Differential Equation:
An eq involving independent and dependent variables and the derivatives of the dependent variables with respect to one or more independent variables is called a Diff Eq. If \( \frac{dy}{dx} \) or \( \frac{dz}{dx} \) are dependent variables and \( x, y \) are independent variables.

Ordinary Diff Eq:
An eq involving only derivatives of one or more dependent variables, with respect to a single independent variable, is called an ordinary diff eq. e.g. \( \frac{dy}{dx} + xy \left( \frac{dy}{dx} \right) = 0 \) is ODE, but \( \frac{dy}{dx} + xy \left( \frac{d^2y}{dx^2} \right) = 0 \) is not ODE.

Partial Diff Eq:
An eq involving partial derivatives of one or more dependent variables with respect to two or more independent variables is called partial diff eq.

Order of Diff Eq:
The order of a diff eq is the order of the highest derivative that occurs in the eq.

Degree of Diff Eq:
The degree of a diff eq is the power of highest order derivative involved in a diff eq.

1. \( \frac{dy}{dx} + y \cos x = \sin x \) Ordinary Diff Eq order 1 Degree 1
2. \( \frac{dy}{dx} + y \left( \frac{dy}{dx} \right) = 0 \) Ordinary Diff Eq order 2 Degree 1
3. \( \left( y \frac{dy}{dx} \right)^2 = \frac{dy}{dx} \) Ordinary Diff Eq order 1 Degree 2
4. \( x \frac{d^2z}{dx^2} + \frac{dz}{dx} = 0 \) Partial Diff Eq order 1 Degree 1
5. \( \frac{2U}{\partial x} + 2 \frac{U}{\partial y} + \frac{2U}{\partial z} = 0 \) Ordinary Diff Eq order 2 Degree 1
6. \( \frac{dy}{dx} + \frac{dy}{dx} = \frac{dy}{dx} \) Ordinary Diff Eq order 1 Degree 1

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Let \((y^2)^{3/2} = x + y\)

cubing both sides \((y^2)^3 = (x + y)^3\)

\[ y^6 = x^3 + 3x^2y + 3xy^2 + y^3 \]

6x \frac{d^2y}{dx^2} + 3\sin \frac{d^2y}{dx^2} - \cos xy

**Linear Diff Eq.**

A diff eq is said to be linear if:

1) The dependent variable \(y\) and its derivatives are all of degree one only.

2) No product of \(y\) and its derivatives are present.

3) No transcendental function \(y\) or its derivatives are present.

\[ 5 \frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 3y = 0 \]

\[ \frac{dy}{dx} - x^4 y = \cos x \]

A diff eq which is not linear is called **Non-Linear Diff Eq.**

\[ \frac{dy}{dx} + 4y = 0 \quad \text{(Power of } y \neq 1) \]

\[ \frac{dy}{dx} + 7y \frac{dy}{dx} + 12y = 0 \quad \text{(} \frac{dy}{dx} \text{ involves product of } y \text{ and } y \text{ derivatives)} \]

\[ \frac{dy}{dx} + \sin xy = 0 \quad \text{(involves transcendental function } y \text{ and } x \text{ dependent variables)} \]

\[ 5 \left( \frac{dy}{dx} \right)^3 + 2 \left( \frac{dy}{dx} \right)^2 + 3y = 0 \quad \text{(} \text{degree of } \frac{dy}{dx} \text{ is not 1}) \]

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Exercise 9.1

1. Classify each of the following eqs as ordinary or partial diff eq, state the order and degree of each eq and determine whether the eq is linear or non-linear.

\( \frac{d^3 y}{dx^3} + 4 \frac{d^{1.5} y}{dx^{1.5}} - 5 \frac{dy}{dx} + 3y = \cos x \)

Ordinary Diff Eq, order 3, degree 1,

It is Linear Diff Eq.

\( x \frac{dy}{dx} + y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} + y = 0 \)

Ordinary Diff Eq, order 1, degree 1.

It is non-linear eq. \( x \) power \( y \) = 1.

\( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \)

Partial Diff Eq, order 2, degree 1.

It is Linear Diff Eq.

\( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + u \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} + u = 0 \)

It is Partial Diff Eq, order 2, degree 1.

Non-linear Diff Eq \( u \) \( \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \) constant.

\( \left( \frac{dy}{dx} \right)^3 = \left( \frac{d^{1.5} y}{dx^{1.5}} + y \right)^{3/2} \)

Ordinary Diff Eq, order 2, degree 3.

\( \left( \frac{dy}{dx} \right)^{3/2} = \left( \frac{d^{1.5} y}{dx^{1.5}} + y \right)^{3/2} \)

\( \left( \frac{dy}{dx} \right)^{3} = \left( \frac{d^{1.5} y}{dx^{1.5}} + y \right)^{3} \)

Non-linear Diff Eq \( u \) Degree \( \neq 1 \).
General Solution or (Integral) or (Complete Primitive):

A sol of a diff eq which contains the number of arbitrary constants equal to the order of the eq is called general sol.

Particular Solution:

A sol obtained from the general sol by giving particular values to the constants is called a particular sol or integral.

Examples. The general sol of diff eq \( \frac{dy}{dx} = 0 \) is \( y = mx + c \).

whereas \( y = 3x + 5 \) is obtained by taking particular values \( m \neq 3 \) and \( c = 5 \).

Singular Sol: \( (S.S) \)

A sol of a diff eq which cannot be obtained from the general sol by any choice of independent arbitrary constant is called singular sol.

e.g. the general sol of \( y = \sqrt{x} \) is \( 2\sqrt{x} + c \) and \( S.S \) is \( y = 0 \).

Note: The arbitrary constants appearing in the general sol of a diff eq must be independent and to check this we show that they cannot be replaced by or reduced to a smaller number of const.

e.g. \( y = \sin(x + \alpha) + m\cos x \) is the sol of \( \frac{dy}{dx} + y = 0 \) it seems to contain three const \( b, m, \alpha \). But they are not independent as they can be reduced to two,

\[
y = \sin(x + \alpha) + m\cos x
\]

\[
= \sin x \cos \alpha + \cos x \sin \alpha + m\sin x
\]

\[
= (\cos \alpha) \sin x + (m + \sin \alpha) \cos x
\]

or \( y = A \sin x + BC \cos x \) so two arbitrary independent const.

Initial Value Condition is a condition on the sol of a diff eq at one pt.

i.e. \( x = a \) \( y(a) = \alpha \), \( \alpha(\beta) = \beta \) i.e. at \( x = a \) \( y = \alpha \), \( y = \beta \)

Boundary Value Condition is a cond on the sol of a diff eq at more than one pt i.e., \( x \), \( y(a) = \alpha \), \( y(b) = \beta \)
Formation of a differential eq.

A diff eq is formed by the elimination of arbitrary constants from a relation of the form $f(x, y) = 0$.

Since to eliminate one constant we need two eqs, and to eliminate two constants we need three eqs and so on. $N$

Now we shall be given one eq of the form $f(x, y) = 0$ and the remaining required number of eqs will be formed by differentiating given eq the required number of times.

This also shows that the order of the required diff eq cannot exceed the number of constants to be eliminated.

Thus we shall not diff the given eq more than the number of constants. E.g to form diff eq from

$$y = Cx \quad \text{(1)}$$

$$\frac{dy}{dx} = C \quad \text{(2)}$$

Required diff eq will be obtained by eliminating $C$ between (1) and (2)

So from (1)\(\quad\) $y = x (2 \frac{dy}{dx})$

Note: As there is just one const, so the required diff eq is to be of order one. I.e we should not diff (2) again to eliminate $C$ as

$$\text{Diff (1)} \quad 2y \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} \cdot \frac{dy}{dx} = 0 \quad C \text{ eliminated}$$

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9.1-5
(i) \( y = x + 3e^x \)
\[
\frac{dy}{dx} = 1 + 3e^x
\]
\[
= 1 + (y - x)
\]
\[
\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}(y - x) = \frac{dy}{dx} - 1
\]
\[
\therefore \frac{d^2y}{dx^2} = 1 + 3e^x - 1
\]
\[
\therefore \frac{d^2y}{dx^2} = 3e^x
\]
\[
\therefore y'' = 3e^x
\]

(ii) \( y = (x^3 + c)e^{3x}, \) \( c \) being arbitrary const.
\[
\frac{dy}{dx} = 3x^2e^{3x} + (x^3 + c)(-3e^{3x})
\]
\[
= 3x^2e^{3x} + (-3) \gamma
\]
\[
\therefore \gamma = (x^3 + c)e^{3x}
\]
\[
\therefore \gamma'' = 3e^{3x}
\]

(iii) \( a + bny = y + b \)
\[
\frac{dy}{dx} = \frac{y'}{y} = \frac{y'}{y} + \frac{1}{y}
\]
\[
\frac{d^2y}{dx^2} = \left(\frac{y'}{y}ight)' = \frac{y''y - y'y''}{y^2}
\]
\[
- \frac{(y')^2 + y''}{y}
\]
\[
- \frac{y' + y''y}{y}
\]
\[
- \frac{y'}{y} + y'' = 2y''
\]
\[
- \frac{y'}{y} + y''(y - y) = 0
\]

(iv) \( y = ae^x + b\ln x + c + dx \)
\[
\frac{dy}{dx} = ae^x + \frac{b}{x} + c
\]
\[
\frac{d^2y}{dx^2} = ae^x - \frac{b}{x^2}
\]
\[
\frac{d^3y}{dx^3} = ae^x + \frac{2b}{x^3}
\]
\[
\frac{d^4y}{dx^4} = ae^x - \frac{6b}{x^4}
\]
\( x^2 + y^2 + 2\gamma + 2\gamma' + C = 0 \)

1. \( 2x + 2y' + 2\gamma + 2\gamma' = 0 \)
   \[ x + y' + \gamma + \gamma' = 0 \]
   \[ (x + \gamma) + (y + \gamma)' = 0 \] — (1)

2. \( 1 + (y + \gamma)' + \gamma' = 0 \)

3. \( (y + \gamma)' + 3\gamma' = 0 \)
   \[ (y + \gamma)' = -3\gamma'' \] — (2)

4. \( 1 + (-3\gamma' + \gamma')' = 0 \)
   \[ (1 + y')' = 3\gamma' \]
   \[ (1 + y')' = 3y' \]
   \[ 3y' - (1 + y')'' = 0 \]

5. \( u = f(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}} \)

6. \( \frac{\partial u}{\partial x} = -\frac{1}{2} (x^2 + y^2 + z^2)^{-\frac{3}{2}} (2x) \)
7. \( \frac{\partial u}{\partial y} = -\frac{x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} y' \)
8. \( \frac{\partial u}{\partial z} = -\frac{1}{2} \left( \frac{3x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \right) \)
   \[ = \frac{1}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \left( x^2 + y^2 + z^2 - 3x \right) \]

9. Similarly
   \( \frac{\partial^2 u}{\partial y^2} = \frac{2y - x - z}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \) — (10)
   \( \frac{\partial^2 u}{\partial z^2} = \frac{2z - x - y}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \) — (11)

Add \( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \)

\( \frac{\partial^2 u}{\partial x^2} = -\frac{2x - y - z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \) — (12)
vii) \( u = f(x-a\gamma) + g(x+a\gamma) \)  \( f, g \) are twice diff fun

\[
\frac{2\dot{u}}{x} = f'(x-a\gamma) + g'(x+a\gamma)
\]

\[
\frac{1\dot{u}}{2a} = f''(x-a\gamma) + g''(x+a\gamma)
\]

\[
\frac{2\ddot{u}}{a} = f'(x-a\gamma) + g'(x+a\gamma)
\]

\[
\frac{2\ddot{u}}{a} = f'(x-a\gamma)(x) + g'(x+a\gamma)(x)
\]

\[
\frac{2\ddot{u}}{a} = f''(x-a\gamma)(x) + g''(x+a\gamma)(x)
\]

\[
\frac{2\ddot{u}}{a} = a^2 [f''(x-a\gamma) + g''(x+a\gamma)]
\]

\[
\frac{2u}{a} = a^2 \frac{\ddot{u}}{x}
\]

\[\text{[9.1-10]}\]

vi) \( \text{Find the differential of } f \) \( \text{of all circles } g \) \( \text{radius } a \). (a constant)

\( f \) \( g \) circle of radius \( a \): \( (x-h)^2 + (y-k)^2 = a^2 \)

\[
2(x-h) + 2(y-k) = 0
\]

\[
(x-h) + (y-k) = 0
\]

\[
1 + (y-k) + (y-k)' = 0
\]

\[
(y-k)'' = -1 - (y-k)'^2
\]

\[
(y-k)'' = -\frac{(1+y^2)}{y''}
\]

Put \( x-h = \frac{(1+y^2)^3}{y''} \)

\[
(x-h) = \frac{(1+y^2)^3}{y''} \]

Squaring and adding \( \text{Appendix E} \) to eliminate const:

\[
(x-h)^2 + (y-k)^2 = \left(\frac{1+y^2}{y''}\right)^2 + \left(\frac{1+y^2}{y''}\right)^2
\]

\[
\alpha^2 = \left(\frac{1+y^2}{y''}\right)^2 \left(\frac{y''^2}{y''} + 1\right)
\]

\[
\alpha^2 \left(\frac{y''}{y''}ight)^2 = (1+y^2)(1+y^2)
\]

\[
\alpha \left(\frac{y''}{y''}ight) = (1+y^2)^3
\]
(ii) Find the diff eq. of circles that pass through origin.

Eq. 8 all circles passing through origin is

\[ x^2+y^2+2gx+2fy = 0 \]  \( \text{---} \) (1) Two Const. of

So, diff twice

Diff.

\[ 2x + 2gy + 2g + 2fy' = 0 \]
\[ x + gy + g + fy' = 0 \]
\[ (x+y) + y'(1+y) = 0 \] \( \text{---} \) (2)

Diff.

\[ 1 + (y+f)y'' + y'f = 0 \]
\[ (y+f) = - \left( \frac{1+y^2}{y''} \right) \] \( \text{---} \) (3)

Put (3) in (2)

\[ (x+y) + y' \left( - \left( \frac{1+y^2}{y''} \right) \right) = 0 \]
\[ (x+y) = y' \left( \frac{1+y^2}{y''} \right) \] \( \text{---} \) (4)

Multiply (4) by \( x \) & (3) by \( y \) and adding

\[ x(x+y) + y'(1+y) = x'y' \left( \frac{1+y^2}{y''} \right) - y'(1+y) \]
\[ x^2 + x'y + y' + xy' = (x'y - y) \left( \frac{1+y^2}{y''} \right) \]
\[ x^2 + y' + (gx + fy) = (x'y - y) \left( \frac{1+y^2}{y''} \right) \] \( \text{---} \) (5)

\[ x^2 + y' + \left( \left( \frac{x+y}{x+y} \right) \right) = (x'y - y) \left( \frac{1+y^2}{y''} \right) \] \( \text{using (5)} \)

\[ \frac{x^2+y'^2}{1} = (x'y - y) \left( \frac{1+y^2}{y''} \right) \]
\[ (x^2+y')y'' = 2 (x'y - y) \left( 1+y' \right) \]

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iii) Find the diff eq of ellipses in standard form.

Ellipses in standard form \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \] can be written because the constant \( a, b, \)

\[
\frac{2x}{a^2} + \frac{2y}{b^2} y' = 0
\]

\[
\frac{x}{a^2} + \frac{y}{b^2} y' = 0
\]

\[
\frac{x}{a^2} + \frac{y}{b^2} y'' + y''' = 0
\]

\[
\Rightarrow \frac{x}{a^2} = -\frac{x(y^2 + y')}{b^2}
\]

\[
\Rightarrow \frac{x}{a^2} = -\frac{x(y^2 + y')}{b^2}
\]

\[
\Rightarrow -x y'' - x y' + y' = 0
\]

\[
xy'' + x y' - y' = 0
\]

iv) Find the diff eq of parabolas each \( y \) which has a latus rectum 4a and whose axis are \( \perp \) to \( x \)-axis.

Eq of given parabola is \( (y - K)^2 = 4a(x - h) \) can be written because two constant \( h, k \).

\[
2(y - K) y' = 4a
\]

\[
(y - K) y' = 2a
\]

\[
y'' + (y - K) y''' = 0
\]

\[
y'' + (y - K) y''' = 0
\]

\[
(y - K) = -\frac{y''}{y''} = \frac{y''}{y''}
\]

Put in \( \Rightarrow \)

\[
\left[ \frac{y''}{y''} \right] y' = 2a
\]

\[
y'^3 = 2a y''
\]

\[
0 = 2a y' + y'
\]
(v) Find the eq. of hyperbolas in standard form.

Standard eq. of hyperbolas is \[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]  

Diag. liney = \( \pm \frac{b}{a} \), two const. \( a, b \)

\[ \frac{2x}{a^2} = \frac{y}{b^2} = 0 \]

\[ \frac{x}{a^2} - \frac{y}{b^2} = 0 \]  

\[ \frac{y}{b^2} - (y\frac{y'' + y'}{b}) = 0 \]

\[ x \cdot y = x \cdot y \frac{y'' + y'}{b} = 0 \]

\[ y = x \frac{y'' + y'}{b} \]

Put (2) \( \rightarrow \)

\[ y = \frac{y'' + y'}{b} \]

vi) Find the eq. of cones which coincide with the axes of coordinates.

\[ ax^2 + by^2 = 1 \]

Diag. lines because two const. \( a, b \)

\[ 2ax + 2by = 0 \]

\[ a \cdot x + b \cdot y = 0 \]  

\[ a + b \cdot (y'' + y') = 0 \]

Eliminating \( a, b \) from (1), (2), (3)

\[ \begin{vmatrix} x^2 & y^2 & 1 \\ x & y & 0 \\ 1 & y & y' \end{vmatrix} = 0 \]

\[ \begin{vmatrix} x & y & 1 \\ 1 & y & y' \end{vmatrix} = 0 \]

\[ x (y' + y') - y' = 0 \]

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(i) \( \frac{dy}{dx} = -x \), \( y(3) = 4 \)

- Sol: \( x + y = c \)
  \[ 3^2 + 4^2 = c^2 \implies \left( \frac{c}{\sqrt{3}} \right)^2 = 5 \]
  \( x + y = 25 \) is reqd sol.

(ii) \( \frac{dy}{dx} + y = 2xe^x \), \( y(-1) = e + 3 \)

- Sol: \( y = (x^2 + c)e^x \)
  \[ e + 3 = (1 + c)e^{(-1)} \implies y(-1) = c + 3 \]
  \( c + 3 = e + ce \implies \left( \frac{c}{e} \right) = \frac{3}{e} \)
  \( y = \left( x^2 + \frac{3}{e} \right)e^x \) is Particular Sol

(iii) \( \frac{dy}{dx} + 2y - 12 = 0 \), \( y(0) = -2 \), \( y'(0) = 6 \)

- Sol: \( y = Ae^x + Be^{3x} \) \[ (-2) = Ae^0 + Be^0 \implies y(0) = -2 \]
  \[ -2 = A + B \] \[ (i) \]
  \[ -2x + 3Be^{3x} = 6 \]
  \[ 6 = 4A + 3Be^6 \implies y'(0) = 6 \]
  \[ 6 = 4A - 3B \] \[ (ii) \]

(a) \( A = \frac{1}{7} \)
(b) \( B = -2 \)

- Sol: \( y = -2e^{-3x} \) is P. Sol.

(iv) \( x \frac{dy}{dx} + 2y = 4x^2 \), \( y(1) = 2 \)

- Sol: \( y = \frac{2}{x^2} + \frac{c}{x^2} \)
  \[ 1 = 1 + \frac{c}{x^2} \implies y(1) = 2 \]
  \( \frac{c}{x^2} = 1 \)
  \( \therefore P. Sol \) is \( y = \frac{x^2 + 1}{x^2} \)
12

\[
\frac{d^3 y}{dx^3} - 3 \frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} - 6y = 0, \quad y(2) = 0, \quad y'(2) = 2, \quad y''(2) = 6
\]

[Solution]

- \( y = C_1 x + C_2 x^2 + C_3 x^3 \)  \( \quad \text{(i)} \)
- \( y' = C_1 + 2C_2 x + 3C_3 x^2 \)  \( \quad \text{(ii)} \)
- \( y'' = 2C_2 + 6C_3 x \)  \( \quad \text{(iii)} \)

- From (i)
  - 0 = 2C_1 + 4C_2 + 8C_3 \Rightarrow y(1) = 0
- From (ii)
  - 2 = C_1 + 4C_2 + 12C_3 \Rightarrow y'(2) = 2
- From (iii)
  - 6 = 2C_2 + 12C_3 \Rightarrow y''(2) = 6

- From (i)
  - 0 = C_1 + 2C_2 + 4C_3
- From (ii)
  - 2 = C_1 + 4C_2 + 12C_3 \hspace{1cm} \text{Subtracting}
  - 2 = -2C_2 - 8C_3
- From (iii)
  - 6 = 2C_2 + 12C_3 \hspace{1cm} \text{Adding}
  - 4 = 0 + 4C_3

- From (i)
  - \( \frac{1}{C_3} = -1 \)
- From (ii)
  - \( \frac{2}{C_3} = -3 \)
- From (iii)
  - \( \frac{3}{C_3} = -2 \)

\[ \text{P.Sol is } y = 2x - 3x^2 + x^3 \]

5(i) Solve boundary value problem (using given boundary conditions)

\[
\frac{d^2 y}{dx^2} + y = 0 \quad \text{, } y(0) = 1 \quad y'(\pi) = -1
\]

- Solution: \( y = C_1 \sin x + C_2 \cos x \)  \( \quad \text{(i)} \)
- \( \text{P.Sol is } y = C_1 \sin x + C_2 \cos x \)  \( \quad \text{(i)} \)

- From (i)
  - \( l = C_1 \)
  - \( -1 = y(\pi) \)
- From (ii)
  - \( l = y(\pi) \)

\[ \text{P.Sol is } y = C_1 \sin x + C_2 \cos x \]
(iii) \( \frac{d^2 y}{dx^2} - dy/dx + 2y = 0 \), \( y(0) = 0 \), \( y(1) = 1 \)

\( y = c_1 e^x + c_2 e^{-x} \) is the Gen.

\( y(0) = 0 \) \( \Rightarrow \)

\( 0 = c_1 e^0 + c_2 e^0 \)

\( 0 = c_1 + c_2 \) \( \Rightarrow \)

\( c_1 = -c_2 \)

\( y(1) = 1 \) \( \Rightarrow \)

\( 1 = c_1 e + c_2 e^{-1} \)

Subtract (i) from (ii)

\[ c_1 + c_2 = 0 \]

\[ c_1 + c_2 e = \frac{1}{e} \]

\[ c_2 e - c_2 e = -\frac{1}{e} \]

\[ c_2 (1 - e^x) = -\frac{1}{e} \]

\[ \Rightarrow c_2 = -\frac{\frac{1}{e}}{(1 - e^x)} \]

\[ \Rightarrow c_2 = \frac{1}{e(1 - e^x)} \]

\[ c_1 = \frac{1}{c_2} \]

\[ y = \frac{1}{e(1 - e^x)} e^x + (\frac{1}{e(1 - e^x)}) e^{-x} \]

\[ = \frac{1}{e(1 - e^x)} \left( e^x - \frac{e^{-x}}{3^x} \right) \text{ Ans.} \]

\( \therefore c_1 \) or \( c_2 \) has two different values.

as \( c_1 = -1 \) \& \( c_2 = 2 \) then we cannot determine \( c_2 \), hence No Solution exist.

See Example 7.

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