

**The Law of Cosine:**

$$\begin{aligned} \circ a^2 &= b^2 + c^2 - 2bc \cos \alpha & \circ b^2 &= c^2 + a^2 - 2ca \cos \beta & \circ c^2 &= a^2 + b^2 - 2ab \cos \gamma \\ \circ \cos \alpha &= \frac{b^2 + c^2 - a^2}{2bc} & \circ \cos \beta &= \frac{c^2 + a^2 - b^2}{2ca} & \circ \cos \gamma &= \frac{a^2 + b^2 - c^2}{2ab} \end{aligned}$$

**The Law of Sine:**

$$\circ \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

**The Law of Tangent:**

$$\circ \frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)} \quad \circ \frac{b-c}{b+c} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)} \quad \circ \frac{c-a}{c+a} = \frac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{\gamma+\alpha}{2}\right)}$$

**Half Angles Formulas:**

$$\begin{aligned} \circ \sin \frac{\alpha}{2} &= \sqrt{\frac{(s-b)(s-c)}{bc}} & \circ \sin \frac{\beta}{2} &= \sqrt{\frac{(s-c)(s-a)}{ca}} & \circ \sin \frac{\gamma}{2} &= \sqrt{\frac{(s-a)(s-b)}{ab}} \\ \circ \cos \frac{\alpha}{2} &= \sqrt{\frac{s(s-a)}{bc}} & \circ \cos \frac{\beta}{2} &= \sqrt{\frac{s(s-b)}{ca}} & \circ \cos \frac{\gamma}{2} &= \sqrt{\frac{s(s-c)}{ab}} \\ \circ \tan \frac{\alpha}{2} &= \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} & \circ \tan \frac{\beta}{2} &= \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} & \circ \tan \frac{\gamma}{2} &= \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}, \end{aligned}$$

where  $s = \frac{a+b+c}{2}$ .

**Area of Triangle ( $= \Delta$ ):**

$$\begin{aligned} \circ \Delta &= \frac{1}{2}bc \sin \alpha = \frac{1}{2}ca \sin \beta = \frac{1}{2}ab \sin \gamma \\ \circ \Delta &= \frac{a^2 \sin \beta \sin \gamma}{2 \sin \alpha} = \frac{b^2 \sin \gamma \sin \alpha}{2 \sin \beta} = \frac{c^2 \sin \alpha \sin \beta}{2 \sin \gamma} \\ \circ \Delta &= \sqrt{s(s-a)(s-b)(s-c)} \quad (\text{Heron's Formula}) \end{aligned}$$

**Circumradius ( $= R$ ):**

$$\circ R = \frac{a}{2 \sin \alpha} = \frac{b}{2 \sin \beta} = \frac{c}{2 \sin \gamma} \quad \circ R = \frac{abc}{4\Delta}$$

**Inradius ( $= r$ ):**

$$\circ r = \frac{\Delta}{s}$$

**Escribed Circle:**

$$\circ r_1 = \frac{\Delta}{s-a} \quad \circ r_2 = \frac{\Delta}{s-b} \quad \circ r_3 = \frac{\Delta}{s-c}$$