Calculus and Analytic Geometry

MATHEMATICS 12

(Punjab Textbook Board)

SHORT TERM PREPARATION

IMPORTANT MCQs

IMPORTANT Short Questions

IMPORTANT Long Questions

EXERCISE WISE

M.SALMAN SHERAZI (03337727666/03067856232)

This document contains all the important MCQs, Short Questions and Long Questions of Mathematics HSSC-II (FSc Part 2) from the Calculus and Analytic Geometry, MATHEMATICS 12. It has been done to help the students and teachers at no cost by M Salman Sherazi.

EXERCISE 1.1

Tick (✔) the correct answer.

1. **The notation $y = f(x)$ was invented by**
   (a) Lebnitz (b) ✔ Euler (c) Newton (d) Lagrange
2. **If $f(x) = x^2 - 2x + 1$, then $f(0) =$**
   (a) -1 (b) 0 (c) ✔ 1 (d) 2
3. **When we say that f is function from set X to set Y, then X is called**
   (a) ✔Domain of f (b) Range of f (c) Codomain of f (d) None of these
4. **The term "Function" was recognized by______ to describe the dependence of one quantity to another.**
   (a) ✔Lebnitz (b) Euler (c) Newton (d) Lagrange
5. **If $f(x) = x^2$ then the range of f is**
   (a) ✔ [0,∞) (b) (-∞, 0] (c) $(0, \infty)$ (d) None of these
6. **If $f(x) = \frac{x}{x^2-4}$ then domain of f is**
   (a) R (b) $R - \{0\}$ (c) ✔ $R - \{\pm 2\}$ (d) Q
7. $Cosh^2x - Sinh^2x =$
   (a) -1 (b) 0 (c) ✔1 (d) None of these
8. $cosechx$ **is equal to**
   (a) $\frac{2}{e^x+e^{-x}}$ (b) $\frac{1}{e^x-e^{-x}}$ (c) ✔ $\frac{2}{e^x-e^{-x}}$ (d) $\frac{2}{e^{-x}+e^x}$
9. **The domain and range of identity function, $I: X \to X$ is**
   (a) ✔X (b) +iv real numbers (c) –iv real numbers (d) integers
10. **The linear function $f(x) = ax + b$ is identity function if**
    (a) $a \neq 0, b = 1$ (b) $a = 1, b = 0$ (c) $a = 1, b = 1$ (d) $a = 0$
11. **The linear function $f(x) = ax + b$ is constant function if**
    $a \neq 0, b = 1$ (b) $a = 1, b = 0$ (c) $a = 1, b = 1$ (d) ✔ $a = 0$

SHORT QUESTIONS

i. **Define Even and Odd function**
ii. **Find $f(-2)$ if $f(x) = \sqrt{x+4}$**
iii. **Express the perimeter P of square as a function of its area A.**
iv. **Show that $x = acos\theta$, $y = bsin\theta$ represent the equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$**
v. **Determine $f(x) = x^{\frac{2}{3}} + 6$ is even or odd.**
vi. **Express the volume V of a cube as a function of the area A of its base.**
vii. **Find $\frac{f(a+h)-f(a)}{h}$ and simplify $f(x) = cosx$**

LONG QUESTIONS

Given $f(x) = x^3 - ax^2 + bx + 1$ If $f(2) = -3$ and $f(-1) = 0$. Find a and b.

EXERCISE 1.2

Tick (✔) the correct answer.

1. **If $f(x) = 2x + 3$, $g(x) = x^2 - 1$, then $(gof)(x) =$**
   (a) $2x^2 - 1$ (b) ✔ $4x^2 + 4x$ (c) $4x + 3$ (d) $x^4 - 2x^2$
2. **If $f(x) = 2x + 3$, $g(x) = x^2 - 1$, then $(fof)(x) =$**
   (b) $2x^2 - 1$ (b) $4x^2 + 4x$ (c) ✔ $4x + 3$ (d) $x^4 - 2x^2$
3. **If $f(x) = 2x + 3$, $g(x) = x^2 - 1$, then $(gog)(x) =$**
   (c) $2x^2 - 1$ (b) $4x^2 + 4x$ (c) $4x + 3$ (d) ✔ $x^4 - 2x^2$

4. **The inverse of a function exists only if it is**
   (a) an into function (b) an onto function (c) ✔ (1-1) and into function (d) None of these

5. **If $f(x) = 2 + \sqrt{x-1}$, then domain of $f^{-1} =$**
   (a)]2,∞[(b) ✔ [2,∞[(c) [1,∞[(d)]1,∞[

SHORT QUESTIONS

i. **Define inverse of a function.**
ii. **Find $fof(x)$ and $gog(x)$ if $f(x) = \sqrt{x+1}$ and $g(x) = \frac{1}{x^2}, x \neq 0$**
iii. **Without finding the inverse, state the domain and range of $f(x) = (x-5)^2, x \geq 5$**
iv. **Let $f: R \to R$ be the function defined by $f(x) = 2x + 1$. Find $f^{-1}(x)$**

LONG QUESTIONS

For the real valued function, f defined below, find $f^{-1}(x)$ and verify

$$f\left(f^{-1}(x)\right) = (f^{-1}(f(x)) = x \text{ if } f(x) = -2x + 8$$

EXERCISE 1.3

Tick (✔) the correct answer.

1. $\lim_{x \to \infty} e^x =$
   (a) 1 (b) ∞ (c) ✔ 0 (d) -1
2. $\lim_{x \to 0} \frac{\sin(x-3)}{x-3} =$

(a) ✔1 (b) ∞ (c) $\frac{sin3}{3}$ (d) -3

3. $\lim\limits_{x\to 0}\frac{\sin(x-a)}{x-a}=$

(a) ✔1 (b) ∞ (c) $\frac{sina}{a}$ (d) -3

4. $f(x)=x^3+x$ **is :**

(a) Even (b) ✔ Odd (c) Neither even nor odd (d) None

5. $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=$

(a) ✔e (b) e^{-1} (c) 0 (d) 1

6. If $f: X\to Y$ is a function , then elements of x are called

(a) Images (b) ✔ Pre-Images (c) Constants (d) Ranges

7. $\lim\limits_{x\to 0}\left(\frac{x}{1+x}\right)=$

(a) e (b) ✔ e^{-1} (c) e^2 (d) $\sqrt{e}$

8. $\lim\limits_{x\to 0}\frac{a^x-1}{x}$ **is equal to**

(a) log_{e^x} (b)log_{a^x} (c) a (d) ✔ log_{e^a}

9. $\lim\limits_{x\to 0}\frac{Sinx^\circ}{x}=$

(a) ✔$\frac{\pi}{180^\circ}$ (b) $\frac{180^\circ}{\pi}$ (c) $180\ \pi$ (d) 1

SHORT QUESTIONS

i. **Show that** $\lim\limits_{x\to 0}\frac{a^x-1}{x}=\log_e a$

ii. **Evaluate** $\lim\limits_{x\to 0}\frac{Sin7x}{x}$

iii. **Evaluate** $\lim\limits_{n\to +\infty}\left(1+\frac{1}{n}\right)^{\frac{n}{2}}$

iv. **Evaluate** $\lim\limits_{\theta\to 0}\frac{1-cos\theta}{\theta}$

v. **Evaluate** $\lim\limits_{x\to 0}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, x>0$

vi. **Evaluate** $\lim\limits_{x\to 0}\frac{x^n-a^n}{x^m-a^m}$

vii. **Evaluate** $\lim\limits_{x\to\infty}\left(\frac{x}{1+x}\right)^x$

LONG QUESTIONS

Prove that $if\ \theta\ is\ measured\ in\ radian\ , then\ \lim\limits_{\theta\to 0}\frac{Sin\theta}{\theta}=1$

Evaluate $\lim\limits_{\theta\to 0}\frac{tan\theta-sin\theta}{sin^3\theta}$.

EXERCISE 1.4

Tick (✔) the correct answer.

1. **A function is said to be continuous at $x = c$ if**
   (a) $\lim_{x\to c} f(x)$ exists (b) $f(c)$is defined (c) $\lim_{x\to c} f(x) = f(c)$ (d) ✔ All of these
2. **The function $f(x) = \frac{x^2-1}{x-1}$ is discontinuous at**
   (a) ✔1 (b) 2 (c) 3 (d) 4
3. **L.H.L of $f(x) = |x-5|$ at $x = 5$ is**
   (a) 5 (b) ✔0 (c) 2 (d) 4

SHORT QUESTIONS

i. **Define the continuous function.**
ii. **Find L.H.L and R.H.L when $x \to c$ if $f(x) = 2x^2 + x - 5$, $c = 1$**
iii. **Discuss the continuity of the function at $x = 3$** $g(x) = \frac{x^2-9}{x-3}$ if $x \neq 3$
iv. **Discuss the continuity of $f(x)$ at $x = c$:** $f(x) = \begin{cases} 2x+5 \; if \; x \le 2 \\ 4x+1 \; if \; x > 2 \end{cases}, c = 2$
v. **Discuss the continuity of $f(x)$ at 3, when** $f(x) = \begin{cases} x-1, \; if \; x \le 3 \\ 2x+1 \; if \; 3 \le x \end{cases}$

LONG QUESTIONS

Find the values of m and n , so that given function f is continuous at $x = 3$

$$f(x) = \begin{cases} mx & if & x < 3 \\ n & if & x = 3 \\ -2x+9 & if & x > 3 \end{cases}$$

If $f(x) = \begin{cases} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} , & x \neq 2 \\ k , & x = 2 \end{cases}$ **Find the value of k so that f is continuous at $x = 2$.**

EXERCISE 2.1

Tick (✔) the correct answer.

1. **The change in variable x is called increment of x.It is denoted by δx which is**
   (a) +iv only (b) –iv only (c) ✔ +iv or –iv (d) none of these

2. **The notation $\frac{dy}{dx}$ or $\frac{df}{dx}$ is used by**
   (a) ✔Leibnitz (b) Newton (c)Lagrange (d) Cauchy
3. **The notation $\dot{f}(x)$ is used by**
   (a) Leibnitz (b) ✔ Newton (c) Lagrange (d) Cauchy

4. **The notation $f'(x)$ or y' is used by**
   (a) Leibnitz (b) Newton (c) ✔ Lagrange (d) Cauchy
5. **The notation $Df(x)$ or Dy is used by**
   (a) Leibnitz (b) Newton (c) Lagrange (d) ✔ Cauchy
6. $\lim\limits_{x\to a}\frac{f(x)-f(a)}{x-a} =$
   (a) ✔$f'(x)$ (b) $f'(a)$ (c) $f(0$ (d) $f(x-a)$
7. $\frac{d}{dx}(x^n) = nx^{n-1}$ **is called**
   (a) ✔ Power rule (b) Product rule (c) Quotient rule (d) Constant
8. **The derivative of a constant function is**
   (a) one (b) ✔ zero (c) undefined (d) None of these
9. **The process of finding derivatives is called**
   (a) ✔Differentiation (b) differential (c) Increment (d) Integration
10. **The increment of y is denoted by**
   (a) ✔δy (b) dy (c) $f'(y)$ (d) None of these

SHORT QUESTIONS

i. **Find the derivative of the given function by definition $f(x) = x^2$**
ii. **Find the derivative of the given function by definition $f(x) = \frac{1}{\sqrt{x}}$**

LONG QUESTIONS

Find by definition , the derivative $w.r.t$ 'x' $f(x) = x^n$ where $n \in Z$

EXERCISE 2.2

Tick (✔) the correct answer.

The derivative of $\frac{1}{(ax+b)^n}$ is

(a) $-n(ax+b)^{n-1}$ (b) $na(ax+b)^{-n+1}$ (c) ✔ $\frac{-na}{(ax+b)^{n+1}}$ (d) $\frac{-na}{(ax+b)^{n-1}}$

LONG QUESTIONS

Find from first Principles , the derivative $w.r.t$ 'x' $(ax+b)^3$

Find from first principles the derivative of $\frac{1}{(az-b)^7}$.

EXERCISE 2.3

Tick (✔) the correct answer.

1. $\frac{d}{dx}[f(x)+g(x)] =$
   (a) ✔ $f'(x)+g'(x)$ (b) $f'(x)-g'(x)$ (c) $f(x)g'(x)+g(x)f'(x)$ (d) $f(x)g'(x)-g(x)f'(x)$
2. $[f(x)g(x)]'$= **Remember that** $[f(x)g(x)]'$=$\frac{d}{dx}[f(x)g(x)]$
   (a) $f'(x)+g'(x)$ (b) $f'(x)-g'(x)$ (c) ✔ $f(x)g'(x)+g(x)f'(x)$ (d) $f(x)g'(x)-g(x)f'(x)$
3. $\frac{d}{dx}\left(\frac{1}{g(x)}\right) =$
   (a) $\frac{1}{[g(x)]^2}$ (b) $\frac{1}{g'(x)}$ (c) $\frac{g'(x)}{[g(x)]^2}$ (d) ✔ $\frac{-g'(x)}{[g(x)]^2}$
4. If $f(x) = \frac{1}{x}$, *then* $f''(a) =$
   (a) $-\frac{2}{(a)^3}$ (b) $-\frac{1}{a^2}$ (c) $\frac{1}{a^2}$ (d) ✔ $\frac{2}{a^3}$
5. $(fog)'(x) =$
   (a) $f'g'$ (b) $f'g(x)$ (c) ✔ $f'(g(x))g'(x)$ (d) cannot be calculated

6. $\frac{d}{dx}(g(x))^n =$
   (a) $n[g(x)]^{n-1}$ (b) $n[(g(x)]^{n-1}g(x)$ (c) ✔ $n[(g(x)]^{n-1}g'(x)$ (d) $[g(x)]^{n-1}g'(x)$

11. $\frac{d}{dx}\left(3x^{\frac{4}{3}}\right) =$
   (a) $4x^{\frac{2}{3}}$ (b) ✔ $4x^{\frac{1}{3}}$ (c) $2x^{\frac{1}{3}}$ (d) $3x^{\frac{1}{3}}$
12. If $\sqrt{x}-\frac{1}{\sqrt{x}}$ then $2x\frac{dy}{dx}+y =$
   (a) $2x$ (b) $2x^3$ (c) ✔ $2\sqrt{x}$ (d) $2x^2$

SHORT QUESTIONS

i. **Find the derivative of** $y = (2\sqrt{x}+2)(x-\sqrt{x})$ $w.r.t\ 'x'$
ii. **Differentiate** $\frac{2x^3-3x^2+5}{x^2+1}$ $w.r.t\ 'x'$
iii. **If** x^4+2x^2+2, **Prove that** $\frac{dy}{dx} = 4x\sqrt{y-1}$
iv. **Differentiate** $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$ $w.r.t\ 'x'$.
v. **Differentiate** $(x-5)(3-x)$

LONG QUESTIONS

Differentiate $\sqrt{\frac{a-x}{a+x}}$ $w.r.t\ 'x'$.

Find $\frac{dy}{dx}$ if $y = \frac{(1+\sqrt{x})(x-x^{\frac{3}{2}})}{\sqrt{x}}$

EXERCISE 2.4

Tick (✔) the correct answer.

1. **The derivative of $(x^3+1)^9$ $w.r.t$ 'x' is**
   (a) ✔$27x^2(x^3+1)^8$ (b) $27x(x^3+1)^8$ (c) $27(x^3+1)^8$ (d) $(x+1)^8$
2. **If $x = at^2$ and $y = 2at$ then $\frac{dy}{dx} =$**
   (a) $\frac{2}{ya}$ (b) $\frac{y}{2a}$ (c) ✔ $\frac{2a}{y}$ (d) $\frac{2}{y}$

SHORT QUESTIONS

i. **Find $\frac{dy}{dx}$ if $x = \theta + \frac{1}{\theta}$, $y = \theta + 1$**
ii. **Find $\frac{dy}{dx}$ by making some suitable substitution if $y = \sqrt{x+\sqrt{x}}$**
iii. **Differentiate $x^2 + \frac{1}{x^2}$ $w.r.t$ $x - \frac{1}{x}$**
iv. **Find $\frac{dy}{dx}$ if $y^2 - xy - x^2 + 4 = 0$**
v. **Find $\frac{dy}{dx}$ if $x^2 + y^2 = 4$**
vi. **Find $\frac{dy}{dx}$ if $y = x^n$ where $n = \frac{p}{q}, q \neq 0$**
vii. **If $y = (ax+b)^n$ where n is negative integer , find $\frac{dy}{dx}$ using quotient theorem.**
viii. **Find $\frac{dy}{dx}$ if $xy + y^2 = 2$**
ix. **Differentiate $(1+x^2)$ $w.r.t$ x^2**
x. **Find $\frac{dy}{dx}$ if $3x + 4y + 7 = 0$**

LONG QUESTIONS

Prove that $y\frac{dy}{dx} + x = 0$ if $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$

Differentiate $\frac{ax+b}{cx+d}$ $w.r.t$ $\frac{ax^2+b}{ax^2+d}$

EXERCISE 2.5

Tick (✔) the correct answer.

1. $\frac{d}{dx}(tan^{-1}x - cot^{-1}x) =$
   (a) $\frac{2}{\sqrt{1+x^2}}$ (b) ✔ $\frac{2}{1+x^2}$ (c) 0 (d) $\frac{-2}{1+x^2}$

2. **If $Sin\sqrt{x}$, then $\frac{dy}{dx}$ is equal to**
   (a) ✔ $\frac{cos\sqrt{x}}{2\sqrt{x}}$ (b) $\frac{cos\sqrt{x}}{\sqrt{x}}$ (c) $cos\sqrt{x}$ (d) $\frac{cosx}{\sqrt{x}}$
3. $\frac{d}{dx}sec^{-1}x =$
   (a) ✔ $\frac{1}{|x|\sqrt{x^2-1}}$ (b) $\frac{-1}{|x|\sqrt{x^2-1}}$ (c) $\frac{1}{|x|\sqrt{1+x^2}}$ (d) $\frac{-1}{|x|\sqrt{1+x^2}}$
4. $\frac{d}{dx}cosec^{-1}x =$
   (a) $\frac{1}{|x|\sqrt{x^2-1}}$ (b) ✔ $\frac{-1}{|x|\sqrt{x^2-1}}$ (c) $\frac{1}{|x|\sqrt{1+x^2}}$ (d) $\frac{-1}{|x|\sqrt{1+x^2}}$
5. **Differentiating sin^3x $w.r.t$ cos^2x is**
   (a) ✔ $-\frac{3}{2}sinx$ (b) $\frac{3}{2}sinx$ (c) $\frac{2}{3}cosx$ (d) $-\frac{2}{3}cosx$
6. **If $\frac{y}{x} = Tan^{-1}\frac{x}{y}$ then $\frac{dy}{dx} =$**
   (a) $\frac{x}{y}$ (b) $-\frac{x}{y}$ (c) ✔ $\frac{y}{x}$ (d) $-\frac{y}{x}$
7. **If $tany(1 + tanx) = 1 - tanx$, show that $\frac{dy}{dx} =$**
   (a) 0 (b) 1 (c) ✔ -1 (d) 2
8. $\frac{d}{dx}(Sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$ **is valid for**
   (a) $0 < x < 1$ (b) $-1 < x < 0$ (c) ✔ $-1 < x < 1$ (d) None of these
9. **If $y = xsin^{-1}\left(\frac{x}{a}\right) + \sqrt{a^2 - x^2}$ then $\frac{dy}{dx} =$**
   (a) $Cos^{-1}\frac{x}{a}$ (b) $Sec^{-1}\frac{x}{a}$ (c) ✔ $Sin^{-1}\frac{x}{a}$ (d) $Tan^{-1}\frac{x}{a}$

SHORT QUESTIONS

i. **Find $\frac{dy}{dx}$ $if\ y = xcosy$**
ii. **Differentiate sin^2x $w.r.t$ cos^2x**
iii. **If $tany(1 + tanx) = 1 - tanx, show\ that \frac{dy}{dx} = -1$**
iv. **If $y = \sqrt{tanx + \sqrt{tanx + \sqrt{tanx} + \cdots\infty}}$, prove that $(2y - 1)\frac{dy}{dx} = sec^2x$**
v. **Differentiate $w.r.t$ the variable involved $tan^3\theta sec^2\theta$**
vi. **Differentiate sin^3x $w.r.t$ cos^2x**
vii. **If $y = tan(pTan^{-1}x), show\ that\ (1 + x^2)\,y_1 - p(1 + y^2) = 0$**

LONG QUESTIONS

Show that $\frac{dy}{dx} = \frac{y}{x}$ $if\ \frac{y}{x} = Tan^{-1}\frac{x}{y}$

Differentiate $cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Differentiate $\sqrt{tanx}$ from first principles.

If $x = acos^3\theta, y = bsin^3\theta$, show that $a\frac{dy}{dx} + btan\theta = 0$

EXERCISE 2.6

Tick (✔) the correct answer.

1. **If $y = e^{-ax}$, $then y \frac{dy}{dx} =$**
   (a) ✔ $-ae^{-2ax}$ (b) $-a^2e^{ax}$ (c) a^2e^{-2ax} (d) $-a^2e^{-2ax}$

2. $\frac{d}{dx}(10^{sinx}) =$
   (a) 10^{cosx} (b) ✔ $10^{sinx}.cosx.ln10$ (c) $10^{sinx}.ln10$ (d) $10^{cosx}.ln10$

3. **If $y = e^{ax}$ then $\frac{dy}{dx} =$**
   (a) $\frac{1}{e^x}$ (b) ✔ ae^{ax} (c) e^{ax} (d) $\frac{1}{a}e^{ax}$

4. $\frac{d}{dx}(a^x) =$
   (a) a^x (b) $e^x lna$ (c) ✔ $a^x.lna$ (d) $x^a.lna$

5. **The function $f(x) = a^x, a > 0, a \neq 0$, and x is any real number is called**
   (a) ✔ Exponential function (b) logarithmic function (c) algebraic function (d) composite function

6. **If $a > 0$, $a \neq 1$, and $x = a^y$ then the function defined by $y = loga^x$ $(x > 0)$ is called a logarithmic function with base**
   (a) 10 (b) e (c) ✔ a (d) x

7. $log_{a^a} =$
   (a) ✔ 1 (b) e (c) a^2 (d) not defined

8. $\frac{d}{dx} log_{a^x} =$
   (a) $\frac{1}{x}\log a$ (b) ✔ $\frac{1}{xlna}$ (c) $\frac{lnx}{xlnx}$ (d) $\frac{lna}{xlnx}$

9. $\frac{d}{dx} ln[f(x)] =$
   (a) $f'(x)$ (b) $ln f'(x)$ (c) ✔ $\frac{f'(x)}{f(x)}$ (d) $f(x).f'(x)$

10. **If $y = log\ 10^{(ax^2+bx+c)}$ then $\frac{dy}{dx} =$**
    (a) ✔ $\frac{1}{(ax^2+bx+c)\ln 10}$ (b) $\frac{2ax+b}{(ax^2+bx+c)}$ (c) $10^{ax^2+bx+c}\ ln10$ (d) $\frac{2ax+b}{(ax^2+bx+c)lna}$

11. $ln\ a^e =$
    (a) lna (b) ✔ $\frac{1}{lna}$ (c) $\frac{1}{\ln\ e}$ (d) $\ln e^e$

SHORT QUESTIONS

i. **Find $f'(x)$ if $f(x) = ln(e^x + e^{-x})$**
ii. **Find $f'(x)$ if $f(x) = e^x(1 + lnx)$**
iii. **Differentiate $(lnx)^x$ $w.r.t$ $'x'$**
iv. **Find $\frac{dy}{dx}$ if $y = a^{\sqrt{x}}$**
v. **Find $\frac{dy}{dx}$ if $y = 5e^{3x-4}$**
vi. **Find $\frac{dy}{dx}$ if $y = (x + 1)^x$**
vii. **Find $\frac{dy}{dx}$ if $y = xe^{sinx}$**

viii. Find $\frac{dy}{dx}$ if $y = (ln\ tanhx)$

ix. Find $\frac{dy}{dx}$ if $y = sinh^{-1}(\frac{x}{2})$

x. Find $\frac{dy}{dx}$ if $y = tanh^{-1}(sinx)$ $\quad , -\frac{\pi}{2} < x < \frac{\pi}{2}$

LONG QUESTIONS

Find $f'(x)$ if $f(x) = \sqrt{ln(e^{2x} + e^{-2x})}$

Find $\frac{dy}{dx}$ if $y = ln(x + \sqrt{x^2 + 1})$

Find $f'(x)$ if $f(x) = \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$

EXERCISE 2.7

Tick (✔) the correct answer.

1. If $y = e^{2x}$, then $y_4 =$
   (a) ✔ $16e^{2x}$ (b) $8e^{2x}$ (c) $4e^{2x}$ (d) $2e^{2x}$
2. If $f(x) = e^{2x}$, then $f'''(x) =$
   (a) $6e^{2x}$ (b) $\frac{1}{6}e^{2x}$ (c) ✔ $8e^{2x}$ (d) $\frac{1}{8}e^{2x}$
3. If $f(x) = x^3 + 2x + 9$ then $f''(x) =$
   (a) $3x^2 + 2$ (b) $3x^2$ (c) ✔ $6x$ (d) $2x$
4. If $cos(ax + b)$, then $y_2 =$
   (a) $a^2 \sin(ax + b)$ (b) $-a^2\sin(ax + b)$ (c) ✔ $-a^2\cos(ax + b)$ (d) $a^2\cos(ax + b)$
5. Fifth order derivative of $x^3 + 2x + 6$ is
   (a) $3x + 2$ (b) $3x$ (c) ✔ 0 (d) 6
6. If $y = x^7 + x^6 + x^5$ then $D^8(y) =$
   (a) $7!$ (b) $7!\,x$ (c) $7! + 6!$ (d) ✔ 0

SHORT QUESTIONS

i. If $y = Sin^{-1}\frac{x}{a}$, then show that $y_2 = x(a^2 - x^2)^{-\frac{3}{2}}$

ii. Find y_2 if $y = x^2 . e^{-x}$

iii. Find y_2 if $x = acos\theta, y = sin\theta$

iv. Find y_2 if $x^3 - y^3 = a^3$

v. Find the first four derivatives of $cos(ax + b)$

LONG QUESTIONS

If $y = acos(lnx) + bsin(lnx)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

If $y = e^x sinx$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

If $y = (cos^{-1}x)^2$, prove that $(1 - x^2)y_2 - xy_1 - 2 = 0$

EXERCISE 2.8

Tick (✔) the correct answer.

1. **$1 - x + x^2 - x^3 + x^4 + \cdots . + (-1)^n x^n + \cdots$ is the expansion of**
(a) $\frac{1}{1-x}$ (b) ✔ $\frac{1}{1+x}$ (c) $\frac{1}{\sqrt{1-x}}$ (d) $\frac{1}{\sqrt{1+x}}$
2. **$f(x) = f(0) + xf'(x) + \frac{x^2}{2!}f''(x) + \frac{x^3}{3!}f'''(x) + \cdots \ldots . + \frac{x^n}{n!}f^n(x) \ldots$ is called_____ series.**
(a) ✔Machlaurin's (b) Taylor's (c) Convergent (d) Divergent
3. **$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} +$............. is an expression of**
(a) e^x (b) $Sinx$ (c) ✔$Cosx$ (d) e^{-x}
4. **$a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots$ is**
(a) Maclaurin's series (b) Taylor Series (c) ✔ Power Series (d) Bionomial Series

SHORT QUESTIONS

i. **Apply Maclaurin's Series expansion to prove that $e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \cdots$**
ii. **Apply Maclaurin's Series expansion to prove that $e^x = 1 + x + \frac{x^2}{2!} + \cdots$**
iii. **State Taylor's series expansion.**
iv. **Expand $cosx$ by Maclaurin's series expansion.**

LONG QUESTIONS

Show that $2^{x+h} = 2x[1 + (ln2)h + (ln2)^2\frac{h^2}{2!} + (ln2)^3\frac{h^3}{3!} + \cdots$

Show that $cos(x + h) = cosx - hsinx + \frac{h^2}{2!}cosx + \frac{h^3}{3!}sinx + \cdots$ and evaluate $cos61°$

EXERCISE 2.9

Tick (✔) the correct answer.

1. **A function $f(x)$ is such that , at a point $x = c$, $f'(x) > 0$ at $x = c$, then f is said to be**
(a) ✔Increasing (b) decreasing (c) constant (d) 1-1 function
2. **A function $f(x)$ is such that , at a point $x = c$, $f'(x) < 0$ at $x = c$, then f is said to be**

(a) Increasing (b) ✔ decreasing (c) constant (d) 1-1 function

(b) **A function $f(x)$ is such that , at a point $x = c$, $f'(x) = 0$ at $x = c$, then f is said to be**

(a) Increasing (b) decreasing (c) ✔ constant (d) 1-1 function

3. **A stationary point is called ______ if it is either a maximum point or a minimum point**

(a) Stationary point (b) ✔ turning point (c) critical point (d) point of inflexion

4. **If $f'(c) = 0$ or $f'(c)$ is undefined , then the number c is called critical value and the corresponding point is called_______**

(a) Stationary point (b) turning point (c) ✔ critical point (d) point of inflexion

5. **If $f'(c)$ does not change before and after $x = c$, then this point is called_______**

(a) Stationary point (b) turning point (c) critical point (d) ✔ point of inflexion

6. **Let f be a differentiable function such that $f'(c) = 0$ then if $f'(x)$ changes sign from +iv to –iv i.e., before and after $x = c$, then it occurs relative ______ at $x = c$**

(a) ✔Maximum (b) minimum (c) point of inflexion (d) none

7. **Let f be a differentiable function such that $f'(c) = 0$ then if $f'(x)$ changes sign from -iv to +iv i.e., before and after $x = c$, then it occurs relative ______ at $x = c$**

(b) Maximum (b) ✔ minimum (c) point of inflexion (d) none

8. **Let f be a differentiable function such that $f'(c) = 0$ then if $f'(x)$ does not change sign i.e., before and after $x = c$, then it occurs ______ at $x = c$**

(c) Maximum (b) minimum (c) ✔point of inflexion (d) none

9. **Let f be differentiable function in neighborhood of c and $f'(c) = 0$ then $f(x)$ has relative maxima at c if**

(a) $f''(c) > 0$ (b) ✔ $f''(c) < 0$ (c) $f''(c) = 0$ (d) $f''(c) \neq 0$

10. **$y = x^x$ has the value**

(a) Minimum at $x = e$ (b) Maximum at $x = e$ (c) ✔ Minimum at $x = \frac{1}{e}$ (d) Maximum at $x = \frac{1}{e}$

SHORT QUESTIONS

i. **Define Increasing and decreasing functions.**

ii. **Determine the interval in which $f(x) = x^2 + 3x + 2; x \in [-4, 1]$**

iii. **Determine the interval in which $f(x) = Cosx$; $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$**

iv. **Find the extreme values of the function $f(x) = 3x^2 - 4x + 5$**

v. **Find the extreme values of the function $f(x) = 1 + x^3$**

LONG QUESTIONS

Show that $y = \frac{lnx}{x}$ has maximum value at $x = e$.

Show that $y = x^x$ has minimum value at $x = \frac{1}{e}$.

EXERCISE 2.10

LONG QUESTIONS

Divide 20 into two parts so that the sum of their squares will be minimum.

Find the dimensions of a rectangle of largest area having parimeter 120 centimeters.

Find the point on the curve $y = x^3 + 1$ that is closest to the point $(18, 1)$

EXERCISE 3.1

Tick (✔) the correct answer.

1. **If $y = f(x)$, then differential of y is**

(a) $dy = f'(x)$ (b) ✔ $dy = f'(x)dx$ (c) $dy = f(x)dx$ (d) $\frac{dy}{dx}$

2. **If $\int f(x)dx = \varphi(x) + c$,then $f(x)$ is called**

(a) Integral (b) differential (c) derivative (d) ✔ integrand

3. **Inverse of $\int \ldots . dx$ is:**

(a) ✔$\frac{d}{dx}$ (b) $\frac{dy}{dx}$ (c) $\frac{d}{dy}$ (d) $\frac{dx}{dy}$

4. **Differentials are used to find:**

(a) ✔Approximate value (b) exact value (c) Both (a) and (b) (d) None of these

5. **$xdy + ydx =$**

(a) $d(x + y)$ (b) ✔$d\left(\frac{x}{y}\right)$ (c) $d(x - y)$ (d) $d(xy)$

6. **If $dy = cosxdx$ then $\frac{dx}{dy} =$**

(a) $sinx$ (b) $cosx$ (c) $cscx$ (d) ✔ $secx$

7. **If $\int f(x)dx = \varphi(x) + c$,then $f(x)$ is called**

(b) Integral (b) differential (c) derivative (d) ✔ integrand

8. **If $y = f(x)$, then differential of y is**

(b) $dy = f'(x)$ (b) ✔ $dy = f'(x)dx$ (c) $dy = f(x)dx$ (d) $\frac{dy}{dx}$

9. **The inverse process of derivative is called:**

(a) Anti-derivative (b) ✔ Integration (c) Both (a) and (b) (d) None of these

SHORT QUESTIONS

i. **Find δy and dy if $y = x^2 + 2x$ when x changes from 2 to 1.8**

ii. **Use differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the following equations.**

(a) $xy + x = 4$ **(b) $xy - lnx = c$**

iii. **Find the approximate increase in the volume of a cube if the length of its each edge changes from 5 to 5.02**

iv. **Find the approximate increase in the area of a circular disc if its diameter is increased form 44cm to 44.4cm.**

v. Define integration.

LONG QUESTIONS

Use differentials, find the approximate value of $sin46°$.

Use differentials to approximate the values of $\sqrt[4]{17}$.

EXERCISE 3.2

Tick (✔) the correct answer.

1. **If $n \neq 1$, then $\int (ax+b)^n dx =$**
(a) $\frac{n(ax+b)^{n-1}}{a}+c$ (b) $\frac{n(ax+b)^{n+1}}{n}+c$ (c) $\frac{(ax+b)^{n-1}}{n+1}+c$ (d) ✔ $\frac{(ax+b)^{n+1}}{a(n+1)}+c$

2. **$\int \sin(ax+b)\,dx$=**
(a) ✔$\frac{-1}{a}\cos(ax+b)+c$ (b) $\frac{1}{a}\cos(ax+b)+c$ (c) $a\cos(ax+b)+c$ (d)$-a\cos(ax+b)+c$

3. **$\int e^{-\lambda x}dx =$**
(a) $\lambda e^{-\lambda x}+c$ (b) $-\lambda e^{-\lambda x}+c$ (c) $\frac{e^{-\lambda x}}{\lambda}+c$ (d) ✔ $\frac{e^{-\lambda}}{-\lambda}+c$

4. **$\int a^{\lambda x}dx =$**
(a) $\frac{a^{\lambda x}}{\lambda}$ (b) $\frac{a^{\lambda x}}{lna}$ (c) ✔ $\frac{a^{\lambda x}}{alna}$ (d) $a^{\lambda x}\lambda.lna$

5. **$\int [f(x)]^n f'(x)dx =$**
(a) $\frac{f^n(x)}{n}+c$ (b) $f(x)+c$ (c) ✔ $\frac{f^{n+1}(x)}{n+1}+c$ (d) $nf^{n+1}(x)+c$

6. **$\int \frac{f'(x)}{f(x)}dx =$**
(a) $f(x)+c$ (b) $f'(x)+c$ (c) ✔$\ln|x|+c$ (nd) $\ln|f'(x)|+c$

7. **$\int \frac{dx}{\sqrt{x+a}+\sqrt{x}}$ can be evaluated if**
(a) ✔$x>0, a>0$ (b) $x<0, a>0$ (c) $x<0, a<0$ (d) $x>0, a<0$

8. **$\int \frac{x}{\sqrt{x^2+3}}dx =$**
(a) ✔$\sqrt{x^2+3}+c$ (b) $-\sqrt{x^2+3}+c$ (c) $\frac{\sqrt{x^2+3}}{2}+c$ (d) $-\frac{1}{2}\sqrt{x^2+3}+c$

SHORT QUESTIONS

i.	**Evaluate**	$\int (\sqrt{x}+1)^2 dx$
ii.	**Evaluate**	$\int \frac{\sqrt{y}(y+1)}{y}dx$
iii.	**Evaluate**	$\int \frac{3-cos2x}{1+cos2x}dx$
iv.	**Evaluate**	$\int x\sqrt{x^2-1}\,dx$
v.	**Prove that**	$\int [f(x)^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1}+c\,, n \neq -1$
vi.	**Evaluate**	$\int \frac{(1+e^x)^3}{e^x}dx$
vii.	**Evaluate**	$\int (lnx)\times\frac{1}{x}dx$

viii.	**Evaluate**	$\int \frac{\sin x+\cos^3 x}{\cos^2 x \sin x}dx$
ix.	**Evaluate**	$\int \frac{1-x^2}{1+x^2}dx$
x.	**Evaluate**	$\int \frac{\cos 2x-1}{1+\cos 2x}dx$
xi.	**Evaluate**	$\int \sqrt{1-\cos 2x}\,dx$
xii.	**Evaluate**	$\int (a-2x)^{\frac{3}{2}}dx$

EXERCISE 3.3

Tick (✔) the correct answer.

1. $\int \frac{dx}{x\sqrt{x^2-1}} =$

(a) ✔ $Sec^{-1}x + c$ (b) $Tan^{-1}x + c$ (c) $Cot^{-1}x + c$ (d) $Sin^{-1}x + c$

2. $\int \frac{dx}{x\ln x} =$

(a) ✔ $\ln \ln x + c$ (b) $x + c$ (c) $\ln f'(x) + c$ (d) $f'(x)\ln f(x)$

3. **In $\int (x^2-a^2)^{\frac{1}{2}}dx$, the substitution is**

(a) $x = a\tan\theta$ (b) ✔ $x = a\sec\theta$ (c) $x = a\sin\theta$ (d) $x = 2a\sin\theta$

4. **The suitable substitution for $\int \sqrt{2ax-x^2}dx$ is:**

(a) $x-a = a\cos\theta$ (b) ✔ $x-a = a\sin\theta$ (c) $x+a = a\cos\theta$ (d) $x+a = a\sin\theta$

5. $\int \frac{x+2}{x+1}dx =$

(a) $\ln(x+1)+c$ (b) $\ln(x+1)-x+c$ (c) ✔ $x+\ln(x+1)+c$ (d) None

6. **The suitable substitution for $\int \sqrt{a^2+x^2}dx$ is:**

(b) ✔ $x = a\tan\theta$ (b) $x = a\sin\theta$ (c) $x = a\cos\theta$ (d)None of these

SHORT QUESTIONS

i.	**Evaluate**	$\int \frac{1}{x\ln x}dx$
ii.	**Evaluate**	$\int \frac{x^2}{4+x^2}dx$
iii.	**Evaluate**	$\int \frac{e^x}{e^x+3}dx$
iv.	**Evaluate**	$\int \frac{\sec^2 x}{\sqrt{\tan x}}dx$
v.	**Evaluate**	$\int \frac{\cos x}{\sin x \ln \sin x}dx$
vi.	**Evaluate**	$\int \frac{\sqrt{2}}{\sin x+\cos x}dx$
vii.	**Evaluate**	$\int \cos x\left(\frac{\ln \sin x}{\sin x}\right)dx$
viii.	**Evaluate**	$\int \frac{dx}{x(\ln 2x)^3}, (x>0)$
ix.	**Find**	$\int a^{x^2}.x\,dx, (a>0, a\neq 1)$
x.	**Evaluate**	$\int \frac{1}{(1+x^2)Tan^{-1}x}dx$

LONG QUESTIONS

Show that $\int \sqrt{a^2-x^2}dx = \frac{a^2}{2}Sin^{-1}\frac{x}{2} + \frac{x}{2}\sqrt{a^2-x^2} + c$

Show that $\int \frac{dx}{\sqrt{x^2-a^2}} = ln\left(x+\sqrt{x^2-a^2}\right) + c$

EXERCISE 3.4

Tick (✔) the correct answer.

1. $\int udv$ equals:

(a) $udu - \int vu$ (b) $uv + \int vdu$ (c) ✔ $uv - \int vdu$ (d) $udu + \int vdu$

2. $\int xcosxdx =$

(a) $sinx + cosx + c$ (b) $cosx - sinx + c$ (c) ✔ $xsinx + cosx + c$ (d) None

3. $\int \frac{e^{Tan^{-1}x}}{1+x^2}dx =$

(a) $e^{Tanx} + c$ (b) $\frac{1}{2}e^{Tan^{-1}x} + c$ (c) $x\,e^{Tan^{-1}x} + c$ (d) ✔ $e^{Tan^{-1}x} + c$

4. $\int e^x\left[\frac{1}{x} + lnx\right] =$

(a) $e^x\frac{1}{x} + c$ (b) $-e^x\frac{1}{x} + c$ (c) ✔ $e^x lnx + c$ (d) $-e^x lnx + c$

5. $\int e^x\left[\frac{1}{x} - \frac{1}{x^2}\right] =$

(a) ✔ $e^x\frac{1}{x} + c$ (b) $-e^x\frac{1}{x} + c$ (c) $e^x lnx + c$ (d) $-e^x\frac{1}{x^2} + c$

SHORT QUESTIONS

i. Evaluate $\int lnxdx$

ii. Evaluate $\int x^3 lnxdx$

iii. Evaluate $\int xTan^{-1}x\,dx$

iv. Evaluate $\int \frac{xSin^{-1}x}{\sqrt{1-x^2}}dx$

v. Evaluate $\int x^2e^{ax}dx$

vi. Evaluate $\int tan^4 x$

vii. Evaluate $\int e^{ax}\left[aSec^{-1}x + \frac{1}{x\sqrt{x^2-1}}\right]$

viii. Evaluate $\int \frac{e^{mTan^{-1}x}}{(1+x^2)}dx$

ix. Evaluate $\int e^x\left(\frac{1}{x} + lnx\right)dx$

x. Evaluate $\int \left(\frac{1-sinx}{1-cosx}\right)e^x dx$

LONG QUESTIONS

Evaluate $\int sin^4 xdx$

Find $\int e^{ax} cosbxdx$

Evaluate $\int \sqrt{4-5x^2}dx$

Show that $\int e^{ax} sinbx = \frac{1}{\sqrt{a^2+b^2}} e^{ax} \sin\left(bx - Tan^{-1}\frac{b}{a}\right) + c$

Evaluate $\int e^{2x} cos3xdx$

EXERCISE 3.5

Tick (✔) the correct answer.

1. $\int \frac{2a}{x^2-a^2} dx =$

(a) $\frac{x-a}{x+a} + c$ (b) ✔ $\ln\frac{x-a}{x+a} + c$ (c) $ln\frac{x+a}{x-a} + c$ (d) $\ln|x-a| + c$

SHORT QUESTIONS

i. **Evaluate** $\int \frac{2a}{a^2-x^2} dx$

ii. **Evaluate** $\int \frac{5x+8}{(x+3)(2x-1)} dx$

iii. **Evaluate** $\int \frac{(a-b)x}{(x-a)(x-b)} dx$

LONG QUESTIONS

Evaluate $\int \frac{x-2}{(x+1)(x^2+1)} dx$

Evaluate $\int \frac{2x^2}{(x-1)^2(2x+3)} dx$

EXERCISE 3.6

Tick (✔) the correct answer.

1. $\int_{\pi}^{-\pi} sinxdx =$

(a) ✔2 (b) -2 (c) 0 (d) -1

2. $\int_{-1}^{2} |x|dx =$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{5}{2}$ (d) ✔ $\frac{3}{2}$

3. $\int_{0}^{1} (4x+k)dx = 2 \; then \; k =$

(a) 8 (b) -4 (c) ✔ 0 (d) -2

4. $\int_{0}^{3} \frac{dx}{x^2+9} =$

(a) $\frac{\pi}{4}$ (b) ✔ $\frac{\pi}{12}$ (c) $\frac{\pi}{2}$ (d) None of these

5. $\int_{0}^{-\pi} sinxdx$ **equals to:**

(a) -2 (b) 0 (c) ✔ 2 (d) 1

6. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} costdt =$

(a) ✔$\frac{\sqrt{3}}{2}-\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}+\frac{1}{2}$ (c) $\frac{1}{2}-\frac{\sqrt{3}}{2}$ (d) None

7. $\int_a^a f(x) =$

(a) ✔0 (b) $\int_b^a f(x)dx$ (c) $\int_b^a f(x)dx$ (d) $\int_a^a f(x)dx$

8. $\int_0^2 2xdx$ **is equal to**

(a) 9 (b) 7 (c) ✔ 4 (d) 0

SHORT QUESTIONS

i. **Evaluate** $\int_0^3 \frac{dx}{x^2+9}$

ii. **Evaluate** $\int_1^2 \frac{x}{x^2+2}dx$

iii. **Evaluate** $\int_1^2 lnxdx$

iv. **Evaluate** $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} costdt$

v. **Evaluate** $\int_0^{\frac{\pi}{6}} xcosxdx$

vi. **Evaluate** $\int_0^2 (e^{\frac{x}{2}} - e^{-\frac{x}{2}})dx$

vii. **Evaluate** $\int_{-1}^5 |x-3|dx$

viii. **Evaluate** $\int_{-2}^1 \frac{1}{(2x-1)^2}dx$

ix. **Evaluate** $\int_2^3 \left(x-\frac{1}{x}\right)^2 dx$

x. **Evaluate** $\int_1^2 \left(x+\frac{1}{x}\right)^{\frac{1}{2}}\left(1-\frac{1}{x^2}\right)dx$

LONG QUESTIONS

Evaluate $\int_0^{\frac{\pi}{4}} (1+cos^2\theta)tan^2\theta\, d\theta$

Evaluate $\int_0^1 \frac{3x}{\sqrt{4-3x}}dx$

Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} cos^2\theta cot^2\theta d\theta$

EXERCISE 3.7

Tick (✔) the correct answer.

1. To determine the area under the curve by the use of integration , the idea was given by

(a) Newton (b) ✔ Archimedes (c) Leibnitz (d) Taylor

SHORT QUESTIONS

i. **Find the area bounded by the curve $y = x^3 + 3x^2$ and the $x - axis$.**

ii. **Find the area between the $x - axis$ and the curve $y^2 = 4 - x$ in the first quadrant from $x = 0$ to $x = 3$.**

iii. **Find the area bounded by cos function from $y = -\frac{\pi}{2}$ to $\frac{\pi}{2}$.**

iv. **Find the area between the $x - axis$ and the curve $y = cos\frac{1}{2}x$ form $-\pi$ to π.**

LONG QUESTIONS

Find the area between the curve $y = x(x-1)(x+1)$ and the $x - axis$.

Find the area between the $x - axis$ and the curve $y = \sqrt{2ax - x^2}$ when $a > 0$.

Find the area between bounded by $y = x(x^2 - 4)$ and the $x - axis$.

EXERCISE 3.8

Tick (✔) the correct answer.

1. **The order of the differential equation : $x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2 = 0$**

(a) 0 (b) 1 (c) ✔ 2 (d) more than 2

2. **The equation $y = x^2 - 2x + c$ represents (c being a parameter)**

(a) One parabola (b) ✔ family of parabolas (c) family of line (d) two parabolas

3. **Solution of the differential equation : $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$**

(a) ✔$y = \sin^{-1} x + c$ (b) $y = \cos^{-1} x + c$ (c) $y = \tan^{-1} x + c$ (d) None

4. **The general solution of differential equation $\frac{dy}{dx} = -\frac{y}{x}$ is**

(a) $\frac{x}{y} = c$ (b) $\frac{y}{x} = c$ (c) ✔ $xy = c$ (d) $x^2y^2 = c$

5. **Solution of differential equation $\frac{dv}{dt} = 2t - 7$ is :**

(a) $v = t^2 - 7t^3 + c$ (b) $v = t^2 + 7t + c$ (c) $v = t - \frac{7t^2}{2} + c$ (d) ✔ $v = t^2 - 7t + c$

6. **The solution of differential equation $\frac{dy}{dx} = sec^2 x$ is**

(a) $y = cosx + c$ (b) ✔$y = tanx + c$ (c) $y = sinx + c$ (d) $y = cotx + c$

SHORT QUESTIONS

i. Solve $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$

ii. Solve $\frac{1}{x}\frac{dy}{dx} - 2y = 0$

iii. Solve $\frac{dy}{dx} = \frac{3}{4}x^2 + x - 3$, if $y = 0$ and $x = 2$

iv. Solve $\frac{dy}{dx} = \frac{y}{x^2}$, $(y > 0)$

v. Solve $\frac{dy}{dx} = \frac{1-y}{y}$

vi. Solve $(e^x + e^{-x})\frac{dy}{dx} = e^x - e^{-x}$

vii. Solve $secx + tany\frac{dy}{dx} = 0$

viii. Solve $1 + cosx\, tany\frac{dy}{dx} = 0$

ix. Solve $\frac{dy}{dx} = -y$

x. Define "Differential Equation" and "Order of Differential Equation".

LONG QUESTIONS

Find the general solution of the equation $\frac{dy}{dx} - x = xy^2$. Also find the particular solution if $y = 1$ when $x = 0$.

Solve the differential equation $\frac{ds}{dt} + 2st = 0$. Also find the particular solution if $s = 4e$, when $t = 0$.

EXERCISE 4.1

Tick (✔) the correct answer.

1. **$If\ x < 0, y < 0$ then the point $P(x, y)$ lies in the quadrant**

(a) I (b) II (c) ✔ III (d) IV

2. **The point P in the plane that corresponds to the ordered pair (x, y) is called:**

(a) ✔ $graph\ of\ (x, y)$ (b) mid-point of x, y (c) $abscissa\ of\ x, y$ (d) ordinate of x, y

3. **If $x < 0$, $y > 0$ then the point $P(-x, -y)$ lies in the quadrant**

(a) I (b) II (c) III (d) ✔ IV

4. **The straight line which passes through one vertex and though the mid-point of the opposite side is called:**

(a) ✔Median (b) altitude (c) perpendicular bisector (d) normal

5. **The straight line which passes through one vertex and perpendicular to opposite side is called:**

(a) Median (b) ✔ altitude (c) perpendicular bisector (d) normal

6. **The point where the medians of a triangle intersect is called_________ of the triangle.**

(a) ✔Centroid (b) centre (c) orthocenter (d) circumference

7. **The point where the altitudes of a triangle intersect is called_________ of the triangle.**

(a) Centroid (b) centre (c) ✔ orthocenter (d) circumference

8. The centroid of a triangle divides each median in the ration of

(a) ✔2:1 (b) 1:2 (c) 1:1 (d) None of these

9. The point where the angle bisectors of a triangle intersect is called_________ of the triangle.

(a) Centroid (b) ✔in centre (c) orthocenter (d) circumference

SHORT QUESTIONS

i. **Show that the points $A(3,1), B(-2,-3)$ and $C(2,2)$ are vertices of an isosceles triangle.**
ii. **Find the mid-point of the line segment joining the vertices $A(-8,3), B(2,-1)$.**
iii. **Show that the vertices $A(-1,2)$, $B(7,5), C(2,-6)$ are vertices of a right triangle.**
iv. **Find the points trisecting the join of $A(-1,-4)$ and $B(6,2)$.**
v. **Find h such that $A(-1,h)$, $B(3,2), and\ C(7,3)$ are collinear.**
vi. **Describe the location in the plane of point $P(x,y)$ for whcih $x = y$.**
vii. **The point $C(-5,3)$ is the centre of a circle and $P(7,-2)$ lies on the circle. What is the radius of the circle?**
viii. **Find the point three-fifth of the way along the line segment from $A(-5,8)$ to $B(5,3)$.**

LONG QUESTIONS

Find h such that the quadrilateral with vertices $A(-3,0)$, $B(1,-2)$, $C(5,0)$ and $D(1,h)$ is parallelogram. Is it a square?

Show that the points $A(5,2), B(-2,3), C(-3,-4)$ and $D(4,-5)$ are the vertices of a $||^m$. Is the $||^m$ a square.

Find the points that divide the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ into four equal parts.

EXERCISE 4.2

SHORT QUESTIONS

i. **The two points P and O' are given in xy −coordinate system. Find the XY-coordinates of P referred to the translated axes $O'X$ and $O'Y$ if $P(-2,6)$ and $O'(-3,2)$.**
ii. **The xy-coordinate axes are translated through point O' whose coordinates are given in xy −coodinate system. The coordinates of P are given in the XY −coodinate system. Find the coordinates of P in xy-coordiante system if $P(-5,-3)$, $O'(-2,3)$.**
iii. **What are translated axes.**

iv. **What are rotated axes.**

LONG QUESTIONS

The xy −coordiante axes are rotated about the origin through the indicated angle. The new axes are $O'X$ and $O'Y$.Find the XY-cooridiantes of the point P with the given xy-coordinates if $P(15, 10)$ and $\theta = arctan\frac{1}{3}$

The xy −coordinte axes are rotated about the origin through the indicated angle and the new axes are OX and OY. Find the xy −coordinates of P and with the given XY-coordiantes if $P(-5, 3)$ and $\theta = 30°$

EXERCISE 4.3

Tick (✔) the correct answer.

1. **The two intercepts form of the equation of the straight line is**
(a) $y = mx + c$ (b) $y - y_1 = m(x - x_1)$ (c) ✔ $\frac{x}{a} + \frac{y}{b} = 1$ (d) $xcos\alpha + ycos\alpha = p$
2. **The Normal form of the equation of the straight line is**
(a) $y = mx + c$ (b) $y - y_1 = m(x - x_1)$ (c) $\frac{x}{a} + \frac{y}{b} = 1$ (d) ✔ $xcos\alpha + ycos\alpha = p$
3. **In the normal form $xcos\alpha + ycos\alpha = p$ the value of p is**
(a) ✔ Positive (b) Negative (c) positive or negative (d) Zero
4. **If α is the inclination of the line l then $\frac{x-x_1}{cos\alpha} = \frac{y-y_1}{sin\alpha} = r(say)$**
(a) Point-slope form (b) normal form (c) ✔ symmetric form (d) none of these
5. **The slope of the line $ax + by + c = 0$ is**
(a) $\frac{a}{b}$ (b) ✔ $-\frac{a}{b}$ (c) $\frac{b}{a}$ (d) $-\frac{b}{a}$
6. **The slope of the line perpendicular to $ax + by + c = 0$**
(a) $\frac{a}{b}$ (b) $-\frac{a}{b}$ (c) ✔ $\frac{b}{a}$ (d) $-\frac{b}{a}$
7. **The general equation of the straight line in two variables x and y is**
(a) ✔ $ax + by + c = 0$ (b) $ax^2 + by + c = 0$ (c) $ax + by^2 + c = 0$ (d) $ax^2 + by^2 + c = 0$
8. **The $x - intercept$ $4x + 6y = 12$ is**
(a) 4 (b) 6 (c) ✔ 3 (d) 2
9. **The lines $2x + y + 2 = 0$ and $6x + 3y - 8 = 0$ are**
(a) ✔ Parallel (b) perpendicular (c) neither (d) non coplanar
10. **The point $(-2, 4)$ lies ____ the line $2x + 5y - 3 = 0$**
(a) ✔ Above (b) below (c) on (d) none of these

SHORT QUESTIONS

i. **Show that the points $A(-3, 6), B(3, 2)$ and $C(6, 0)$ are collinear.**

ii. **Find an equation of the straight line if its slope is 2 and $y-axis$ is 5.**
iii. **Find the slope and inclination of the line joining the points $(-2,4)$; $(5,11)$**
iv. **Find k so that the line joining $A(7,3)$; $B(k,-6)$ and the line joining $C(-4,5)$; $D(-6,4)$ are perpendicular.**
v. **Find an equation of the line bisecting the I and III quadrants.**
vi. **Find an equation of the line for $x-intercept: -3$ and $y-intercept: 4$**
vii. **Find the distance from the point $P(6,-1)$ to the line $6x-4y+9=0$**
viii. **Find whether the given point $(5,8)$ lies above or below the line $2x-3y+6=0$**
ix. **Check whether the lines are concurrent or not.**
$$3x-4y-3=0;\ 5x+12y+1=0\ ;\ 32x+4y-17=0$$
x. **Transform the eqution $5x-12y+39=0$ to "Two-intercept form".**

LONG QUESTIONS

Find the distance between the line given parallel lines. Sketch the lines. Also find an equation of the parallel line lying midway between them.

$$3x-4y+3=0 \quad ; \quad 3x-4y+7=0$$

The points $A(-1,2), B(6,3)$ and $C(2,-4)$ are vertices of a triangle. Show that the line joining the midpoint D of AB and the midpoint E of AC is parallel to BC and $DE=\frac{1}{2}BC$.

EXERCISE 4.4

Tick (✔) the correct answer.

1. **If φ be an angle between two lines l_1 and l_2 when slopes m_1 and m_2, then angle from l_1 to l_2**

(a) $tan\varphi=\frac{m_1-m_2}{1+m_1m_2}$ (b) ✔ $tan\varphi=\frac{m_2-m_1}{1+m_2m_1}$ (c) $tan\varphi=\frac{m_1+m_2}{1+m_1m_2}$ (d) $tan\varphi=\frac{m_2+m_1}{1+m_1m_2}$

2. **If φ be an acute angle between two lines l_1 and l_2 when slopes m_1 and m_2, then acute angle from l_1 to l_2**

(a) $|tan\varphi=\frac{m_1-m_2}{1+m_1m_2}|$ (b) ✔ $|tan\varphi=\frac{m_2-m_1}{1+m_2m_1}|$ (c) $|tan\varphi=\frac{m_1+m_2}{1+m_1m_2}|$ (d) $|tan\varphi=\frac{m_2+m_1}{1+m_1m_2}|$

3. **Two lines l_1 and l_2 with slopes m_1 and m_2 are parallel if**

(a) ✔ $m_1-m_2=0$ (b) $m_1+m_2=0$ (c) $m_1m_2=0$ (d) $m_1m_2=-1$

4. **Two lines l_1 and l_2 with slopes m_1 and m_2 are perpendicular if**

(b) $m_1-m_2=0$ (b) $m_1+m_2=0$ (c) $m_1m_2=0$ (d) ✔ $m_1m_2=-1$

5. **The lines represented by $ax^2+2hxy+by^2=0$ are orthogonal if**

(a) $a-b=0$ (b) ✔ $a+b=0$ (c) $a+b>0$ (d) $a-b<0$

6. The lines lying in the same plane are called

(a) Collinear (b) ✔ coplanar (c) non-collinear (d) non-coplanar

7. The distance of the point $(3,7)$ from the $x-axis$ is

(a) ✔7 (b) -7 (c) 3 (d) -3

8. Two lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are parallel if

(a) ✔$\frac{a_1}{a_2}=\frac{b_1}{b_2}$ (b) $\frac{a_1}{b_1}=-\frac{a_2}{b_2}$ (c) $\frac{a_1}{c_1}=\frac{a_2}{c_2}$ (d) $\frac{b_1}{c_1}=\frac{b_2}{c_2}$

SHORT QUESTIONS

i. Find the point of intersection of the lines $x-2y+1=0$ and $2x-y+2=0$

ii. Find an equation of the line through the point $(2,-9)$ and the intersection of the lines $2x+5y-8=0$ and $3x-4y-6=0$.

iii. Determine the value of p such that the lines $2x-3y-1=0, 3x-y-5=0$ and $3x+py+8=0$ meet at a point.

iv. Find the angle measured from the line l_1 to the line l_2 where
l_1: Joining $(2,7)$ and $(7,10)$ l_2: Joining $(1,1)$ and $(-5,5)$

v. Express the given system of equations in matrix form

$$2x+3y+4=0; x-2y-3=0; 3x+y-8=0$$

vi. Find the angle from the line with slope $-\frac{7}{3}$ to the line with slope $\frac{5}{2}$.

LONG QUESTIONS

Find the interior angles of the triangle whose vertices are

$$A(6,1), B(2,7), C(-6,-7)$$

Find the area of the region bounded by the triangle whose sides are

$$7x-y-10=0 \quad ; 10x+y-41=0 \quad ; \quad 3x+2y+3=0$$

Find the interior angles of the quadrilateral whose vertices are

$A(5,2), B(-2,3), C(-3,-4)$ and $D(4,-5)$

EXERCISE 4.5

Tick (✔) the correct answer.

1. The equation $y^2-16=0$ represents two lines.

(a) ✔Parallel to $x-axis$ (b) Parallel $y-axis$ (c) not || to $x-axis$ (d) not || to $y-axis$

2. The perpendicular distance of the line $3x+4y+10=0$ from the origin is

(a) 0 (b) 1 (c) ✔2 (d) 3

3. The lines represented by $ax^2+2hxy+by^2=0$ are orthogonal if

(b) $a-b=0$ (b) ✔ $a+b=0$ (c) $a+b>0$ (d) $a-b<0$

4. Every homogenous equation of second degree $ax^2+bxy+by^2=0$ represents two straight lines

(a) ✔Through the origin (b) not through the origin (c) two || line (d) two ⊥ar lines

5. The equation $10x^2 - 23xy - 5y^2 = 0$ is homogeneous of degree

(a) 1 (b) ✔ 2 (c) 3 (d) more than 2

6. The equation $y^2 - 16 = 0$ represents two lines.

(a) ✔Parallel to $x - axis$ (b) Parallel $y - axis$ (c) not || to $x - axis$ (d) not || to $y - axis$

SHORT QUESTIONS

i. Find an equation of each of the lines represented by $20x^2 + 17xy - 24y^2 = 0$

ii. Define Homogenous equation.

iii. Write down the joint equation.

iv. Find a joint equation of the straight lines through the origin perpendicular to the lines represented by $x^2 + xy - 6y^2 = 0.$

v. Find measure of angle between the lines represented by $x^2 - xy - 6y^2 = 0.$

LONG QUESTIONS

Find the lines represented by $x^2 + 2xy\sec\alpha + y^2 = 0$ and also find measure of the angle between them.

Find a join equation of the lines through the origin and perpendicular to the lines: $x^2 - 2xy\tan\alpha - y^2 = 0$

Find a joint equation of the lines through the origin and perpendicular to the lines $ax^2 + 2hxy + by^2 = 0$

EXERCISE 5.1

Tick (✔) the correct answer.

1. (0,0) is satisfied by

(a) $x - y < 10$ (b) $2x + 5y > 10$ (c) ✔ $x - y \geq 13$ (d) None

2. The point where two boundary lines of a shaded region intersect is called _____ point.

(a) Boundary (b) ✔ corner (c) stationary (d) feasible

3. If $x > b$ then

(a) $-x > -b$ (b) $-x < b$ (c) $x < b$ (d) ✔ $-x < -b$

4. The symbols used for inequality are

(a) 1 (b) 2 (c) 3 (d) ✔4

5. A linear inequality contains at least _________ variables.

(a) ✔One (b) two (c) three (d) more than three

6. An inequality with one or two variables has ________ solutions.

(a) One (b) two (c) three (d) ✔infinitely many

7. $ax + by < c$ is not a linear inequality if

(a) ✔ $a = 0, b = 0$ (b) $a \neq 0, b \neq 0$ (c) $a = 0, b \neq 0$ (d) $a \neq 0, b = 0, c = 0$

8. The graph of corresponding linear equation of the linear inequality is a line called________

(a) ✔ Boundary line (b) horizontal line (c) vertical line (d) inclined line

9. The graph of a linear equation of the form $ax + by = c$ is a line which divides the whole plane into ______ disjoints parts.

(a) ✔ Two (b) four (c) more than four (d) infinitely many

10. The graph of the inequality $x \leq b$ is

(a) Upper half plane (b) lower half plane (c) ✔ left half plane (d) right half plane

11. The graph of the inequality $y \leq b$ is

(b) Upper half plane (b) ✔ lower half plane (c) left half plane (d) right half plane

SHORT QUESTIONS

i. **Define "Corner Point" or "Vertex".**

ii. **Graph the solution set of linear inequality $3x + 7y \geq 21$.**

iii. **Indicate the solution set of $3x + 7y \geq 21$; $x - y \leq 2$**

iv. **What is "Corresponding equation".**

v. **Graph the inequality $x + 2y < 6$.**

LONG QUESTIONS

Graph the following system of inequalities

$$2x + y \geq 2 ; x + 2y \leq 10 ; y \geq 0$$

Graph the following system of inequalities and find the corner points

$$x + y \leq 5 \; ; \; -2x + y \leq 0 ; y \geq 0$$

Graph the solution region of the following system of linear inequalities by shading

$$2x + 3y \leq 18 ; 2x + y \leq 10 ; \; -2x + y \leq 10$$

EXERCISE 5.2

Tick (✔) the correct answer.

1. The feasible solution which maximizes or minimizes the objective function is called

(a) Exact solution (b) ✔ optimal solution (c) final solution (d) objective function

2. Solution space consisting of all feasible solutions of system of linear in inequalities is called

(a) Feasible solution (b) Optimal solution (c) ✔ Feasible region (d) General solution

3. Corner point is also called

(a) Origin (b) Focus (c) ✔ Vertex (d) Test point

4. For feasible region:

(a) ✔ $x \geq 0, y \geq 0$ (b) $x \geq 0, y \leq 0$ (c) $x \leq 0, y \geq 0$ (d) $x \leq 0, y \leq 0$

5. $x = 0$ is in the solution of the inequality

(a) $x < 0$ (b) $x + 4 < 0$ (c) ✔ $2x + 3 > 0$ (d) $2x + 3 < 0$

6. Linear inequality $2x - 7y > 3$ is satisfied by the point

(a) (5,1) (b) (-5,-1) (c) (0,0) (d) ✔ (1,-1)

7. The non-negative constraints are also called

(a) ✔ Decision variable (b) Convex variable (c) Decision constraints (d) concave variable

8. If the line segment obtained by joining any two points of a region lies entirely within the region , then the region is called

(a) Feasible region (b) ✔ Convex region (c) Solution region (d) Concave region

SHORT QUESTIONS

i. **Graph the feasible region of** $x + y \le 5$; $-2x + y \le 0$ $x \ge 0; y \ge 0$

ii. **Graph the feasible region of** $5x + 7y \le 35; x - 2y \le 4$ $x \ge 0; y \ge 0$

iii. **What is "Convex".**

iv. **Define "Feasible region".**

v. **Graph the feasible region of** $2x - 3y \le 6; 2x + y \ge 2$ $x \ge 0; y \ge 0$

LONG QUESTIONS

Graph the feasible region and find the corner points of

$$2x + y \le 10; x + 4y \le 12; x + 2y \le 10 \quad x \ge 0; y \ge 0$$

Graph the feasible region and find the corner points of

$$2x + y \le 20 \; ; 8x + 15y \le 120 \; ; x + y \le 11 \quad x \ge 0 \; ; y \ge 0$$

EXERCISE 5.3

Tick (✔) the correct answer.

1. A function which is to be maximized or minimized is called:

(a) Linear function (b) ✔ Objective function (c) Feasible function (d) None of these

2. For optimal solution we evaluate the objective function at

(a) Origen (b) Vertex (c) ✔ Corner Points (d) Convex points

3. We find corner points at

(a) Origen (b) Vertex (c) ✔ Feasible region (d) Convex region

LONG QUESTIONS

Maximize $f(x, y) = x + 3y$ subject to constraints

$$2x + 5y \le 30 \; ; 5x + 4y \le 20 \quad x \ge 0 \; ; y \ge 0$$

Minimize $z = 3x + y$ subject to constraints

$$3x + 5y \geq 15 \ ; x + 6y \geq 9 \qquad x \geq 0\,; y \geq 0$$

Maximize $f(x, y) = 2x + 5y$ subject to constraints

$$2y - x \leq 8 \ ; \ x - y \leq 4 \qquad x \geq 0\,; y \geq 0$$

EXERCISE 6.1

Tick (✔) the correct answer.

1. **The locus of a revolving line with one end fixed and other end on the circumference of a circle of a circle is called:**
   (a) a sphere (b) a circle (c) ✔ a cone (d) a conic
2. **The set of points which are equal distance from a fixed point is called:**
   (a) ✔ Circle (b) Parabola (c) Ellipse (d) Hyperbola
3. **The circle whose radius is zero is called:**
   (a) Unit circle (b) ✔ point circle (c) circumcircle (d) in-circle
4. **The circle whose radius is 1 is called:**
   (a) ✔ Unit circle (b) point circle (c) circumcircle (d) in-circle

5. **The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents the circle with centre**
   (a) (g, f) (b) ✔ $(-g, -f)$ (c) $(-f, -g)$ (d) $(g, -f)$
6. **The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents the circle with centre**
   (a) ✔ $\sqrt{g^2 + f^2 - c}$ (b) $\sqrt{g^2 + f^2 + c}$ (c) $\sqrt{g^2 + c^2 - f}$ (d) $\sqrt{g + f - c}$

SHORT QUESTIONS

i. **Write the equation of the circle with centre $(-3, 5)$ and radius.**
ii. **Find the equation of the circle with ends of a diameter at $(-3, 2)$ and $(5, -6)$.**
iii. **Find the centre and radius of the circle of $x^2 + y^2 + 12x - 10y = 0$**

LONG QUESTIONS

Find an equation of the circle passing through $A(3, -1), B(0, 1)$ and having centre at $4x - 3y - 3 = 0$

Show that the circles $x^2 + y^2 + 2x - 8 = 0$ and $x^2 + y^2 - 6x + 6y - 46 = 0$ touch internally.

Find the equation of the circle of radius 2 and tangent to the line $x - y - 4 = 0$ at $A(1, -3)$

Show that the lines $3x - 2y = 0$ and $2x + 3y - 13 = 0$ are tangents to the circle $x^2 + y^2 + 6x - 4y = 0$

EXERCISE 6.2

LONG QUESTIONS

Find the length of the chord cut off from the line $2x + 3y = 13$ by the circle $x^2 + y^2 = 26$

Find the length of the tangent drawn from the point $(-5, 4)$ to the circle

$$5x^2 + 5y^2 - 10x + 15y - 131 = 0$$

Find an equation of the chord of contact of the tangents drawn from $(4, 5)$ to the circle $2x^2 + 2y^2 - 8x + 12y + 21 = 0$

EXERCISE 6.3

LONG QUESTIONS

Prove that length of a diameter of the circle $x^2 + y^2 = a^2$ is $2a$.

Prove that the midpoint of the hypotenuse of a right triangle is the circumference of the triangle.

The perpendicular at the outer end of a radial segment is tangent to the circle.

EXERCISE 6.4

Tick (✔) the correct answer.

1. **The ratio of the distance of a point from the focus to distance from the directrix is denoted by**

(a) ✔r (b) R (c) E (d) e

2. **Standard equation of Parabola is :**

(a) $y^2 = 4a$ (b) $x^2 + y^2 = a^2$ (c) ✔ $y^2 = 4ax$ (d) $S = vt$

3. **The focal chord is a chord which is passing through**

(a) ✔Vertex (b) Focus (c) Origin (d) None of these

4. **The curve $y^2 = 4ax$ is symmetric about**

(a) ✔$y - axis$ (b) $x - axis$ (c) Both (a) and (b) (d) None of these

5. **Latusrectum of $x^2 = -4ay$ is**

(a) $x = a$ (b) $x = -a$ (c) $y = a$ (d) ✔ $y = -a$

6. **Eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is**

(a) $\frac{a}{c}$ (b) ac (c) ✔ $\frac{c}{a}$ (d) None of these

7. **Focus of $y^2=-4ax$ is**

(a) $(0,a)$ (b) ✔ $(-a,0)$ (c) $(a,0)$ (d) $(0,-a)$

8. **A type of the conic that has eccentricity greater than 1 is**

(a) An ellipse (b) A parabola (c) ✔ A hyperbola (d) A circle

9. **$x^2+y^2=-5$ represents the**

(a) Real circle (b) ✔ Imaginary circle (c) Point circle (d) None of these

10. **Which one is related to circle**

(a) $e=1$ (b) $e>1$ (c) $e<1$ (d) ✔ $e=0$

11. **Circle is the special case of :**

(a) Parabola (b) Hyperbola (c) ✔ Ellipse (d) None of these

12. **Equation of the directrix of $x^2=-4ay$ is:**

(a) $x+a=0$ (b) $x-a=0$ (c) $y+a=0$ (d) ✔ $y-a=0$

SHORT QUESTIONS

- i. **Define Parabola.**
- ii. **Analyze the parabola $x^2=-16y$**
- iii. **The point of a parabola which is closest to the focus is the vertex of the parabola.**
- iv. **Find the focus , vertex and directrix of the parabola**
  $y^2=8x$, $x^2=4(y-1)$, $y^2=-8(x-3)$
- v. **Write an equation of the parabola with given elements**
  Focus $(-3,1)$; directrix $x=3$ **directrix $x=-2$, Focus $(2,2)$**
  Directrix $y=3$; vertex $(2,2)$

LONG QUESTIONS

Find an equation of the parabola having its focus at the origin and directrix parallel to the (i) $x-axis$ (ii) $y-axis$

Show that the ordinate at any point P of the parabola is a mean proportional between the length of the letusrectum and the abscissa of P.

EXERCISE 6.5

Tick (✔) the correct answer.

1. **The midpoint of the foci of the ellipse is its**

(a) Vertex (b) ✔ Centre (c) Directrix (d) None of these

2. **Focus of the ellipse always lies on the**

(a) Minor axis (b) ✔ Major axi (c) Directrix (d) None of these

3. **Length of the major axis of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$ is**

(a) ✔ $2a$ (b) $2b$ (c) $\frac{2b^2}{a}$ (d) None of these

4. **In the cases of ellipse it is always true that:**

(a) ✔ $a^2>b^2$ (b) $a^2<b^2$ (c) $a^2=b^2$ (d) $a<0, b<0$

5. **Two conics always intersect each other in ______ points**

(a) No (b) one (c) two (d) ✔ four

6. **The eccentricity of ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is**

(a) ✔ $\frac{\sqrt{7}}{4}$ (b) $\frac{7}{4}$ (c) 16 (d) 9

7. **The foci of an ellipse are $(4,1)$ and $(0,1)$ then its centre is:**

(a) $(4,2)$ (b) ✔ $(2,1)$ (c) $(2,0)$ (d) $(1,2)$

SHORT QUESTIONS

i. **Analyze the equation $4x^2+9y^2=36$**

ii. **Find the equation of the ellipse with given data :**
Foci $(\pm 3,0)$ and minor axis of length 10
Vertices $-1,1),(5,1)$; Foci $(4,1)$ and $(0,1)$
Centre $(0,0)$, focus $(0,-3)$, vertex $(0,4)$

iii. **Find the centre , foci , eccentricity , vertices and directrices of the ellipse whose equations are given : $9x^2+y^2=18$, $25x^2+9y^2=225$**

LONG QUESTIONS

Prove that the letusrectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$.

Let a be a positive number and $0<c<a$. Let $F(-c,0)$ and $F'(c,0)$ be two given points. Prove that the locus of points $P(x,y)$ such that

$|PF|+|PF'|=2a$, is an ellipse.

EXERCISE 6.6

Tick (✔) the correct answer.

1. **The foci of hyperbola always lie on :**

(a) $x-axis$ (b) ✔ $Transverse\ axis$ (c) $y-axis$ (d) Conjugate axis

2. **Length of transverse axis of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is**

(a) ✔ $2a$ (b) $2b$ (c) a (d) b

3. **$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is symmetric about the:**

(a) $y-axis$ (b) $x-axis$ (c) ✔ Both (a) and (b) (d) None of these

4. **If $c=\sqrt{65}$, $b=7$ and $a=4$ then the eccentricity of hyperbola is :**

(a) ✔ $\frac{\sqrt{65}}{4}$ (b) $\frac{65}{16}$ (c) $\frac{\sqrt{65}}{7}$ (d) $\frac{7}{4}$

SHORT QUESTIONS

i. **Define Hyperbola.**

ii. **Discuss $25x^2 - 16y^2 = 400$**

iii. **Find the equation of hyperbola with given data : Foci $(\pm 5, 0)$, vertex $(3, 0)$ Foci $(0, \pm 6), e = 2$, Foci $(5, -2), (5, 4)$ and one vertex $(5, 3)$**

iv. **Find the centre ,foci , eccentricity , vertices and directrix of $x^2 - y^2 = 9$ $\frac{y^2}{4} - x^2 = 1$, $\frac{y^2}{16} - \frac{x^2}{9} = 1$**

LONG QUESTIONS

For any point on the hyperbola the difference of its distances from the points $(2, 2)$ and $(10, 2)$ is 6. Find the equation of hyperbola

Let $0 < a < c$ and $F'(-c, 0), F(c, 0)$ be two fixed points . Show that th set of points $P(x, y)$ such that

$|PF| - |PF'| = \pm 2a$ is the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{c^2 - a^2} = 1$

EXERCISE 7.1

Tick (✔) the correct answer.

1. **Two vectors are said to be negative of each other if they have the same magnitude and _________direction.**

(a) Same (b) ✔ opposite (c) negative (d) parallel

2. **Parallelogram law of vector addition to describe the combined action of two forces, was used by**

(a) Cauchy (b) ✔ Aristotle (c) Alkhwarzmi (d) Leibnitz

3. **The vector whose initial point is at the origin and terminal point is P , is called**

(a) Null vector (b) unit vector (c) ✔position vector (d) normal vector

4. **If R be the set of real numbers, then the Cartesian plane is defined as**

(a) $R^2 = \{(x^2, y^2): x, y \in R\}$ (b) ✔ $R^2 = \{(x, y): x, y \in R\}$ (c) $R^2 = \{(x, y): x, y \in R, x = -y\}$ (d) $R^2 = \{(x, y): x, y \in R, x = y\}$

5. **The element $(x, y) \in R^2$ represents a**

(a) Space (b) ✔ point (c) vector (d) line

6. **If $\underline{u} = [x, y]$ in R^2, then $|\underline{u}|$ =?**

(a) $x^2 + y^2$ (b) ✔ $\sqrt{x^2 + y^2}$ (c) $\pm\sqrt{x^2 + y^2}$ (d) $x^2 - y^2$

7. **If $|\underline{u}| = \sqrt{x^2 + y^2} = 0$, then it must be true that**

(a) $x \geq 0, y \geq 0$ (b) $x \leq 0, y \leq 0$ (c) $x \geq 0, y \leq 0$ (d) ✔ $x = 0, y = 0$

8. **Each vector $[x, y]$in R^2 can be uniquely represented as**

(a) $x\underline{i} - y\underline{j}$ (b) ✔ $x\underline{i} + y\underline{j}$ (c) $x + y$ (d) $\sqrt{x^2 + y^2}$

9. **The lines joining the mid-points of any two sides of a triangle is always _____to the third side.**

(a) Equal (b) ✔ Parallel (c) perpendicular (d) base

SHORT QUESTIONS

i. **Write the vector $\overrightarrow{PQ}$ in the form of $x\underline{i} + y\underline{j}$ if $P(2, 3), Q(6, -2)$**

ii. **Find the sum of the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$, given the four points $A(1,-1), B(2,0), C(-1,3)$ and $D(-2,2)$**

iii. **Find the unit vector in the direction of vector given $\underline{v} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$**

iv. **If $\overrightarrow{AB} = \overrightarrow{CD}$. Find the coordinates of the points A when points B, C, D are $(1,2), (-2,5), (4,11)$ respectively.**

v. **If B, C and D are respectively $(4,1), (-2,3)$ and $(-8,0)$. Use vector method to find the coordinates of the point A if $ABCD$ is a parallelogram.**

vi. **Define Parallel vectors.**

LONG QUESTIONS

Prove that the line segment joining the mid points of two sides of a triangle is parallel to the third side and half as long.

Prove that the line segments joining the mid points of the sides of a quadrilateral taken in order form a parallelogram.

EXERCISE 7.2

Tick (✔) the correct answer.

1. **If $\underline{u} = 3\underline{i} - \underline{j} + 2\underline{k}$ then [3,-1,2] are called ____________ of $\underline{u}$.**

(a) Direction cosines (b) ✔ direction ratios (c) direction angles (d) elements

2. **Which of the following can be the direction angles of some vector**

(a) $45°, 45°, 60°$ (b) $30°, 45°, 60°$ (c) ✔$45°, 60°, 60°$ (d) obtuse

3. **Measure of angle θ between two vectors is always.**

(a) $0 < \theta < \pi$ (b) $0 \le \theta \le \frac{\pi}{2}$ (c) ✔$0 \le \theta \le \pi$ (d) obtuse

4. **If the dot product of two vectors is zero, then the vectors must be**

(a) Parallel (b) ✔ orthogonal (c) reciprocal (d) equal

5. **If the cross product of two vectors is zero, then the vectors must be**

(a) ✔ Parallel (b) orthogonal (c) reciprocal (d) Non coplanar

6. **If θ be the angle between two vectors $\underline{a}$ and $\underline{b}$, then $cos\theta =$**

(a) $\frac{\underline{a}\times\underline{b}}{|\underline{a}||\underline{b}|}$ (b) ✔ $\frac{\underline{a}.\underline{b}}{|\underline{a}||\underline{b}|}$ (c) $\frac{\underline{a}.\underline{b}}{|\underline{a}|}$ (d) $\frac{\underline{a}.\underline{b}}{|\underline{b}|}$

7. **If θ be the angle between two vectors $\underline{a}$ and $\underline{b}$, then projection of $\underline{b}$ along $\underline{a}$ is**

(a) $\frac{\underline{a}\times\underline{b}}{|\underline{a}||\underline{b}|}$ (b) $\frac{\underline{a}.\underline{b}}{|\underline{a}||\underline{b}|}$ (c) ✔ $\frac{\underline{a}.\underline{b}}{|\underline{a}|}$ (d) $\frac{\underline{a}.\underline{b}}{|\underline{b}|}$

8. **If θ be the angle between two vectors $\underline{a}$ and $\underline{b}$, then projection of $\underline{a}$ along $\underline{b}$ is**

(a) $\frac{\underline{a}\times\underline{b}}{|\underline{a}||\underline{b}|}$ (b) $\frac{\underline{a}.\underline{b}}{|\underline{a}||\underline{b}|}$ (c) $\frac{\underline{a}.\underline{b}}{|\underline{a}|}$ (d) ✔ $\frac{\underline{a}.\underline{b}}{|\underline{b}|}$

9. **Let $\underline{u} = a\underline{i} + b\underline{j} + c\underline{k}$ then projection of $\underline{u}$ along $\underline{i}$ is**

(a) ✔a (b) b (c) c (d) u

SHORT QUESTIONS

i. **Find α, so that $\left|\alpha\underline{i} + (\alpha+1)\underline{j} + 2\underline{k}\right| = 3$**

ii. **Find a vector whose magnitude is 4 and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$.**

iii. **Find a and b so that the vectors $3\underline{i} - \underline{j} + 4\underline{k}$ and $a\underline{i} + b\underline{j} - 2\underline{k}$ are parallel.**

iv. **Find the direction cosines for the given vector: $\underline{v} = 3\underline{i} - \underline{j} + 2\underline{k}$**

v. **Find Two vectors of length 2 parallel to the vector $\underline{v} = 2\underline{i} - 4\underline{j} + 4\underline{k}$.**

LONG QUESTIONS

The position vectors of the points A, B, C and D are $2\underline{i} - \underline{j} + \underline{k}$, $3\underline{i} + \underline{j}$, $2\underline{i} + 4\underline{j} - 2\underline{k}$ and $-\underline{i} - 2\underline{j} + \underline{k}$ respectively. Show that $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$.

EXERCISE 7.3

Tick (✔) the correct answer.

1. **In any ΔABC , the law of cosine is**

(a) ✔ $a^2 = b^2 + c^2 - 2bcCosA$ (b) $a = bCosC + cCosB$ (c) $a.b = 0$ (d) $a - b = 0$

2. **In any ΔABC , the law of projection is**

(a) $a^2 = b^2 + c^2 - 2bcCosA$ (b) ✔ $a = bCosC + cCosB$ (c) $a.b = 0$ (d) $a - b = 0$

3. **If $\underline{u}$ is a vector such that $\underline{u}.\underline{i} = 0, \underline{u}.\underline{j} = 0, \underline{u}.\underline{k} = 0$ then $\underline{u}$ is called**

(a) Unit vector (b) ✔ null vector (c) $[\underline{i}, \underline{j}, \underline{k}]$ (d) none of these

4. **Cross product or vector product is defined**

(a) In plane only (b) ✔ in space only (c) everywhere (d) in vector field

5. **If $\underline{u}$ and $\underline{v}$ are two vectors , then $\underline{u} \times \underline{v}$ is a vector**

(a) Parallel to $\underline{u}$and $\underline{v}$ (b) parallel to $\underline{u}$ (c) ✔ perpendicular to $\underline{u}$ and $\underline{v}$ (d) orthogonal to $\underline{u}$

6. **If $\underline{u}$ and $\underline{v}$ be any two vectors, along the adjacent sides of ||gram then the area of ||gram is**

(a) $\underline{u} \times \underline{v}$ (b) ✔ $|\underline{u} \times \underline{v}|$ (c) $\frac{1}{2}(\underline{u} \times \underline{v})$ (d) $\frac{1}{2}|\underline{u} \times \underline{v}|$

7. **If $\underline{u}$ and $\underline{v}$ be any two vectors, along the adjacent sides of triangle then the area of triangle is**

(a) $\underline{u} \times \underline{v}$ (b) $|\underline{u} \times \underline{v}|$ (c) $\frac{1}{2}(\underline{u} \times \underline{v})$ (d) ✔ $\frac{1}{2}|\underline{u} \times \underline{v}|$

8. **The scalar triple product of $\underline{a}$,$\underline{b}$ and $\underline{c}$ is denoted by**

(a) $\underline{a}.\underline{b}.\underline{c}$ (b) ✔ $\underline{a}.\underline{b} \times \underline{c}$ (c) $\underline{a} \times \underline{b} \times \underline{c}$ (d) $(\underline{a} + \underline{b}) \times \underline{c}$

SHORT QUESTIONS

i. **Calculate the projection of $\underline{a}$ along $\underline{b}$ if $\underline{a} = \underline{i} - \underline{k}$, $\underline{b} = \underline{j} + \underline{k}$**

ii. **Find a real number α so that the vectors $\underline{u}$ and $\underline{v}$ are perpendicular $\underline{u} = 2\alpha\underline{i} + \underline{j} - \underline{k}$, $\underline{v} = \underline{i} + \alpha\underline{j} + 4\underline{k}$**

iii. **If $\underline{v}$ is a vector for which $\underline{v}.\underline{i} = 0$, $\underline{v}.\underline{j} = 0$, $\underline{v}.\underline{k} = 0$ find $\underline{v}$.**

iv. **Find the angle between the vectors $\underline{u} = 2\underline{i} - \underline{j} + \underline{k}$ and $\underline{v} = \underline{-i} + \underline{j}$**

v. **Define Orthogonal Vectors.**

LONG QUESTIONS

Prove that the angle in semi circle is a right angle.

Prove that $cos(\alpha+\beta)=\beta cos\alpha sin\beta - sin\alpha cos\beta$

Prove that the altitudes of a triangle are concurrent.

EXERCISE 7.4

Tick (✔) the correct answer.

1. Cross product or vector product is defined

(b) In plane only (b) ✔ in space only (c) everywhere (d) in vector field

2. If $\underline{u}$ and $\underline{v}$ are two vectors , then $\underline{u}\times\underline{v}$ is a vector

(b) Parallel to $\underline{u}$and $\underline{v}$ (b) parallel to $\underline{u}$ (c) ✔ perpendicular to $\underline{u}$ and $\underline{v}$ (d) orthogonal to $\underline{u}$

3. If $\underline{u}$ and $\underline{v}$ be any two vectors, along the adjacent sides of ||gram then the area of ||gram is

(b) $\underline{u}\times\underline{v}$ (b) ✔ $|\underline{u}\times\underline{v}|$ (c) $\frac{1}{2}(\underline{u}\times\underline{v})$ (d) $\frac{1}{2}|\underline{u}\times\underline{v}|$

4. If $\underline{u}$ and $\underline{v}$ be any two vectors, along the adjacent sides of triangle then the area of triangle is

(b) $\underline{u}\times\underline{v}$ (b) $|\underline{u}\times\underline{v}|$ (c) $\frac{1}{2}(\underline{u}\times\underline{v})$ (d) ✔ $\frac{1}{2}|\underline{u}\times\underline{v}|$

5. Two non zero vectors are perpendicular iff

(a) $\underline{u}.\underline{v}=1$ (b) $\underline{u}.\underline{v}\neq 1$ (c) $\underline{u}.\underline{v}\neq 0$ (d) ✔ $\underline{u}.\underline{v}=0$

SHORT QUESTIONS

i. **If $\underline{u}=2\underline{i}-\underline{j}+\underline{k}$ and $\underline{v}=4\underline{i}+2\underline{j}-\underline{k}$, find $\underline{u}\times\underline{v}$ and $\underline{v}\times\underline{u}$**

ii. **Find the area of triangle, determined by the point $P(0,0,0)$; $Q(2,3,2)$; $R(-1,1,4)$**

iii. **Find the area of $||^m$, whose vertices are:**
$A(1,2,-1); B(4,2,-3); C(6,-5,2); D(9,-5,0)$

iv. **Which vectors, if any, are perpendicular or parallel**
$$\underline{u}=\underline{i}+2\underline{j}-\underline{k}\,;\ \underline{v}=\underline{-i}+\underline{j}+\underline{k};\ \underline{w}=-\frac{\pi}{2}\underline{i}-\pi\underline{j}+\frac{\pi}{2}\underline{k}$$

v. **If $\underline{a}+\underline{b}+\underline{c}=0$, then prove that $\underline{a}\times\underline{b}=\underline{b}\times\underline{c}=\underline{c}\times\underline{a}$**

vi. **If $\underline{a}\times\underline{b}=0$ and $\underline{a}.\underline{b}=0$, what conclusion can be drawn about $\underline{a}$ or $\underline{b}$?**

LONG QUESTIONS

Prove that : $sin(\alpha-\beta)=sin\alpha cos\beta - cos\alpha sin\beta$

Prove that : $\underline{a}\times(\underline{b}+\underline{c})+\underline{b}\times(\underline{c}+\underline{a})+\underline{c}\times(\underline{a}+\underline{b})=0$

EXERCISE 7.5

Tick (✔) the correct answer.

1. **The scalar triple product of $\underline{a}$, $\underline{b}$ and $\underline{c}$ is denoted by**

(b) $\underline{a}.\underline{b}.\underline{c}$ (b) ✔ $\underline{a}.\underline{b}\times\underline{c}$ (c) $\underline{a}\times\underline{b}\times\underline{c}$ (d) $(\underline{a}+\underline{b})\times\underline{c}$

2. **The vector triple product of $\underline{a}$, $\underline{b}$ and $\underline{c}$ is denoted by**

(a) $\underline{a}.\underline{b}.\underline{c}$ (b) $\underline{a}.\underline{b}\times\underline{c}$ (c) ✔ $\underline{a}\times\underline{b}\times\underline{c}$ (d) $(\underline{a}+\underline{b})\times\underline{c}$

3. **Notation for scalar triple product of $\underline{a}$, $\underline{b}$ and $\underline{c}$ is**

(a) $\underline{a}.\underline{b}\times\underline{c}$ (b) $\underline{a}\times\underline{b}.\underline{c}$ (c) $[\underline{a}.\underline{b}.\underline{c}]$ (d) ✔ all of these

4. **If the scalar product of three vectors is zero, then vectors are**

(a) Collinear (b) ✔ coplanar (c) non coplanar (d) non-collinear

5. **If any two vectors of scalar triple product are equal, then its value is equal to**

(a) 1 (b) ✔0 (c) -1 (d) 2

6. **Moment of a force about a point is:**

(a) ✔Vector quantity (b) scalar quantity (c) zero (d) None of these

7. **Two vectors lying in the same plane are called:**

(a) Collinear vectors (b) perpendicular vectors (c) ✔ coplanar vectors (d) parallel vectors

8. **Moment of a force $\underline{F}$ about a point is given by:**

(a) Dot product (b) ✔cross product (c) both (a) and (b) (d) None of these

SHORT QUESTIONS

i. **What are coplanar vectors?**

ii. **A force $\underline{F}=7\underline{i}+4\underline{j}-3\underline{k}$ is applied at $P(1,-2,3)$. Find its moment about the point $Q(2,1,1)$.**

iii. **Find work done by $\underline{F}=2\underline{i}+4\underline{j}$ if its points of application to a body moves if from $A(1,1)$ to $B(4,6)$.**

iv. **Prove that the vectors $\underline{i}-2\underline{j}+\underline{k}, -2\underline{i}+3\underline{j}-4\underline{k}$ and $\underline{i}-3\underline{j}+5\underline{k}$ are coplanar.**

v. **If $\underline{a}=3\underline{i}-\underline{j}+5\underline{k}, \underline{b}=4\underline{i}+3\underline{j}-2\underline{k}$ and $\underline{c}=2\underline{i}+5\underline{j}+\underline{k}$ fine $\underline{a}.\underline{b}\times c$**

vi. **Find the volume of tetrahedron with the vertices $A(0,1,2)$, $B(3,2,1)$, $C(1,2,1)$ and $D(5,5,6)$.**

vii. **Find the value of $2\underline{i}\times 2\underline{j}.\underline{k}$ and $[k\ \ i\ \ j]$**

viii. **Prove that $\underline{u}.(\underline{v}\times\underline{w})+\underline{v}.(\underline{w}\times\underline{u})+\underline{w}.(\underline{u}\times\underline{v})=3\underline{u}.(\underline{v}\times\underline{w})$**

ix. **Write down the volume of tetrahedron.**

x. **Find the value of , so that $\alpha\underline{i}+\underline{j}, \underline{i}+\underline{j}+3\underline{k}$ and $2\underline{i}+\underline{j}-2\underline{k}$ are coplanar.**

LONG QUESTIONS

Prove that the points whose position vectors are $\left(-6\underline{i}+3\underline{j}+2\underline{k}\right)$, $B\left(3\underline{i}-2\underline{j}+4\underline{k}\right)$, $C\left(5\underline{i}+7\underline{j}+3\underline{k}\right)$ and $D\left(-13\underline{i}+17\underline{j}-\underline{k}\right)$ are coplanar.

A force of magnitude 6 units acting parallel to $2\underline{i}-2\underline{j}+\underline{k}$ displces, the point of application from $(1,2,3)$ to $(5,3,7)$. Find the work done.