Exercise 4.3

① $3x^2+2x-\sqrt{3x^2+2x-1}=3$

Suppose $y=3x^2+2x$

$$y-\sqrt{y-1}=3 \quad \text{— ①}$$

$$y-3=\sqrt{y-1}$$

$$(y-3)^2=\left(\sqrt{y-1}\right)^2$$

$$y^2-6y+9=y-1$$

$$y^2-6y+9-y+1=0$$

$$y^2-7y+10=0$$

$$y^2-5y-2y+10=0$$

$$y(y-5)-2(y-5)=0$$

$$(y-2)(y-5)=0$$

$ab=0$
$\Rightarrow a=0$ or $b=0$.

$y-2=0$

$y=2$

Put in ①

L.H.S $y-\sqrt{y-1}=2-\sqrt{2-1}=2-1=1$

R.H.S 3

L.H.S ≠ R.H.S.

$y-5=0$

$y=5.$

Put in ①

L.H.S $y-\sqrt{y-1}=5-\sqrt{5-1}=3$

R.H.S 3

L.H.S = R.H.S.

We solve only

$y=5.$

$3x^2+2x-5=0 \Rightarrow 3x^2+5x-3x-5=0$

$x(3x+5)-1(3x+5)=0 \Rightarrow (x-1)(3x+5)=0$

$x-1=0$, $3x+5=0$

$x=1$, $x=-5/3$

S.S. $=\{1, -\frac{5}{3}\}$.

(2) $x^2 - \frac{x}{2} - 7 = x - 3\sqrt{2x^2-3x+2}$

Multiply by 2

$2x^2 - x - 14 = 2x - 6\sqrt{2x^2-3x+2}$

$2x^2 - 3x - 14 = -6\sqrt{2x^2-3x+2}$

Let $y = 2x^2 - 3x$

$y - 14 = -6\sqrt{y+2}$ — ①

$(y-14)^2 = (-6\sqrt{y+2})^2$

$y^2 - 28y + 196 = 36(y+2)$

$y^2 - 28y + 196 = 36y + 72$

$y^2 - 28y - 36y + 196 - 72 = 0$

$y^2 - 64y + 124 = 0$

$y^2 - 62y - 2y + 124 = 0$

$y(y-62) - 2(y-62) = 0$

$(y-2)(y-62) = 0$

For videos, visit YouTube
Suppose Math.

$y - 2 = 0$

$y = 2$

Put in ①

L.H.S $y - 14 = 2 - 14 = -12$

R.H.S $-6\sqrt{y+2} = -6\sqrt{2+2} = -12$

L.H.S = R.H.S

$y - 62 = 0$

$y = 62$

Put in ①

L.H.S $y - 14 = 62 - 14 = 48$

R.H.S $-6\sqrt{y+2} = -6\sqrt{62+2} = -48$

L.H.S ≠ R.H.S.

We solve only $y = 2$

$2x^2 - 3x - 2 = 0$

$2x^2 - 4x + x - 2 = 0 \Rightarrow 2x(x-2) + 1(x-2) = 0$

$(2x+1)(x-2) = 0$

$2x + 1 = 0$, $x - 2 = 0$

$x = -\frac{1}{2}$, $x = 2$ $\Rightarrow$ S.S. $= \{-\frac{1}{2}, 2\}$.

③ $\sqrt{2x+8} + \sqrt{x+5} = 7$ — ①

$$\left(\sqrt{2x+8}\right)^2 = \left(7 - \sqrt{x+5}\right)^2$$

$$2x+8 = 49 + x + 5 - 14\sqrt{x+5}$$

$$2x+8-49-x-5 = -14\sqrt{x+5}$$

$$(x-46)^2 = \left(-14\sqrt{x+5}\right)^2$$

$$x^2 + 2116 - 92x = 196(x+5)$$

$$x^2 - 92x + 2116 - 196x - 196(5) = 0$$

$$x^2 - 288x + 1136 = 0$$

$$x = \frac{-(-288) \pm \sqrt{(-288)^2 - 4(1)(1136)}}{2(1)} = \frac{288 \pm \sqrt{78400}}{2}$$

$$x = \frac{288 \pm 280}{2}$$

$$x = \frac{288+280}{2}, \qquad x = \frac{288-280}{2}$$

$$x = \frac{\cancel{568}^{284}}{\cancel{2}} = 284, \qquad x = \frac{8}{2} = 4.$$

Put in ①

L.H.S $\sqrt{2x+8} + \sqrt{x+5}$

$= \sqrt{2(284)+8} + \sqrt{284+5}$

$= 24 + 17 = 41$

R.H.S 7

L.H.S ≠ R.H.S

Put in ①

L.H.S $\sqrt{2x+8} + \sqrt{x+5}$

$= \sqrt{2(4)+8} + \sqrt{4+5}$

$= 4 + 3 = 7$ = R.H.S.

L.H.S = R.H.S.

We solve only $x = 4$

So

$$S.S = \{4\}.$$

④ $\sqrt{3x+4} = 2+\sqrt{2x-4}$ — ①

$$\left(\sqrt{3x+4}\right)^2 = \left(2+\sqrt{2x-4}\right)^2$$

$$3x+4 = 4+2x-4+4\sqrt{2x-4}$$

$$3x+4-2x = 4\sqrt{2x-4}$$

$$(x+4)^2 = \left(4\sqrt{2x-4}\right)^2$$

$$x^2+16+8x = 16(2x-4)$$

$$x^2+16+8x = 32x-64$$

$$x^2+16+8x-32x+64 = 0$$

$$x^2-24x+80 = 0$$

$$x^2-20x-4x+80 = 0$$

$$x(x-20)-4(x-20) = 0$$

$$(x-4)(x-20) = 0$$

$x-4=0$,

$x=4$ ✓

$x-20=0$

$x=20.$ ✓

Put in ①

L.H.S $\sqrt{3x+4}$

$= \sqrt{3(4)+4} = 4$

R.H.S $2+\sqrt{2x-4} = 2+\sqrt{2(4)-4}$

$= 2+\sqrt{4} = 2+2 = 4$

L.H.S = R.H.S.

Put in ①

L.H.S $\sqrt{3x+4}$

$= \sqrt{3(20)+4} = 8$

R.H.S $2+\sqrt{2x-4}$

$= 2+\sqrt{2(20)-4} = 2+\sqrt{36}$

$= 2+6 = 8.$

L.H.S = R.H.S.

S.S. = $\{4, 20\}$.

(5) $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$ — (1)

$$\left(\sqrt{x+7}+\sqrt{x+2}\right)^2=\left(\sqrt{6x+13}\right)^2$$

$$x+7+x+2+2\sqrt{x+7}\sqrt{x+2}=6x+13$$

$$2x+9+2\sqrt{x^2+2x+7x+14}=6x+13$$

$$2\sqrt{x^2+9x+14}=6x+13-2x-9$$

$$2\sqrt{x^2+9x+14}=4x+4$$

Divide by 2

$$\left(\sqrt{x^2+9x+14}\right)^2=(2x+2)^2$$

$$x^2+9x+14=4x^2+4+8x$$

$$x^2+9x+14-4x^2-4-8x=0$$

$$-3x^2+x+10=0$$

$$3x^2-x-10=0$$

$$3x^2-6x+5x-10=0$$

$$3x(x-2)+5(x-2)=0$$

$$(3x+5)(x-2)=0$$

$3x+5=0$

$x=-\frac{5}{3}$

Put in (1)

L.H.S $\sqrt{x+7}+\sqrt{x+2}$

$=\sqrt{-\frac{5}{3}+7}+\sqrt{-\frac{5}{3}+2}=\sqrt{\frac{-3+21}{3}}+\sqrt{\frac{-5+6}{3}}$

$=\sqrt{\frac{18}{3}}+\sqrt{\frac{1}{3}}=\sqrt{6}+\sqrt{\frac{1}{3}}=\sqrt{2}\sqrt{3}+\frac{1}{\sqrt{3}}$

R.H.S $\sqrt{6x+13}=\sqrt{6\left(-\frac{5}{3}\right)+13}=\sqrt{3}$

L.H.S ≠ R.H.S

$x-2=0$

$x=2$. ✓

Put in (1)

L.H.S $\sqrt{x+7}+\sqrt{x+2}$

$=\sqrt{2+7}+\sqrt{2+2}$

$=3+2=5$

R.H.S $\sqrt{6x+13}$

$=\sqrt{12+13}=5$

L.H.S = R.H.S.

So S.S. $=\{2\}$.