Mathematics HSSC-I (Annual 2017)

Federal Board of Intermediate and Secondary Education, Islamabad

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Section A (Marks 20)

Question 1: Circle the correct option. i.e. A, B, C, D, each part carries one mark.

1. Reference angle lies in quadrant:

   (A) IV (B) I (C) II (D) III

2. The set $0, 1$ is closed w.r.t:

   (A) Division (B) Addition (C) Subtraction (D) Multiplication

3. $\sqrt{-5}$ belong to the set of:

   (A) Rational numbers (B) Real numbers (C) Complex numbers (D) Integers

4. The set of integers Z is group under

   (A) Addition (B) Subtraction (C) Division (D) Multiplication

5. A declarative statement which may be true or false but not both is called:

   (A) Tautology (B) Proposition (C) Deduction (D) Induction

6. If $A = \begin{bmatrix} x & 1 \\ 1 & 1 \end{bmatrix}$ and A is singular matrix then $x =$

   (A) 3 (B) 0 (C) 1 (D) 2

7. The product of all forth roots of unity is:

   (A) 2 (B) 1 (C) 0 (D) -1

8. A fraction in which the degree of numerator is less than the degree of denominator is called:

   (A) Algebraic relation (B) Improper fraction (C) Proper fraction (D) Equation

9. $1^3 + 2^3 + 3^3 + ... + n^3 =$

   (A) $\dfrac{n^2(n+1)^2}{4}$ (B) $\dfrac{n(n+1)(2n+1)}{6}$ (C) $\dfrac{n(n+1)}{2}^3$ (D) $\dfrac{n(n+1)(2n+1)}{3}$

10. An infinite geometric series converges only if:

    (A) $r = -1$ (B) $r = 1$ (C) $|r| > 1$ (D) $|r| < 1$

11. An event E is said to be sure if:

(A) $P(E) = \infty$ (B) $P(E) = 0$ (C) $P(E) = 1$ (D) $P(E) = -1$

12. Numbers of terms in the expansion of $(a + b)^n$ is:

(A) $n^2 + 1$ (B) $n + 1$ (C) $n - 1$ (D) n

13. The sun of odd coefficients in the expansion of $(1 + x)^n$ is:

(A) 2^{n+1} (B) n^2 (C) 2^n (D) 2^{n-1}

14. $tan(\frac{3\pi}{2} - \theta) =$

(A) $-cot\theta$ (B) $tan\theta$ (C) $-tan\theta$ (D) $cot\theta$

15. If $cot\theta < 0$ and $cos\theta > 0$, then the terminal arm of angle lies in the quadrant:

(A) IV (B) I (C) II (D) III

16. $sin3\alpha =$

(A) $4sin\alpha - 3sin^3\alpha$ (B) $4cos^3\alpha - 3cos\alpha$ (C) $3cos^3\alpha - 4cos\alpha$ (D) $3sin\alpha - 4sin^3\alpha$

17. The period of $3sin3x$ is:

(A) 6π (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$

18. The range of $cotx$ is:

(A) R^- (B) R (C) $[-1, 1]$ (D) R^+

19. The circle passes through the vertices of the triangle is called:

(A) Unit circle (B) Circum circle (C) In-circle (D) Escribed circle

20. The domain of principal cosine function is:

(A) $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (B) $[0, \frac{\pi}{2}]$ (C) $[0, \pi]$ (D) $[0, \frac{3\pi}{2}]$

ANSWERS

1. A 2. B 3. C 4. A 5. B 6. C 7. D 8. C 9. A 10. D 11. C
12. B 13. C 14. D 15. A 16. D 17. D 18. B 19. B 20. B

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Section B (Marks 40)

Question 2: Attempt any TEN parts. All parts carry equal marks. (10 × 4 = 40)

(i) Specify by using De Moivre's theorem $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3$.

(ii) Give logical prove of the theorem $(A\cup B)'=A'\cap B'$.

(iii) Without expansion verify that $\begin{vmatrix} -a & 0 & c \\ 0 & a & -b \\ b & -c & 0 \end{vmatrix}=0$.

(iv) Find the value of a and b if -2 and 2 are the roots of the polynomial x^3-4x^2+ax+b.

(v) Resolve into partial fraction $\frac{x^2+1}{x^3+1}$.

(vi) Insert four harmonic between $\frac{7}{3}$ and $\frac{7}{11}$.

(vii) Find the values of n and r, when ${}^{n-1}C_{r-1} : {}^{n}C_{r} : {}^{n+1}C_{r+1} = 3:6:11$

(viii) Show that the middle term of $(1+x)^{2n}$ is $\frac{1.3.5...(2n-1)}{n!}2^n x^n$.

(ix) Prove that $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\tan\theta+\sec\theta$.

(x) Without using table or calculator prove that $\sin 19^\circ \cos 11^\circ + \sin 71^\circ \sin 11^\circ = \frac{1}{2}$

(xi) Find the period of cosine function.

(xii) The sides of the triangle are x^2+x+1, $2x+1$ and x^2-1. Prove that the greatest angle of the triangle is 120°.

(xiii) Show that $\cos^{-1}(-x)=\pi-\cos^{-1}(x)$.

(xiv) Solve $\sin x+\cos x=0$.

Solutions

(i)
$$\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3=\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^2\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$
$$=\left[\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}i\right)^2+2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right)\right]\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$
$$=\left[\frac{1}{4}-\frac{3}{4}-\frac{\sqrt{3}}{2}i\right]\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=\left[-\frac{2}{4}-\frac{\sqrt{3}}{2}i\right]\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

$$=\left[-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right]\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=\left(-\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}i\right)^2=\frac{1}{4}-\frac{3}{4}i^2$$

$$=\frac{1}{4}+\frac{3}{4}=\frac{4}{4}=1$$

(ii) $(A\cup B)'=A'\cap B'$

The corresponding formula of logic is

$$\sim(p\vee q)=\sim p\wedge\sim q$$

We construct truth table of the two sides.

p	q	$\sim p$	$\sim q$	$p\vee q$	$\sim(p\vee q)$	$\sim p\wedge\sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

The last two columns of the above table show that $\sim(p\wedge q)=\sim p\vee\sim q$.

and hence $(A\cup B)'=A'\cap B'$.

(iii) L.H.S $=\begin{vmatrix}-a & 0 & c\\ 0 & a & -b\\ b & -c & 0\end{vmatrix}$

Multiply c_1 by c, c_2 by b and c_3 by a, we have

$$\text{L.H.S}=\frac{1}{abc}\begin{vmatrix}-ac & 0 & ac\\ 0 & ab & -ab\\ bc & -bc & 0\end{vmatrix}$$

$$=\frac{1}{abc}\begin{vmatrix}0 & 0 & ac\\ 0 & ab & -ab\\ 0 & -bc & 0\end{vmatrix} \qquad \text{by } c_1+(c_2+c_3)$$

$$=\frac{1}{abc}(0) \qquad \text{(Since all entries of } c_1 \text{ are zero)}$$

= R.H.S

(iv) Suppose $f(x)=x^3-4x^2+ax+b$

By synthetic division, we have

$$\begin{array}{r|llll} -2 & 1 & -4 & a & b\\ & \downarrow & -2 & 12 & -2a-24\\ \hline 2 & 1 & -6 & a+12 & \underline{|b-2a-24}\\ & \downarrow & 2 & -8 & \\ \hline & 1 & -4 & \underline{|a+4} & \end{array}$$

Since -2 and 2 are the roots of $f(x)$ then reminder is equal to zero, that is,

$a+4=0 \quad \Rightarrow \boxed{a=-4}$

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$b-2a-24=0 \quad \Rightarrow b-2(-4)-24=0$

$\Rightarrow \; b+8-24=0 \;\Rightarrow\; b-16=0 \;\Rightarrow\; \boxed{b=16}$

(v) $\dfrac{x^2+1}{x^3+1} = \dfrac{x^2+1}{(x+1)(x^2-x+1)} \qquad \because \; x^3+1=(x+1)(x^2-x+1)$

Now consider

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \quad \ldots\ldots\ldots\ldots\text{(I)}$$

$\Rightarrow \; x^2+1 = A\left(x^2-x+1\right)+\left(Bx+C\right)\left(x+1\right)$

$\Rightarrow \; x^2+1 = A\left(x^2-x+1\right)+\left(Bx^2+Bx+Cx+C\right)$

$\Rightarrow \; x^2+1 = \left(A+B\right)x^2+\left(B+C-A\right)x+\left(A+C\right)$

By comparing the coefficients of x^2, x and x^0, we get

$A+B=1 \quad \ldots\ldots (1)$

$B+C-A=0 \quad \ldots\ldots (2)$

$A+C=1 \quad \ldots\ldots (3)$

From (3), we have

$C=1-A$

Put the value of C in (2).

$-2A+B=-1 \quad \ldots\ldots (4)$

Subtract (4) from (1)

$$\begin{array}{rcl} A+B &=& 1 \\ -2A+B &=& -1 \\ \scriptstyle + \quad - & & \scriptstyle + \\ \hline 3A &=& 2 \end{array} \quad \Rightarrow \quad \boxed{A=\frac{2}{3}}$$

Put the value of A in (1), we have

$\dfrac{2}{3}+B=1 \;\Rightarrow\; B=1-\dfrac{2}{3} \;\Rightarrow\; \boxed{B=\dfrac{1}{3}}$

Put the value of A in (3), we have

$\dfrac{2}{3}+C=1 \;\Rightarrow\; C=1-\dfrac{2}{3} \;\Rightarrow\; \boxed{C=\dfrac{1}{3}}$

Put the values of A, B *and* C in (I), We have

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{2}{3\left(x+1\right)} + \frac{\left(x+1\right)}{3\left(x^2-x+1\right)}.$$

(vi) Let H_1, H_2, H_3, H_4 are four H.Ms between $\dfrac{7}{3}$ and $\dfrac{7}{11}$.

Then $\dfrac{7}{3}, H_1, H_2, H_3, H_4, \dfrac{7}{11}$ are H.P.

So $\dfrac{3}{7}, \dfrac{1}{H_1}, \dfrac{1}{H_2}, \dfrac{1}{H_3}, \dfrac{1}{H_4}, \dfrac{11}{7}$ are A.P.

Here $a_1=\dfrac{3}{7}$ and $a_6=\dfrac{11}{7}$

$$\Rightarrow a_1 + 5d = \frac{11}{7} \quad \Rightarrow \frac{3}{7} + 5d = \frac{11}{7}$$

$$\Rightarrow 5d = \frac{11}{7} - \frac{3}{7}$$

Now $\frac{1}{H_1} = a_2 = a_1 + d = \frac{3}{7} + \frac{8}{35} = \frac{15+8}{35} = \frac{23}{35} \qquad \Rightarrow H_1 = \frac{35}{23}$

$$\frac{1}{H_2} = a_3 = a_1 + 2d = \frac{3}{7} + \frac{16}{35} = \frac{15+16}{35} = \frac{31}{35} \qquad \Rightarrow H_2 = \frac{35}{31}$$

$$\frac{1}{H_3} = a_4 = a_1 + 3d = \frac{3}{7} + \frac{24}{35} = \frac{15+24}{35} = \frac{39}{35} \qquad \Rightarrow H_3 = \frac{35}{39}$$

$$\frac{1}{H_4} = a_5 = a_1 + 4d = \frac{3}{7} + \frac{32}{35} = \frac{15+32}{35} = \frac{47}{35} \qquad \Rightarrow H_3 = \frac{35}{47}$$

Hence $\frac{35}{23}, \frac{35}{31}, \frac{35}{39}, \frac{35}{47}$ are H.Ms between $\frac{7}{3}$ and $\frac{7}{11}$.

(vii) $^{n-1}C_{r-1} : {}^{n}C_r : {}^{n+1}C_{r+1} = 3 : 6 : 11$

First consider

$$^{n-1}C_{r-1} : {}^{n}C_r = 3 : 6$$

$$\Rightarrow \frac{(n-1)!}{(n-1-r+1)!\,(r-1)!} : \frac{n!}{(n-r)!\,r!} = 3 : 6$$

$$\Rightarrow \frac{(n-1)!}{(n-r)!\,(r-1)!} : \frac{n!}{(n-r)!\,r!} = 3 : 6 \qquad \Rightarrow \frac{\frac{(n-1)!}{(n-r)!\,(r-1)!}}{\frac{n!}{(n-r)!\,r!}} = \frac{3}{6}$$

$$\Rightarrow \frac{(n-1)!}{(n-r)!\,(r-1)!} \times \frac{(n-r)!\,r!}{n!} = \frac{1}{2}$$

$$\Rightarrow \frac{(n-1)!}{(r-1)!} \times \frac{r!}{n!} = \frac{1}{2} \qquad \Rightarrow n = 2r \text{ .... (i)}$$

Now consider $^{n}C_r : {}^{n+1}C_{r+1} = 6 : 11$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} : \frac{(n+1)!}{(n+1-r-1)!\,(r+1)!} = 6 : 11$$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} : \frac{(n+1)!}{(n-r)!\,(r+1)!} = 6 : 11$$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} : \frac{(n+1)!}{(n+1-r-1)!\,(r+1)!} = 6 : 11 \qquad \Rightarrow \frac{\frac{n!}{(n-r)!\,r!}}{\frac{(n+1)!}{(n-r)!\,(r+1)!}} = \frac{6}{11}$$

$$\Rightarrow \frac{n!}{(n-r)!\,r!}\times\frac{(n-r)!\,(r+1)!}{(n+1)!}=\frac{6}{11} \qquad \Rightarrow \frac{n!}{r!}\times\frac{(r+1)!}{(n+1)!}=\frac{6}{11}$$

$$\Rightarrow \frac{n!}{r!}\times\frac{(r+1)\,r!}{(n+1)\,n!}=\frac{6}{11} \qquad \Rightarrow \frac{(r+1)}{(n+1)}=\frac{6}{11}$$

$$\Rightarrow 11(r+1)=6(n+1)$$

$$\Rightarrow 11(r+1)=6(2r+1) \qquad \because\ n=2r$$

$$\Rightarrow 11r+11=12r+6$$

$$\Rightarrow 11r-12r=6-11 \qquad \Rightarrow -r=-5 \quad \Rightarrow \boxed{r=5}$$

Putting value of r in equation (i)

$$\boxed{n=10}$$

(viii) Since $2n$ is even so the middle term is $\dfrac{2n+2}{2}=n+1$ and

$$a=1\ ,\quad x=x\ ,\quad n=2n\ ,\quad r+1=n+1 \quad \Rightarrow \quad r=n$$

Now $T_{r+1}=\binom{n}{r}a^{n-r}x^{r}$

$$\Rightarrow T_{n+1}=\binom{2n}{n}(1)^{2n-n}x^{n}$$

$$\Rightarrow T_{n+1}=\frac{(2n)!}{(2n-n)!\cdot n!}(1)^{n}x^{n}=\frac{(2n)!}{n!\cdot n!}x^{n}$$

$$=\frac{2n(2n-1)(2n-2)(2n-3)(2n-4)\cdot\ldots\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{n!\cdot n!}x^{n}$$

$$=\frac{\left[2n\,(2n-2)(2n-4)\cdot\ldots\cdot 4\cdot 2\right]\left[(2n-1)(2n-3)\cdot\ldots\cdot 5\cdot 3\cdot 1\right]}{n!\cdot n!}x^{n}$$

$$=\frac{2^{n}\left[n\,(n-1)(n-2)\cdot\ldots\cdot 2\cdot 1\right]\left[(2n-1)(2n-3)\cdot\ldots\cdot 5\cdot 3\cdot 1\right]}{n!\cdot n!}x^{n}$$

$$=\frac{2^{n}\,n!\,\left[(2n-1)(2n-3)\cdot\ldots\cdot 5\cdot 3\cdot 1\right]}{n!\cdot n!}x^{n}$$

$$=\frac{2^{n}\left[1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)\right]}{n!}x^{n}$$

$$=\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}{n!}\,2^{n}x^{n}$$

(ix) L.H.S. $=\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$

$$=\frac{\dfrac{\sin\theta}{\cos\theta}+\dfrac{1}{\cos\theta}-1}{\dfrac{\sin\theta}{\cos\theta}-\dfrac{1}{\cos\theta}+1} \quad = \frac{\dfrac{\sin\theta+1-\cos\theta}{\cos\theta}}{\dfrac{\sin\theta-1+\cos\theta}{\cos\theta}}$$

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$$=\frac{\sin\theta+1-\cos\theta}{\sin\theta-1+\cos\theta}=\frac{\sin\theta+1-\cos\theta}{\sin\theta-1+\cos\theta}\times\frac{\sin\theta+1+\cos\theta}{\sin\theta+1+\cos\theta}$$

$$=\frac{\sin^2\theta+2\sin\theta+1-\left(1-\sin^2\theta\right)}{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}=\frac{\sin^2\theta+2\sin\theta+1-1+\sin^2\theta}{1+2\sin\theta\cos\theta-1}$$

$$=\frac{2\sin^2\theta+2\sin\theta}{2\sin\theta\cos\theta}=\frac{2\sin\theta\left(\sin\theta+1\right)}{2\sin\theta\cos\theta}$$

$$=\frac{\left(\sin\theta+1\right)}{\cos\theta}=\tan\theta+\sec\theta=R.H.S$$

(x) $\sin 19^\circ\cos 11^\circ+\sin 71^\circ\sin 11^\circ=\frac{1}{2}$

$L.H.S.=\sin 19^\circ\cos 11^\circ+\sin 71^\circ\sin 11^\circ$

$$==\frac{1}{2}\left[2\sin 19^\circ\cos 11^\circ+2\sin 71^\circ\sin 11^\circ\right]$$

$$=\frac{1}{2}\left[\left\{\sin\left(19^\circ+11^\circ\right)+\sin\left(19^\circ-11^\circ\right)\right\}-\left\{\cos\left(71^\circ+11^\circ\right)-\cos\left(71^\circ-11^\circ\right)\right\}\right]$$

$$=\frac{1}{2}\left[\sin 30^\circ+\sin 8^\circ-\cos 82^\circ+\cos 60^\circ\right]$$

$$=\frac{1}{2}\left[\frac{1}{2}+\sin 8^\circ-\cos 82^\circ+\frac{1}{2}\right]$$

$$=\frac{1}{2}\left[\frac{1}{2}+\sin 8^\circ-\cos\left(90^\circ-8^\circ\right)+\frac{1}{2}\right]\quad\left(\because\cos 82^\circ=\cos\left(90^\circ-8^\circ\right)=\sin 8^\circ\right)$$

$$=\frac{1}{2}\left[\frac{1}{2}+\sin 8^\circ-\sin 8^\circ+\frac{1}{2}\right]$$

$$=\frac{1}{2}\left[\frac{1}{2}+\frac{1}{2}\right]=\frac{1}{2}=R.H.S.$$

(xi) Suppose P is the period of cosine function, then

$\cos\left(\theta+P\right)=\cos\theta\quad\forall\theta\in\mathbb{R}$ …. (1)

Now put $\theta=0,$ we have

$\cos\left(0+P\right)=\cos 0\quad\Rightarrow\quad\cos P=1$

$\Rightarrow\quad P=0,\pm\pi,\pm2\pi,...$

(i) If $P=\pi$, then from (1)

$\cos\left(\theta+\pi\right)=\cos\theta$ (not true)

$\because\quad\cos\left(\theta+\pi\right)=\cos\left(2\cdot\frac{\pi}{2}+\theta\right)=-\cos\theta$

$\therefore\ \pi$ is not the period of $\cos\theta$

(ii) If $P=2\pi$, then from (1)

$\cos\left(\theta+2\pi\right)=\cos\theta$ (true)

$\therefore\ 2\pi$ is the period of $\cos\theta$

(xii) Let $a = x^2 + x + 1, \quad b = 2x + 1, \quad c = x^2 - 1$

Since $a = x^2 + x + 1$ is greatest side. Therefore α is the greatest angle.

Now $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$$= \frac{(2x+1)^2 + (x^2-1)^2 - (x^2+x+1)^2}{2(2x+1)(x^2-1)}$$

$$= \frac{4x^2 + 4x + 1 + x^4 - 2x^2 + 1 - (x^4 + x^2 + 1 + 2x^3 + 2x + 2x^2)}{2(2x^3 - 2x + x^2 - 1)}$$

$$= \frac{4x^2 + 4x + 1 + x^4 - 2x^2 + 1 - x^4 - x^2 - 1 - 2x^3 - 2x - 2x^2}{2(2x^3 - 2x + x^2 - 1)}$$

$$= \frac{-2x^3 - x^2 + 2x + 1}{2(2x^3 - 2x + x^2 - 1)} = \frac{-1(2x^3 - 2x + x^2 - 1)}{2(2x^3 - 2x + x^2 - 1)} = \frac{-1}{2}$$

$$\Rightarrow \quad \cos\alpha = -\frac{1}{2} \quad \Rightarrow \quad \alpha = \cos^{-1}\left(-\frac{1}{2}\right) \quad \Rightarrow \quad \alpha = 120^\circ$$

(xiii) $\cos^{-1}(-x) = \pi - \cos^{-1} x$

Suppose $y = \pi - \cos^{-1} x$ … (i)

$$\Rightarrow \pi - y = \cos^{-1} x \quad \Rightarrow \cos(\pi - y) = x$$

$$\Rightarrow \cos\pi\cos y + \sin\pi\sin y) = x \quad \Rightarrow (-1)\cos y + (0)\sin y) = x$$

$$\Rightarrow -\cos y + 0 = x \quad \Rightarrow -\cos y = x$$

$$\Rightarrow \cos y = -x \quad \Rightarrow y = \cos^{-1}(-x) \; \ldots \text{(ii)}$$

From (i) and (ii)

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

(xiv) $\sin x + \cos x = 0$

$\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\cos x} = 0 \quad (\textit{Dividing by } \cos x \neq 0)$

$\tan x + 1 = 0 \quad \Rightarrow \tan x = -1$

$\because$ $\tan x$ is –ve in II and IV Quadrants with the reference angle $x = \dfrac{\pi}{4}$

$\therefore$ $x = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$, *where* $x \in [0, \pi]$

As π is the period of $\tan x$

$\therefore$ General value of x is $\dfrac{3\pi}{4} + n\pi, \quad n \in \mathbb{Z}$

Hence Solution set $= \left\{\dfrac{3\pi}{4} + n\pi\right\}, n \in \mathbb{Z}.$

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Section C (Marks 40)

Note: Attempt any five questions. All question carries equal marks.

Question 3 Use matrix to solve the following system

$$x+y=2$$
$$2x-z=1$$
$$2y-3z=-1$$

Solution $A=\begin{bmatrix}1 & 1 & 0\\ 2 & 0 & -1\\ 0 & 2 & -3\end{bmatrix}$ $B=\begin{bmatrix}2\\ 1\\ -1\end{bmatrix}$

$$|A|=a_{11}\,A_{11}+a_{12}\,A_{12}+a_{13}\,A_{13}$$
$$=1(0+2)-1(-6-0)+0=2+6=8$$

Cofactors of $A=\begin{bmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{bmatrix}$

$A_{11}=(-1)^{1+1}\begin{vmatrix}0 & -1\\ 2 & -3\end{vmatrix}=(-1)^{2}(0+2)=2$, $A_{12}=(-1)^{1+2}\begin{vmatrix}2 & -1\\ 0 & -3\end{vmatrix}=(-1)^{3}(-6)=6$

$A_{13}=(-1)^{1+3}\begin{vmatrix}2 & 0\\ 0 & 2\end{vmatrix}=(-1)^{4}(4)=4$, $A_{21}=(-1)^{2+1}\begin{vmatrix}1 & 0\\ 2 & -3\end{vmatrix}=(-1)^{3}(-3)=3$

$A_{22}=(-1)^{2+2}\begin{vmatrix}1 & 0\\ 0 & -3\end{vmatrix}=(-1)^{4}(-3)=-3$, $A_{23}=(-1)^{2+3}\begin{vmatrix}1 & 1\\ 0 & 2\end{vmatrix}=(-1)^{5}(2)=-2$

$A_{31}=(-1)^{3+1}\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}=(-1)^{4}(-1)=-1$, $A_{32}=(-1)^{3+2}\begin{vmatrix}1 & 0\\ 2 & -1\end{vmatrix}=(-1)^{5}(-1)=1$

$A_{33}=(-1)^{3+3}\begin{vmatrix}1 & 1\\ 2 & 0\end{vmatrix}=(-1)^{6}(-2)=-2$

Cofactors of $A=\begin{bmatrix}2 & 6 & 4\\ 3 & -3 & -2\\ -1 & 1 & -2\end{bmatrix}$

$$Adj\,A=(cofactor\ of\ A)^{t}=\begin{bmatrix}2 & 3 & -1\\ 6 & -3 & 1\\ 4 & -2 & -2\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}Adj\,A=\frac{1}{8}\begin{bmatrix}2 & 3 & -1\\ 6 & -3 & 1\\ 4 & -2 & -2\end{bmatrix}$$

$$X=A^{-1}B\ \Rightarrow\ \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{8}\begin{bmatrix}2 & 3 & -1\\ 6 & -3 & 1\\ 4 & -2 & -2\end{bmatrix}\begin{bmatrix}2\\ 1\\ -1\end{bmatrix}$$

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$$=\frac{1}{8}\begin{bmatrix}4+3+1\\12-3-1\\8-2+2\end{bmatrix} \quad =\frac{1}{8}\begin{bmatrix}8\\8\\8\end{bmatrix} \quad =\frac{8}{8}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix} \quad \Rightarrow \quad x=1, \quad y=1, \quad z=1$$

Question 4 Show that the roots of the equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are real. Also show that the roots will be equal only if $a=b=c$.

Solution

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$$

Simplyfying

$$x^2-bx-ax+ab+x^2-cx-bx+bc+x^2-ax-cx+ac=0$$

$$3x^2-2ax-2bx-2cx+ab+bc+ac=0$$

$$3x^2-2(a+b+c)x+ab+bc+ac=0$$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-[-2(a+b+c)]\pm\sqrt{[-2(a+b+c)]^2-4(3)(ab+bc+ac)}}{2(3)}$$

$$x=\frac{2(a+b+c)\pm 2\sqrt{(a+b+c)^2-3(ab+bc+ac)}}{6}$$

$$x=\frac{(a+b+c)\pm\sqrt{a^2+b^2+c^2+2ab+2bc+2ca-3ab-3bc-3ca}}{3}$$

$$x=\frac{(a+b+c)\pm\sqrt{a^2+b^2+c^2-ab-bc-ca}}{3}$$

$$\left\{\frac{(a+b+c)\pm\sqrt{a^2+b^2+c^2-ab-bc-ca}}{3}\right\}$$

When $a=b=c$ then

$$x=\frac{3a\pm\sqrt{3a^2-3a^2}}{3}\Rightarrow x=\frac{3a\pm\sqrt{0}}{3}\Rightarrow x=a$$

Question 5 Show that the sum of n A.Ms between a and b is equal to n times their A.M.

Solution

Let $A_1, A_2, A_3, \ldots, A_n$ be the n A.Ms between a & b

then $a, A_1, A_2, A_3, \ldots, A_n, b$ are in A.P

Here $a_1 = a$

$a_{n+2} = b$

$\Rightarrow a_1 + (n+2-1)d = b$

$\Rightarrow a_1 + (n+1)d = b$

$\Rightarrow (n+1)d = b - a$

$\Rightarrow d = \frac{b-a}{n+1}$

Now

$A_1 = a_2 = a_1 + d = a + \frac{b-a}{n+1}$

$A_2 = a_3 = a_1 + 2d = a + 2\left(\frac{b-a}{n+1}\right)$

$A_3 = a_4 = a_1 + 3d = a + 3\left(\frac{b-a}{n+1}\right)$

$\vdots$

$A_n = a_{n+1} = a_1 + nd = a + n\left(\frac{b-a}{n+1}\right)$

Now Sum of n A.Ms

$= A_1 + A_2 + A_3 + \cdots + A_n$

$= a + \frac{b-a}{n+1} + a + 2\left(\frac{b-a}{n+1}\right) + a + 3\left(\frac{b-a}{n+1}\right) + \cdots + a + n\left(\frac{b-a}{n+1}\right)$

$= (a+a+a+\cdots+a) + \frac{b-a}{n+1} + 2\left(\frac{b-a}{n+1}\right) + 3\left(\frac{b-a}{n+1}\right) + \cdots + n\left(\frac{b-a}{n+1}\right)$

$= na + \frac{b-a}{n+1}(1+2+3+\cdots+n)$

$a_1 = 1,\ d = 2-1 = 1,\ n = n$

$= na + \frac{b-a}{n+1}\left[\frac{n}{2}\{2(1) + (n-1)(1)\}\right]$

$= na + \frac{b-a}{n+1}\left\{\frac{n}{2}(2+n-1)\right\}$

$= na + \frac{b-a}{(n+1)}\left[\frac{n}{2}(n+1)\right]$

$= na + (b-a)\left(\frac{n}{2}\right)$

$= n\left[a + \frac{b-a}{2}\right]$

$= n\left[\frac{2a+b-a}{2}\right]$

$= n\left(\frac{a+b}{2}\right)$

$= n$ (A.Ms between a & b)

Hence sum of n A.Ms between a & b is n times their A.Ms.

Question 6 Expand $\dfrac{(4+2x)^{\frac{1}{2}}}{2-x}$ up to 4 terms.

Solution.

$$\frac{(4+2x)^{\frac{1}{2}}}{2-x} = (4+2x)^{\frac{1}{2}}(2-x)^{-1} = (4)^{\frac{1}{2}}\left(1+\frac{2x}{4}\right)^{\frac{1}{2}}(2)^{-1}\left(1-\frac{x}{2}\right)^{-1}$$

$$= (4)^{\frac{1}{2}}\left(1+\frac{x}{2}\right)^{\frac{1}{2}}(2)^{-1}\left(1-\frac{x}{2}\right)^{-1} = 2\left(1+\frac{x}{2}\right)^{\frac{1}{2}}\frac{1}{2}\left(1-\frac{x}{2}\right)^{-1}$$

$$=\left(1+\frac{x}{2}\right)^{\frac{1}{2}}\left(1-\frac{x}{2}\right)^{-1} \quad =\left(1+\frac{x}{2}\right)^{\frac{1}{2}}\left(1-\frac{x}{2}\right)^{-1}$$

$$=\left(1+\frac{1}{2}\left(\frac{x}{2}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{x}{2}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{x}{2}\right)^3+\ldots\right)$$

$$\times\left(1+(-1)\left(-\frac{x}{2}\right)+\frac{(-1)(-1-1)}{2!}\left(-\frac{x}{2}\right)^2+\frac{(-1)(-1-1)(-1-2)}{3!}\left(-\frac{x}{2}\right)^3+\ldots\right)$$

$$=\left(1+\frac{x}{4}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}\left(\frac{x^2}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3\cdot 2}\left(\frac{x^3}{8}\right)+\ldots\right)$$

$$\times\left(1+\frac{x}{2}+\frac{(-1)(-2)}{2}\left(\frac{x^2}{4}\right)+\frac{(-1)(-2)(-3)}{3\cdot 2}\left(-\frac{x^3}{8}\right)+\ldots\right)$$

$$=\left(1+\frac{x}{4}-\frac{x^2}{32}+\frac{x^3}{128}+\ldots\right)\times\left(1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}+\ldots\right)$$

$$=1+\left(\frac{x}{4}+\frac{x}{2}\right)+\left(-\frac{x^2}{32}+\frac{x^2}{8}+\frac{x^2}{4}\right)+\left(\frac{x^3}{128}-\frac{x^3}{64}+\frac{x^3}{16}+\frac{x^3}{8}\right)+\ldots$$

$$=1+\frac{3x}{4}+\frac{11x^2}{32}+\frac{23x^3}{128}+\ldots$$

Question 7 Prove that $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4\pi}{9}=\frac{3}{16}$

Solution L.H.S $=\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{\pi}{3}\sin\frac{4\pi}{9}$

$$=\sin\frac{180^\circ}{9}\sin\frac{2(180^\circ)}{9}\sin\frac{(180^\circ)}{3}\sin\frac{4(180^\circ)}{9} \quad \because \ \pi=180^\circ$$

$$=\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ \quad =\sin 20^\circ \sin 40^\circ \frac{\sqrt{3}}{2}\sin 80^\circ$$

$$=\frac{\sqrt{3}}{2}\sin 80^\circ \sin 40^\circ \sin 20^\circ \quad =-\frac{\sqrt{3}}{4}\left(-2\sin 80^\circ \sin 40^\circ\right)\sin 20^\circ$$

$$=-\frac{\sqrt{3}}{4}\left(\cos(80^\circ+40^\circ)-\cos(80^\circ-40^\circ)\right)\sin 20^\circ$$

$$=-\frac{\sqrt{3}}{4}\left(\cos 120^\circ-\cos 40^\circ\right)\sin 20^\circ \quad =-\frac{\sqrt{3}}{4}\left(-\frac{1}{2}-\cos 40^\circ\right)\sin 20^\circ$$

$$=\frac{\sqrt{3}}{8}\sin 20^\circ+\frac{\sqrt{3}}{4}\cos 40^\circ \sin 20^\circ \quad =\frac{\sqrt{3}}{8}\sin 20^\circ+\frac{\sqrt{3}}{8}\left(2\cos 40^\circ \sin 20^\circ\right)$$

$$=\frac{\sqrt{3}}{8}\sin 20^\circ+\frac{\sqrt{3}}{8}\left(\sin(40+20)-\sin(40-20)\right)$$

$$=\frac{\sqrt{3}}{8}\sin 20^\circ+\frac{\sqrt{3}}{8}\left(\sin 60^\circ-\sin 20^\circ\right)=\frac{\sqrt{3}}{8}\sin 20^\circ+\frac{\sqrt{3}}{8}\left(\frac{\sqrt{3}}{2}-\sin 20^\circ\right)$$

$$=\frac{\sqrt{3}}{8}\sin 20^\circ+\frac{3}{16}-\frac{\sqrt{3}}{8}\sin 20^\circ=\frac{3}{16}\quad=\text{R.H.S}$$

Question 8 Prove that in an equilateral triangle $r:R:r_1=1:2:3$

Solution In equilateral triangle all the sides are equal so $a=b=c$

$$s=\frac{a+b+c}{2}=\frac{a+a+a}{2}=\frac{3a}{2}$$

$$s-a=\frac{3a}{2}-a=\left(\frac{3}{2}-1\right)a=\frac{1}{2}a$$

Now $\Delta=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{s(s-a)(s-a)(s-a)}=\sqrt{s(s-a)^3}$

$$=\sqrt{\frac{3a}{2}\left(\frac{1}{2}a\right)^3}=\sqrt{\frac{3a}{2}\left(\frac{a^3}{8}\right)}=\sqrt{\frac{3a^4}{16}}=\frac{\sqrt{3}\,a^2}{4}$$

Now $r=\dfrac{\Delta}{s}=\dfrac{\sqrt{3}\,a^2/4}{3a/2}=\dfrac{\sqrt{3}\,a^2}{4}\cdot\dfrac{2}{3a}=\dfrac{\sqrt{3}\,a}{6}$

$$R=\frac{abc}{4\Delta}=\frac{a\cdot a\cdot a}{4\sqrt{3}a^2/4}=\frac{a}{\sqrt{3}}=\frac{a}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}\,a}{3}$$

$$r_1=\frac{\Delta}{s-a}=\frac{\sqrt{3}a^2/4}{\frac{1}{2}a}=\frac{\sqrt{3}\,a^2}{4}\cdot\frac{2}{a}=\frac{\sqrt{3}\,a}{2}$$

$$r:R:r_1=\frac{\sqrt{3}\,a}{6}:\frac{\sqrt{3}\,a}{3}:\frac{\sqrt{3}\,a}{2}=\frac{1}{6}:\frac{1}{3}:\frac{1}{2}\qquad \div\text{ing by }\sqrt{3}\,a$$

$$=1:2:3\qquad \times\text{ing by }6$$

Question 9 Solve the equation $\operatorname{cosec} x=\sqrt{3}+\cot x$

Solution $\operatorname{cosec} x=\sqrt{3}+\cot x$

$$\Rightarrow \frac{1}{\sin x}=\sqrt{3}+\frac{\cos x}{\sin x}\quad\Rightarrow 1=\sqrt{3}\sin x+\cos x$$

$$\Rightarrow 1=\sqrt{3}\sin x+\cos x\quad\Rightarrow 1-\cos x=\sqrt{3}\sin x$$

Square on both sides, we have

$$\Rightarrow (1-\cos x)^2=\left(\sqrt{3}\sin x\right)^2\quad\Rightarrow \cos^2 x+1-2\cos x=3\sin^2 x$$

$$\Rightarrow \cos^2 x+1-2\cos x=3\left(1-\cos^2 x\right)\quad\Rightarrow \cos^2 x+1-2\cos x=3-3\cos^2 x$$

$$\Rightarrow \cos^2 x+1-2\cos x-3+3\cos^2 x=0\quad\Rightarrow 4\cos^2 x-2\cos x-2=0$$

$$\Rightarrow 2\cos^2 x-\cos x-1=0\quad\Rightarrow (2\cos x+1)(\cos x-1)=0$$

$$\Rightarrow 2\cos x+1=0\text{ or }\cos x-1=0$$

$\Rightarrow \quad \cos x = -\dfrac{1}{2} \quad$ or $\quad \cos x = 1$

(i) If $\cos x = -\dfrac{1}{2}$

Since $\cos x$ is –ve in II and III Quadrants with the reference angle $x = \dfrac{\pi}{3}$,

$x = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3} \quad \textit{and} \quad x = \pi + \dfrac{\pi}{3} = \dfrac{4\pi}{3} \quad \textit{where} \quad x \in [0, 2\pi]$

Now $x = \dfrac{4\pi}{3}$ does not satisfy the given equation (i).

$\therefore \quad x = \dfrac{2\pi}{3}$ is the only solution.

Since 2π is the period of $\cos x$, therefore $x = \dfrac{2\pi}{3} + 2n\pi, \quad n \in \mathbb{Z}$.

(ii) If $\cos x = 1$, then

$x = 0 \quad \textit{and} \quad x = 2\pi \quad \textit{where} \quad x \in [0, 2\pi]$

Now both $\csc x$ *and* $\cot x$ are not defined for $x = 0$ *and* $x = 2\pi$

$\therefore \quad x = 0$ *and* $x = 2\pi$ are not admissible.

Hence solution set $= \left\{ \dfrac{2\pi}{3} + 2n\pi \right\}, \quad n \in \mathbb{Z}$.

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