Mathematics HSSC-I (Supplementary 2016)

Federal Board of Intermediate and Secondary Education, Islamabad

Section A (Marks 20)

Question 1: Circle the correct option. i.e. A, B, C, D, each part carries one mark.

1. The solution set of $\sec x = -2$ is:

   (A) Empty set (B) $\{\frac{\pi}{6}+2x\pi\}\cup\{\frac{11\pi}{6}+2x\pi\}, n\in\mathbb{Z}$ (C) $\{\frac{2\pi}{3}+2x\pi\}\cup\{\frac{4\pi}{3}+2x\pi\}, n\in\mathbb{Z}$
   (D) $\{\frac{\pi}{3}+2x\pi\}\cup\{\frac{5\pi}{3}+2x\pi\}, n\in\mathbb{Z}$

2. Multiplicative inverse of $-3-5i$ is:

   (A) $\frac{-3-5i}{8}$ (B) $\frac{3-5i}{34}$ (C) $\frac{3+5i}{34}$ (D) $\frac{-3+5i}{34}$

3. $(A-B)\cap B =$

   (A) Universal set (B) ϕ (C) A (D) B

4. The value of the determinant $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$

   (A) 4 (B) 10 (C) 0 (D) 13

5. The matrix $\begin{bmatrix} 0 & 2-3i \\ -2-3i & 0 \end{bmatrix}$

   (A) Identity matrix (B) Singular matrix (C) Skew-Hermitian matrix (D) Symmetric matrix

6. If ω is a cube root of unity, then an equation whose roots are 3ω and $2\omega^2$ will be

   (A) $x^2+3x+6=0$ (B) $x^2-9x-3=0$ (C) $x^2-3x-9=0$ (D) $x^2+3x+9=0$

7. The formulation of partial fractions of $\frac{x^4}{1-x^4}$ will be:

   (A) $\frac{A}{1-x}+\frac{B}{1+x}+\frac{Cx+D}{1+x^2}$ (B) $A+\frac{B}{1-x}+\frac{C}{1+x}+\frac{Dx+E}{1+x^2}$ (C) $\frac{A}{1+x^2}+\frac{B}{1+x^2}$
   (D) $\frac{A}{1-x}+\frac{B}{1+x}+\frac{C}{1+x^2}$

8. If $a_{n-2}=3n-11$, then $8th$ term will be

   (A) 24 (B) 13 (C) 20 (D) 19

9. The sum of infinite G.P. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...$ is:(answer is 2 but not in options)

   (A) 9 (B) 0 (C) 4 (D) 1

10. If ${}^nC_8 -{}^n C_{12}$, then $n=$

    (A) 10 (B) 20 (C) 4 (D) 96

11. The factorial form of $\frac{(n+1)n(n-1)}{3.2.1}$ is:

    (A) $\frac{(n-1)!}{3!}$ (B) $\frac{(n+1)!}{3!n!}$ (C) $\frac{(n+1)!}{3!(n-2)!}$ (D) $\frac{(n-1)!}{3!(n+1)!}$

12. The $6th$ term from end in the expansion of $(\frac{3}{2}x-\frac{1}{3x})^{11}$ is:

    (A) $9th$ term (B) $12th$ term (C) $6th$ term (D) $5th$ term

13. The middle term from end in the expansion of $(2x-\frac{1}{2x})^{2m+1}$ is:

    (A) $(m+2)th$ term (B) $(m+1)th$ and $(m+2)th$ term (C) $2mth$ term (D) $(m+1)th$ term

14. If $\tan\theta=\frac{8}{15}$, terminal arm lies in III quadrant, then $\sec\theta$

    (A) $\frac{8}{17}$ (B) $\frac{-17}{15}$ (C) $\frac{-17}{8}$ (D) $\frac{-15}{17}$

15. $\cos^2 2\theta=$

    (A) $4\cos^3\theta-3\cos\theta$ (B) $\frac{1+\cos 4\theta}{2}$ (C) $4\cos^2\theta\sin^2\theta$ (D) $\cos 2\theta-\sin 2\theta$

16. $\cos(\frac{3\pi}{2}-\theta)=$

    (A) $-\cos\theta$ (B) $\cos\theta$ (C) $\sin\theta$ (D) $\cos(\frac{\pi}{2}+\theta)$

17. Period of $3\cos\frac{x}{5}$ is:

    (A) 30π (B) 2π (C) 10π (D) 6π

18. $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=$

    (A) 0 (B) $2\sec^2\theta$ (C) $2\cos^2\theta$ (D) $\frac{1}{\cos\theta}$

19. $90°\pm\theta, 180°\pm\theta, 270°\pm\theta, 360°\pm\theta$ are called:

    (A) Obtuse angle (B) Supplementary angle (C) Allied angles (D) Acute angles

20. $\cos^{-1}(-x)-\cos^{-1}(x)$=?

    (A) $\sin^{-1}x$ (B) π (C) 0 (D) 1

ANSWERS

1. C 2. D 3. B 4. C 5. C 6. D 7. A 8. D 10. B 11. C 12. C
13. B 14. B 15. B 16. A 17. C 18. B 19. C 20. B

Section B (Marks 40)

Question 2: Attempt any TEN parts. All parts carry equal marks. (10 × 4 = 40)

(i) Separate into real and imaginary parts $\dfrac{(-2+3i)^2}{(1+i)}$.

(ii) Convert to logical form and prove by constructing the truth table $(\mathrm{A}\cap\mathrm{B})' = \mathrm{A}'\cup\mathrm{B}'$.

(iii) Without expansion verify that $\begin{vmatrix} 1 & a^2 & \dfrac{a}{bc} \\ 1 & b^2 & \dfrac{b}{ca} \\ 1 & c^2 & \dfrac{c}{ab} \end{vmatrix} = 0$.

(iv) If the roots of $px^2+qx+q=0$ are α and β then prove that $\sqrt{\dfrac{\alpha}{\beta}}+\sqrt{\dfrac{\beta}{\alpha}}+\sqrt{\dfrac{q}{p}}=0$.

(v) Resolve into partial fraction $\dfrac{3x-11}{(x^2+1)(x+3)}$.

(vi) Which term of the sequence $x^2-y^2,\ x+y,\ \dfrac{x+y}{x-y},...$ is $\dfrac{x+y}{(x-y)^9}$?

(vii) In how many ways can letter of the word "MISSISSIPPI" be arranged, when all letters are to be used?

(viii) Find the term independent of x in the expansion of $\left(x-\dfrac{2}{x}\right)^{10}$.

(ix) Prove the identity $\dfrac{1+\cos\theta}{1-\cos\theta}=(\cos ec\,\theta+\cot\theta)$.

(x) Prove that $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$.

(xi) Prove that $\dfrac{1}{r}=\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}$.

(xii) Prove that $\tan^{-1}\dfrac{120}{119}=2\cos^{-1}\dfrac{12}{13}$.

(xiii) Prove that $\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{7}=\dfrac{\pi}{4}$.

(xiv) Solve the equation $\sin x=\dfrac{1}{2}$.

Solutions

(i) $$\frac{(-2+3i)^2}{(1+i)} = \frac{(-2)^2+(3i)^2+2(-2)(3i)}{1+i}$$

$$= \frac{4+9i^2-12i}{1+i} = \frac{4-9-12i}{1+i} = \frac{-5-12i}{1+i} = \frac{-5-12i}{1+i}\times\frac{1-i}{1-i}$$

$$= \frac{-5+5i-12i+12i^2}{1^2-i^2} = \frac{-5-7i-12}{1-(-1)}$$

$$= \frac{-17-7i}{1+1} = \frac{-17-7i}{2} = \frac{-17}{2}-\frac{7}{2}i$$

(ii) The corresponding formula of logic is

$$\sim(p\wedge q) = \sim p \vee \sim q$$

p	q	$\sim p$	$\sim q$	$p\wedge q$	$\sim(p\wedge q)$	$\sim p\vee\sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

The last two columns of the above table shows that $\sim(p\wedge q) = \sim p\vee\sim q$

and hence $(A\cap B)' = A'\cup B'$.

(iii)

$$L.H.S. = \begin{vmatrix} 1 & a^2 & \frac{a}{bc} \\ 1 & b^2 & \frac{b}{ca} \\ 1 & c^2 & \frac{c}{ab} \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} 1 & a^2 & abc\frac{a}{bc} \\ 1 & b^2 & abc\frac{b}{ca} \\ 1 & c^2 & abc\frac{c}{ab} \end{vmatrix}$$ Multiplying c_3 by abc.

$$= \frac{1}{abc}\begin{vmatrix} 1 & a^2 & a^2 \\ 1 & b^2 & b^2 \\ 1 & c^2 & c^2 \end{vmatrix} = \frac{1}{abc}(0) \quad (\because c_2 = c_3)$$

$$= 0 \qquad = R.H.S.$$

(iv) $px^2+qx+q=0$

$a=p \qquad b=q \qquad c=q$

Let α,β be the roots of the equations. Then

$$\alpha+\beta=-\frac{q}{p} \qquad \alpha\beta=\frac{q}{p}$$

L.H.S. $= \sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{q}{p}} \quad =\frac{\alpha+\beta}{\sqrt{\alpha\beta}}+\sqrt{\frac{q}{p}}$

$$=\frac{-\frac{q}{p}}{\sqrt{\frac{q}{p}}}+\sqrt{\frac{q}{p}} \quad =-\sqrt{\frac{q}{p}}+\sqrt{\frac{q}{p}}=0=R.H.S$$

(v) Let $\frac{3x-11}{(x^2+1)(x+3)}=\frac{Ax+B}{(x^2+1)}+\frac{C}{(x+3)}$

$$3x-11=(Ax+B)(x+3)+C(x^2+1) \text{ .... } (1)$$

$$3x-11=(A+C)x^2+(3A+B)x+(3B+C) \text{ .... } (2)$$

Putting $x+3=0 \quad \Rightarrow x=-3$, Using it in (1), we get

$$-9-11=C(9+1) \quad \Rightarrow C=-2$$

Equating the coefficients of x^2 and x in (2), we obtain

$$0=A+C \quad \Rightarrow A=-C \quad \Rightarrow A=2$$

and $3=3A+B \quad \Rightarrow B=3-3A \quad \Rightarrow B=3-6 \quad \Rightarrow B=-3$

Hence partial differential equation are: $\frac{2x-3}{x^2+1}-\frac{2}{x+3}$

(vi) $a_1=x^2-y^2$

$$r=\frac{x+y}{x^2-y^2}=\frac{x+y}{(x+y)(x-y)}=\frac{1}{x-y}$$

$$n=? \qquad a_n=\frac{x+y}{(x-y)^9}$$

Since $a_n=a_1r^{n-1}$

$$\Rightarrow \frac{x+y}{(x-y)^9}=x^2-y^2\left(\frac{1}{x-y}\right)^{n-1} \qquad \Rightarrow \frac{x+y}{(x-y)^9}=(x-y)(x+y)\left(\frac{1}{x-y}\right)^{n-1}$$

$$\Rightarrow \frac{1}{(x-y)^9}=(x-y)\frac{1}{(x-y)^{n-1}} \qquad \Rightarrow \frac{1}{(x-y)^9}=\frac{1}{(x-y)^{n-2}}$$

$$\Rightarrow (x-y)^9=(x-y)^{n-2} \qquad \Rightarrow 9=n-2 \qquad \Rightarrow n=11$$

(vii) Numbers of letters in $MISSISSIPPI=11$
In $MISSISSIPPI$
I is repeated 4 times
S repeated 4 times

P is repeated 2 times
M comes only once.

Required number of Permutation = $\begin{pmatrix} 11 \\ 4,4,2,1 \end{pmatrix}$

$$= \frac{(11!)}{(4)!\times(4)!\times(2)!\times(1)!} = 34650 \text{ ways}$$

(viii) $\left(x - \frac{2}{x}\right)^{10}$

Since $T_{r+1} = \binom{n}{r} a^{n-r} x^r$,

Here $a = x$, $x = \frac{2}{x}$, $n = 10$ so

$$T_{r+1} = \binom{10}{r} x^{10-r} \left(\frac{2}{x}\right)^r$$

$$= \binom{10}{r} x^{10-r}.2^r.x^{-r} \qquad = \binom{10}{r} x^{10-2r}.2^r.$$

For term independent of x, We must have

$x^{10-2r} = x^0 \Rightarrow 10 - 2r = 0$

$\Rightarrow 2r = 10 \Rightarrow r = 5$

So $T_{5+1} = \binom{10}{5} x^0 x^5 \qquad \Rightarrow T_6 = 252 \times 32 = 8064$

(ix)

R.H.S. = $(\operatorname{cosec}\theta + \cot\theta)^2$

$$= \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta \cot\theta = \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} + 2\frac{1}{\sin\theta}\frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}{\sin^2\theta} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1+\cos\theta}{1-\cos\theta} = L.H.S$$

(x) $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

$\sin 3\alpha = \sin(2\alpha + \alpha)$

$= \sin 2\alpha \cos\alpha + \cos 2\alpha \sin\alpha$

$= 2\sin\alpha\cos\alpha\cos\alpha + (1 - 2\sin^2\alpha)\sin\alpha$

$= 2\sin\alpha\cos^2\alpha + \sin\alpha - 2\sin^3\alpha$

$= 2\sin\alpha(1-\sin^2\alpha) + \sin\alpha - 2\sin^3\alpha$

$= 2\sin\alpha - 2\sin^3\alpha + \sin\alpha - 2\sin^3\alpha$

$= 3\sin\alpha - 4\sin^3\alpha = R.H.S.$

(xi) $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

$$\text{R.H.S} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$= \frac{1}{\frac{\Delta}{s-a}} + \frac{1}{\frac{\Delta}{s-b}} + \frac{1}{\frac{\Delta}{s-c}} = \frac{s-a}{\Delta} + \frac{s-b}{\Delta} + \frac{s-c}{\Delta}$$

$$= \frac{s-a+s-b+s-c}{\Delta} = \frac{3s-(a+b+c)}{\Delta}$$

$$= \frac{3s-2s}{\Delta} \qquad \because \; 2s = a+b+c$$

$$= \frac{s}{\Delta} = \frac{1}{\Delta/s} = \frac{1}{r} = \text{L.H.S}$$

(xii) Suppose $\alpha = \tan^{-1}\frac{120}{119}$ ………………… (i)

$$\Rightarrow \tan\alpha = \frac{120}{119}$$

Now $\sec\alpha = \sqrt{1+\tan^2\alpha}$

$$= \sqrt{1+\left(\frac{120}{119}\right)^2} = \sqrt{1+\frac{14400}{14161}} = \sqrt{\frac{28561}{14161}} = \frac{169}{119}$$

So $\cos\alpha = \frac{1}{\sec\alpha} = \frac{1}{169/119} = \frac{119}{169}$

Now $\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+119/169}{2}} = \sqrt{\frac{288/169}{2}} = \sqrt{\frac{288}{2\times169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

$$\Rightarrow \frac{\alpha}{2} = \cos^{-1}\frac{12}{13} \Rightarrow \alpha = 2\cos^{-1}\frac{12}{13} \quad \text{………………… (ii)}$$

From (i) and (ii)

$$\tan^{-1}\frac{120}{119} = 2\cos^{-1}\frac{12}{13}$$

(xiii) L.H.S $= 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$

$$= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{3}}{1-\frac{1}{3}\frac{1}{3}}\right) + \tan^{-1}\frac{1}{7} \quad = \tan^{-1}\left(\frac{\frac{1+1}{3}}{1-\frac{1}{9}}\right) + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{\frac{2}{3}}{\frac{9-1}{9}}\right) + \tan^{-1}\frac{1}{7} \quad = \tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right) + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{2\times 9}{3\times 8}\right) + \tan^{-1}\frac{1}{7} \quad = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right) \quad = \tan^{-1}\left(\frac{\frac{21+4}{28}}{1-\frac{3}{28}}\right) \quad = \tan^{-1}\left(\frac{\frac{25}{28}}{\frac{28-3}{28}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{25}{28}}{\frac{25}{28}}\right) \quad = \tan^{-1}(1) \quad = \frac{\pi}{4} \quad = \text{R.H.S}$$

(xiv) $\sin x = \frac{1}{2}$

$\because$ $\sin x$ is positive in I and II Quardrants with the reference angle $x = \frac{\pi}{6}$

$\therefore$ $x = \frac{\pi}{6}$ and $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ where $x \in [0, 2\pi]$

Since 2π is the period of $\sin x$

$\therefore$ General values of x are $\frac{\pi}{6} + 2n\pi$ and $\frac{5\pi}{6} + 2n\pi$ $, n \in Z$

Hence solution set = $\left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\}$ $, n \in Z$

Section C (Marks 40)

Note: Attempt any five questions. All questions carry equal marks.

Question 3 Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$

Solution $A=\begin{bmatrix}1 & 2 & -3\\ 0 & -2 & 0\\ -2 & -2 & 2\end{bmatrix}$

Cofactors of $A=\begin{bmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{bmatrix}$

$$A_{11}=(-1)^{1+1}\begin{vmatrix}-2 & 0\\ 2 & 2\end{vmatrix}=(-1)^2(-4-0)=-4$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}0 & 0\\ -2 & 2\end{vmatrix}=(-1)^3(0)=0$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}0 & -2\\ -2 & -2\end{vmatrix}=(-1)^4(-4)=-4$$

$$A_{21}=(-1)^{2+1}\begin{vmatrix}2 & -3\\ -2 & 2\end{vmatrix}=(-1)^3(4-6)=2$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}1 & -3\\ -2 & 2\end{vmatrix}=(-1)^4(2-6)=-4$$

$$A_{23}=(-1)^{2+3}\begin{vmatrix}1 & 2\\ -2 & -2\end{vmatrix}=(-1)^5(-2+4)=-2$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix}2 & -3\\ -2 & 0\end{vmatrix}=(-1)^4(-6)=-6$$

$$A_{32}=(-1)^{3+2}\begin{vmatrix}1 & -3\\ 0 & 0\end{vmatrix}=(-1)^5(0)=0$$

$$A_{33}=(-1)^{3+3}\begin{vmatrix}1 & 2\\ 0 & -2\end{vmatrix}=(-1)^6(-2)=-2$$

Cofactors of $A=\begin{bmatrix}-4 & 0 & -4\\ 2 & -4 & -2\\ -6 & 0 & -2\end{bmatrix}$

$$Adj\,A=(cofactor\ of\ A)^t=\begin{bmatrix}-4 & 2 & -6\\ 0 & -4 & 0\\ -4 & -2 & -2\end{bmatrix}$$

$$\begin{aligned}|A|&=a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}\\ &=1(-4)+2(0)+(-3)(-4)\\ &=-4+12=8\end{aligned}$$

$$A^{-1}=\frac{1}{|A|}Adj\,A=\frac{1}{8}\begin{bmatrix}-4 & 2 & -6\\ 0 & -4 & 0\\ -4 & -2 & -2\end{bmatrix}=\begin{bmatrix}-\frac{1}{2} & \frac{1}{4} & -\frac{3}{4}\\ 0 & -\frac{1}{2} & 0\\ -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{4}\end{bmatrix}$$

Question 4 Solve the equation $(x+1)(x+2)(x+3)(x+4)=24$

Solution $(x+1)(x+2)(x+3)(x+4)=24$

$\Rightarrow (x^2+3x+2)(x+3)(x+4)=24 \qquad \Rightarrow (x^3+6x^2+11x+6)(x+4)=24$

$\Rightarrow x^4+6x^3+11x^2+6x+4x^3+24x^2+44x+24=24$

$\Rightarrow x^4+10x^3+35x^2+50x+24-24=24-24 \qquad \Rightarrow x^4+10x^3+35x^2+50x=0$

$\Rightarrow x(x^3+10x^2+35x+50)=0$

$\Rightarrow x^3+10x^2+35x+50=0$ or $x=0$

Now $x^3+10x^2+35x+50=0$

$\Rightarrow (x+5)(x^2+5x+10)=0$

$\Rightarrow x+5=0$ *or* $x^2+5x+10=0$

$\Rightarrow x=-5$ *or* $x^2+5x+10=0$

Now $x^2+5x+10=0$

$$\Rightarrow x=\frac{-5\pm\sqrt{25-40}}{2} \Rightarrow x=\frac{-5\pm\sqrt{15}i}{2}$$

Hence S.Set = $\left\{-5,0,\frac{-5\pm\sqrt{15}i}{2}\right\}$

Question 5 Find four members in A.P. Whose sum is 32 and the sums of whose squares is 276

Solution.

Consider four numbers $a-3d,\ a-d,\ a+d,\ a+3d$ are in A.P.

Then according to first condition

$a-3d+a-d+a+d+a+3d=32 \Rightarrow 4a=32 \Rightarrow \boxed{a=8}$

According to second condition

$(a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2=32$

$a^2-6ad+9d^2+a^2-2ad+d^2+a^2+2ad+d^2+a^2+6ad+9d^2=276$

$4a^2+20d^2=276$

Put $a=8$ in above

$4(8)^2+20d^2=276 \Rightarrow 256+20d^2=276$

$\Rightarrow 20d^2=276-256 \Rightarrow 20d^2=20 \Rightarrow d^2=1 \Rightarrow d=\pm1$.

When $a=8,\ d=1$

$a-3d=8-3(1)=8-3=5$

$a-d=8-1=7$

$a+d=8+1=9$

$a+3d=8+3(1)=8+3=11$

When $a=8,\ d=-1$

$a-3d=8-3(-1)=8+3=11$

$a-d=8+1=9$

$a+d=8+(-1)=8-1=7$

$a+3d=8+3(-1)=8-3=5$

Hence 5,7,9,11 or 11,9,7,5 are required numbers.

Question 6 If $y=\frac{2}{5}+\frac{1.3}{2!}\left(\frac{2}{5}\right)^2+\frac{1.3.5}{3!}\left(\frac{2}{5}\right)^3+...$ then prove that $y^2+2y-4=0$

Solution

We have given $y=\frac{2}{5}+\frac{1\cdot 3}{2!}\left(\frac{2}{5}\right)^2+\frac{1\cdot 3\cdot 5}{3!}\left(\frac{2}{5}\right)^3+\ldots$

Adding 1 on both sides

$$1+y=1+\frac{2}{5}+\frac{1\cdot 3}{2!}\left(\frac{2}{5}\right)^2+\frac{1\cdot 3\cdot 5}{3!}\left(\frac{2}{5}\right)^3+\ldots$$

Let the given series be identical with

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\ldots$$

This implies

$$nx=\frac{2}{5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$\frac{n(n-1)}{2!}x^2=\frac{1\cdot 3}{2!}\left(\frac{2}{5}\right)^2 \quad \ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) $nx=\frac{2}{5} \quad \Rightarrow \quad x=\frac{2}{5n} \quad \ldots\ldots\ldots\ldots\ldots \text{(iii)}$

Putting value of x in (ii)

$$\frac{n(n-1)}{2!}\left(\frac{2}{5n}\right)^2=\frac{1\cdot 3}{2!}\left(\frac{2}{5}\right)^2 \quad \Rightarrow \quad \frac{n(n-1)}{2}\left(\frac{4}{25n^2}\right)=\frac{3}{2}\left(\frac{4}{25}\right)$$

$$\Rightarrow \frac{n-1}{n}=3 \quad \Rightarrow n-1=3n \quad \Rightarrow n-3n=1.$$

$$\Rightarrow -2n=1 \quad \Rightarrow \boxed{n=-\frac{1}{2}}$$

Putting value of n in equation (iii)

$$x=\frac{2}{5\left(-\frac{1}{2}\right)} \quad \Rightarrow \boxed{x=-\frac{4}{5}}$$

So $(1+x)^n=\left(1-\frac{4}{5}\right)^{-\frac{1}{2}} \quad =\left(\frac{1}{5}\right)^{-\frac{1}{2}} \quad =(5)^{\frac{1}{2}} \quad =\sqrt{5}$

This implies

$$1+y=\sqrt{5}$$

On squaring both sides

$$(1+y)^2=\left(\sqrt{5}\right)^2$$

$$\Rightarrow 1+2y+y^2=5 \quad \Rightarrow 1+2y+y^2-5=0$$

$$\Rightarrow y^2+2y-4=0$$ Proved.

Question 7 If α, β, γ are the angles of ΔABC , then prove $\tan\alpha + \tan\beta + \tan\gamma = \text{an}\,\alpha\tan\beta\tan\gamma$.

Solution

As α, β, γ are the angles of ΔABC,

$\therefore\ \alpha + \beta + \gamma = 180^\circ \quad \Rightarrow \alpha + \beta = 180^\circ - \gamma$

$\therefore\ \tan(\alpha + \beta) = \tan(180^\circ - \gamma)$

$$\Rightarrow\ \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = -\tan\gamma$$

$\Rightarrow\ \tan\alpha + \tan\beta = -\tan\gamma + \tan\alpha\tan\beta\tan\gamma$

$\Rightarrow\ \tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma$ Proved.

Question 8 Prove that $\cos 20^\circ\ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \dfrac{1}{16}$

Solution L.H.S $= \dfrac{\sin 3\theta}{\cos\theta} + \dfrac{\cos 3\theta}{\sin\theta} = \dfrac{\sin 3\theta\sin\theta + \cos 3\theta\cos\theta}{\sin\theta\cos\theta}$

$$= \frac{\cos(3\theta - \theta)}{\sin\theta\cos\theta} = \frac{\cos 2\theta}{\sin\theta\cos\theta} = \frac{2\cos 2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{2\cos 2\theta}{\sin 2\theta} = 2\cot 2\theta = \text{R.H.S}$$