

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

Ex 1.2

(i)

$$f(x) = 2x + 1$$

$$g(x) = \frac{3}{x-1}, \quad x \neq 1$$

$$f(x) = 2x + 1, \quad g(x) = \frac{3}{x-1}, \quad x \neq 1$$

$$\begin{aligned} f \circ g(x) &= f(g(x)) = 2g(x) + 1 \\ &= 2\left(\frac{3}{x-1}\right) + 1 \\ &= \frac{6+x-1}{x-1} = \frac{5+x}{x-1} \end{aligned}$$

$$\begin{aligned} g \circ f(x) &= g(f(x)) = \frac{3}{f(x)-1} \\ &= \frac{3}{2x+1-1} = \frac{3}{2x} \end{aligned}$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

Ex 1.2

(i)

$$f(x) = 2x + 1$$

$$g(x) = \frac{3}{x-1}, \quad x \neq 1$$

$$\begin{aligned} f \circ f(x) &= f(f(x)) = 2f(x) + 1 \\ &= 2(2x + 1) + 1 \\ &= 4x + 2 + 1 = 4x + 3 \end{aligned}$$

$$\begin{aligned} g \circ g(x) &= g(g(x)) = \frac{3}{g(x) - 1} \\ &= \frac{3}{\frac{3}{x-1} - 1} = \frac{3}{\left(\frac{3-x+1}{x-1}\right)} \\ &= \frac{3(x-1)}{4-x} \end{aligned}$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

Ex 1.2

(ii)

$$f(x) = \sqrt{x+1}$$

$$g(x) = \frac{1}{x^2}, \quad x \neq 0$$

$$f(x) = \sqrt{x+1}, \quad g(x) = \frac{1}{x^2}, \quad x \neq 0$$

$$f \circ g(x) = f(g(x)) = \sqrt{g(x)+1}$$

$$= \sqrt{\frac{1}{x^2} + 1} = \sqrt{\frac{1+x^2}{x^2}}$$

$$= \frac{\sqrt{1+x^2}}{x}$$

$$g \circ f(x) = g(f(x)) = \frac{1}{f(x)^2} = \frac{1}{(\sqrt{x+1})^2}$$

$$= \frac{1}{x+1}$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

Ex 1.2

(ii)

$$f(x) = \sqrt{x+1}$$

$$g(x) = \frac{1}{x^2}, \quad x \neq 0$$

$$\begin{aligned} f \circ f(x) &= f(f(x)) = \sqrt{f(x)+1} \\ &= \sqrt{\sqrt{x+1}+1} \end{aligned}$$

$$\begin{aligned} g \circ g(x) &= g(g(x)) = \frac{1}{g(x)^2} \\ &= \frac{1}{\left(\frac{1}{x^2}\right)^2} = \frac{1}{\left(\frac{1}{x^4}\right)} \\ &= \frac{1 \cdot x^4}{1} = x^4 \end{aligned}$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

Chapter 1
FUNCTIONS AND
LIMITS



Ex 1.2

(iii)

$$f(x) = \frac{1}{\sqrt{x-1}}, \quad x \neq 1$$

$$g(x) = (x^2 + 1)^2$$

$$f(x) = \frac{1}{\sqrt{x-1}}, \quad x \neq 1, \quad g(x) = (x^2 + 1)^2$$

$$\begin{aligned} f \circ g(x) &= f(g(x)) = \frac{1}{\sqrt{g(x)-1}} = \frac{1}{\sqrt{(x^2+1)^2-1}} \\ &= \frac{1}{\sqrt{x^4+2x^2+\cancel{1}-\cancel{1}}} = \frac{1}{\sqrt{x^2(x^2+2)}} \\ &= \frac{1}{x\sqrt{x^2+2}} \end{aligned}$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

Ex 1.2

(iii)

$$f(x) = \frac{1}{\sqrt{x-1}}, \quad x \neq 1$$

$$g(x) = (x^2 + 1)^2$$

$$\begin{aligned} g \circ f(x) &= g(f(x)) = (f(x) + 1)^2 \\ &= \left[\left(\frac{1}{\sqrt{x-1}} \right) + 1 \right]^2 \\ &= \left[\frac{1}{x-1} + 1 \right]^2 \\ &= \left(\frac{\cancel{1} + x - \cancel{1}}{x-1} \right)^2 \\ &= \left(\frac{x}{x-1} \right)^2 = \frac{x^2}{(x-1)^2} \\ &= \frac{x^2}{x^2 - 2x + 1} \end{aligned}$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

Ex 1.2

(iii)

$$f(x) = \frac{1}{\sqrt{x-1}}, \quad x \neq 1$$

$$g(x) = (x^2 + 1)^2$$

$$\begin{aligned} f \circ f(x) &= f(f(x)) = \frac{1}{\sqrt{f(x)-1}} \\ &= \frac{1}{\sqrt{\frac{1}{\sqrt{x-1}} - 1}} \\ &= \sqrt{\frac{1}{\left(\frac{1-\sqrt{x-1}}{\sqrt{x-1}}\right)}} = \sqrt{\frac{\sqrt{x-1}}{1-\sqrt{x-1}}} \\ &= \frac{\sqrt{\sqrt{x-1}}}{\sqrt{1-\sqrt{x-1}}} \end{aligned}$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

Ex 1.2

$$\begin{aligned}g \circ g(x) &= g(g(x)) \\&= (g(x)^2 + 1)^2 \\&= [(x^2 + 1)^2 + 1]^2 \\&= [(x^2 + 1)^4 + 1]^2\end{aligned}$$

(iii)

$$f(x) = \frac{1}{\sqrt{x-1}}, \quad x \neq 1$$

$$g(x) = (x^2 + 1)^2$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

Ex 1.2

$$f(x) = 3x^4 - 2x^2, \quad g(x) = \frac{2}{\sqrt{x}}, \quad x \neq 0$$

$$f \circ g(x) = f(g(x)) = 3(g(x))^4 - 2(g(x))^2$$

$$= 3\left(\frac{2}{\sqrt{x}}\right)^4 - 2\left(\frac{2}{\sqrt{x}}\right)^2$$

$$= 3\left(\frac{2^4}{(x^{1/2})^4}\right) - 2\left(\frac{4}{x}\right)$$

$$= 3\left(\frac{16}{x^2}\right) - \frac{8}{x} = \frac{48}{x^2} - \frac{8}{x}$$

$$= \frac{48 - 8x}{x^2} = \frac{8(6-x)}{x^2}$$

(iv)

$$f(x) = 3x^4 - 2x^2$$

$$g(x) = \frac{2}{\sqrt{x}}, \quad x \neq 0$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

Ex 1.2

$$\begin{aligned}g \circ f(x) &= g(f(x)) = \frac{2}{\sqrt{f(x)}} \\&= \frac{2}{\sqrt{3x^4 - 2x^2}} \\&= \frac{2}{\sqrt{x^2(3x^2 - 2)}} \\&= \frac{2}{x\sqrt{3x^2 - 2}}.\end{aligned}$$

(iv)

$$f(x) = 3x^4 - 2x^2$$

$$g(x) = \frac{2}{\sqrt{x}}, \quad x \neq 0$$

1

The real valued functions f and g are defined below. Find

(a) $f \circ g(x)$

(b) $g \circ f(x)$

(c) $f \circ f(x)$

(d) $g \circ g(x)$

Ex 1.2

$$f \circ f(x) = f(f(x))$$

$$= 3f(x)^4 - 2f(x)^2$$

$$= 3[3x^4 - 2x^2]^4 - 2[3x^4 - 2x^2]^2$$

$$g \circ g(x) = g(g(x))$$

$$= \frac{2}{\sqrt{g(x)}} = \frac{2}{\sqrt{\frac{2}{\sqrt{x}}}} = \frac{2}{\left(\frac{\sqrt{2}}{\sqrt{\sqrt{x}}}\right)}$$

$$= \frac{2\sqrt{\sqrt{x}}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}\sqrt{\sqrt{x}}}{\cancel{2}} = \sqrt{2\sqrt{x}}$$

(iv)

$$f(x) = 3x^4 - 2x^2$$

$$g(x) = \frac{2}{\sqrt{x}}, \quad x \neq 0$$

2For the real valued function f defined below, find

(a) $f^{-1}(x)$

(b) $f^{-1}(-1)$

and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ **Ex 1.2**

$$f(x) = -2x + 8$$

Let $y = f(x) \Rightarrow x = f^{-1}(y)$ ✓

$$y = -2x + 8$$

$$2x = 8 - y$$

$$x = \frac{8 - y}{2}$$

$$f^{-1}(y) = \frac{8 - y}{2}$$

Interchange y with x ,

$$f^{-1}(x) = \frac{8 - x}{2}$$

(i)

$$f(x) = -2x + 8$$

2For the real valued function f defined below, find

(a) $f^{-1}(x)$

(b) $f^{-1}(-1)$

and verify

$f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Ex 1.2

$$f^{-1}(-1) = \frac{8 - (-1)}{2} = \frac{8 + 1}{2} = \frac{9}{2}$$

$$\begin{aligned} f(f^{-1}(x)) &= -2f^{-1}(x) + 8 = -2\left(\frac{8-x}{2}\right) + 8 \\ &= -8 + x + 8 = x \end{aligned}$$

$$\begin{aligned} f^{-1}(f(x)) &= \frac{8 - f(x)}{2} = \frac{8 - (-2x + 8)}{2} \\ &= \frac{8 + 2x - 8}{2} = \frac{2x}{2} = x \end{aligned}$$

Hence proved that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.**(i)**

$$f(x) = -2x + 8$$

2For the real valued function f defined below, find

(a) $f^{-1}(x)$

(b) $f^{-1}(-1)$

and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ **Ex 1.2**

$$f(x) = 3x^3 + 7$$

Let $y = f(x) \Rightarrow x = f^{-1}(y) \checkmark$

$$y = 3x^3 + 7$$

$$y - 7 = 3x^3$$

$$\left(\frac{y-7}{3}\right)^{1/3} = (x^3)^{1/3}$$

$$x = \left(\frac{y-7}{3}\right)^{1/3}$$

$$f^{-1}(y) = \left(\frac{y-7}{3}\right)^{1/3}$$

Interchange

 y with x

$$f^{-1}(x) = \left(\frac{x-7}{3}\right)^{1/3}$$

(ii)

$$f(x) = 3x^3 + 7$$

2For the real valued function f defined below, find

(a) $f^{-1}(x)$

(b) $f^{-1}(-1)$

and verify

$f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Ex 1.2

$$f^{-1}(-1) = \left(\frac{-1-7}{3}\right)^{1/3} = \left(-\frac{8}{3}\right)^{1/3}$$

$$\begin{aligned} f(f^{-1}(x)) &= 3[f^{-1}(x)]^3 + 7 \\ &= 3\left[\left(\frac{x-7}{3}\right)^{1/3}\right]^3 + 7 = \cancel{3}\left(\frac{x-7}{\cancel{3}}\right) + 7 \\ &= x - \cancel{7} + \cancel{7} = x \end{aligned}$$

$$\begin{aligned} f^{-1}(f(x)) &= \left(\frac{f(x)-7}{3}\right)^{1/3} = \left(\frac{3x^3 + \cancel{7} - \cancel{7}}{3}\right)^{1/3} = \left(\frac{\cancel{3}x^3}{\cancel{3}}\right)^{1/3} \\ &= (x^3)^{1/3} = x \end{aligned}$$

Hence $f(f^{-1}(x)) = f^{-1}(f(x)) = x.$

(ii)

$$f(x) = 3x^3 + 7$$

2For the real valued function f defined below, find

(a) $f^{-1}(x)$

(b) $f^{-1}(-1)$

and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ **Ex 1.2**

$$f(x) = (-x + 9)^3$$

Let

$$y = f(x) \Rightarrow x = f^{-1}(y) \checkmark$$

$$y = (-x + 9)^3$$

$$y^{1/3} = -x + 9$$

$$x = 9 - y^{1/3}$$

$$f^{-1}(y) = 9 - y^{1/3}$$

Replace

 y with x ,

$$f^{-1}(x) = 9 - x^{1/3}$$

(iii)

$$f(x) = (-x + 9)^3$$

2For the real valued function f defined below, find

(a) $f^{-1}(x)$

(b) $f^{-1}(-1)$

and verify

$f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Ex 1.2

$$f^{-1}(-1) = 9 - (-1)^{\frac{1}{3}}$$

$$f(f^{-1}(x)) = (-f^{-1}(x) + 9)^3$$

$$= \left[-\left(9 - x^{\frac{1}{3}}\right) + 9 \right]^3 = \left[-\cancel{9} + x^{\frac{1}{3}} + \cancel{9} \right]^3$$

$$= \left(x^{\frac{1}{3}} \right)^3 = x$$

$$f^{-1}(f(x)) = 9 - f(x)^{\frac{1}{3}} = 9 - \left[(-x + 9)^3 \right]^{\frac{1}{3}}$$

$$= \cancel{9} + x - \cancel{9} = x$$

Hence

$f(f^{-1}(x)) = f^{-1}(f(x)) = x.$

(iii)

$f(x) = (-x + 9)^3$

2For the real valued function f defined below, find

(a) $f^{-1}(x)$

(b) $f^{-1}(-1)$

and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ **Ex 1.2**

$$f(x) = \frac{2x+1}{x-1}, \quad x > 1$$

Let $y = f(x) \Rightarrow x = f^{-1}(y)$ ✓

$$y = \frac{2x+1}{x-1}$$

$$xy - y = 2x + 1$$

$$xy - 2x = y + 1$$

$$x(y-2) = y+1$$

$$x = \frac{y+1}{y-2}$$

$$f^{-1}(y) = \frac{y+1}{y-2}$$

Replace y with x ,

$$f^{-1}(x) = \frac{x+1}{x-2}$$

(iv)

$$f(x) = \frac{2x+1}{x-1}, \quad x > 1$$

2

For the real valued function f defined below, find

(a) $f^{-1}(x)$

(b) $f^{-1}(-1)$

and verify

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

Ex 1.2

$$\begin{aligned}
 f^{-1}(-1) &= \frac{-1+1}{-1-2} = \frac{0}{-3} = 0 \\
 f(f^{-1}(x)) &= \frac{2f^{-1}(x)+1}{f^{-1}(x)-1} \\
 &= \frac{2\left(\frac{x+1}{x-2}\right)+1}{\frac{x+1}{x-2}-1} = \frac{\left(\frac{2(x+1)+x-2}{x-2}\right)}{\left(\frac{x+1-x+2}{x-2}\right)} \\
 &= \frac{2x+2+x-2}{3} \\
 &= \frac{3x}{3} = x
 \end{aligned}$$

(iv)

$$f(x) = \frac{2x+1}{x-1}, \quad x > 1$$

$$\begin{aligned}
 f^{-1}(f(x)) &= \frac{f(x) + 1}{f(x) - 2} \\
 &= \frac{\frac{2x+1}{x-1} + 1}{\frac{2x+1}{x-1} - 2} = \frac{\left(\frac{2x+1+x-1}{x-1}\right)}{\left(\frac{2x+1-2x+2}{x-1}\right)} \\
 &= \frac{2x+x}{1+2} = \frac{3x}{3} = x
 \end{aligned}$$

Hence

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x.$$

(iv)

$$f(x) = \frac{2x + 1}{x - 1}, \quad x > 1$$

LECTURES BY

AKHTAR ABBAS

UNIVERSITY OF JHANG

03326297570

UCADEMY