In case (ii), let \( b \) be the radius of the circle. Then the time \( t_2 \) required by \( P \) to return to its initial position is given by

\[
t_2 = \frac{T}{\pi} \cos^{-1} \frac{x}{b} = \frac{T}{\pi} \tan^{-1} \sqrt{\frac{b^2 - x^2}{x}}.
\]

Since by equation (8.19),

\[
v = \sqrt{\frac{\lambda}{x} \left(b^2 - x^2\right)},
\]

it follows that the time \( t_2 \) is given by

\[
t_2 = \frac{T}{\pi} \tan^{-1} \frac{v}{\sqrt{\lambda x}} = \frac{T}{\pi} \tan^{-1} \frac{T_0}{2\pi x},
\]

because \( T = \frac{2\pi}{\sqrt{\lambda}} \).

**Exercises Set 8**

1. Obtain the equations of motion (8.5), (8.6) and (8.10) by graphical method.

2. A particle moving in a straight line starts from rest and is accelerated uniformly to attain a velocity of 60 miles per hour in 4 seconds. Find the acceleration of motion and the distance travelled by the particle in the last three seconds.

   [Ans. 22 ft./sec², 165 ft.]

3. Two particles start simultaneously from a point \( O \) and move in a straight line; one with a velocity of 45 miles per hour and an acceleration of 2 ft./sec², and the other with a velocity of 90 miles per hour and a retardation (the rate of decrease of velocity) of 8 ft./sec². Find the time after which the velocities of the particles are the same and the distance of \( O \) from the point where they meet again.

   [Ans. 66 sec., 1045.44 ft.]

4. A particle moving along a straight line starts from rest and is accelerated uniformly till it attains a velocity \( v \). The motion is then retarded and the particle comes to rest after traversing a total distance \( x \). If the acceleration is \( f \), find the retardation
and the total time taken by the particle from rest to rest.

\[
\text{[Ans. } \frac{\sqrt{\frac{2}{f} (2x^2 - 2x^2)} - 2x}{v} \text{]}
\]

5. Two particles travel along a straight line. Both start at the same time and are accelerated uniformly at different rates. The motion is such that when a particle attains the maximum velocity \( v \), its motion is retarded uniformly. The two particles come to rest simultaneously at a distance \( x \) from the starting point. If the acceleration of the first is \( a \) and that of the second is \( \frac{1}{2} a \), find the distance between the points where the two particles attain their maximum velocities.

\[
\text{[Ans. } \frac{r^2}{2a} \text{]}
\]

6. A particle is projected vertically upwards with a velocity \( \sqrt{2gh} \) and another is let fall from a height \( h \) at the same time. Find the height of the point where they meet each other.

\[
\text{[Ans. } \frac{3h}{4} \text{]}
\]

7. Two particles are projected simultaneously in the vertically upward direction with velocities \( \sqrt{2gh} \) and \( \sqrt{2gk} \) \((k > h)\). After a time \( t \), when the two particles are still in flight, another particle is projected upwards with a velocity \( u \). Find the condition so that the third particle may meet the first two during their upward flight.

\[
\text{[Ans. } t < \sqrt{\frac{2h}{g}}, \quad u > \frac{k}{\sqrt{\frac{2h}{g}}} + \frac{1}{2} (\sqrt{\frac{2gh}{g}}) \text{]}
\]

8. A gunner detects a plane at \( t = 0 \) approaching him with a velocity \( v \), the horizontal and the vertical distances of the plane being \( k \) and \( h \) respectively. His gun can fire a shell vertically upwards with an initial velocity \( u \). Find the time when he should fire the gun and the condition on \( u \) so that he may be
able to hit the plane if it continues its flight in the same horizontal line.

\[
\text{Ans. } t = \frac{h}{v} - \frac{1}{x}, \text{ where } x = \frac{u \pm \sqrt{u^2 - 2gh}}{g}, \ u^2 > 2gh
\]

9. A particle is projected vertically upwards. After a time \( t \), another particle is sent up from the same point with the same velocity and meets the first at height \( h \) during the downward flight of the first. Find the velocity of projection.

\[
\text{Ans. } \frac{\sqrt{8gh + a^2vt^2}}{2}
\]

10. Discuss the motion of a particle moving in a straight line if it starts from rest at \( t = 0 \) and its acceleration is equal to \((i) \ t^2; \ (ii) \ a \cos t + b \sin t; \ (iii) \ -a^2x\).

11. A particle starts with a velocity \( u \) and moves in a straight line. If it suffers a retardation equal to the square of the velocity, find the distance travelled by the particle in a time \( t \).

\[
\text{Ans. } b \log (bt + 1)
\]

12. Discuss the motion of a particle moving in a straight line if it starts from rest at a distance \( a \) from a point \( O \) and moves with an acceleration equal to \( \mu \) times its distance from \( O \).

\[
\text{Ans. } v = \sqrt{a \left( x^2 - a^2 \right)}, \ x = a \cosh \sqrt{\mu} t
\]

13. A particle moving in a straight line starts with a velocity \( u \) and has acceleration \( v^2 \), where \( v \) is the velocity of the particle at time \( t \). Find the velocity and the time as functions of the distance travelled by the particle.

\[
\text{Ans. } v = \frac{u}{1 - u^2}, \ t = \frac{x}{2u} (2 - ux)
\]

14. The acceleration of a particle falling freely under the gravitational pull is equal to \( \frac{k}{x^2} \), where \( x \) is the distance of the particle from the centre of the earth. Find the velocity of the particle if it is let fall from an altitude \( R \), on striking the surface of the earth if the
radius of earth is \( r \) and the air offers no resistance to motion.

\[
\text{Ans. } \sqrt{2\pi} \left( \frac{1}{r} - \frac{1}{R} \right)
\]

15. A particle describes simple harmonic motion with frequency \( N \). If the greatest velocity is \( V \), find the amplitude and the maximum value of the acceleration of the particle.

Also show that the velocity \( v \) at a distance \( x \) from the centre of motion is given by 

\[
v = 2\pi N \sqrt{a^2 - x^2}, \text{ where } a \text{ is the amplitude.}
\]

\[
\text{Ans. } a = \frac{V}{2\pi N^2}, \text{ Max. accel. } = 2\pi N V
\]

16. A particle describing simple harmonic motion has velocities 5 ft./sec. and 4 ft./sec. when its distances from the centre are 12 ft. and 13 ft. respectively. Find the time period of motion.

\[
\text{Ans. } \frac{10\pi}{3}
\]

17. The maximum velocity that a particle executing simple harmonic motion of amplitude \( a \) attains, is \( a \).

If it is disturbed in such a way that its maximum velocity becomes \( na \), find the change in the amplitude and the time period of motion.

\[
\text{Ans. } (n-1)a, \text{ no change.}
\]

18. A point describes simple harmonic motion in such a way that its velocity and acceleration at a point \( P \) are \( v \) and \( f \) respectively and the corresponding quantities at another point \( Q \) are \( u \) and \( g \). Find the distance \( PQ \).

\[
\text{Ans. } \frac{u^2 - v^2}{f + g}
\]

19. If a point \( P \) moves with a velocity \( v \) given by

\[a^2 = n^2(ax^2 + 2bx + c),\]

show that \( P \) executes a simple harmonic motion. Find the centre, the amplitude and the time period of the motion.

\[
\text{Ans. } x = -\frac{b}{a}; \frac{\sqrt{b^2 - ac}}{a}; \frac{2\pi}{\sqrt{a}}
\]
We deal with constant acceleration. Let \( u \) be the initial acceleration velocity, whereas \( v \) denotes final velocity attained by particle in time \( t \).

The acceleration is constant, so velocity-time graph must be a straight line.

\[
\begin{align*}
OA &= u \\
OB &= t \\
BD &= v
\end{align*}
\]

\[
\Rightarrow CD = BD - BC = BD - OA = v - u \quad (i)
\]

\[
AC = OB = t
\]

Thus acceleration = slope of \( AD = \tan \theta = \frac{CD}{AC} \)

\[
\Rightarrow \frac{v-u}{t} = a \quad \text{i.e.} \quad v-u = at \quad \text{or} \quad \boxed{v = u + at} \quad (ii)
\]

The area of rectangle \( OACB = ut \)

\( \& \) The area of \( \Delta ACD = \frac{1}{2} (CD \times AC) = \frac{1}{2} (v-u) t \)

\[
= \frac{1}{2} at^2
\]

Considering figure we use we find that:

The area under \( AD = \) Total distance \( x \) covered by particle in time \( t \).

\( x = \text{Area of rectangle } OACB + \text{area of } \Delta ACD \)

\[
= ut + \frac{1}{2} at^2
\]

\( \Rightarrow \boxed{x = ut + \frac{1}{2} at^2} \quad (iv) \)

Now

\[
\frac{AC}{CD} = \cot \theta = \frac{1}{\tan \theta} = \frac{1}{a}
\]
\[ AC = \frac{CD}{a} = \frac{v-u}{a} \]

The area of rectangle OACB:
\[ = OA \cdot AC = U \cdot (\frac{v-u}{a}) = \frac{U(V-U)}{a} \]

The area of \( \triangle ACD = \frac{1}{2} AC \cdot CD \]
\[ = \frac{1}{2} (\frac{v-u}{a}) (v-u) = \frac{(v-u)^2}{2a} \]

Thus \( x \) (Distance covered by particle in time \( t \))
- Total area under \( AD \)
- Area of rectangle OACB + Area of \( \triangle ACD \)
\[ x = \frac{UV-U^2 + (v-u)^2}{2a} \]
\[ = \frac{2UV-2V^2 + V^2 + U^2 - 2UV}{2a} \]
\[ = \frac{V^2-U^2}{2a} \quad \text{i.e.} \quad \frac{V^2-U^2}{2a} = \frac{x}{a} \quad \text{(v)} \]

**Sol: 2**

\[ u = 0 \, \text{ft/sec} \]
\[ v = 60 \, \text{mile/hour} = \frac{60 \times 1760 \times 3}{60 \times 60} \, \text{ft/sec} \]
\[ \Rightarrow \quad v = 88 \, \text{ft/sec} \]
\[ a = \ ? \quad \text{where} \quad t = 4 \, \text{sec} \]
We know \( v = u + at \)
\[ 88 = 0 + a \cdot 4 \quad \Rightarrow \quad a = \frac{88}{4} \]
\[ \Rightarrow \quad \left[ a = 22 \, \text{ft/sec}^2 \right] \]

Let \( x_1 \) be distance traveled in \( t = 4 \) so
\[ x_1 = ut + \frac{1}{2}at^2 = 0 \cdot t + \frac{1}{2} \cdot 22 \cdot 16 \]
\[ \left[ x_1 = 176 \, \text{ft} \right] \]

& Let \( x_2 \) be distance traveled by first minute \( t = 1 \)
so
\[ x_2 = ut + \frac{1}{2}at^2 = 0 \cdot t + \frac{1}{2} \cdot 22 \cdot 1 \]
\[ \left[ x_2 = 11 \right] \]

Distance covered by last three minutes,
\[ x = x_1 - x_2 = (176 - 11) \, \text{ft} \]
\[ x = 165 \, \text{ft} \]
Solution 3: Motion of a first particle for which:

\[ u = 45 \text{ miles/hour} = \frac{45 \times 1760 \times 3}{60 \times 60} = 66 \text{ ft/sec} \]

\[ a = 2 \text{ ft/sec}^2 \]

\[ v = v \quad \text{and} \quad t = t' \]

Put values in equation \( v = u + at \) \( \quad \text{(i)} \)

\[ \Rightarrow \quad v = 66 + 2t' \quad \text{(ii)} \]

Now for second particle:

\[ u = 90 \text{ miles/hour} = \frac{90 \times 1760 \times 3}{60 \times 60} = 132 \text{ ft/sec} \]

\[ v = v \quad \text{and} \quad t = t' \quad \text{and} \quad a = -8 \text{ ft/sec}^2 \]

Put values in \( \text{(ii)} \)

\[ v = 132 + 8t' \quad \text{(iii)} \]

When the velocities of particle same then comparision eq. \( \text{(ii)} \) & \( \text{(iii)} \)

\[ 66 + 2t' = 132 - 8t' \]

\[ 10t' = 66 \Rightarrow t' = \frac{66}{10} = 6.6 \text{ sec} \quad \text{(iv)} \]

If the particle meets each other after covering distance \( x \) in time \( t' \)

Now taking eq. \( \quad x = ut' + \frac{1}{2}at'^2 \quad \text{(v)} \)

Considering case of 1st particle

\[ u = 66 \text{ ft/sec} \quad \text{and} \quad t = t' \]

\[ a = 2 \text{ ft/sec}^2 \quad \text{and} \quad x = x \]

\[ \Rightarrow \quad x = 66t' + \frac{1}{2} \times 2 \times t'^2 = 66t' + t'^2 \quad \text{(vi)} \]

For motion of 2nd particle:

\[ u = 132 \text{ ft/sec} \quad \text{and} \quad t = t' \]

\[ a = -8 \text{ ft/sec}^2 \quad \text{and} \quad x = x \]

\[ \Rightarrow \quad x = 132t' - \frac{1}{2} \times 8 \times t'^2 = 132t' - 4t'^2 \quad \text{(vii)} \]

Comparing eq. \( \text{(vi)} \) & \( \text{(vii)} \)

\[ 66t' + t'^2 = 132t' - 4t'^2 \]

\[ 5t'^2 - 66t' = 0 \quad \Rightarrow \quad 5t' - 66 = 0 \]

\[ t' = \frac{66}{5} \quad \text{and} \quad 5t' = 66 \Rightarrow t' = \frac{66}{5} \text{ sec} \]

Time will not be zero so, \[ t = \frac{66}{5} \]
Putting \( t' = 13.2 \text{ sec} \) in (vi) we get,

\[
x = 66 \times 66 + \left( \frac{66}{5} \right)^2
\]

\[
= 871.20 + 174.24
\]

\[
x = 1045.44 \text{ ft}
\] (vii)

Required distance covered after which they meet each other.

**Solution 4**

Let us consider particle at rest attains velocity \( v \) after travelling distance \( x_1 \) with acceleration \( f \).

Putting values in \( i \)

\[
V^2 - 0 = 2fx_1
\]

\[
x_1 = \frac{V^2}{2f}
\] (ii)

Let same particle having velocity \( V \) comes to rest.

so put \( V' = 0 \), \( u = V \), \( a = -V \) (retardation) \( \& x = x_2 \) in \( i \)

we get

\[
a^2 - V'^2 = -2g^2X_2 \Rightarrow X_2 = \frac{V^2}{2V}
\] (iii)

The particle after covering total distance \( x \) comes to rest so adding (ii) \& (iii).

\[
x = x_1 + x_2 = \frac{V^2}{2f} + \frac{V^2}{2V}
\]

\[
x = V^2 \left( \frac{1}{2f} + \frac{1}{2V} \right) \Rightarrow \frac{2x}{V^2} = \frac{1}{f} + \frac{1}{V}
\]

\[
\frac{1}{f} = \frac{2x}{V^2} - \frac{1}{f} \Rightarrow \frac{2fx - V^2}{V^2 \cdot f} \]

Required retardation

Now applying the Eq. of motion i.e.

\[
v = u + at \quad \text{(vi)}
\]

Case (i) : \( \frac{f}{2} \) \( t = t_1 \)

\[
v = V, \ u = 0, \ a = f
\]

\[
v = 0 + ft_1 \Rightarrow t_1 = \frac{V}{f}
\] (vii)
Case (ii) If \( t = t_2 \), \( v = 0 \), \( u = v \) & \( a = -\dot{r} \).

Putting values in \((vi)\):

\[
\sigma = v - rt_2 \quad \Rightarrow \quad t_2 = \frac{v}{r}
\]

Adding \((vii)\) & \((viii)\), we get

\[
t_1 + t_2 = \frac{v}{f} + \frac{v}{r} = \frac{v}{f} + \frac{\sqrt{(2fx - v^2)}}{v^2f} = \frac{v^2 + 2fx - v^2}{v^2f} = \frac{2fx}{v^2f} = \frac{2x}{v}\]

\( \text{(Required total time covered by particle from rest to rest)} \)

**Case (iii)**: we have

\[
v^2 - u^2 = 2ax \quad \text{(i)}
\]

Case (i): Let us considering that 1st particle having acceleration \( a \) attain max velocity \( v \) after covering distance \( x_1 \); we have:

\[
u = 0 \quad \Rightarrow \quad \text{putting values in (i)}
\]

we get:

\[
v^2 - 0^2 = 2ax_1
\]

\[
x_1 = \frac{v^2}{2a} \quad \text{(ii)}
\]

Case (vii):

Let 2nd particle attain max. velocity \( v \) after distance \( x_2 \), showing acceleration \( \ddot{a} \).

The eq.\(s \) become:

\[
v^2 - 0^2 = 2a, \quad x_2
\]

\[
\Rightarrow \quad x_2 = \frac{v^2}{a} \quad \text{(iii)}
\]

Then required distance between two points where two particle attain their max velocity is given by:

\[
x_2 - x_1 = \frac{v^2}{a} - \frac{v^2}{2a} = \frac{2v^2 - v^2}{2a} = \frac{v^2}{2a}
\]

\( \text{(iv)} \)

\( \text{Required Distance} \)
Sol: 6. \[ x = ut + \frac{1}{2}at^2 \]

Let us consider that both particles meet each other at point C with height \( x \) from point A after time \( t \).

Case (i). Upward motion of 1st particle.

\[
U = \sqrt{2gh}, \quad x = x = AC \quad t = t, \quad a = -g.
\]

Put values in i) so,
\[
x = \sqrt{2gh} t - \frac{1}{2}gt^2 \quad (i)\]

Case (ii). Downward motion of 2nd particle.

Here, \( u = 0 \), \( x = h - x' = BC \)

\[
t = t, \quad a = g.
\]

Put values in i) so
\[
-h - x = 0 \cdot t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 \quad (ii)
\]

Adding (i) & (ii),
\[
x + h - x = \sqrt{2gh} t - \frac{1}{2}gt^2 + \frac{1}{2}gt^2
\]
\[
h = \sqrt{2gh} t \Rightarrow t = \frac{h}{\sqrt{2gh}} = \frac{\sqrt{h}}{\sqrt{2g}} \quad (iv)
\]

Putting values of \( t \) in (i) we have,
\[
x = \sqrt{2gh} \cdot \frac{\sqrt{h}}{\sqrt{2g}} = \frac{1}{2}g \left( \frac{\sqrt{h}}{\sqrt{2g}} \right)^2 = h - \frac{g}{4} = h - \frac{h}{4}
\]

\[
x = \frac{3}{4}h = AC \quad (v)
\]

Required point: distance where two particles meet.

Sol: 7. \[ v^2 - u^2 = 2ax \quad (i)\]

Let \( H \) be max. height attained by 1st particle whereas \( u = \sqrt{2gh}, \quad v = 0, \quad a = -g\)

\[
0^2 - \left( \sqrt{2gh} \right)^2 = 2(-g)H
\]

\[
\Rightarrow H = \frac{\sqrt{2gh}}{-\sqrt{g}} = h \quad (ii)
\]

Similarly,

The max. height attained by 2nd particle = \( k \) \quad (iii)

\[
k > h \quad (\text{given}) \quad \& \quad 3rd \text{ particle has to}\]
Ex. set. VII

meet both particles during their upward flight so time \( t \) should be the time when 1st particle has not yet attain max. height \( h \).

If \( T \) be time taken by 1st particle to cover distance \( h \) so applying the equation i.e.,

\[
\begin{align*}
\text{we get } & \quad 0 = \frac{1}{2} g t^2 - h \\
\Rightarrow & \quad T = \frac{\sqrt{2h}}{g} = \sqrt{\frac{2h}{g}} \quad \text{(vii)}
\end{align*}
\]

\( t < \sqrt{\frac{2h}{g}} \) \( \Rightarrow \; K > h \) \( \text{(given)} \)

\[
\begin{align*}
\Rightarrow & \quad \sqrt{\frac{2h}{g}} < \sqrt{\frac{2K}{g}} \quad \text{so}
\end{align*}
\]

\( t < \sqrt{\frac{2h}{g}} \) \( < \sqrt{\frac{2K}{g}} \quad \text{(vii)} \) \( \text{and 2nd particle also does not attain max. height} \; K \)

Set \( t' = \sqrt{\frac{2h}{g}} - t \) and 3rd particle projected with velocity \( u \) must covered distance \( K \) in \( t' \) so putting values in equation i.e.,

\[
\begin{align*}
X &= ut' + \frac{1}{2} gt'^2 \quad \text{(vii)}
\end{align*}
\]

we get

\[
\begin{align*}
K &= u(t' + \frac{1}{2} gt') \\
\Rightarrow & \quad K = u(\sqrt{\frac{2h}{g}} - t) = \frac{1}{2} g (\sqrt{\frac{2h}{g}} - t)^2 - \text{ i.e.,}
\end{align*}
\]

\[
\begin{align*}
U(\sqrt{\frac{2h}{g}} - t) &= K + \frac{1}{2} g (\sqrt{\frac{2h}{g}} - t)^2
\end{align*}
\]

\[
\begin{align*}
U &= \frac{K}{\sqrt{\frac{2h}{g}} - t} + \frac{1}{2} g (\sqrt{\frac{2h}{g}} - t)^2
\end{align*}
\]

\[
\begin{align*}
U &= \frac{K}{\sqrt{\frac{2h}{g}} - t} + \frac{1}{2} (2gh - gt) \quad \text{(viii)}
\end{align*}
\]

Hence the 3rd particle must be projected with velocity \( U \geq \frac{K}{\sqrt{\frac{2h}{g}} - t} + \frac{1}{2} \sqrt{2gh} - gt \)

to meet both particles during their upward flight.
The time taken by plane moving with velocity \( v \) to cover distance \( h \) is given by (or to reach over head gunner)

\[ h = vt' \]

\[ t' = \frac{h}{v} \]  

The time taken by gun fire with initial velocity \( U \) to reach pt B which is at height \( K \) over head gunner is given by

\[ K = Ut - \frac{1}{2}gt^2 \]  

\[ gt^2 = 2Ut - 2K \]  

\[ t = \frac{2U + \sqrt{(2U)^2 - 4g(2K)}}{2g} = \frac{U + \sqrt{U^2 - 2gK}}{g} \]

Thus required time of fire i.e. \( t'' = t' - t \)

\[ t'' = \frac{h}{v} - \frac{U + \sqrt{U^2 - 2gK}}{g} \]

The roots of eq are real when

Discriminant \( \geq 0 \)

\[ (-2U)^2 - 4g(2K) \geq 0 \]

\[ 4(U^2 - 2gK) \geq 0 \]

\[ U^2 - 2gK \geq 0 \]

\[ U^2 \geq 2gK \]

Required condition to hit plane
Let us consider that 2nd particle meets 1st particle at pt. C which lies at height $h$ from A.

Then apply the equation of motion, i.e.
$$x = ut + \frac{1}{2}at^2 \quad \text{(i)}$$

where $x = h$, $t = T$, $a = -g$

$$u = v$$

Putting values in (i) we get
$$h = uT - \frac{1}{2}gT^2 \Rightarrow \quad \frac{1}{2}gT^2 - 2uT + 2h = 0 \quad \text{(ii)}$$

It is quadratic in $T$ and so gives two values say $t_1, t_2$ of $T$.

$$T = \frac{2u \pm \sqrt{(-2u)^2 - 4 \cdot \frac{1}{2}g \cdot 2h}}{2 \cdot \frac{1}{2}g} = \frac{U \pm \sqrt{U^2 - 2gh}}{g}$$

So there are two values $t_1, t_2$ of $T$ so take
$$t_1 = \frac{U + \sqrt{U^2 - 2gh}}{g}, \quad t_2 = \frac{U - \sqrt{U^2 - 2gh}}{g}$$

If $t$ is difference in times $t_1, t_2$, so
$$t = t_1 - t_2 = \frac{U + \sqrt{U^2 - 2gh}}{g} - \frac{U - \sqrt{U^2 - 2gh}}{g}$$

$$t = \frac{2U}{g} \quad \text{i.e.}$$

$$\frac{g}{4}t^2 = \sqrt{U^2 - 2gh}$$

Squaring $\frac{g}{4}t^2$ simplifying
$$\frac{g}{4}t = U^2 - 2gh$$

$$\Rightarrow U^2 = \frac{g}{4}t^2 + 2gh = \frac{g^2t^2 + 8gh}{4} \quad \text{i.e.}$$

$$U = \frac{\sqrt{8gh + g^2t^2}}{2} \quad \text{(iv)}$$

Required velocity of projection.
(iii) \(-nx^2\) 

\[ v = n^2x \quad (i) \]

Separating the variable and integrating,

\[ \int v\,dv = \int n^2x\,dx \]

\[ \frac{v^2}{2} = n^2 \frac{x^2}{2} + c_1 \quad (ii) \]

where \(c_1\) is a constant of integration.

Applying conditions i.e.,

\[ x = 0 \quad \text{when} \quad v = 0 \]

\[ c_1 = 0 \]

Thus

\[ \frac{v^2}{2} = n^2 \frac{x^2}{2} \quad \Rightarrow \quad v^2 = n^2x^2 \quad \Rightarrow \quad v = nx \]

Separating the variable and integrating,

\[ \int \frac{dx}{x} = \int n\,dt \quad \Rightarrow \quad \log x = nt + c_2 \quad (iii) \]

where \(c_2\) is another constant of integration.

---

(Sol: 11) The equation of motion is given by

\[ \frac{dv}{dt} = \pm v^2 \quad (i) \]

where the sign indicates retardation.

Separating the variable and then integrating,

\[ \int \frac{dv}{v^2} = \int \pm dt \]

\[ \frac{v^{-1}}{-1} = t + c_1 \]

\[ \Rightarrow \frac{1}{v} = t + c_1 \quad (ii) \]

where \(c_1\) is a constant of integration.

Applying initial condition i.e., \(v = u\) at \(t = 0\)

\[ \frac{1}{u} = 0 + c_1 \quad \Rightarrow \quad c_1 = 0 \]

Then

\[ \frac{1}{v} = t + \frac{1}{u} \quad \Rightarrow \quad v = \frac{ut + 1}{u} \]

\[ \frac{dx}{dt} = v = \frac{u}{ut+1} \quad (iii) \]
Separating the variable & integrating
\[ \int dx = \int \frac{v}{u} \, dt \]

\[ x = \log (ut+1) + c_2 \quad (i) \]

where \( c_2 \) is another constant of integration.

\[ x = 0 \quad \text{if} \quad t = 0 \]

\[ 0 = \log (u+1) + c_2 \Rightarrow c_2 = -\log 2 \]

Thus, \( x = \log (ut+1) \quad (ii) \)

Required distance travelled by particle in time \( t \)

**Sol: \( 12 \)**

The acceleration of particle is given by

\[ v \frac{dv}{dx} = 4x \]

Separating the variable \( v \) then integrating

\[ \int v \, dv = \int 4x \, dx \]

\[ \frac{v^2}{2} = 4 \frac{x^2}{2} + c_1 \quad (iii) \]

where \( c_1 \) is the constant of integration.

Applying condition, i.e.

\[ v = 0 \quad \text{at} \quad x = a \]

\[ 0 = 4 \frac{a^2}{2} + c_1 \]

\[ \Rightarrow c_1 = -4 \frac{a^2}{2} \]

Then

\[ \frac{v^2}{2} = 4 \frac{x^2}{2} - 4 \frac{a^2}{2} \Rightarrow v^2 = 4 (x^2 - a^2) \]

\[ \frac{dx}{dt} = \sqrt{4(x^2-a^2)} \]

Separating the variable \( x \) then integrating

\[ \int \frac{dx}{\sqrt{x^2-a^2}} = \int dt \]

\[ \Rightarrow \cosh^{-1} \frac{x}{a} = \sqrt{4} \, t + c_2 \quad (iv) \]

Where \( c_2 \) is another constant of integration.
Applying condition \( x = a \) at \( t = 0 \):
\[
\cosh^{-1}\frac{a}{x} = c_2 = c_2 
\Rightarrow c_2 = \cosh^{-1}1 = 0
\]

Thus
\[
\cosh^{-1}\frac{x}{a} = \sqrt{v} t
\]
\[
\frac{x}{a} = \cosh \sqrt{v} t
\]
\[
\Rightarrow x = a \cosh \sqrt{v} t
\]

**Sol:13:** The acceleration of particle is given by
\[
\frac{dv}{dt} = v^3
\]
Separating the variable \( \sqrt{v} \) then integrating,
\[
\int v\,dv = \int 1\,dx
\]
\[
\Rightarrow \int v^2 \,dv = x + c_1 \quad \text{i.e.}
\]
\[
v = x + c_1 \Rightarrow \frac{1}{v} = x + c_1
\]
where \( -c_1 \) is a constant of integration.

Applying condition i.e. \( v = 0 \) at \( x = 0 \)
\[
\Rightarrow \frac{1}{v} = 0 + c_1 = c_1
\]
Then
\[
\frac{1}{v} = x - \frac{1}{u} = \frac{u x - 1}{u}
\]
\[
\Rightarrow \frac{1}{v} = \frac{1-ux}{u} \quad \text{i.e.}
\]
\[
v = \frac{u}{1-ux}
\]

It shows \( v \) is a function of distance \( x \) travelled by particle.
Now \( \frac{dx}{dt} = v = \frac{u}{1-ux} \quad \text{i.e.}
\]

Separating the variable \( \sqrt{v} \) integrating,
\[
\int 1 \,dt = \int \frac{1-ux}{u} \,dx \Rightarrow t = \int \left( \frac{1}{u} - \frac{ux}{u} \right) \,dx \Rightarrow
\]
\[
t = \frac{1}{u} x - \frac{x^2}{2} + c_2 \quad \text{(iv)}
\]

Applying condition i.e. when
\[
t = 0 \Rightarrow x = 0 \quad \text{we get: } 0 = 0 + c_2 \Rightarrow c_2 = 0
\]
Then
\[
t = \frac{x}{u} - \frac{x^2}{2u} = \frac{2x - ux^2}{2u} \Rightarrow \frac{x}{u}
\]
\[ t = \frac{x}{2u} (2 - ux) - (v) \]

It shows that \( t \) is a function of distance \( x \) covered by the particle.

\[ x \quad x \quad x \]

**Sol: 14**

The acceleration due to gravity is given by

\[ v \cdot \frac{dv}{dx} = -\frac{k}{x^2} \quad \ldots \quad (i) \]

where the sign shows that \( x \) is measured against direction in which gravitational acceleration increases.

Separating the variable and then integrating,

\[ \int v \, dv = -\int \frac{k}{x^2} \, dx \quad \therefore \quad \frac{v^2}{2} = -k \int x^{-2} \, dx = -k \left( \frac{x^{-1}}{-1} \right) + c_1 \]

\[ \Rightarrow \quad \frac{v^2}{2} = \frac{k}{x} + c_1 \quad \ldots \quad (ii) \]

where \( c_1 \) is a constant of integration.

Applying condition i.e.

\[ v = 0 \quad \text{at} \quad x = R \]

\[ 0 = \frac{k}{R} + c_1 \quad \Rightarrow \quad c_1 = -\frac{k}{R} \]

Then

\[ \frac{v^2}{2} = \frac{k}{x} - \frac{k}{R} \Rightarrow \]

\[ v^2 = 2k \left( \frac{1}{x} - \frac{1}{R} \right) \quad \ldots \quad (iii) \]

\[ \Rightarrow \quad \text{The required velocity on each surface} \]

i.e. (when \( x = y = (A) \)) become

\[ v^2 = 2k \left( \frac{1}{y} - \frac{1}{R} \right) \]

\[ \Rightarrow \quad v = \sqrt{2k \left( \frac{1}{y} - \frac{1}{R} \right)} \quad \ldots \quad (iv) \]

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The velocity of a particle executing simple harmonic motion is given by

\[ v = \sqrt{\lambda (a^2 - x^2)} \quad \text{(ii)} \]

where \( a \) is amplitude.

The velocity is max (or greatest) if \( x = 0 \) so

\[ V(\text{greatest velocity}) = \sqrt{\lambda a} \quad \text{(ii)} \]

\[ \text{Time period} = \frac{2\pi}{\sqrt{\lambda}} \]

\[ N(\text{Frequency}) = \frac{1}{T} = \frac{1}{\frac{2\pi}{\sqrt{\lambda}}} \]

\[ \Rightarrow \frac{\sqrt{\lambda}}{T} = 2\pi N \quad \text{(iii)} \]

Now \[ \text{acceleration} = \frac{dv}{dx} = -\lambda x \]

It is max when \( x = a \)

\[ \Rightarrow \text{The required max value of acceleration} \]

\[ \lambda a = (\sqrt{\lambda})^2 a \]

\[ (2\pi N)^2 = \left( \frac{V}{2\pi N} \right)^2 \quad \text{(vi)} \]

It is known that

\[ v \cdot \frac{dv}{dx} = -\lambda x \]

\[ \Rightarrow \int v \, dv = -\lambda \int x \, dx \quad \text{i.e.} \]

\[ \frac{v^2}{2} = -\lambda \frac{x^2}{2} + c_1 \quad \text{(vii)} \]

where \( c_1 \) is a constant of integration.

If \( x = a \), \( v = 0 \)

\[ \Rightarrow 0 = -\frac{\lambda a^2}{2} + c_1 \Rightarrow c_1 = \frac{2\lambda a^2}{2} \]

Thus

\[ \frac{v^2}{2} = -\frac{\lambda x^2}{2} + \frac{2\lambda a^2}{2} \]

\[ v^2 = \lambda (a^2 - x^2) \]

\[ \Rightarrow v = \sqrt{\lambda (a^2 - x^2)} \quad \text{(viii)} \]

\[ \sqrt{\lambda} = 2\pi N \quad \text{(ix)} \]

\[ \Rightarrow v = 2\pi N \sqrt{\frac{a^2 - x^2}{\lambda}} \quad \text{(ix)} \]

(Proved)
\[ E_2 - 15 \]

**Sol. 16.** It is known that for a particle describing S.H.M.,
\[ v^2 = \lambda (a^2 - x^2) \quad \text{--- (i)} \]

**Case i.** If \( v = 5 \text{ ft/sec} \), \( a = 12 \text{ ft} \)
Putting values in (i)
\[ (5)^2 = \lambda (a^2 - (12)^2) \]
\[ 25 = \lambda (a^2 - 144) \quad \text{--- (ii)} \]

**Case ii.** If \( v = 4 \text{ ft/sec} \), \( a = 13 \text{ ft} \)
Putting values in (i)
\[ (4)^2 = \lambda (a^2 - (13)^2) \]
\[ 16 = \lambda (a^2 - 169) \quad \text{--- (iii)} \]

Subtracting (iii) from (ii)
\[
\begin{align*}
25 &= \lambda a^2 - 144 \\
16 &= \lambda a^2 - 169
\end{align*}
\]
\[ 9 = 25 \lambda - 169 \lambda \]
\[ 9 = 25 \lambda - 169 \lambda \]
\[ \lambda = \frac{9}{25} \Rightarrow \sqrt{\lambda} = \frac{9}{5} \quad \text{--- (iv)} \]

**Time period** i.e.
\[ T = \frac{2\pi}{\sqrt{\lambda}} = \frac{2\pi}{\sqrt{\frac{9}{5}}} = \frac{2\pi \cdot 5}{3} \]
\[ \Rightarrow T = \frac{10\pi}{3} \quad \text{--- (v)} \]

**Sol. 17.** If the particle executes S.H.M. so
The max. velocity = \( \sqrt{\lambda} \) amplitude --- (i)
where, \( a \) = amplitude \( = a \quad \text{given,} \)
\[ \max \text{ velocity} = v \]
\[ \Rightarrow v = \sqrt{\lambda} a \quad \text{--- (vi)} \]
\[ \sqrt{\lambda} = \frac{v}{a} \quad \text{--- (vii)} \]

If the particle is disturbed so its max. velocity is \( nv \) and amplitude \( a' \) (say) then:
\[ nv = \sqrt{\lambda} a' \quad \text{--- (viii)} \]
\[ \Rightarrow a' = \frac{nv}{\sqrt{\lambda}} = \frac{nv}{\sqrt{\frac{9}{5}}} \quad \text{--- (ix)} \]
Case (i)

Thus the required change in amplitude

\[ a' = a - a = na - a = (n - 1)a \]  

\[ \text{(New amplitude)} \]

Case (iii)

Time period = \( \frac{2\pi}{\sqrt{a}} \)

where as \( \sqrt{a} \) is same in both cases and \( 2\pi \) is also constant, so there is no change in time period.

### Solution : 18

Let \( O \) be centre of motion of particle, where \( a \) its acceleration at pt \( P \) and \( b \) is acceleration at pt \( Q \) at another pt \( Q \) are \( v \) and \( \xi \), then

Case (i)

\[ v^2 = \lambda (a^2 - x_1^2) \]  

\[ f = -\lambda x_1 \]  

\[ \text{where } \quad OP = x_1. \]

Case (ii)

\[ v^2 = \lambda (a^2 - x_2^2) \]  

\[ \xi = -\lambda x_2 \]  

\[ \text{where } \quad OQ = x_2. \]

Subtracting (iii) from (i)

\[ u^2 - v^2 = \lambda (x_1^2 - x_2^2) \]  

\[ \text{from (ii) & (iv)} \]

\[ f + \xi = -\lambda (x_1 + x_2) \]

\[ \Rightarrow \quad x_1 + x_2 = \frac{f + \xi}{-\lambda} \]

Thus \( (u + v)(u - v) = \lambda (x_2 - x_1)(x_2 + x_1) \)
\[(u + v)(u - v) = \lambda(x_2 - x_1)(x_2 + x_1)
\]
\[(u + v)(u - v) = \lambda(x_2 - x_1) \frac{f + \xi}{f + \xi} \quad \text{i.e.}
\]
\[x_2 - x_1 = -\frac{(u^2 - v^2)}{f + \xi}
\]
\[PQ = x_2 - x_1 = -\frac{(u^2 - v^2)}{f + \xi}
\]
\[\text{or } |PQ| = \frac{u^2 - v^2}{f + \xi} \quad \text{(viii)}
\]
\[\text{Required distance}
\]

\[\text{Sol: 19.} \]

\[v^2 = n^2(ax^2 + 2bx + c) \quad \text{(i)}\]

differentiate \(v^2 = n^2(2ax + 2b) + 0\)
\[2v \cdot \frac{dv}{dx} = n^2(2ax + 2b) \cdot \frac{b}{a}
\]
\[\Rightarrow v \cdot \frac{dv}{dx} = an^2(x + \frac{b}{a}) \quad \text{(ii)}
\]
\[\text{or } v \cdot \frac{dv}{dx} = an^2x
\]

where \(x = x + \frac{b}{a}\)

It clearly describes a s.h.m. and \(v = \frac{dv}{dx} = 0\)
at the centre of motion: i.e.
\[an^2x = 0
\]
\[\Rightarrow x = 0 \quad \text{r.e.}
\]
\[x + \frac{b}{a} = 0 \Rightarrow x = -\frac{b}{a} \quad \text{(iii)}
\]
\[\text{is centre of motion}
\]

\[0(-\frac{b}{a}, 0) \delta(0, 0)
\]

we have \(\sqrt{\lambda} = \frac{an}{\sqrt{\alpha} \cdot \alpha}
\]

Time period i.e.
\[T = \frac{2\pi}{\sqrt{\lambda}} = \frac{4\pi}{\sqrt{\alpha}} \quad \text{(iv)}
\]

If \(v = 0\), \(n^2(ax^2 + 2bx + c) = 0^2 = 0\)
\[ ax^2 + 2bx + c = 0 \quad \text{i.e.} \]

\[ x = \frac{-2b \pm \sqrt{(2b)^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - ac}}{a} \]

\[ x = \frac{-b}{a} \pm \frac{\sqrt{b^2 - ac}}{a} \quad \text{(v)} \]

\Rightarrow \text{The distance of each of these two points } -\frac{b}{a} \pm \frac{\sqrt{b^2 - ac}}{a} \text{ from centre of motion.}

\Rightarrow \text{The distance of each of these two points } -\frac{b}{a} \pm \frac{\sqrt{b^2 - ac}}{a} \text{ is } \frac{\sqrt{b^2 - ac}}{a}.

\Rightarrow \text{Required amplitude } = \frac{\sqrt{b^2 - ac}}{a} \quad \text{(vi)}.

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