

Objective

Q.1 Four possible answers to each statement are given below. Tick (✓) the correct answer.

(1) $\sqrt{\frac{5}{16}}$ is :

- (a) Rational Number
(c) Prime number

- (b) Irrational Number
(d) Whole Number

(2) $\{x/x \in E \text{ and } 4 < x < 6\}$ equals:

- (a) $\{4\}$
(c) $\{6\}$

- (b) $\{5\}$
(d) f

(3) The multiplicative inverse of $-i$ is :

- (a) i
(c) 1

- (b) $-i$
(d) -1

(4) If A is a matrix of order $m \times n$ then the number of elements in each row of A is:

- (a) M
(c) $m + n$

- (b) N
(d) $m - n$

(5) If A is a square matrix of order 3×3 then $|kA|$ equals :

- (a) $k|A|$
(c) $k^3|A|$

- (b) $k^2|A|$
(d) $k^4|A|$

(6) If $4^x = 2$ then x equals:

- (a) 2
(c) $\frac{1}{2}$

- (b) $\frac{-1}{2}$
(d) 1

(7) If a and b are the roots of $3x^2 - 2x + 4 = 0$ then the equation whose roots are $2a, 2b$ is:

- (a) $2x^2 + 6x + 8 = 0$
(c) $3x^2 - 4x + 16 = 0$

- (b) $4x^2 - 2x + 3 = 0$
(d) $3x^2 + 16x - 4 = 0$

(8) An infinite arithmetic series consisting of non-zero terms is:

- (a) Convergent
(c) Neither Convergent nor Divergent

- (b) Divergent
(d) Oscillatory

(9) The product of n geometric means between a and b equals ____ A, G, H, have their usual meanings:

- (a) A^n
(c) G^n

- (b) H^n
(d) nG

(10) 4P_3 equals:

- (a) 4P_1
(c) 4P_4

- (b) 4P_2
(d) 5P_4

(11) P(E) equals:

(a) $1 + P(E)$

(c) $2 - P(E)$

(b) $P(E) - 1$

(d) $1 - P(E)$

(12) If n is a positive integer then $n^2 + n$ is divisible by :

(a) 2

(c) 4

(b) 3

(d) 5

(13) If n is even positive integer, then $\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots + \binom{n}{n-1}$ equals:

(a) 2^n

(c) 2^{n-1}

(b) 2^{n+1}

(d) 3^n

(14) If n is a positive integer and $|x| < 1$ then $1 + nx + \frac{n(n-1)}{2!}x^2 + \dots$ is :

(a) Arithmetic Series

(c) Harmonic Series

(b) Geometric Series

(d) Binomial Series

(15) Which one is true :

(a) $1 \text{ radian} < 1^0$

(c) $1 \text{ radian} = 1^0$

(b) $1 \text{ radian} > 1^0$

(d) $5 \text{ radian} < 2^0$

(16) $\sin\left(\frac{p}{2} - q\right)$ equals :

(a) $\cos(q)$

(c) $-\cos(q)$

(b) $\sin(q)$

(d) $\sin(q)$

(17) $2 \sin a \cos b$ equals:

(a) $\sin(a + b) - \sin(a - b)$

(c) $\sin(a + b) + \sin(a - b)$

(b) $\cos(a + b) + (a - b)$

(d) $\cos(a + b) - (a - b)$

(18) $2 \sin 12^0 \sin 46^0$ equals:

(a) $\cos 34^0 + \cos 58^0$

(c) $\sin 34^0 + \sin 58^0$

(b) $\sin 34^0 - \sin 58^0$

(d) $\cos 34^0 - \cos 58^0$

(19) With usual notations for triangle R equals :

(a) $\frac{b}{2 \sin a}$

(c) $\frac{c}{2 \sin g}$

(b) $\frac{a}{2 \sin b}$

(d) $\frac{\Delta}{5}$

(20) The solution set of $\sin x = 0$ is given by _____ $n \in Z$:

(a) $\{2np\}$

(c) $\{np\}$

(b) $\{p + 2np\}$

(d) $\{3np\}$

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Provided and Composed by: Haji Asif ALI (asif.mathematics@gmail.com)

Section – I

Note: All questions are to be attempted on answer book.

Q # 2: Write any TWENTY-FIVE short answers of the following questions:

- (i) State De-Moivre's theorem.
- (ii) Prove that $z\bar{z} = |z|^2, \quad \forall z \in C$
- (iii) Is there any set which has no proper subset? If so, name that set.
- (iv) Define order of a set.
- (v) State Commutative and associative property of binary operation.
- (vi) Let $p \rightarrow p$ be a given conditional logic, then find its converse and inverse.
- (vii) Find the inverse of the matrix $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$
- (viii) Define (i) Consistent system. (ii) Inconsistent system
- (ix) If $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, show that $A^2 = I_2$
- (x) Show that $\begin{vmatrix} 2 & 3 & 0 \\ 3 & 9 & 6 \\ 2 & 15 & 1 \end{vmatrix} = 9 \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$
- (xi) Prove that each complex cube root of unity is square of the other.
- (xii) Use factor theorem, to prove that $x-a$ is a factor of $x^n - a^n$.
- (xiii) If a, b are the roots of $x^2 - px - p - c = 0$ prove that $(1+a)(1+b) = 1-c$
- (xiv) Show that the roots of $(p+q)x^2 - px - q = 0$ will be rational.
- (xv) If 5 and 8 are the two A.M's between a and b, find a and b.
- (xvi) Find two G.M's between 1 and 8.
- (xvii) If 5 is H.M, between 2 and b, find b.
- (xviii) Find n if ${}^n P_2 = 30$
- (xix) How many diagonals can be formed by joining the vertices of polygon having 5 sides?
- (xx) A fair coin is tossed three times. What is the probability that no head appears.
- (xxi) A die is thrown, what is the probability the number is prime?
- (xxii) Show $\frac{5^{2n} - 3^{2n}}{2}$ is an integer for $n = 2, 3$.
- (xxiii) Find 3rd term in the expansion of $\left(2a - \frac{x}{a}\right)^7$
- (xxiv) Expand up to 3rd term $(1-x)^{-3}$
- (xxv) Evaluate $\sqrt{17}$ up to two decimals.
- (xxvi) Convert $\left(22\frac{1}{2}\right)^\circ$ to radians using $p = 3.1416$
- (xxvii) Show $(\tan q + \cot q)^2 = \sec^2 q \operatorname{cosec}^2 q$
- (xxviii) If a, b, g are the angles of a triangle A B C. Prove that $\operatorname{Cos}\left(\frac{a+b}{2}\right)\operatorname{Sin}\left(\frac{g}{2}\right)$
- (xxix) Prove that $\tan(45^\circ + A)\tan(45^\circ - A) = 1$
- (xxx) Prove that $\frac{\operatorname{Sin}3x - \operatorname{Sin}x}{\operatorname{Cos}x - \operatorname{Cos}3x} = \operatorname{Cot}2x$

- (xxxix) Write domain and range of $\cos x$.
- (xxxix) Find the period of $\sin \frac{x}{3}$
- (xxxix) If $a = 3, c = 5$, and $A = 120^\circ$, find "a" by using law of cosine.
- (xxxix) Find the area of the triangle ABC, when $a = 200, b = 120, g = 150^\circ$
- (xxxix) For the triangle ABC, $a = 13, b = 14, c = 15$ find R
- (xxxix) Define trigonometric equation.
- (xxxix) Solve the equation $\cot^2 q = \frac{1}{3}$

Section - II

Note: Attempt any THREE questions.

Q # 3 (a) Show that
$$\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$$

(b) Show that $(1+w)(1+w^2)(1+w^4)(1+w^8) \dots 2n \text{ factors} = 1$

Q # 4 (a) Resolve into partial fractions $\frac{9x-7}{(x^2+1)(x+3)}$

(b) Find n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be H.M between "a" and "b".

Q # 5 (a) Find the values of n and r when ${}^{n-1}C_{r-1} : {}^nC_r = {}^nC_{r+1} = 3 : 6 : 11$

(b) If x is so small that its square and higher powers can be neglected, then show that

$$\frac{(9+7x)^2 - (16+3x)^{\frac{1}{4}}}{4+5x} = \frac{1}{4} - \frac{17}{384}x$$

Q # 6 (a) Prove that $\sin^6 q - \cos^6 q = (\sin^2 q - \cos^2 q)(1 - \sin^2 q \cos^2 q)$

(b) If a, b, g are the angles of triangle, prove that $\tan a + \tan b + \tan g = \tan a \tan b \tan g$

Q # 7 (a) Show that $\frac{1}{2rR} = \frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$

(b) Prove that $\sin^{-1} A + \sin^{-1} B = \sin^{-1} (A\sqrt{1-B^2} + B\sqrt{1-A^2})$

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(asif.mathematics@gmail.com)