In case *(ii)*, let b be the radius of the circle. Then the time t_2 required by P to return to its initial position is given by

$$t_2 = \frac{T}{\pi} \cos^{-1} \frac{x}{b}$$

$$= \frac{T}{\pi} \tan^{-1} \frac{\sqrt{b^2 - x^2}}{x}.$$

Since by equation (8.19),

$$v = \sqrt{\lambda (b^2 - x^2)},$$

it follows that the time t_2 is given by

$$t_2 = \frac{T}{\pi} \tan^{-1} \frac{v}{\sqrt{\lambda}\, x} = \frac{T}{\pi} \tan^{-1} \frac{Tv}{2\pi x},$$

because $\quad T = \dfrac{2\pi}{\sqrt{\lambda}},$

Exercises Set 8

1. Obtain the equations of motion (8.5), (8.6) and (8.10) by graphical method.

2. A particle moving in a straight line starts from rest and is accelerated uniformly to attain a velocity of 60 miles per hour in 4 seconds. Find the acceleration of motion and the distance travelled by the particle in the last three seconds.

   [**Ans.** 22 ft./sec^2., 165 ft.]

3. Two particles start simultaneously from a point O and move in a straight line; one with a velocity of 45 miles per hour and an acceleration of 2 ft./sec^2. and the other with a velocity of 90 miles per hour and a retardation (the rate of decrease of velocity) of 8 ft./sec^2. Find the time after which the velocities of the particles are the same and the distance of O from the point where they meet again.

   [**Ans.** 6·6 sec., 1045·44 ft.]

4. A particle moving along a straight line starts from rest and is accelerated uniformly till it attains a velocity v. The motion is then retarded and the particle comes to rest after traversing a total distance x. If the acceleration is f, find the retardation

and the total time taken by the particle from rest to rest.

$$\left[\textbf{Ans.}\quad \frac{v^2 f}{2fx - v^2},\ \frac{2x}{v}\right]$$

5. Two particles travel along a straight line. Both start at the same time and are accelerated uniformly at different rates. The motion is such that when a particle attains the maximum velocity v, its motion is retarded uniformly. The two particles come to rest simultaneously at a distance x from the starting point. If the acceleration of the first is a and that of the second is $\frac{1}{2}a$, find the distance between the points where the two particles attain their maximum velocities.

$$\left[\textbf{Ans.}\quad \frac{v^2}{2a}\right]$$

6. A particle is projected vertically upwards with a velocity $\sqrt{2gh}$ and another is let fall from a height h at the same time. Find the height of the point where they meet each other.

$$\left[\textbf{Ans.}\quad \frac{3h}{4}\right]$$

7. Two particles are projected simultaneously in the vertically upward direction with velocities $\sqrt{2gh}$ and $\sqrt{2gk}$ $(k>h)$. After a time t, when the two particles are still in flight, another particle is projected upwards with a velocity u. Find the condition so that the third particle may meet the first two during their upward flight.

$$\left[\textbf{Ans.}\quad t < \sqrt{\frac{2h}{g}},\ u > \frac{k}{\sqrt{\frac{2h}{g}} - t} + \frac{1}{2}\left(\sqrt{2gh} - gt\right)\right]$$

8. A gunner detects a plane at $t=0$ approaching him with a velocity v, the horizontal and the vertical distances of the plane being h and k respectively. His gun can fire a shell vertically upwards with an initial velocity u. Find the time when he should fire the gun and the condition on u so that he may be

able to hit the plane if it continues its flight in the same horizontal line.

$$\left[\textbf{Ans.}\quad t=\frac{h}{v}-x, \text{ where } x=\frac{u\pm\sqrt{u^2-2gk}}{g},\ u^2>2gk\right]$$

9. A particle is projected vertically upwards. After a time t, another particle is sent up from the same point with the same velocity and meets the first at height h during the downward flight of the first. Find the velocity of projection.

$$\left[\textbf{Ans.}\quad \frac{\sqrt{8gh+g^2t^2}}{2}\right]$$

10. Discuss the motion of a particle moving in a straight line if it starts from rest at $t=0$ and its acceleration is equal to (*i*) t^n, (*ii*) $a\cos t+b\sin t$, (*iii*) $-n^2x$.

11. A particle starts with a velocity u and moves in a straight line. If it suffers a retardation equal to the square of the velocity, find the distance travelled by the particle in a time t.

$$[\textbf{Ans.}\quad \log(ut+1)].$$

12. Discuss the motion of a particle moving in a straight line if it starts from rest at a distance a from a point O and moves with an acceleration equal to μ times its distance from O.

$$[\textbf{Ans.}\quad v=\sqrt{\mu(x^2-a^2)},\ x=a\cosh\sqrt{\mu}\,t]$$

13. A particle moving in a straight line starts with a velocity u and has acceleration v^3, where v is the velocity of the particle at time t. Find the velocity and the time as functions of the distance travelled by the particle.

$$\left[\textbf{Ans.}\quad v=\frac{u}{1-ux},\ t=\frac{x}{2u}(2-ux)\right]$$

14. The acceleration of a particle falling freely under the gravitational pull is equal to $\frac{k}{x^2}$, where x is the distance of the particle from the centre of the earth. Find the velocity of the particle if it is let fall from an altitude R, on striking the surface of the earth if the

radius of earth is r and the air offers no resistance to motion.

[**Ans.** $\sqrt{2k\left(\frac{1}{r}-\frac{1}{R}\right)}$]

15. A particle describes simple harmonic motion with frequency N. If the greatest velocity is V, find the amplitude and the maximum value of the acceleration of the particle.
Also show that the velocity v at a distance x from the centre of motion is given by $v=2\pi N\sqrt{a^2-x^2}$, where a is the amplitude.

[**Ans.** $a=\frac{V}{2\pi N}$, Max. accel. $=2\pi NV$]

16. A particle describing simple harmonic motion has velocities 5 ft./sec. and 4 ft./sec. when its distances from the centre are 12 ft. and 13 ft. respectively. Find the time-period of motion.

[**Ans.** $\frac{10\pi}{3}$].

17. The maximum velocity that a particle executing simple harmonic motion of amplitude a attains, is v. If it is disturbed in such a way that its maximum velocity becomes nv, find the change in the amplitude and the time period of motion.

[**Ans.** $(n-1)\,a$, no change].

18. A point describes simple harmonic motion in such a way that its velocity and acceleration at a point P are u and f respectively and the corresponding quantities at another point Q are v and g. Find the distance PQ.

[**Ans.** $\frac{u^2-v^2}{f+g}$]

19. If a point P moves with a velocity v given by

$$v^2=n^2(ax^2+2bx+c),$$

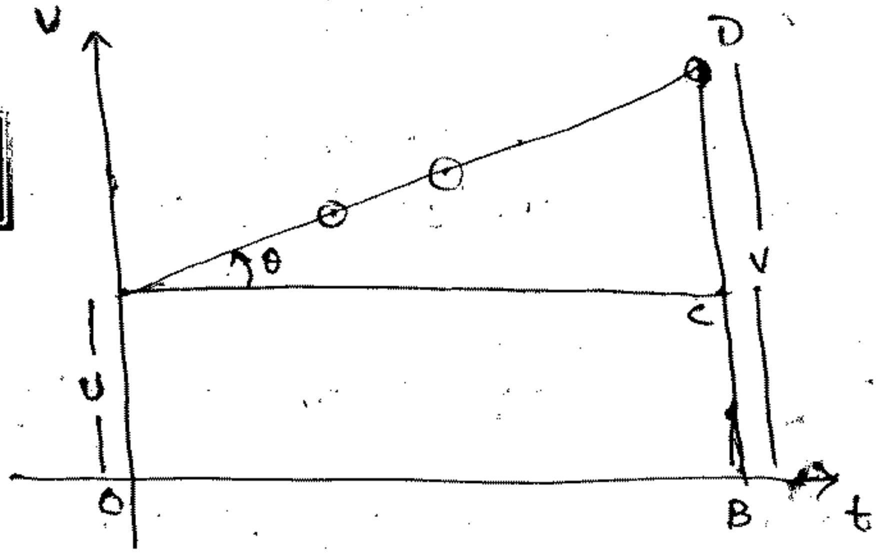
show that P executes a simple harmonic motion. Find the centre, the amplitude and the time-period of the motion.

[**Ans.** $x=\frac{-b}{a}$; $\frac{\sqrt{b^2-ac}}{a}$; $\frac{2\pi}{n\sqrt{a}}$]

Sunday
19-7-98
ES-1

Ex : set. VII

Sol : 1



We deal with constant acceleration & let u be the intial ~~acceleration~~ velocity, where as v denotes final velocity attaine by particle in time t
∵ The acceleration is constant. so velocity time-graph must be a st. line

$$\left.\begin{aligned} OA &= u \\ OB &= t \\ BD &= v \end{aligned}\right\} \text{—— (i)}$$

$\Rightarrow \quad CD = BD - BC = BD - OA = v - u$ —— (ii)

$AC = OB = t$

Thus acceleration = slope of $AD = \tan\theta = \dfrac{CD}{AC}$

$\Rightarrow \quad \dfrac{v-u}{t} = a$ — i.e.

$v - u = at$ or $\boxed{v = u + at}$ —— (iii)

The area of rectangle $OACB = ut$

& The area of $\Delta ACD = \frac{1}{2}(CD \cdot AC) = \frac{1}{2}\{(v-u)\}t$

$= \frac{1}{2}at^2$

Considering figure ~~use~~ we find that :-

The area under AD = Total distance x covered by particle in time t. ⇒

$x =$ Area of rectangle OACB + area of ΔACD

$= ut + \frac{1}{2}at^2$

$\Rightarrow \quad x = ut + \frac{1}{2}at^2$ —— (iv)

Now

$\dfrac{AC}{CD} = \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{a}$

$\Rightarrow \quad AC = \frac{CD}{a} = \frac{v-u}{a}$

$\because$ The area of rectangle OACB

$= OA \cdot AC = u \cdot \left(\frac{v-u}{a}\right) = \frac{vu - u^2}{a}$

&

The area of $\Delta ACD = \frac{1}{2} AC \cdot CD$

$= \frac{1}{2}\left(\frac{v-u}{a}\right) \cdot (v-u) = \frac{(v-u)^2}{2a}$

Thus x (Distance covered by particle in time t)

= Total area und AD

= Area of rectangle OACB + area of ΔACD

$\Rightarrow \quad x = \frac{uv - u^2}{a} + \frac{(v-u)^2}{2a}$

$= \frac{2uv - 2u^2 + v^2 + u^2 - 2uv}{2a}$

$x = \frac{v^2 - u^2}{2a}$ — i.e. $\boxed{v^2 - u^2 = 2ax}$ —(v)

Sol: 2

$u = 0$ ft/sec

$v = 60$ mile/hour $= \frac{60 \times 1760 \times 3}{60 \times 60}$ ft/sec

$\Rightarrow \quad v = 88$ ft/sec

$a = ?$ where $t = 4$ sec

We know $v = u + at$

$88 = 0 + a \cdot 4 \Rightarrow a = \frac{88}{4}$

$\Rightarrow \quad \boxed{a = 22 \text{ ft/sec}^2}$.

Let x_1 be distance traveled in $t = 4$ so

$x_1 = ut + \frac{1}{2}at^2 = 0 \cdot t + \frac{1}{2} \times 22 \times 16$

$\boxed{x_1 = 176 \text{ ft}}$.

& Let x_2 be distance traveled by first minut $t = 1$

so $x_2 = ut + \frac{1}{2}at^2 = 0 \cdot t + \frac{1}{2} \cdot 22 \cdot 1$

$\boxed{x_2 = 11}$

Distance covered by last three minute.

$x = x_1 - x_2 = (176 - 11)$ ft

$\boxed{x = 165 \text{ ft}}$.

Ex: set. 8

E8-3

Sol: 3. Motion of a first particle for which:

$u = 45$ miles/hour $= \dfrac{45 \times 1760 \times 3}{60 \times 60} = 66$ ft/sec

$a = 2$ ft/sec^2

$v = v$ & $t = t$

put values in equation $v = u + at$ ——— (i)

so $v = 66 + 2t$ ——— (ii)

Now for second particle.

$u = 90$ miles/hour $= \dfrac{90 \times 1760 \times 3}{60 \times 60}$

$u = 132$ ft/sec

$v = v$ & $t = t$ & $a = -8$ ft/sec^2

put values in (i)

$v = 132 - 8t$ ——— (iii)

When the velocities of particle same then comparing eq. (ii) & (iii)

$66 + 2t = 132 - 8t$

$10t = 66 \Rightarrow \boxed{t = 6.6 \text{ sec}}$ ——— (iv)

If the particle meets each other after covering distance x in time t'.

Now taking eq. $x = ut + \frac{1}{2}at^2$ ——— (v)

Considering case of 1st particle

$u = 66$ ft/sec , $t = t'$

$a = 2$ ft/sec^2 , $x = x$

$\therefore x = 66t' + \frac{1}{2}\cdot 2 \cdot t'^2 = 66t' + t'^2$ ——— (vi)

For motion of 2nd particle.

$u = 132$ ft/sec $t = t'$

$a = -8$ ft/sec^2 $x = x$

$\therefore x = 132t' - \frac{1}{2}\cdot 8 \cdot t'^2 = 132t' - 4t'^2$ ——— (vii)

Comparing vi) & vii)

$66t' + t'^2 = 132t' - 4t'^2$

$\Rightarrow 5t'^2 - 66t' = 0$ i.e.

$t'(5t' - 66) = 0$

$t = 0$ & $5t' = 66 \Rightarrow t' = \frac{66}{5}$ s

time will not be zero so, $\boxed{t' = 13.2 \text{ s}}$

putting $t' = 13.2$ sec in (vi) we get

$$x = 66 \times \frac{66}{5} + \left(\frac{66}{5}\right)^2$$
$$= 871.20 + 174.24$$
$$\boxed{x = 1045.44 \text{ ft}} \quad \text{(viii)}$$

Required distance covered after which they meets each other.

—— x —— x ——

Sol. 4 ∵ we have

$$v^2 - u^2 = 2ax \quad \text{(i)}$$

Let us consider particle at rest attains velocity v after travelling distance x_1 with acceleration f

∴ putting values in (i):

$$v^2 - 0 = 2f x_1$$
$$\Rightarrow \quad x_1 = \frac{v^2}{2f} \quad \text{(ii)}$$

Let same particle having velocity v comes to rest.

so put $v = 0$, $u = v$

$a = -r$ (retardation) & $x = x_2$ in (i)

we get

$$0^2 - v^2 = -2r^2 x_2 \quad \Rightarrow \quad \boxed{x_2 = \frac{v^2}{2r}} \quad \text{(iii)}$$

The particle after covering total distance x comes to rest so adding (ii) & (iii)

$$x = x_1 + x_2 = \frac{v^2}{2f} + \frac{v^2}{2r}$$

$$x = v^2\left(\frac{1}{2f} + \frac{1}{2r}\right) \Rightarrow \frac{2x}{v^2} = \frac{1}{f} + \frac{1}{r}.$$

$$\frac{1}{r} = \frac{2x}{v^2} - \frac{1}{f} \Rightarrow \boxed{r = \frac{2fx - v^2}{v^2 f}} \quad \text{(v)}$$

Required retardation

Now applying the Eq. of motion i.e.

$$v = u + at \quad \text{(vi)}$$

Case (i): If $t = t_1$, $v = v$, $u = 0$, $a = f$

$$v = 0 + f t_1 \Rightarrow t_1 = \frac{v}{f} \quad \text{(vii)}$$

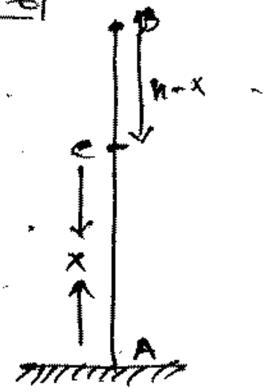
case (ii) If $t = t_2$, $v = 0$, $u = v$ & $a = -r$.
Putting values in (vi)
$$0 = v - rt_2 \Rightarrow t_2 = \frac{v}{r}.$$
Adding (vii) & (viii) we get.
$$t_1 + t_2 = \frac{v}{f} + \frac{v}{r}$$
$$= \frac{v}{f} + \cancel{v}\left(\frac{2fx - v^2}{v^{\cancel{2}}f}\right) = \frac{\cancel{v^2} + 2fx - \cancel{v^2}}{vf}$$
$$= \frac{2\cancel{f}x}{v\cancel{f}} = \frac{2x}{v} \text{ — ix}$$
(Required total time covered by particle from rest to rest).

— x — x —

$\boxed{\text{Sol: 5}}$. we have
$$v^2 - u^2 = 2ax \text{ — (i)}$$
case(i) Let us considering that 1st particle having acceleration 'a' attain max. velocity v after covering distance x_1; we have
$u = 0$ ∴ putting values in (i)
we get $v^2 - 0^2 = 2ax_1$
$$\Rightarrow x_1 = \frac{v^2}{2a} \text{ — (ii)}$$
Case (ii).
Let 2nd particle attain max; velocity v after distance x_2; having acceleration $\frac{a}{2}$
∴ The eq (i) become.
$$v^2 - 0^2 = 2 \cdot \frac{a}{2} \cdot x_2$$
$$\Rightarrow x_2 = \frac{v^2}{a} \text{ — (iii)}$$
Then required distance between two points where two particle attain their max velocity is given by.
$$x_2 - x_1 = \frac{v^2}{a} - \frac{v^2}{2a} = \frac{2v^2 - v^2}{2a}$$
$$\boxed{= \frac{v^2}{2a}} \text{ — (iv)}$$
: required Distance.

— x —

E8-6



Sol: 6. $\because$ we have

$$x = ut + \frac{1}{2}at^2$$

Let us consider that both particles meets each other at pt. C with height x from A after time t.

Case (i). Upward motion of 1st particle.

$u = \sqrt{2gh}$, $x = x = AC$

$t = t$, $a = -g$

put values in i) so

$$x = \sqrt{2gh}\, t - \frac{1}{2}gt^2 \quad \text{—— (ii)}$$

Case (ii) Downward motion of 2nd particle

Here $u = 0$, $x = h - x = BC$

$t = t$, $a = g$

put values in i) so

$$h - x = 0 \cdot t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 \quad \text{—— (iii)}$$

Adding (ii) & (iii)

$$\cancel{x} + h - \cancel{x} = \sqrt{2gh}\, t - \cancel{\frac{1}{2}gt^2} + \cancel{\frac{1}{2}gt^2}$$

$$h = \sqrt{2gh}\, t \Rightarrow t = \frac{h}{\sqrt{2gh}} = \sqrt{\frac{h}{2g}} \quad \text{—— (iv)}$$

Putting values of t in ii we have

$$x = \sqrt{2gh} \cdot \sqrt{\frac{h}{2g}} - \frac{1}{2}g\left(\sqrt{\frac{h}{2g}}\right)^2 = h - \frac{\cancel{g}h}{4\cancel{g}} = h - \frac{h}{4}$$

$$x = \frac{3}{4}h = AC \quad \text{—— (v)}$$

Required ~~partcl~~ distance where two particle meets.

Sol: 7. $\because$ we have

$$v^2 - u^2 = 2ax \quad \text{—— (i)}$$

Let H be max. height attain by 1st particle whereas $u = \sqrt{2gh}$, $v = 0$, $a = -g$

So $0^2 - (\sqrt{2gh})^2 = 2(-g)H$

$$\Rightarrow H = \frac{\cancel{-2g}h}{\cancel{-2g}} = h \quad \text{—— (ii)}$$

Similarly

~~Thus~~ max. height attained by 2nd particle $= k$ —— (iii)

$\because k > h$ (given), & 3rd particle has to

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meet both particle during their upward flight so time t should be the time when 1st particle has not yet attain max height h.

If T be time taken by 1st particle to cover distance h so applying the equation i.e,

$$V = U + at \quad \text{--- (iv)}$$

we get $0 = \sqrt{2gh} - gT$

$$\Rightarrow T = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}} \quad \text{--- (v)}$$

$\therefore$ $t < \sqrt{\frac{2h}{g}}$ & $k > h$ (given)

$$\Rightarrow \sqrt{\frac{2h}{g}} < \sqrt{\frac{2k}{g}} \quad \text{so}$$

$$t < \sqrt{\frac{2h}{g}} < \sqrt{\frac{2k}{g}} \quad \text{--- (vi)}$$ and 2nd particle also does not attain max heigh k.

Let $t' = \sqrt{\frac{2h}{g}} - t$ and 3rd particle projected with velocity u must covered distance k in t' so putting values in equation i.e,

$$x = ut' + \frac{1}{2}at' \quad \text{--- (vii)}$$

we get $k = uk' - \frac{1}{2}gt^2$

$$\Rightarrow k = u\left(\sqrt{\frac{2k}{g}} - t\right) - \frac{1}{2}g\left(\sqrt{\frac{2h}{g}} - t\right)^2 \text{ --- i.e,}$$

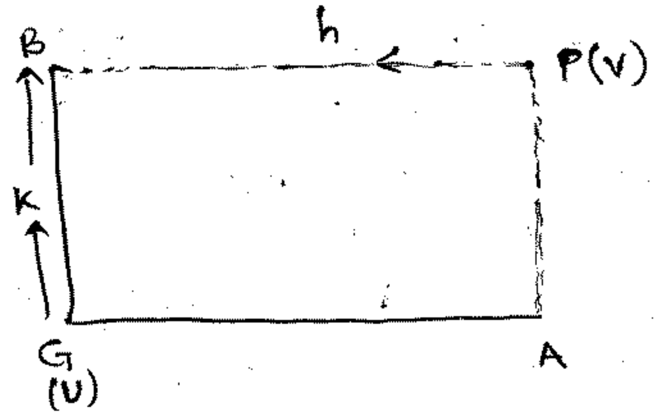
$$u\left(\sqrt{\frac{2h}{g}} - t\right) = k + \frac{1}{2}g\left(\sqrt{\frac{2k}{g}} - t\right)^2$$

$$u = \frac{k}{\sqrt{\frac{2h}{g}} - t} + \frac{1}{2}\frac{g\left(\sqrt{\frac{2h}{g}} - t\right)^2}{\left(\sqrt{\frac{2h}{g}} - t\right)}$$

$$u = \frac{k}{\sqrt{\frac{2k}{g}} - t} + \frac{1}{2}(2gh - gt) \quad \text{--- (viii)}$$

Hence the third particle must be projected with velocity $u > \frac{k}{\sqrt{\frac{2h}{g}} - 1} + \frac{1}{2}\sqrt{2gh} - gt$ to meet both particle during their upward flight

Sol: 8



The time taken by plane moving with velocity v to cover distance h is given by (or to reach over head gunner)

$$\left.\begin{aligned} h &= vt' \\ \Rightarrow \quad t' &= \frac{h}{v} \end{aligned}\right\} \text{—— (i)}$$

The time taken by gun fire with initial velocity u to reach pt B which is at height K over head gunner is given by

$$k = ut - \frac{1}{2}gt^2 \text{ —— (ii)}$$

$$\Rightarrow \quad gt^2 - 2ut + 2k = 0 \text{ —— (iii)}$$

$$\therefore \quad t = \frac{2u \pm \sqrt{(-2u)^2 - 4(g)(2k)}}{2g} = \frac{\cancel{2}\left(u \pm \sqrt{u^2 - 2gk}\right)}{\cancel{2}g}$$

$$t = \frac{u \pm \sqrt{u^2 - 2gk}}{g} \text{ —— (iv)}$$

Thus required time of fire i.e

$$t'' = t' - t$$

$$t'' = \frac{h}{v} - \frac{u \pm \sqrt{u^2 - 2gk}}{g} \text{ —— (v)}$$

The roots of eq are real when

Discriminant ≥ 0

$$\Rightarrow \quad (-2u)^2 - 4g \cdot (2k) \geq 0$$

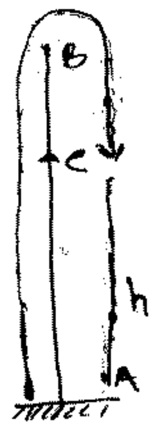
$$4(u^2 - 2gk) \geq 0$$

$$u^2 - 2gk \geq 0 \quad \text{i.e}$$

$$u^2 \geq 2gk$$

Required condition to hit plane

—— x —— x ——



Sol. 9. Let us consider that 2nd particle meets 1st particle at pt. C which lies at height h from A.

Then apply the equation of motion. i.e

$$x = ut + \frac{1}{2}at^2 \quad \text{—— (i)}$$

where $x = h$, $t = T$, $a = -g$, $u = u$,

Putting values in (i) we get

$$h = uT - \frac{1}{2}gT^2 \Rightarrow$$

$$gT^2 - 2uT + 2h = 0 \quad \text{—— (ii)}$$

It is quadric in T and so gives two values say. t_1, t_2 of T.

$$T = \frac{2u \pm \sqrt{(-2u)^2 - 4g \cdot 2h}}{2g} = \frac{2(u \pm \sqrt{u^2 - 2gh})}{2g}$$

$\because$ There are two values t_1, t_2 of T so take

$$t_1 = \frac{u + \sqrt{u^2 - 2gh}}{g}, \quad t_2 = \frac{u - \sqrt{u^2 - 2gh}}{g}$$

If t is difference in times t_1 & t_2

so $t = t_1 - t_2$

$$= \frac{u + \sqrt{u^2 - 2gh}}{g} - \frac{u - \sqrt{u^2 - 2gh}}{g}$$

$$t = \frac{2\sqrt{u^2 - 2gh}}{g} \quad \text{—— i.e}$$

$$\frac{gt}{2} = \sqrt{u^2 - 2gh}$$

Squaring & simplifying

$$\frac{g^2t^2}{4} = u^2 - 2gh$$

$$\Rightarrow u^2 = \frac{g^2t^2}{4} + 2gh = \frac{g^2t^2 + 8gh}{4} \quad \text{— i.e.}$$

$$u = \frac{\sqrt{8gh + g^2t}}{2} \quad \text{—— (iv)}$$

Required Velocity of projection

Sol: 10 (i) & (ii) are easy.

(iii) $-nx^2$ **Correction** $= +nx^2$

$\because \quad v\dfrac{dv}{dx} = n^2 x$ —— (i)

Separating the variable & integrating.

$\int v\,dv = +n^2\int x\,dx$

$\Rightarrow \quad \dfrac{v^2}{2} = n^2\dfrac{x^2}{2} + c_1$ —— (ii)

where c_1 is a constant of integration.

Applying conditions i.e.

$x = 0 \quad$ when $\quad v = 0$

$\therefore \quad 0 = 0 + c_1 \Rightarrow c_1 = 0$

Thus $\dfrac{v^2}{\cancel{2}} = \dfrac{n^2x^2}{\cancel{2}} \Rightarrow v^2 = n^2x^2 \Rightarrow v = nx$.

Separating the variable & integrating.

$\int \dfrac{dx}{x} = n\int dt \quad$ i.e

$\log_e x = nt + c_2$ —— (iv)

where c_2 is another constt of integration.

Sol: 11 The equation of motion is given by.

$-\dfrac{dv}{dt} = v^2$ —— (i)

where $-$ve sign indicates retardation

$\because$ separating the variable and then integrating.

$-\int \dfrac{dv}{v^2} = \int 1\cdot dt$

$\Rightarrow \quad -\int v^{-2}dv = \int 1\cdot dt$

$-\dfrac{v^{-1}}{(-1)} = t + c_1$

$\Rightarrow \quad \dfrac{1}{v} = t + c_1$ —— (ii)

where c_1 is constt. of integration.

Applying intial condition i.e. $v = u$ at $t=0$

$\therefore \quad \dfrac{1}{u} = 0 + c_1 = c_1$

Then $\dfrac{1}{v} = t + \dfrac{1}{u} = \dfrac{ut+1}{u}$

$\dfrac{dx}{dt} = v = \dfrac{u}{ut+1}$ —— (iii)

Ex. No. 8.

∴ Separating the variable & integrating

$$\int dx = \int \frac{v}{vt+1}\, dt \quad \text{i.e.}$$

$$x = \log(vt+1) + c_2 \quad \text{——(iv)}$$

where c_2 is another constt. of integration.

∵ $x = 0$ if $t = 0$

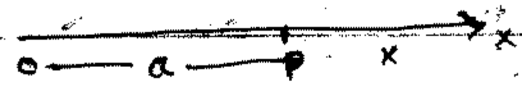
∴ $0 = \log(0+1) + c_2 = c_2$ ∵ $\log 1 = 0$

Thus $x = \log(vt+1)$ ——(v)

Required distance travelled by particle in time t.

—— x —— x —— x ——

Sol: 12



The acceleration of particle is given by

$$v\frac{dv}{dx} = \mu x \quad \text{——(i)}$$

Separating the variable & then integrating

$$\int v\,dv = \mu \int x\, dx$$

$$\frac{v^2}{2} = \mu\frac{x^2}{2} + c_1 \quad \text{——(ii)}$$

where c_1 is the constt. of integration.

Applying condition i.e.

$v = 0$ at $x = a$

$$\Rightarrow 0 = \mu\frac{a^2}{2} + c_1$$

$$\Rightarrow c_1 = -\mu\frac{a^2}{2}$$

Then

$$\frac{v^2}{\cancel{2}} = \mu\frac{x^2}{\cancel{2}} - \mu\frac{a^2}{\cancel{2}} \Rightarrow v^2 = \mu(x^2 - a^2)$$

$$\frac{dx}{dt} = \boxed{v = \sqrt{\mu(x^2 - a^2)}} \quad \text{——(iii)}$$

Separating the variable & then integrating

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \sqrt{\mu}\int 1\, dt$$

$$\Rightarrow \cosh^{-1}\frac{x}{a} = \sqrt{\mu}\, t + c_2 \quad \text{——(iv)}$$

Where c_2 is anothe constant of Integration.

Applying condition. i.e.

$x = a$ at $t = 0$

$\cosh^{-1}\frac{a}{a} = 0 + c_2 = c_2 \Rightarrow c_2 = \cosh^{-1} 1 = 0$

Thus

$\cosh^{-1}\frac{x}{a} = \sqrt{4}\, t$

$\frac{x}{a} = \cosh\sqrt{4}\, t$

$\Rightarrow \boxed{x = a\cosh\sqrt{4}\, t}$

Sol: 13. The acceleration of particle is given by

$v \cdot \frac{dv}{dx} = v^3$ —— (i)

Seperating the variable & then integrating..

$\int \frac{\cancel{v}\,dv}{v^{\cancel{3}}} = \int 1 \cdot dx$

$\Rightarrow \int v^{-2} dv = x + c_1$ — i.e.

$\frac{v^{-1}}{-1} = x + c_1 \Rightarrow -\frac{1}{v} = x + c_1$ —— (ii)

where c_1 is a constt. of integration.

Applying condition i.e $v = u$ of $x = 0$

$\therefore -\frac{1}{u} = 0 + c_1 = c_1$

Then $-\frac{1}{v} = x - \frac{1}{u} = \frac{ux - 1}{u}$

$\Rightarrow +\frac{1}{v} = \frac{1 - ux}{u}$ — i.e.

$v = \frac{u}{1 - ux}$ —— (iii)

It shows v is a function of distance x travelled by particle.

Now $\frac{dx}{dt} = v = \frac{u}{1 - ux}$ i.e.

Separating the variable & integrating.

$\int 1 \cdot dt = \int \frac{1 - ux}{u} dx \Rightarrow t = \int \left(\frac{1}{u} - \frac{\cancel{u}x}{\cancel{u}}\right) dx$ i.e.

$t = \frac{1}{u} \cdot x - \frac{x^2}{2} + c_2$ —— (iv)

where c_2 is another constt. of integration.

Applying condition. i.e. when

$t = 0,\ x = 0$, we get: $0 = 0 - 0 + c_2 = c_2$

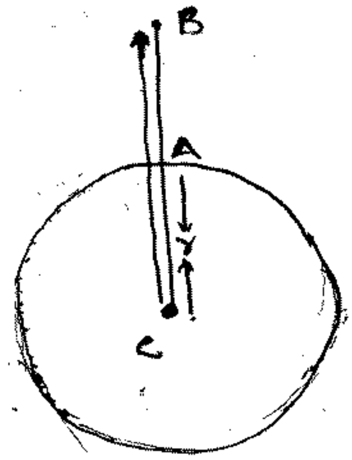
Then $t = \frac{x}{u} - \frac{x^2}{2} = \frac{2x - ux^2}{2u}$ $\Rightarrow$ ~~t=~~

$$\Rightarrow \quad t = \frac{x}{2U}(2 - Ux) \quad \text{—— (v)}.$$

It shows that t is a function of distance x covered by the particle.

—— x —— x —— x ——

Sol: 14



The acceleration due to gravity is given by

$$v \cdot \frac{dv}{dx} = -\frac{k}{x^2} \quad \text{—— (i)}$$

where $-ve$ sign shows that x is measured against direction in which gravitational acceleration increases.

Separating the variable and then integrating. i.e.

$$\int v\,dv = -\int \frac{k}{x^2}\,dx$$

$$\frac{v^2}{2} = -k\int x^{-2}dx = -k\left(\frac{x^{-1}}{-1}\right) + c_1$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{k}{x} + c_1 \quad \text{—— (ii)}$$

where c_1 is a constt. of integration.

Applying condition i.e.

$$v = 0 \quad \text{at} \quad x = R$$

$$\therefore \quad 0 = \frac{k}{R} + c_1 \Rightarrow c_1 = -\frac{k}{R}$$

Then $\frac{v^2}{2} = \frac{k}{x} - \frac{k}{R} \Rightarrow$

$$v^2 = 2k\left(\frac{1}{x} - \frac{1}{R}\right) \quad \text{—— (iii)}$$

$\Rightarrow$ The required velocity on earth surface i.e. (when $x = r = CA$) become

$$v^2 = 2k\left(\frac{1}{r} - \frac{1}{R}\right)$$

$$\Rightarrow \quad v = \sqrt{2k\left(\frac{1}{r} - \frac{1}{R}\right)} \quad \text{—— (iv)}$$

Sol: 15 $\because$ The velocity of a particle executing S.H.M is given by

$$v = \sqrt{\lambda(a^2 - x^2)} \text{ ——— (i)}$$

where a is amplitude.

The velocity is max (or greatest) if $x = 0$ so:-

$\therefore$ V (greatest velocity) $= \sqrt{\lambda}\, a$ —— (ii)

& T (time period) $= \dfrac{2\pi}{\sqrt{\lambda}}$

N (Frequency) $= \dfrac{1}{T}$ or $\dfrac{\sqrt{\lambda}}{2\pi}$

$\Rightarrow \sqrt{\lambda} = 2\pi N$ ——— (iii)

Then

$$a = \frac{V}{\sqrt{\lambda}} = \frac{V}{2\pi N} \text{ —— (iv)}$$

(Required amplitude)

Now $|\text{acceleration}| = |-\lambda x| = \lambda x$ —(v)

It is max when $x = a$

$\Rightarrow$ The required max value of acceleration

$$\lambda a = (\sqrt{\lambda})^2 a$$

$$(2\pi N)^{\cancel{2}} \cdot \left(\frac{V}{\cancel{2\pi N}}\right)$$

$$= 2\pi N V \text{ ——— (vi)}$$

It is know that

$$v \cdot \frac{dv}{dx} = -\lambda x$$

$\Rightarrow \int v\,dv = -\lambda \int x\,dx$ —— i.e

$$\frac{v^2}{2} = -\lambda \cdot \frac{x^2}{2} + c_1 \text{ —— (vii)}$$

where c_1 is a constt of integration.

If $x = a$, $v = 0$

$\therefore 0 = -\lambda a^2 + c_1 \Rightarrow c_1 = \dfrac{\lambda a}{2}$

Thus $\dfrac{v^2}{\cancel{2}} = -\dfrac{\lambda x^2}{\cancel{2}} + \dfrac{\lambda a^2}{\cancel{2}}$ — i.e

$$v^2 = \lambda(a^2 - x^2)$$

$\Rightarrow v = \sqrt{\lambda}\sqrt{a^2 - x^2}$ .... (viii)

$\because \sqrt{\lambda} = 2\pi N$

$\Rightarrow v = 2\pi N\sqrt{a^2 - x^2}$ ——— (ix)

Proved

——— x ——— x ———

[Page: 170] **Ex. No. 8** [E2-15]

Sol: 16 It is known that for a particle discribing S.H.M.

$$v^2 = \lambda(a^2 - x^2) \quad \text{---}(i)$$

Case (i) If $v = 5\,ft/sec$, $x = 12\,ft$

Putting values in (i)

$$(5)^2 = \lambda(a^2 - (12)^2)$$

$$25 = \lambda(a^2 - 144) \quad \text{---}(ii)$$

Case (ii) If $v = 4\,ft/sec$, $x = 13\,ft$

Putting values in (i)

$$(4)^2 = \lambda\{a^2 - (13)^2\}$$

$$16 = \lambda(a^2 - 169) \quad \text{---}(iii)$$

Subtracting (iii) from (ii)

$$25 = \cancel{\lambda a^2} - 144\lambda$$

$$16 = \cancel{\lambda a^2} \mp 169\lambda$$

$$9 = 25\lambda \quad \text{i.e.}$$

$$\lambda = \frac{9}{25} \Rightarrow \sqrt{\lambda} = \frac{3}{5} \quad \text{---}(iv)$$

$\because$ Time period i.e.

$$T = \frac{2\pi}{\sqrt{\lambda}} = \frac{2\pi}{3/5} = 2\pi \cdot \frac{5}{3}$$

$$\Rightarrow T = \frac{10\pi}{3} \quad \text{---}(v)$$

Sol: 17 If the particle excutes S.H.M. so

The max: velocity $= \sqrt{\lambda}$ amplitude ---(i)

where as amplitude $= a$, max velocity $= v$ } given,

$$\Rightarrow v = \sqrt{\lambda}\, a \text{ --- i.e.}$$

$$\left.\begin{aligned}\sqrt{\lambda} &= \frac{v}{a}\\ \& \quad a &= \frac{v}{\sqrt{\lambda}}\end{aligned}\right\} \text{---}(ii)$$

If the particle is disturbed so its max: velocity is nv and amplitude a' (say) Then

$$nv = \sqrt{\lambda}\, a' \quad \text{---}(iii)$$

$$\Rightarrow a' = \frac{nv}{\sqrt{\lambda}} = \frac{nv}{v/a} \text{ --- i.e.}$$

$\because \quad a' = \frac{n\cancel{v}\cdot a}{\cancel{v}} = na$ —— (iv)

(New amplitude)

Case (i) Thus the required change in amplitude

$= a' - a = na - a$

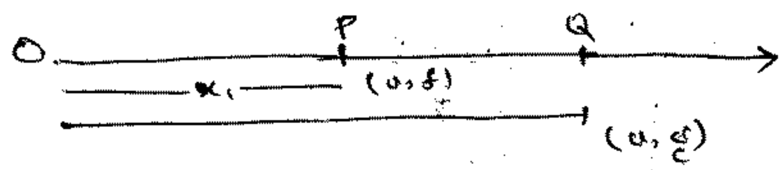
$\boxed{= (n-1)a}$ —— (v)

Case (ii)

Time period $= \dfrac{2\pi}{\sqrt{\lambda}}$ ——

where as $\sqrt{\lambda}$ is same in both cases and 2π is also constant so there is no change in time period.

Sol: 18



Let O be centre of motion of particle; where as its velocity & acceleration at pt P are v & f (respectively) and its corresponding quantities at another pt Q are V & g

Then

Case (i) $\quad v^2 = \lambda(a^2 - x_1^2)$ —— (i)

$f = -\lambda x_1$ —— (ii)

where $OP = x_1$

Case (ii) $\quad V^2 = \lambda(a^2 - x_2^2)$ —— (iii)

$g = -\lambda x_2$ —— (iv)

where $OQ = x_2$

Subtracting (iii) from (i)

$$v^2 = \lambda \cancel{a^2} - \lambda x_1^2$$
$$\underline{-V^2 = -\cancel{\lambda a^2} \mp \lambda x_2^2}$$
$$v^2 - V^2 = \lambda(x_2^2 - x_1^2) \quad \text{—— (v)}$$

from (ii) & (iv)

$$f + g = -\lambda(x_1 + x_2)$$
$$\Rightarrow x_1 + x_2 = \frac{f+g}{-\lambda}$$

Thus $(v+V)(v-V) = \lambda(x_2 - x_1)(x_2 + x_1)$

$$\Rightarrow (u+v)(u-v) = \lambda(x_2-x_1)(x_2+x_1)$$

$$(u+v)(u-v) = \lambda(x_2-x_1)\frac{f+\frac{\sigma}{2}}{(-\lambda)} \text{ — i.e.}$$

$$x_2-x_1 = -\left(\frac{u^2-v^2}{f+\frac{\sigma}{2}}\right)$$

$$\overrightarrow{PQ} = x_2-x_1 = -\frac{(u^2-v^2)}{f+\frac{\sigma}{2}}$$

$$\text{or } |\overrightarrow{PQ}| = \frac{u^2-v^2}{f+\frac{\sigma}{2}} \text{ — (viii)}$$

Required distance.

Sol: 19.

$$v^2 = n^2(ax^2+2bx+c) \text{ — (i)}$$

differentiate (i) w.r.t. x.

$$2v\cdot\frac{dv}{dx} = n^2(2ax+2b\cdot 1+0)$$

$$= 2an^2\left(x+\frac{b}{a}\right)$$

$$\left.\begin{aligned} \Rightarrow v\cdot\frac{dv}{dx} &= an^2\left(x+\frac{b}{a}\right) \\ \text{or } v\cdot\frac{dv}{dx} &= an^2X \end{aligned}\right\} \text{(ii)}$$

where $X = x+\frac{b}{a}$

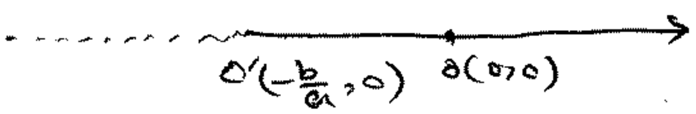
It clearly describes a S.H.M. and $v = \frac{dv}{dx} = 0$ at centre of motion i.e.

$$an^2X = 0$$

$$\Rightarrow X = 0 \quad \text{i.e.}$$

$$x+\frac{b}{a} = 0 \Rightarrow x = -\frac{b}{a} \text{ — (iii)}$$

is centre O′ of motion.



we have $\lambda = an^2$

$$\sqrt{\lambda} = \sqrt{a}\cdot n$$

Time period i.e.

$$T = \frac{2\pi}{\sqrt{\lambda}} = \frac{4\pi}{n\sqrt{a}} \text{ — (iv)}$$

If $v = 0$, $n^2(ax^2+2bx+c) = 0^2 = 0$

$\Rightarrow$ $ax^2 + 2bx + c = 0$ — i.e.

$$x = \frac{-2b \pm \sqrt{(2b)^2 - 4a.c}}{2a} = \frac{2\{-b \pm \sqrt{b^2 - ac}\}}{2a}$$

$$x = -\frac{b}{a} \pm \frac{\sqrt{b^2 - ac}}{a} \quad \text{——— (v)}$$

$\Rightarrow$ The distance of each of these two points $-\frac{b}{a} \pm \frac{\sqrt{b^2 - ac}}{a}$ ——— (v)

$\Rightarrow$ The distance of each of these two points $-\frac{b}{a} \pm \frac{\sqrt{b^2 - ac}}{a}$ from centre of motion.

i.e. $x = -\frac{b}{a}$ is $\frac{\sqrt{b^2 - ac}}{a}$

So

required amplitude $= \boxed{\frac{\sqrt{b^2 - ac}}{a}}$ ——— (vi).

——— × ———

The End.