

**This is a Title with Each ii Word Capitalized and
Support α , β and Break of Line.**



A thesis
submitted in partial fulfillment of the
requirement for the degree of
Master of Science in Mathematics

by
Muhammad Kamran
CIIT/FA14-RMT-001/ATK

COMSATS UNIVERSITY ISLAMABAD
Attock Campus
January 2014

**COMSATS UNIVERSITY ISLAMABAD
ATTOCK CAMPUS**

FINAL APPROVAL

This thesis titled

**This is a Title with Each ii Word Capitalized and Support α , β and
Break of Line.**

submitted to the Department of Mathematics

by

Name	Registration Number
Muhammad Kamran	CIIT/FA14-RMT-001/ATK

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Abstract

A function is convex if the line segment joining two points on the graph lies above the graph. These functions have important properties and applications in mathematics. Specially, they are very important in optimization and minimization problems. Also these functions are used in statistic and functional analysis. A positive function f is logarithmic convex if $\log f$ is convex. It would seem that log convex functions unremarkable because they are so simply related to convex functions. But they have some surprising properties.

In the first chapter we generalize results for logarithmic convexity of Giaccardi's difference for classes of functions with the help of divided difference.

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Student Name

Chapter 1

Introduction

In this chapter we will give the notions and definitions of monotone function, convex function, some important inequalities and properties which will frequently use in the rest of chapters to proof our results.

1.1 Monotone function

• J. E. Pečarić, F. Proschan and Y. L. Tong, *Convex Functions, Partial Orderings and Statistical Applications*, Academic Press, New York, 1992.

The monotone function are such function which maintain the order of inequalities. It is often defined on the interval I , where I is also an order set (see [?, ?]).

Definition 1.1.1. A function $f : I \rightarrow \mathbb{R}$ is said to be nondecreasing (respectively nonincreasing) if $x_1 < x_2$ implies $f(x_1) \leq f(x_2)$ (respectively $f(x_1) \geq f(x_2)$) for $x_1, x_2 \in I$. We say that f is increasing (respectively decreasing) if $x_1 < x_2$ implies $f(x_1) < f(x_2)$ (respectively $f(x_1) > f(x_2)$) for all $x_1, x_2 \in I$.

Example 1.1.2. The function $f : [0, \infty) \rightarrow \mathbb{R}$ given by $f(x) = x^2$ is increasing on $[0, \infty)$, while $f : (-\infty, 0] \rightarrow \mathbb{R}$ given by $f(x) = x^2$ is decreasing on $(-\infty, 0]$

Definition 1.1.3. A function is said to be monotone on an interval I , if it is either increasing or decreasing.

The following criteria is often used to investigate the monotonicity of function (see [?, ?]).

Proposition 1.1.4. *Suppose that a function f is continuous on the closed interval $[a, b]$ and has a derivative at each point of the open interval (a, b) .*

1. *If $f'(x)$ is positive for all x in (a, b) , then f is increasing function on $[a, b]$.*
2. *If $f'(x)$ is negative for all x in (a, b) , then f is decreasing function on $[a, b]$.*

1.2 Convex function

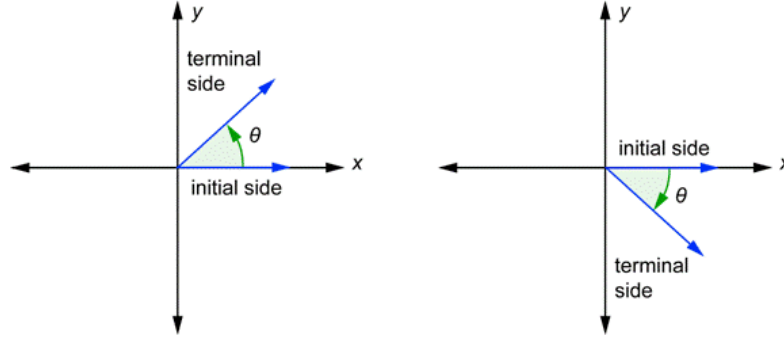


Figure 1.1:

$$\frac{f(x_1) - f(x_2)}{x_1 - x_2} \leq \frac{f(x_2) - f(x_3)}{x_2 - x_3} \quad (1.1)$$

Convex geometry as a new field of mathematics takes its origin from the publication of the book by Minkowski [?]. This book influenced the formation of a new field in mathematics, viz., functional analysis [?]

Definition 1.2.1. A function $f : I \rightarrow \mathbb{R}$ is said to be convex if

$$f(tx + (1 - t)y) \leq tf(x) + (1 - t)f(y) \quad (1.2)$$

for all $x, y \in I$ and $t \in [0, 1]$.

A function f is said to be strictly convex on I if (1.2) is strict for $x \neq y$.

If f is a convex function on I , then for $x_1 < x_2 < x_3$ the inequality

$$\frac{f(x_1) - f(x_2)}{x_1 - x_2} \leq \frac{f(x_2) - f(x_3)}{x_2 - x_3} \quad (1.3)$$

holds.

Lemma 1.2.2. *This is an example of lemma $x + 2y$ and this is good.*

Proposition 1.2.3. *This is an example of proposition $x + 2y$ and this is good.*

Theorem 1.2.4. *This is an example of thorem $x + 2y$ and $2x - y$ and this is good.*

Corollary 1.2.5. *This is an example of thorem $x + 2y$ and $2x - y$ and this is good.*

(1) This is first

(2) This is second

(3) This is third

Here is one more.

A- This is first

B- This is second

C- This is third

Chapter 2

Title of 2nd chap

good

good

very good

2.1 Title 1st Section

$$x^2 \tag{2.1}$$

$$y^2 \tag{2.2}$$

2.2 Title of 2nd Section

$$z^2 \tag{2.3}$$

[1] [2] [3] [4]

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