EXERCISE # 9.2

❖ **Question # 1:** $\frac{dy}{dx}=\frac{x^2}{y(1+x^3)}$

Solution:

Given equation is

$$\frac{dy}{dx}=\frac{x^2}{y(1+x^3)}$$

$$\Rightarrow ydy=\frac{x^2dx}{1+x^3}\quad \text{(by separating var.)}$$

Integrating both sides, we have

$$\int ydy=\int\frac{x^2}{1+x^3}dx$$

$$\Rightarrow \frac{y^2}{2}=\frac{1}{3}\int\frac{3x^2}{1+x^3}dx$$

$$\Rightarrow \frac{y^2}{2}=\frac{1}{3}\ln(1+x^3)+c_1$$

$$\Rightarrow 3y^2=2\ln(1+x^3)+6c_1$$

$$\Rightarrow 3y^2-2\ln(1+x^3)+c \quad \because 6c_1=c$$

is required solution.

❖ **Question # 2:** $\frac{dy}{dx}+y^2\sin x=0$

Solution:

Given equation is

$$\frac{dy}{dx}+y^2\sin x=0$$

$$\Rightarrow \frac{dy}{dx}=-y^2\sin x$$

$$\Rightarrow \frac{dy}{y^2}=-\sin x dx$$

Integrating both sides

$$\int\frac{dy}{y^2}=-\int\sin x dx$$

$$\Rightarrow \frac{-1}{y}=-(-\cos x)+c$$

$$\Rightarrow \frac{1}{y}+\cos x=c$$

is required solution.

❖ **Question #3:** $\frac{dy}{dx}=1+x+y^2+xy^2$

Solution:

Given equation is

$$\frac{dy}{dx}=1+x+y^2+xy^2$$

$$\Rightarrow \frac{dy}{dx}=1(1+x)+y^2(1+x)$$

$$\Rightarrow \frac{dy}{dx}=(1+y^2)(1+x)$$

$$\Rightarrow \frac{dy}{1+y^2}=(1+x)dx$$

Integrating both sides

$$\Rightarrow \int\frac{dy}{1+y^2}=\int(1+x)dx$$

$$\Rightarrow \tan^{-1}(y)=\int dx+\int xdx$$

$$\Rightarrow \tan^{-1}(y)-x-\frac{x^2}{2}=c$$

is required solution.

❖ **Question # 4:**

$$(xy+2x+y+2)dx+(x^2+2x)dy=0$$

Solution:

Given equation is

$$(xy+2x+y+2)dx+(x^2+2x)dy=0$$

$$\Rightarrow (xy+2x+y+2)dx=-(x^2+2x)dy$$

$$\Rightarrow y(x+1)+2(x+1)dx=-(x^2+2x)dy$$

$$\Rightarrow (y+2)(x+1)dx=-(x^2+2x)dy$$

$$\Rightarrow \frac{dy}{y+2}=-\frac{x+1}{(x^2+2x)}dx$$

Integrating both sides

$$\int\frac{dy}{y+2}=-\int\frac{x+1}{(x^2+2x)}dx$$

$$\Rightarrow \int\frac{dy}{y+2}=-\frac{1}{2}\int\frac{2x+2}{x^2+2x}dx$$

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$\Rightarrow \ln(y+2) = -\frac{1}{2}\ln(x^2+2x)+c$

is required solution.

❖ **Question #5:-**

$\frac{dy}{dx} = 2x^2 + y + x^2y + xy - 2x - 2$

Solution:

Given equation is

$\frac{dy}{dx} = 2x^2 + y + x^2y + xy - 2x - 2$

$\Rightarrow \frac{dy}{dx} = y(1 - x^2 + x) + 2(x^2 - x - 1)$

$\Rightarrow \frac{dy}{dx} = y(1 - x^2 + x) - 2(1 - x^2 + x)$

$\Rightarrow \frac{dy}{dx} = (y-2)(1 - x^2 + x)$

$\Rightarrow \frac{dy}{y-2} = (1 - x^2 + x)dx$

Integrating both sides

$\Rightarrow \int \frac{dy}{y-2} = \int (1 - x^2 + x)dx$

$\Rightarrow \int \frac{dy}{y-2} = \int dx - \int x^2dx + \int xdx$

$\Rightarrow \ln(y-2) = x - \frac{x^3}{3} + \frac{x^2}{2} + c_1$

$\Rightarrow 6\ln(y-2) = 6x - 2x^3 + 3x^2 + 6c_1$

$\Rightarrow -6\ln(y-2) = -6x + 2x^3 - 3x^2 - 6c_1$

$\Rightarrow \ln(y-2)^{-6} = 2x^3 - 3x^2 - 6x + c_2 \because -6c_1 = c_2$

$\Rightarrow (y-2)^{-6} = e^{2x^3-3x^2-6x+c_2}$

$\Rightarrow (y-2)^{-6} = ce^{2x^3-3x^2-6x}$

is required solution.

❖ **Question # 6: $\mathbf{cosec\,ydx + secxdy = 0}$**

Solution:

Given equation is

$\text{cosec}\,ydx + \sec x dy = 0$

$\Rightarrow \text{cosec}ydx = -\text{cosec}ydx$

$\Rightarrow \frac{dx}{\sec x} = -\frac{dy}{\text{cosec}\,y}$

Integrating both sides

$\Rightarrow \int \frac{dx}{\sec x} = -\int \frac{dy}{\text{cosec}y}$

$\Rightarrow \int \cos x dx = -\int \sin y dy$

$\Rightarrow \sin x = -(-\cos x)$

$\Rightarrow \sin x = \cos x + c$

$\Rightarrow \sin x - \cos x = c$

is required solution.

❖ **Question # 7: $\mathbf{y(1+x)dx + x(1+y)dy}$**

Solution:

Given equation is

$y(1+x)dx + x(1+y)dy$

$\Rightarrow y(1+x)dx = -x(1+y)dy$

$\Rightarrow \frac{(1+x)}{x}dx = -\frac{(1+y)}{y}dy$

Integrating both sides

$\Rightarrow \int \frac{(1+x)}{x}dx = -\int \frac{(1+y)}{y}dy$

$\Rightarrow \int \frac{dx}{x} + \int \frac{x}{x}dx = -\int \frac{dy}{y} - \int \frac{y}{y}dy$

$\Rightarrow \ln x + x = -\ln y - y + c$

$\Rightarrow x + y + \ln x + \ln y = c$

$\Rightarrow x + y + \ln|xy| = c$

is required solution.

❖ **Question # 8: $\mathbf{y\sqrt{1+x^2}dx + x\sqrt{1+y^2}dy = 0}$**

Solution:

Given equation is

$y\sqrt{1+x^2}dx + x\sqrt{1+y^2}dy = 0$

$\Rightarrow y\sqrt{1+x^2}dx = -x\sqrt{1+y^2}dy$

$\Rightarrow \frac{\sqrt{1+x^2}}{x}dx = -\frac{\sqrt{1+y^2}}{y}dy$

Integrating both sides

$\Rightarrow \int \frac{\sqrt{1+x^2}}{x}dx = -\int \frac{\sqrt{1+y^2}}{y}dy \quad --- \quad (1)$

Consider

$$I_1 = \int \frac{\sqrt{1+x^2}}{x} dx$$

$$\text{put } x = \tan\theta$$
$$dx = \sec^2\theta$$

Therefore,

$$I_1 = \int \frac{\sec\theta.\sec^2\theta}{\tan\theta}$$

$$I_1 = \int \frac{\sec\theta.(1+\tan^2\theta)d\theta}{\tan\theta}$$

$$I_1 = \int \frac{\sec\theta}{\tan\theta} d\theta + \int \frac{\sec\theta.\tan^2\theta}{\tan\theta} d\theta$$

$$I_1 = \int \frac{1/\cos\theta}{\sin\theta/\cos\theta} d\theta + \int \sec\theta.\tan\theta\, d\theta$$

$$I_1 = \int \frac{1}{\sin\theta} d\theta + \int \sec\theta.\tan\theta\, d\theta$$

$$I_1 = \int \operatorname{cosec}\theta d\theta + \int \sec\theta.\tan\theta\, d\theta$$

$$I_1 = \ln(\operatorname{cosec}\theta - \cot\theta) + \sec\theta$$

$$x = \tan\theta$$

$$\cot\theta = \frac{1}{x}$$

$$\operatorname{cosec}\theta = \sqrt{\cot^2\theta + 1}$$

$$\operatorname{cosec}\theta = \sqrt{\frac{1}{x^2} + 1}$$

$$\operatorname{cosec}\theta = \frac{\sqrt{1+x^2}}{x}$$

Therefore,

$$I_1 = \ln\left(\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right) + \sec\theta$$

$$\Rightarrow \int \frac{\sqrt{1+x^2}}{x} dx = \ln\left(\frac{\sqrt{1+x^2}-1}{x}\right) + \sqrt{1+x^2}$$

Similarly,

$$\int \frac{\sqrt{1+y^2}}{y} dy = \ln\left(\frac{\sqrt{1+y^2}-1}{y}\right) + \sqrt{1+y^2}$$

Therefore, (1) becomes

$$\ln\left(\frac{\sqrt{1+x^2}-1}{x}\right) + \sqrt{1+x^2}$$
$$= -\ln\left(\frac{\sqrt{1+y^2}-1}{y}\right) - \sqrt{1+y^2}$$
$$+ c$$

$$\Rightarrow \sqrt{1+x^2} + \sqrt{1+y^2}$$
$$+ \ln\left(\frac{(\sqrt{1+x^2}-1)(\sqrt{1+y^2}-1)}{xy}\right)$$
$$= c$$

❖ **Question # 9:** $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

Solution:

Given equation is

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\Rightarrow -dx\left(\sqrt{1-y^2}\right) = dy\left(\sqrt{1-x^2}\right)$$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = -\frac{dy}{\sqrt{1-y^2}}$$

Integrating both sides

$$\Rightarrow \int \frac{dx}{\sqrt{1-x^2}} = -\int \frac{dy}{\sqrt{1-y^2}}$$

$$\Rightarrow \sin^{-1}(x) = -\sin^{-1}(y) + c$$

$$\Rightarrow \sin^{-1}(x) + \sin^{-1}(y) = c$$

is required solution.

Question # 10: $(\mathbf{e^x+1})\mathbf{ydy} = (\mathbf{y+1})\mathbf{e^x dx}$

Solution:

Given equation is

$$(e^x+1)ydy = (y+1)e^x dx$$

$$\Rightarrow \frac{y}{y+1} dy = \frac{e^x}{e^x+1} dx$$

Integrating both sides

$$\int \frac{y}{y+1} dy = \int \frac{e^x}{e^x+1} dx$$

$$\Rightarrow \int \frac{y+1-1}{y+1} dy = \int \frac{e^x}{e^x+1} dx$$

$\Rightarrow \int \frac{y+1}{y+1}dy - \int \frac{dy}{1+y} = \ln(e^x+1)$

$\Rightarrow \int dy - \int \frac{dy}{1+y} = \ln(e^x+1)$

$\Rightarrow y - \ln(y+1) = \ln(e^x+1) + c$

$\Rightarrow c + y = \ln(y+1) + \ln(e^x+1)$

Is required solution.

❖ Question # 11: $\frac{dy}{dx} = \frac{y^3+2y}{x^2+3x}$

Solution:

Given equation is

$\frac{dy}{dx} = \frac{y^3+2y}{x^2+3x}$

$\Rightarrow (x^2+3x)dy = (y^3+2y)dx$

$\Rightarrow \frac{dy}{(y^3+2y)} = \frac{dx}{x^2+3x}$

$\int \frac{dy}{y(y^2+2)} = \int \frac{dx}{x(x+3)} \quad ----(1)$

Consider

$\frac{1}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3}$

$\Rightarrow 1 = A(x+3) + B(x)$

Put $x = 0$ in equation (1), we have

$1 = A(3)$

$\Rightarrow A = \frac{1}{3}$

Put $x + 3 = 0 \Rightarrow x = -3$ in (1), we have

$1 = 0 - 3B$

$\Rightarrow B = -\frac{1}{3}$

$\frac{1}{x^2+3x} = \frac{1}{3x} - \frac{1}{3(x+3)}$

$\Rightarrow \int \frac{dx}{x(x+3)} = \frac{1}{3}\int \frac{dx}{x} - \frac{1}{3}\int \frac{dx}{x+3}$

$\Rightarrow \int \frac{dx}{x(x+3)} = \frac{1}{3}\ln x - \frac{1}{3}\ln(x+3)$

$\Rightarrow \int \frac{dx}{x(x+3)} = \frac{1}{3}(\ln x - \ln(x+3))$

$\Rightarrow \int \frac{dx}{x(x+3)} = \frac{1}{3}\ln\left(\frac{x}{x+3}\right)$

Consider

$\frac{1}{y(y^2+2)} = \frac{C}{y} + \frac{Dy+E}{y^2+2} \quad ---(2)$

$\Rightarrow 1 = C(y^2+2) + (Dy+E)(y)$

$\Rightarrow 1 = C(y^2+2) + D(y^2) + E(y)$

put $y = 0$ then

$\Rightarrow 1 = C(2) \Rightarrow C = \frac{1}{2}$

Now,

Comparing the co-efficient of y^2 & y of (2)

y^2: $\quad 0 = C + D$

$\Rightarrow 0 = \frac{1}{2} + D$

$\Rightarrow D = -\frac{1}{2}$

y: $\quad E = 0$

Therefore, equation (2) will become

$\frac{1}{y(y^2+2)} = \frac{1}{2y} + \frac{\frac{-1}{2}+0}{y^2+2}$

Integrating both sides, we have

$\int \frac{dy}{y(y^2+2)} = \frac{1}{2}\int \frac{dy}{y} - \frac{1}{2}\int \frac{ydy}{y^2+2}$

$\int \frac{dy}{y(y^2+2)} = \frac{1}{2}\ln y - \frac{1}{2\times 2}\int \frac{2ydy}{y^2+2}$

$= \frac{1}{2}\ln y - \frac{1}{4}\ln(y^2+2)$

$= \frac{1}{2}\left(\ln y - \frac{1}{2}\ln(y^2+2)\right)$

$= \frac{1}{2}\left(\ln\left(\frac{y}{\sqrt{y^2+2}}\right)\right)$

Equation (1)becomes

$\frac{1}{2}\left(\ln\left(\frac{y}{\sqrt{y^2+2}}\right)\right) = \frac{1}{3}\ln\left(\frac{x}{x+3}\right) + c$

❖ Question # 12:

$(\sin x + \cos x)dy + (\cos x - \sin x)dx = 0$

Solution:

Given equation is

$(\sin x+\cos x)dy+(\cos x-\sin x)dx=0$

$(\sin x+\cos x)dy=-(\cos x-\sin x)dx$

$dy=-\left(\frac{\cos x-\sin x}{\sin x+\cos x}\right)dx$

Integrating both sides

$\int dy=-\int\left(\frac{\cos x-\sin x}{\sin x+\cos x}\right)$

$\Rightarrow y=-\ln(\sin x+\cos x)+\ln c$

$\Rightarrow y=\ln\left(\frac{c}{\sin x+\cos x}\right)$

$\Rightarrow e^y=\frac{c}{\sin x+\cos x}$

$\Rightarrow e^y(\sin x+\cos x)=c$

is required solution.

Question # 13: $e^x\left(1+\frac{dy}{dx}\right)=xe^{-y}$

Solution:

Given equation is

$e^x\left(1+\frac{dy}{dx}\right)=xe^{-y}$

$\Rightarrow \frac{e^x}{e^{-y}}\left(1+\frac{dy}{dx}\right)=x$

$\Rightarrow e^{x+y}\left(1+\frac{dy}{dx}\right)=x$

put $x+y=z$

$1+\frac{dy}{dx}=\frac{dz}{dx}$

$e^z\left(\frac{dz}{dx}\right)=x$

$e^z dz=xdx$

Integrating both sides

$\int e^z dz=\int xdx$

$\Rightarrow e^z=\frac{x^2}{2}+c$

$\Rightarrow e^{x+y}=\frac{x^2}{2}+c$

Taking log on both sides

$x+y=\ln\left(\frac{x^2}{2}+c\right)$

$\Rightarrow y=\ln\left(\frac{x^2}{2}+c\right)-x$

Question # 14: $xe^{(x^2+y)}dx=ydy$

Solution:

Given equation is

$xe^{(x^2+y)}dx=ydy$

$\Rightarrow xe^{x^2}.e^y dx=ydy$

$\Rightarrow xe^{x^2}dx=\frac{ydy}{e^y}$

Integrating both sides

$\int xe^{x^2}dx=\int ye^{-y}dy$

$\Rightarrow \frac{1}{2}\int e^{x^2}.2xdx=y.\left(\frac{e^{-y}}{-1}\right)-\int\frac{e^{-y}}{-1}dy$

$\Rightarrow \frac{1}{2}e^{x^2}=-ye^{-y}+\int e^{-y}dy$

$\Rightarrow \frac{1}{2}e^{x^2}=-ye^{-y}+\frac{e^{-y}}{-1}+c$

$\Rightarrow \frac{1}{2}e^{x^2}=-ye^{-y}-e^{-y}+c$

is required solution.

❖ **Question # 15:**

$(2x\cos y)dx+x^2(\sec y-\sin y)dy=0$

Solution:

Given equation is

$(2x\cos y)dx+x^2(\sec y-\sin y)dy=0$

$\Rightarrow (2x\cos y)dx=-x^2(\sec y-\sin y)dy$

$\Rightarrow -\frac{2xdx}{x^2}=\frac{\sec y-\sin y}{\cos y}dy$

Integrating both sides

$-2\int\frac{xdx}{x^2}=\int\frac{\left(\frac{1}{\cos y}-\sin y\right)}{\cos y}dy$

$\Rightarrow -2\int\frac{dx}{x}=\int\frac{\left(\frac{1-\sin y\cos y}{\cos y}\right)}{\cos y}dy$

$\Rightarrow -2\ln x=\int\frac{1-\sin y\cos y}{\cos^2 y}dy$

$\Rightarrow -2\ln x=\int\frac{1}{\cos^2 y}dy-\int\frac{\sin y\cos y}{\cos^2 y}dy$

$\Rightarrow -2\ln x = \int \sec^2 y dy - \int \frac{\sin y}{\cos y} dy$

$\Rightarrow -2\ln x = \int \sec^2 y dy - \int \tan y dy$

$\Rightarrow -2\ln x = \tan y - \ln|\sec y|$

$\Rightarrow 2\ln x = \ln|\sec y| - \tan y + c$ **Ans**

Solve the initial value problems:

❖ **Question # 16:**

$\mathbf{2(y-1)dy = (3x^2+4x+2)dx, y(0) = -1}$

Solution:

Given equation is

$2(y-1)dy = (3x^2+4x+2)dx$

Integrating both sides

$\int 2(y-1)dy = \int (3x^2+4x+2)dx$

$\Rightarrow 2\int (y-1)dy = 3\int x^2dx + 4\int xdx + 2\int dx$

$\Rightarrow 2\int ydy - 2\int dy = 3.\frac{x^3}{3} + 4.\frac{x^2}{2} + 2x + c$

$\Rightarrow 2.\frac{y^2}{2} - 2y = x^3 + 2x^2 + 2x + c$

$\Rightarrow y^2 - 2y = x^3 + 2x^2 + 2x + c \quad --- \quad (i)$

Applying $\mathbf{y(0) = -1}$, **we have**

$(-1)^2 - 2(-1) = 0 + 0 + 0 + c$

$\Rightarrow 1 + 2 = c$

$\Rightarrow c = 3$

Equation(i) becomes

$y^2 - 2y = x^3 + 2x^2 + 2x + 3$

is required solution.

❖ **Question # 17:**

$\mathbf{(3x+8)(y^2+4)dx - 4y(x^2+5x+6)dy = 0}$

$\mathbf{y(1) = 2}$

Solution:

Given equation is

$\mathbf{(3x+8)(y^2+4)dx - 4y(x^2+5x+6)dy = 0}$

$\Rightarrow (3x+8)(y^2+4)dx = 4y(x^2+5x+6)dy$

$\Rightarrow \frac{(3x+8)}{x^2+5x+6}dx = \frac{4y}{y^2+4}dy$

Integrating both sides

$\Rightarrow \int \frac{(3x+8)}{x^2+5x+6}dx = 2\int \frac{2y}{y^2+4}dy$

$\Rightarrow \int \frac{(3x+8)}{x^2+5x+6}dx = 2\ln(y^2+4) \quad ---- \quad (1)$

Consider

$I_1 = \int \frac{(3x+8)}{x^2+5x+6}dx$

$I_1 = \int \frac{(3x+8)}{(x+3)(x+2)}$

$\frac{3x+8}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2}$

$\Rightarrow 3x+8 = A(x+2) + B(x+3) \quad --- \quad (i)$

put $x+2 = 0 \Rightarrow x = -2$ in (i), we have

$-6 + 8 = 0 + B$

$\Rightarrow B = 2$

put $x+3 = 0 \Rightarrow x = -2$ in (i), we have

$-9 + 8 = A(-3+2)$

$\Rightarrow -1 = -A$

$\Rightarrow A = 1$

Therefore,

$I_1 = \int \frac{(3x+8)}{x^2+5x+6}dx = \int \frac{1dx}{x+3} + 2\int \frac{dx}{x+2}$

$= \ln(x+3) + 2\ln(x+2)$

From equation (1)

$2\ln(y^2+4) = \ln(x+3) + 2\ln(x+2) + \ln c$

$\Rightarrow \ln(y^2+4)^2 = \ln(x+3) + \ln(x+2)^2 + \ln c$

$\Rightarrow \ln(y^2+4)^2 = \ln[(x+3)\times(x+2)^2 \times c]$

Taking Anti log on both sides

$(y^2+4)^2 = (x+3)(x+2)^2 \times c \quad ---- \quad (2)$

At $x = 1, y = 2$

$((2)^{2+}4)^2 = (1+3)(1+2)^2 \times c$

$\Rightarrow 64 = 4 \times 9 \times c$

$\Rightarrow 64 = 36 \times c \quad \Rightarrow c = \dfrac{16}{9}$

Equation(2)becomes

$(y^2 + 4)^2 = (x + 3)(x + 2)^2 \times {}^{16}/_{9}$

$\Rightarrow 9(y^2 + 4)^2 = 16(x + 3)(x + 2)^2$

is required solution.

❖ Question # 18:

$\mathbf{(1 + 2y^2)dy = ycosxdx, y(0) = 1}$

Solution:

Given equation is

$(1 + 2y^2)dy = ycosxdx$

$\Rightarrow \dfrac{1 + 2y^2}{y}dy = cosxdx$

Integrating both sides

$\Rightarrow \displaystyle\int \frac{1 + 2y^2}{y}dy = \int cosxdx$

$\Rightarrow \displaystyle\int \frac{1}{y}dy + 2\int \frac{y^2}{y}dy = \int cosxdx$

$\Rightarrow \displaystyle\int \frac{1}{y}dy + 2\int ydy = \int cosxdx$

$\Rightarrow lny + 2.\dfrac{y^2}{2} = sinx$

$lny + y^2 = sinx + c \quad - - - - - \quad (1)$

Put $x = 0, y = 1$

$ln(1) + 1^2 = sin(0) + c \quad \Rightarrow c = 1$

Equation (1)becomes

$lny + y^2 = sinx + 1$

is the required solution.

❖ Question # 19:

$\mathbf{8cos^2ydx + cosec^2xdy = 0}, \quad \mathbf{y\left(\frac{\pi}{12}\right) = \frac{\pi}{4}}$

Solution:

Given equation is

$8cos^2ydx + cosec^2xdy = 0$

$cosec^2xdy = -8cos^2ydx$

$\Rightarrow \dfrac{dy}{cos^2y} = -8\dfrac{dx}{cosec^2x}$

$\Rightarrow sec^2ydy = -8sin^2xdx$

Integrating both sides

$\displaystyle\int sec^2ydy = -8\int sin^2xdx$

$\Rightarrow \displaystyle\int sec^2ydy = -8\int \left(\frac{1 - cos2x}{2}\right)dx$

$\Rightarrow \displaystyle\int sec^2ydy = -\frac{8}{2}\int dx - \left(-\frac{8}{2}\right)\int cos2xdx$

$\Rightarrow tany = -4\displaystyle\int dx + 4\int cos2xdx$

$\Rightarrow tany = -4x + 4.\dfrac{sin2x}{2} + c$

$\Rightarrow tany = -4x + 2sin2x + c \quad - - - - \quad (1)$

Using $x = \frac{\pi}{12}, y = \frac{\pi}{4}$

$tan\left(\dfrac{\pi}{4}\right) = -4\left(\dfrac{\pi}{12}\right) + 2sin2\left(\dfrac{\pi}{12}\right) + c$

$1 = -\dfrac{\pi}{3} + 2\left(\dfrac{1}{2}\right) + c$

$\Rightarrow c = 1 - 1 + \dfrac{\pi}{3} \quad \Rightarrow c = \dfrac{\pi}{3}$

So equation (1) becomes

$tany = -4x + 2sin2x + \dfrac{\pi}{3}$

$\Rightarrow 4x - 2sin2x + tany = \dfrac{\pi}{3}$

is required solution.

❖ Question # 20:

$\mathbf{\dfrac{dy}{dx} = \dfrac{x(x^2 + 1)}{4y^3}, y(0) = -\dfrac{1}{\sqrt{2}}}$

Solution:

Given equation is

$\mathbf{\dfrac{dy}{dx} = \dfrac{x(x^2 + 1)}{4y^3}}$

$4y^3dy = x(x^2 + 1)dx$

Integrating both sides

$4\displaystyle\int y^3dy = \int (x^3 + x)dx$

$\Rightarrow 4.\dfrac{y^4}{4} = \dfrac{x^4}{4} + \dfrac{x^2}{2} + c$

$\Rightarrow y^4 = \dfrac{x^4}{4} + \dfrac{x^2}{2} + c \quad - - - - \quad (1)$

At $y = -\frac{1}{\sqrt{2}}$, $x = 0$

$$\left(-\frac{1}{\sqrt{2}}\right)^4 = 0 + 0 + c \quad \Rightarrow c = \frac{1}{4}$$

putting the value of c in equation (1)

$$y^4 = \frac{x^4}{4} + \frac{x^2}{2} + \frac{1}{4}$$

Multiplying both sides by 4

$$4y^4 = x^4 + 2x^2 + 1$$

$$\Rightarrow 4y^4 = (x^2 + 1)^2$$

$$\Rightarrow 2y^2 = (x^2 + 1)$$

$$\Rightarrow y^2 = \frac{(x^2 + 1)}{2}$$

$$\Rightarrow y = \pm\sqrt{\frac{x^2 + 1}{2}}$$

$$\Rightarrow y = -\sqrt{\frac{x^2 + 1}{2}}$$

is the required solution.

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