Roll No. | | | | | | | Answer Sheet No._____

Sig. of Candidate._____ Sig. of Invigilator._____

MATHEMATICS HSSC–II

SECTION – A (Marks 20)

Time allowed: 25 Minutes Version Number | 1 | 7 | 0 | 2 |

NOTE: **Section–A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.**

Q. 1 **Circle the correct option i.e. A / B / C / D. Each part carries one mark.**

(i) Which of the following is a unit vector in the direction of $[2,6]$?
A. $[\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}]$ B. $[\frac{-2}{\sqrt{40}}, \frac{6}{\sqrt{40}}]$ (C.) $[\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}]$ D. $[\frac{1}{\sqrt{40}}, \frac{3}{\sqrt{40}}]$

(ii) What is the value of $f^{-1}(6)$ if $f(x) = 5x + 1$?
A. 31 B. -1 (C.) 1 D. $\frac{7}{5}$

(iii) What is the value of $g(x)$ if $f(x) = \frac{2x}{3x+4}$ and $f[g(x)] = x$?
A. $\frac{2+3x}{4x}$ B. $\frac{4x}{2+3x}$ (C.) $\frac{4x}{2-3x}$ D. $\frac{2-3x}{4}$

(iv) If $f(x) = a + x$ then which of the following options is correct?
A. $f'(0) \cdot f'(1) = 0$ B. $f'(0) < f'(1)$ C. $f'(0) > f'(1)$ D. $f'(0) = f'(1)$

(v) Which of the following defines a function "f" for $f(-x) = -f(x)$?
A. $f(x) = \log x$ B. $f(x) = x^2$ (C.) $f(x) = \sin x$ D. $f(x) = \cos x$

(vi) If $f(x) = (x-1)^2 \sin x$ then which of the following options is correct for $f'(0)$?
(A.) 1 B. -2 C. -1 D. 0

(vii) Which of the following represents $\frac{dy}{dx}$ if $\sin x = e^y$?
A. $-\cot x$ B. $\tan x$ C. $-\tan x$ (D.) $\cot x$

(viii) What evaluates $\int_2^\infty \frac{dx}{x^2}$?
A. 0 B. $-\frac{1}{2}$ (C.) $\frac{1}{2}$ D. $\frac{1}{24}$

(ix) For what value of k, $\int_{-3}^{3}(x^5 + k)dx = 30$?
A. -5 B. $-35\frac{1}{2}$ C. 0 (D.) 5

(x) If $\int_{-2}^{1} f(x)\,dx = 5$ and $\int_1^3 f(x)\,dx = 3$ then what results $\int_{-2}^{3} f(x)dx$?
A. 4 B. 2 (C.) 8 D. 0

(xi) What is the area under the curve $y = \cos x$ from $x = 0$ to $x = \frac{\pi}{2}$?
A. 2 B. π C. 0 (D.) 1

(xii) What is the perpendicular distance of the line $4x - 3y - 25 = 0$ from origin?
A. 1 B. 0 C. $\frac{25}{7}$ (D.) 5

(xiii) Which of the following is an equation of a line passing through $(1,0)$ and $(0,1)$?
A. $x - y = -1$ B. $x - y = 1$ (C.) $x + y = 1$ D. $x + y = -1$

(xiv) For what value of k, lines $4x - y + k = 0$ and $6kx - 3y + 1 = 0$ are parallel?
(A.) 2 B. 0 C. 1 D. -2

(xv) Which of the following ordered pairs does not satisfy $4x - 3y < 2$?
(A.) $(3,0)$ B. $(1,1)$ C. $(-2,1)$ D. $(0,0)$

(xvi) What is the length of the Latus Rectum of the parabola $(y-2)^2 = -4(x-7)$?
A. 8 B. $\frac{1}{4}$ (C.) 4 D. 16

(xvii) Which of the ordered pairs describes x-intercept of the conic $y^2 + 8x - 6y + 24 = 0$?
A. $(0,\ 3+\sqrt{15})$ B. $(3,\ 0)$ C. $(0,\ 3)$ (D.) $(-3,\ 0)$

(xviii) Which of the following is an axis of the parabola $(y+3)^2 = 12\left(x + \frac{1}{2}\right)$?
(A.) $y = -3$ B. $x = -\frac{1}{2}$ C. $x = \frac{1}{2}$ D. $y = 3$

(xix) For what value of α, vectors $5\underline{i} - \underline{j} + \underline{k}$ and $\alpha\underline{i} + 3\underline{j} - 3\underline{k}$ are parallel to each other?
A. -3 (B.) -15 C. 15 D. 3

(xx) What equals $\underline{a} \cdot \underline{b} \times \underline{c}$ if $\underline{a} = \underline{i} + 2\underline{j} + 3\underline{k}$, $\underline{b} = 2\underline{i} + 4\underline{j} + 6\underline{k}$ and $\underline{c} = 3\underline{i} - \underline{j} + \underline{k}$?
A. -64 B. 24 C. 28 (D.) 0

For Examiner's use only:

Total Marks: 20

Marks Obtained:

MATHEMATICS HSSC–II

Time allowed: 2:35 Hours **Total Marks Sections B and C: 80**

NOTE: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Graph paper will be provided on Demand.

SECTION – B (Marks 40)

Q. 2 Attempt any TEN parts. All parts carry equal marks. **(10 x 4 = 40)**

(i) Let the real valued functions f and g be defined by $f(x) = 2x + 1$ and $(x) = x^2 - 1$. Obtain the expressions for $fg(x)$; $gf(x)$; $f^2(x)$; $g^2(x)$

(ii) Evaluate $\lim_{\theta \to 0} \frac{1-\cos p\,\theta}{1-\cos q\,\theta}$

(iii) Find $\frac{dy}{dx}$ if $x = \frac{a(1-t^2)}{1+t^2}$ and $y = \frac{2bt}{1+t^2}$

(iv) Differentiate $\cos\sqrt{x}$ with respect to x by ab-initio method.

(v) A box with a square base and open top is to have a volume of 4 cubic dm. Find the dimensions of the box which will require the least material.

(vi) Show that $\int \sqrt{a^2 - x^2}\,dx = \frac{a^2}{2}\sin^{-1}\frac{x}{a} + \frac{x}{2}\sqrt{a^2 - x^2} + c$

(vii) Evaluate $\int_0^{\frac{\pi}{4}} \cos^4 t\,dt$

(viii) A quadrilateral has the points $A(9,3), B(-7,7), C(-3,-7)$ and $D(5,-5)$ as its vertices. Find the midpoints of its sides. Show that the figure formed by joining the midpoints consecutively is a parallelogram.

(ix) Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ Such that the product of the x and y-intercepts of each is 3

(x) Find equations of two tangents drawn from $(2,3)$ to the circle $x^2 + y^2 = 9$

(xi) Find the foci, eccentricity and vertices of an ellipse $\frac{(2x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$

(xii) Find equations of the tangents to the conic $9x^2 - 4y^2 = 36$ parallel to $5x - 2y + 7 = 0$

(xiii) Find the angle between the vectors $2\underline{i} - \underline{j} + \underline{k}$ and $-\underline{i} + \underline{j}$

(xiv) Use vector method to prove $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

SECTION – C (Marks 40)

Note: Attempt any FIVE questions. All questions carry equal marks. **(5 x 8 = 40)**

Q. 3 If $f(x) = \begin{cases} 3x & if \quad x \le -2 \\ x^2 - 1 & if \quad -2 < x < 2 \\ 3 & if \quad x \ge 2 \end{cases}$ discuss the continuity at $x = 2$ and $x = -2$

Q. 4 Find extreme values of the function $f(x) = \sin x + \cos x$ occurring in the interval $[0, 2\pi]$

Q. 5 Evaluate $\int \frac{x^3 - 6x^2 + 25}{(x+1)^2(x-2)^2}\,dx$

Q. 6 Show that the lines $4x - 3y - 8 = 0$, $3x - 4y - 6 = 0$ and $x - y - 2 = 0$ are concurrent and the third line bisects the angle formed by the first two lines.

Q. 7 Graph the feasible region of the system of linear inequalities $x + 2y \le 14$; $3x + 4y \le 36$; $2x + y \le 10$; $x \ge 0$; $y \ge 0$ and find the corner points.

Q. 8 Find equations of the circle passing through two points $A(1,4)$, $B(-1,8)$ and tangent to the line $x + 3y - 3 = 0$

Q. 9 Prove that the altitudes of a triangle are concurrent.

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MATHEMATICS HSSC–II

SECTION – A (Marks 20)

Time allowed: 25 Minutes Version Number | 1 | 7 | 0 | 5 |

NOTE: Section–A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q. 1 Circle the correct option i.e. A / B / C / D. Each part carries one mark.

(i) What is the evaluated value of $\lim_{h\to 0}\frac{|h|}{h}$?

A. ± 1 B. 1 C. 0 D. Limit does not exist

(ii) What evaluates $\lim_{x\to 0}\frac{\cos\left(\frac{3\pi}{2}-x\right)-\cos\left(\frac{3\pi}{2}\right)}{x}$?

A. $\sqrt{2}$ B. -1 C. 1 D. Limit does not exist

(iii) Which of the following represents $f'(x)$ if $e^{f(x)} = 1 + x^2$?

A. $2x(1+x^2)$ B. $2x\ln(1+x^2)$ C. $2x\,e^{1+x^2}$ D. $\frac{2x}{1+x^2}$

(iv) If $f(x) = x^3 - 2x^2 + 4x - 1$ then which of the following represents $f(1+x)$?

A. $x^3 + x^2 + 3x + 2$ B. $x^3 + x^2 - 3x + 2$

C. $x^3 + x^2 + 3x - 2$ D. $x^3 - x^2 + 3x + 2$

(v) What equals $f'(0)$ if $(x) = \sin^2(3-x)$?

A. $-2\sin 3\cos 3$ B. $2\sin 3\cos 3$ C. $6\sin 3\cos 3$ D. $-2\sin 3$

(vi) If $f(x) = (2x+1)^4$ then what will be the 4th derivative of $f(x)$ at $x = 0$?

A. 0 B. 48 C. 324 D. 384

(vii) What evaluates $\int_0^8 \frac{dx}{\sqrt{1+x}}$?

A. -8 B. 4 C. 6 D. $\frac{52}{81}$

(viii) What is the solution of a differential equation $\frac{dy}{dx} = \tan x$?

A. $\ln|\cos x| + c$ B. $\ln|\sec x| + c$ C. $\sec^2 x + c$ D. $\frac{1}{2}\tan^2 x + c$

(ix) For what value of k, $\int_0^{-1}(k + 3x^2)dx = -3$?

A. -2 B. 4 C. 2 D. -4

(x) What is the evaluated value of $\int_0^1 \frac{1}{1-x}dx$?

A. 0 B. undefined C. 1 D. -1

(xi) What is the perpendicular distance of a point (2,3) from y-axis?

A. 2 B. $\sqrt{13}$ C. 3 D. 5

(xii) At what angle lines $7y = 5x + 8$ and $7x + 5y = 16$ intersect each other?

A. 0 B. $\frac{\pi}{2}$ C. $\frac{\pi}{4}$ D. π

(xiii) What is the slope of a line perpendicular to $13x + 26y - k = 0$?

A. $-\frac{1}{2}$ B. -2 C. $\frac{1}{2}$ D. 2

(xiv) Which point in the following is not a solution of $2x - 3y < 5$?

A. $(-1,-1)$ B. $(2,-2)$ C. (2,2) D. (3,3)

(xv) At what point on the graph a conic $\frac{x^2}{4} + \frac{y^2}{9} = 1$ has its centre?

A. (2,3) B. (0,0) C. (4,9) D. (1,1)

(xvi) What is the length of the minor axis of an ellipse $2x^2 + 3y^2 - 6 = 0$?

A. $2\sqrt{2}$ B. $\sqrt{2}$ C. 2 D. $\sqrt{3}$

(xvii) At what point, centre of a circle $4x^2 + 4y^2 - 12x - 16y - 21 = 0$ lies?

A. $(\frac{3}{2}, -2)$ B. $(2, \frac{3}{2})$ C. $(\frac{3}{2}, 2)$ D. $(-\frac{3}{2}, -2)$

(xviii) For what value of "t" vectors $-\underline{i} + \underline{j} + \underline{k}$ and $-\frac{\pi}{2}\underline{i} - \pi\underline{j} + t\underline{k}$ are perpendicular?

A. π B. $\frac{\pi}{2}$ C. $-\pi$ D. $-\frac{\pi}{2}$

(xix) What is the volume of a parallelepiped determined by $\underline{i} + 2\underline{j} - \underline{k}$, $\underline{i} - 2\underline{j} + 3\underline{k}$ and $\underline{i} - 7\underline{j} - 4\underline{k}$?

A. 20 B. 48 C. 8 D. 38

(xx) What are the direction *cosines* of a vector $\underline{i} + \underline{j}$?

A. $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1$ B. 1,1,0 C. $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ D. $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$

For Examiner's use only:

Total Marks: 20

Marks Obtained:

MATHEMATICS HSSC–II

Time allowed: 2:35 Hours **Total Marks Sections B and C: 80**

NOTE: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Graph paper will be provided on Demand.

SECTION – B (Marks 40)

Q. 2 Attempt any TEN parts. All parts carry equal marks. **(10 x 4 = 40)**

(i) For a real valued function $f(x) = (-x+9)^3$, find $f^{-1}(x)$ and verify $f[f^{-1}(x)] = f^{-1}[f(x)] = x$

(ii) If a function $f(x) = \begin{cases} mx & if\ x < 3 \\ n & if\ x = 3 \\ -2x+9 & if\ x > 3 \end{cases}$ is continuous at $x = 3$. Find the values of "m" and "n"

(iii) If $x = a\cos^3\theta, y = b\sin^3\theta$, show that $a\frac{dy}{dx} + b\tan\theta = 0$

(iv) Find $\frac{dy}{dx}$ if $y = \frac{\sqrt{x^2-1}\,(x+1)}{(x^3+1)^{\frac{3}{2}}}$

(v) If $y = e^{ax}\sin bx$ then show that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2+b^2)y = 0$

(vi) Find the area between the curve $y = x(x-1)(x+1)$ and the x-axis.

(vii) Evaluate $\int x^3\tan^{-1}x\ dx$

(viii) Find a joint equation of the straight lines through the origin, perpendicular to the lines represented by $x^2 + xy - 6y^2 = 0$

(ix) Find the equation of the line through the point of intersection of the lines $x - y - 4 = 0$ and $7x + y + 20 = 0$ and parallel to the line $6x + y - 14 = 0$

(x) Find the foci, eccentricity and vertices of a hyperbola $\frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1$

(xi) Find equations of the tangents to the ellipse $\frac{x^2}{4} + y^2 = 1$ which are parallel to the line $2x - 4y + 5 = 0$

(xii) By transforming the equation $x^2 - y^2 - 6x + 2y + 7 = 0$ referred to a new origin and axes remaining parallel to the original axes, the first degree terms are removed. Find the coordinates of the new origin and the transformed equation.

(xiii) Use vector method to prove $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(xiv) Find area of the parallelogram whose vertices are $A(-1,1,1), B(-1,2,2), C(-3,4,-5)$ and $D(-3,5,-4)$

SECTION – C (Marks 40)

Note: Attempt any FIVE questions. All questions carry equal marks. **(5 x 8 = 40)**

Q. 3 Find the graphical solution of $x = \sin 2x$

Q. 4 Discuss the function defined as $f(x) = \sin x + \frac{1}{2\sqrt{2}}\cos 2x$ for extreme values in the interval $(0, 2\pi)$

Q. 5 Show that $\int e^{ax}\cos bx\ dx = \frac{1}{\sqrt{a^2+b^2}}\ e^{ax}\cos(bx - \tan^{-1}\frac{b}{a}) + c$

Q. 6 Find area of the region bounded by $x + y + 1 = 0$ and $10x^2 - xy - 21y^2 = 0$

Q. 7 Find the maximum and minimum values of the function defined as $f(x, y) = 2x + 3y$ subject to the constraints $-y \le 2\ ;\ x + y \le 4\ ;\ 2x - y \le 6\ ;\ x \ge 0$

Q. 8 Show that tangent at any point P of a parabola makes equal angles with the line PF and the line through P, parallel to the axis of the parabola, F being focus.

Q. 9 Prove that perpendicular bisectors of the sides of a triangle are concurrent.