Solution & Area of Oblique Triangle

The Law of Cosine:

- $a^2 = b^2 + c^2 - 2bc\cos\alpha$
- $b^2 = c^2 + a^2 - 2ca\cos\beta$
- $c^2 = a^2 + b^2 - 2ab\cos\gamma$
- $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$
- $\cos\beta = \dfrac{c^2 + a^2 - b^2}{2ca}$
- $\cos\gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$

The Law of Sine:

- $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

The Law of Tangent:

- $\dfrac{a-b}{a+b} = \dfrac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$
- $\dfrac{b-c}{b+c} = \dfrac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)}$
- $\dfrac{c-a}{c+a} = \dfrac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{\gamma+\alpha}{2}\right)}$

Half Angles Formulas:

- $\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$
- $\sin\dfrac{\beta}{2} = \sqrt{\dfrac{(s-c)(s-a)}{ca}}$
- $\sin\dfrac{\gamma}{2} = \sqrt{\dfrac{(s-a)(s-b)}{ab}}$
- $\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$
- $\cos\dfrac{\beta}{2} = \sqrt{\dfrac{s(s-b)}{ca}}$
- $\cos\dfrac{\gamma}{2} = \sqrt{\dfrac{s(s-c)}{ab}}$
- $\tan\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$
- $\tan\dfrac{\beta}{2} = \sqrt{\dfrac{(s-c)(s-a)}{s(s-b)}}$
- $\tan\dfrac{\gamma}{2} = \sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}$,

where $s = \dfrac{a+b+c}{2}$.

Area of Triangle $(=\Delta)$:

- $\Delta = \dfrac{1}{2}bc\sin\alpha = \dfrac{1}{2}ca\sin\beta = \dfrac{1}{2}ab\sin\gamma$
- $\Delta = \dfrac{a^2\sin\beta\sin\gamma}{2\sin\alpha} = \dfrac{b^2\sin\gamma\sin\alpha}{2\sin\beta} = \dfrac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$
- $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ (*Heron's Formula*)

Circumradius $(=R)$:

- $R = \dfrac{a}{2\sin\alpha} = \dfrac{b}{2\sin\beta} = \dfrac{c}{2\sin\gamma}$
- $R = \dfrac{abc}{4\Delta}$

Inradius $(=r)$:

- $r = \dfrac{\Delta}{s}$

Escribed Circle:

- $r_1 = \dfrac{\Delta}{s-a}$
- $r_2 = \dfrac{\Delta}{s-b}$
- $r_3 = \dfrac{\Delta}{s-c}$