Exercise 4.6

Sum and Product of Roots

$$ax^2+bx+c=0, \qquad a\neq 0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Let $\alpha=\dfrac{-b+\sqrt{b^2-4ac}}{2a}$, $\beta=\dfrac{-b-\sqrt{b^2-4ac}}{2a}$

sum $S=\alpha+\beta=\dfrac{-b+\sqrt{b^2-4ac}}{2a}+\dfrac{-b-\sqrt{b^2-4ac}}{2a}$

$$=\frac{-b+\cancel{\sqrt{b^2-4ac}}-b-\cancel{\sqrt{b^2-4ac}}}{2a}=\frac{-2b}{2a}$$

$$\alpha+\beta=-\frac{b}{a}=-\frac{\text{coefficient of } x}{\text{Coefficient of } x^2}.$$

$P=\alpha\beta=\left(\dfrac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\dfrac{-b-\sqrt{b^2-4ac}}{2a}\right)$

$$=\frac{(-b)^2-\left(\sqrt{b^2-4ac}\right)^2}{4a^2}=\frac{b^2-(b^2-4ac)}{4a^2}$$

$$=\frac{\cancel{b^2}-\cancel{b^2}+4ac}{4a^2}=\frac{\cancel{4}ac}{\cancel{4}a^2}=\frac{c}{a}$$

$$\alpha\beta=\frac{c}{a}=\frac{\text{constant term}}{\text{coefficient of } x^2}.$$

Formation of an equation whose roots are given

Given roots are α and β,

then equation is

$$(x-\alpha)(x-\beta) = 0$$

$$x^2 - \beta x - \alpha x + \alpha\beta = 0$$

$$x^2 - (\alpha+\beta)\, x + \alpha\beta = 0$$

$$x^2 - S\, x + P = 0.$$

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Suppose Math.

Exercise 4.6

Q1 If α, β are the roots of $3x^2 - 2x + 4 = 0$, find the values of

If α, β are roots, then

$$\alpha+\beta = -\frac{(-2)}{3} = \frac{2}{3}$$

$$\alpha\beta = \frac{4}{3}$$

$\alpha+\beta = -\frac{b}{a}$

$\alpha\beta = \frac{c}{a}$

(i) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2+\alpha^2}{\alpha^2\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$

$$= \frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)}{\left(\frac{4}{3}\right)^2} = \frac{\left(\frac{4}{9} - \frac{8}{3}\right)}{\left(\frac{16}{9}\right)}$$

$$= \frac{\left(\frac{4-24}{9}\right)}{\left(\frac{16}{9}\right)} = \frac{-20}{16} = -\frac{5}{4}.$$

(ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\beta\alpha} = \frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha\beta}$

$$= \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)}{\left(\frac{4}{3}\right)}$$

$$= \left(\frac{4}{9} - \frac{8}{3}\right) \div \frac{4}{3}$$

$$= \left(\frac{4-24}{3\cancel{9}}\right) \times \frac{3}{4} = \frac{-\cancel{20}^{5}}{3} \times \frac{1}{\cancel{4}} = -\frac{5}{3}.$$

(iii) $\alpha^4 + \beta^4 = (\alpha^2)^2 + (\beta^2)^2$

$\alpha+\beta = \frac{2}{3}$ ✓

$\alpha\beta = \frac{4}{3}$ ✓

$= (\alpha^2)^2 + (\beta^2)^2 + 2\alpha^2\beta^2 - 2\alpha^2\beta^2$

$= (\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2$

$= \left[\underline{\alpha^2 + \beta^2 + 2\alpha\beta} - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2$

$= \left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2$

$= \left[\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)\right]^2 - 2\left(\frac{4}{3}\right)^2$

$= \left(\frac{4}{9} - \frac{8}{3}\right)^2 - 2\left(\frac{16}{9}\right)$

$= \left(\frac{4-24}{9}\right)^2 - \frac{32}{9}$

$= \left(\frac{-20}{9}\right)^2 - \frac{32}{9} = \frac{400}{81} - \frac{32}{9}$

$= \frac{400 - 9(32)}{81} = \frac{400-288}{81} = \frac{112}{81}.$

1
32
9
288

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Suppose Math.

$$a^3+b^3=(a+b)(a^2+b^2-ab)$$

(iv) $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$

$= (\alpha+\beta)\left[\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta-\alpha\beta\right]$

$= (\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]$

$= \left(\frac{2}{3}\right)\left[\left(\frac{2}{3}\right)^2-3\left(\frac{4}{3}\right)\right]$

$= \left(\frac{2}{3}\right)\left[\frac{4}{9}-\frac{12}{3}\right] = \frac{2}{3}\left(\frac{4-36}{9}\right)$

$= \frac{2}{3}\left(\frac{-32}{9}\right) = -\frac{64}{27}.$

(v) $\frac{1}{\alpha^3}+\frac{1}{\beta^3} = \frac{\alpha^3+\beta^3}{\alpha^3\beta^3} = \frac{\left(-\frac{64}{27}\right)}{\left(\frac{4}{3}\right)^3}$

$= \frac{\left(-\frac{64}{27}\right)}{\left(\frac{64}{27}\right)} = -1.$