt{\csc^2 \theta - 1} \)

\( \Rightarrow \cot \theta = \sqrt{\left(\frac{m^2 + 1}{2m}\right)^2 - 1} = \sqrt{\left(\frac{m^2 + 1}{2m}\right)^2 - 1} \)

\( = \sqrt{\frac{m^4 + 2m^2 + 1}{4m^2} - 1} = \sqrt{\frac{m^4 + 2m^2 + 1 - 4m^2}{4m^2}} = \sqrt{\frac{m^4 - 2m^2 + 1}{4m^2}} \)

\( = \sqrt{\frac{(m^2 - 1)^2}{(2m)^2}} = \frac{m^2 - 1}{2m} \)

\( \Rightarrow \cot \theta = \frac{m^2 - 1}{2m} \)

\( \sin \theta = \frac{1}{\csc \theta} = \frac{1}{\left(\frac{m^2 + 1}{2m}\right)} = \frac{2m}{m^2 + 1} \)

\( \Rightarrow \sin \theta = \frac{2m}{m^2 + 1} \)

Now \( \frac{\cos \theta}{\sin \theta} = \cot \theta \quad \Rightarrow \cos \theta = (\cot \theta)(\sin \theta) \)

\( \Rightarrow \cos \theta = \left(\frac{m^2 - 1}{2m}\right)\left(\frac{2m}{m^2 + 1}\right) \quad \Rightarrow \cos \theta = \left(\frac{m^2 - 1}{m^2 + 1}\right) \)

\( \sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{m^2 - 1}{m^2 + 1}} \quad \Rightarrow \sec \theta = \left(\frac{m^2 + 1}{m^2 - 1}\right) \)

\( \tan \theta = \frac{1}{\cot \theta} = \frac{1}{\frac{m^2 - 1}{2m}} \quad \Rightarrow \tan \theta = \left(\frac{2m}{m^2 - 1}\right) \)

Question # 7
If \( \tan \theta = \frac{1}{\sqrt{7}} \) and the terminal arm of the angle is not in the II quad. Find the value of \( \frac{\csc^2 \theta - \sec^2 \theta}{\csc^2 \theta + \sec^2 \theta} \).

**Solution**

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Since \( \tan \theta \) is +ive and terminal arm is not in the 3rd quadrant, therefore terminal arm lies in 1st quadrant.

Now \( \sec^2 \theta = 1 + \tan^2 \theta \)
\[
\Rightarrow \sec \theta = \pm \sqrt{1 + \tan^2 \theta}
\]
as terminal arm is in the first quadrant so \( \sec \theta \) is +ive.

\[
\sec \theta = \sqrt{1 + \left( \frac{1}{\sqrt{7}} \right)^2} = \sqrt{1 + \frac{1}{7}} = \sqrt{\frac{8}{7}} \quad \Rightarrow \quad \sec \theta = \frac{2\sqrt{2}}{\sqrt{7}}
\]

Now \( \cos \theta = \frac{1}{\sec \theta} = \frac{1}{\frac{2\sqrt{2}}{\sqrt{7}}} \quad \Rightarrow \quad \cos \theta = \frac{\sqrt{7}}{2\sqrt{2}}
\]

Now \( \frac{\sin \theta}{\cos \theta} = \tan \theta \quad \Rightarrow \quad \sin \theta = (\tan \theta)(\cos \theta) \)
\[
\Rightarrow \sin \theta = \left( \frac{1}{\sqrt{7}} \right) \left( \frac{\sqrt{7}}{2\sqrt{2}} \right) \quad \Rightarrow \quad \sin \theta = \frac{1}{2\sqrt{2}}
\]

\[
\csc \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{1}{2\sqrt{2}}} \quad \Rightarrow \quad \csc \theta = 2\sqrt{2}
\]

Now \[ \frac{\csc^2 \theta - \sec^2 \theta}{\csc^2 \theta + \sec^2 \theta} = \frac{(2\sqrt{2})^2 - \left( \frac{2\sqrt{2}}{\sqrt{7}} \right)^2}{(2\sqrt{2})^2 + \left( \frac{2\sqrt{2}}{\sqrt{7}} \right)^2} = \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}} = \frac{\frac{48}{7}}{\frac{64}{7}} = \frac{48}{7} \cdot \frac{7}{64} = \frac{3}{4} \quad \text{Answer}
\]

**Question # 8**

If \( \cot \theta = \frac{5}{2} \) and the terminal arm of the angle is in the 1st quadrant, find the value of \( \frac{3\sin \theta + 4\cos \theta}{\cos \theta - \sin \theta} \)

**Solution**

Since \( \csc^2 \theta = 1 + \cot^2 \theta \)
\[
\Rightarrow \csc \theta = \pm \sqrt{1 + \cot^2 \theta}
\]

As terminal ray is in 1st quadrant so \( \csc \theta \) is +ive.

\[
csc \theta = \sqrt{1 + \cot^2 \theta} = \sqrt{1 + \left( \frac{5}{2} \right)^2} = \sqrt{1 + \frac{25}{4}} = \sqrt{\frac{29}{4}} = \frac{\sqrt{29}}{2}
\]
Now \[ \sin \theta = \frac{1}{\csc \theta} = \frac{1}{\sqrt{29}/2} \quad \Rightarrow \quad \sin \theta = \frac{2}{\sqrt{29}} \]

Now \[ \frac{\cos \theta}{\sin \theta} = \cot \theta \quad \Rightarrow \quad \cos \theta = (\cot \theta)(\sin \theta) \]
\[ \Rightarrow \quad \cos \theta = \left( \frac{5}{2} \right) \left( \frac{2}{\sqrt{29}} \right) \quad \Rightarrow \quad \cos \theta = \frac{5}{\sqrt{29}} \]

Now \[ \frac{3\sin \theta + 4\cos \theta}{\cos \theta - \sin \theta} = \frac{3 \left( \frac{2}{\sqrt{29}} \right) + 4 \left( \frac{5}{\sqrt{29}} \right)}{\sqrt{29} - \frac{2}{\sqrt{29}}} = \frac{6\sqrt{29} + 20}{\sqrt{29} - 2\sqrt{29}} \]
\[ = \frac{6 + 20}{\frac{5 - 2}{\sqrt{29}}} = \frac{26}{\frac{3}{\sqrt{29}}} = \frac{26}{3} \cdot \frac{\sqrt{2}}{3} = \frac{26}{3} \quad \text{Answer} \]

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**Book:** 
*Exercise 9.2 (Page 301)*  
*Text Book of Algebra and Trigonometry Class XI*  
*Punjab Textbook Board, Lahore.*

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