

Bright Career Science Academy, Narowal

Trigonometric Formulas (Edition-4)

Compiled By: Muzzammil Subhan

Different Relations

$$l = r\theta \quad \& \quad A = \frac{1}{2}r^2\theta$$

where θ is measure in radian

$$\bullet 180^\circ = \pi \text{ radian}$$

$$\bullet 1^\circ = \frac{\pi}{180} \text{ radian} \quad \& \quad \bullet 1 \text{ radian} = \frac{180^\circ}{\pi}$$

$$\bullet 1 \text{ radian} \approx 57.296^\circ$$

$$\bullet 1^\circ \approx 0.0175 \text{ radian}$$

$$\bullet \pi \approx 3.1416 \quad \& \quad \bullet e \approx 2.718$$

Trigonometric Ratios

$$\bullet \sin\theta = \frac{p}{h} \quad \bullet \operatorname{cosec}\theta = \frac{h}{p}$$

$$\bullet \cos\theta = \frac{b}{h} \quad \bullet \sec\theta = \frac{h}{b}$$

$$\bullet \tan\theta = \frac{p}{b} \quad \bullet \cot\theta = \frac{b}{p}$$

$$\bullet \sin\theta = \frac{1}{\operatorname{cosec}\theta} \quad \bullet \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\bullet \cos\theta = \frac{1}{\sec\theta} \quad \bullet \sec\theta = \frac{1}{\cos\theta}$$

$$\bullet \tan\theta = \frac{1}{\cot\theta} \quad \bullet \cot\theta = \frac{1}{\tan\theta}$$

$$\bullet \tan\theta = \frac{\sin\theta}{\cos\theta} \quad \bullet \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Trigonometric Co-ratios

$$\bullet \sin\theta \rightleftharpoons \cos\theta$$

$$\bullet \sec\theta \rightleftharpoons \operatorname{cosec}\theta$$

$$\bullet \tan\theta \rightleftharpoons \cot\theta$$

Law of Sine

$$\bullet \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Law of Cosines

$$\bullet a^2 = b^2 + c^2 - 2bc \cos\alpha$$

$$\bullet b^2 = c^2 + a^2 - 2ca \cos\beta$$

$$\bullet c^2 = a^2 + b^2 - 2ab \cos\gamma$$

Law of Tangents

$$\bullet \frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$$

$$\bullet \frac{b-c}{b+c} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)}$$

$$\bullet \frac{c-a}{c+a} = \frac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{\gamma+\alpha}{2}\right)}$$

Types of Angles

$$\bullet \text{Acute Angle: } 0 < \theta < 90^\circ$$

$$\bullet \text{Right Angle: } \theta = 90^\circ$$

$$\bullet \text{Obtuse Angle: } 90^\circ < \theta < 180^\circ$$

$$\bullet \text{Straight Angle: } \theta = 180^\circ$$

$$\bullet \text{Reflex Angle: } 180^\circ < \theta < 360^\circ$$

$$\bullet 1 \text{ Rotation: } \theta = 360^\circ$$

Signs of Trigonometric Functions

$$\bullet \cos(-\theta) = \cos\theta \quad \bullet \sec(-\theta) = \sec\theta$$

$$\bullet \sin(-\theta) = -\sin\theta \quad \bullet \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$$

$$\bullet \tan(-\theta) = -\tan\theta \quad \bullet \cot(-\theta) = -\cot\theta$$

Fundamental Law Of Trigonometry

$$\bullet \cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\bullet \cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$$

$$\bullet \sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

$$\bullet \sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$$

$$\bullet \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$\bullet \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$

Express Products as Sums or Differences

$$\bullet 2 \sin\alpha \cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$$

$$\bullet 2 \cos\alpha \sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$$

$$\bullet 2 \cos\alpha \cos\beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$$

$$\bullet -2 \sin\alpha \sin\beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$$

Express Sums or Differences as Products

$$\bullet \sin\alpha + \sin\beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\bullet \sin\alpha - \sin\beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\bullet \cos\alpha + \cos\beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\bullet \cos\alpha - \cos\beta = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

Area of Triangle

$$\bullet \Delta = \frac{a^2 \sin\beta \sin\gamma}{2 \sin\alpha} = \frac{b^2 \sin\gamma \sin\alpha}{2 \sin\beta} = \frac{c^2 \sin\alpha \sin\beta}{2 \sin\gamma}$$

$$\bullet \Delta = \frac{1}{2}bc \sin\alpha = \frac{1}{2}ca \sin\beta = \frac{1}{2}ab \sin\gamma$$

$$\bullet \Delta = \sqrt{s(s-a)(s-b)(s-c)} \quad (\text{Hero's Formula})$$

$$\text{where } \bullet s = \frac{a+b+c}{2}$$

Inverse Trigonometric Formulas

$$\bullet \sin^{-1}A + \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$$

$$\bullet \sin^{-1}A - \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)$$

$$\bullet \cos^{-1}A + \cos^{-1}B = \cos^{-1}\left(AB - \sqrt{(1-A^2)(1-B^2)}\right)$$

$$\bullet \cos^{-1}A - \cos^{-1}B = \cos^{-1}\left(AB + \sqrt{(1-A^2)(1-B^2)}\right)$$

$$\bullet \tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$\bullet \tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$$

$$\bullet 2 \tan^{-1}A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)$$

Half Angle Formulas

$$\bullet \sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\bullet \sin\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$$

$$\bullet \sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$\bullet \cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

$$\bullet \cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ca}}$$

$$\bullet \cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

$$\bullet \tan\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\bullet \tan\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$$

$$\bullet \tan\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

Different Radii

$$\bullet R = \frac{a}{2\sin\alpha} = \frac{b}{2\sin\beta} = \frac{c}{2\sin\gamma}$$

$$\bullet R = \frac{abc}{4\Delta}$$

$$\bullet r = \frac{\Delta}{s}$$

$$\bullet r_1 = \frac{\Delta}{s-a} \quad \bullet r_2 = \frac{\Delta}{s-b} \quad \bullet r_3 = \frac{\Delta}{s-c}$$

Where R=Circum Radius

r=In Radius

r_1, r_2, r_3 =Escribed Radius

Δ =Area of Triangle

Pythagorean Identities

$$\bullet h^2 = p^2 + b^2$$

$$\bullet \sin^2\theta + \cos^2\theta = 1$$

$$\bullet 1 + \tan^2\theta = \sec^2\theta$$

$$\bullet 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

Half Angle Identities

$$\bullet \sin\theta = 2 \sin\frac{\theta}{2} \cos\frac{\theta}{2}$$

$$\bullet \cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$$

$$\bullet 2 \sin^2\frac{\theta}{2} = 1 - \cos\theta$$

$$\bullet 2 \cos^2\frac{\theta}{2} = 1 + \cos\theta$$

Double Angle Identities

$$\bullet \sin 2\theta = 2 \sin\theta \cos\theta$$

$$\bullet \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\bullet \tan 2\theta = \frac{2 \tan\theta}{1 - \tan^2\theta}$$

$$\bullet 2 \sin^2\theta = 1 - \cos 2\theta$$

$$\bullet 2 \cos^2\theta = 1 + \cos 2\theta$$

Triple Angle Identities

$$\bullet \sin 3\theta = 3 \sin\theta - 4 \sin^3\theta$$

$$\bullet \cos 3\theta = 4 \cos^3\theta - 3 \cos\theta$$

$$\bullet \tan 3\theta = \frac{3 \tan\theta - \tan^3\theta}{1 - 3 \tan^2\theta}$$

Domains, Ranges & Periods of Trigonometric Functions

Function	Domian	Range	Period
$y = \sin x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$	2π
$y = \cos x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$	2π
$y = \tan x$	$-\infty < x < +\infty, x \neq (2n+1)\frac{\pi}{2}$	$-\infty < y < +\infty$	π
$y = \sec x$	$-\infty < x < +\infty, x \neq (2n+1)\frac{\pi}{2}$	$y \geq 1$ or $y \leq -1$	2π
$y = \operatorname{cosec} x$	$-\infty < x < +\infty, x \neq n\pi$	$y \geq 1$ or $y \leq -1$	2π
$y = \cot x$	$-\infty < x < +\infty, x \neq n\pi$	$-\infty < y < +\infty$	π

Ranges of Inverse Trigonometric Functions

Function	Range	Function	Range
$y = \sin^{-1}x$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$	$y = \cos^{-1}x$	$0 \leq y \leq \pi$
$y = \tan^{-1}x$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$	$y = \cot^{-1}x$	$0 < y < \pi$
$y = \operatorname{cosec}^{-1}x$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$	$y = \sec^{-1}x$	$0 \leq y \leq \pi, y \neq \frac{\pi}{2}$

Compiled By: Muzzammil Subhan

Contact No: 0300-7779500

M.Phil. Math (Minhaj University), M.Sc. Math (Quaid-i-Azam University, Islamabad)

M.Ed. (University of Sargodha), B.Sc., B.C.S. & PGD-IT

This page can also be downloaded from our website: WWW.MATHCITY.ORG