Mathematics - A Course
Class: B.Sc Part - I
Book: Calculus with Analytic Geometry

Chapter No: 02
The Derivative
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Newton - Raphson Method:-

Exercise 2.4

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Use the Newton-Raphson method to approximate, up to four places of decimal, a root of each of the following:-

1. $x^3 - 3x - 3 = 0$ with $x_0 = 2$

Sol:-

$x^3 - 3x - 3 = 0 \longrightarrow$ eq (i).

Then differentiate eq (i) w.r.t "x".

$3x^2 - 3$ eq (ii).

Now By using Newton-Raphson Method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Put $n = 0$:-

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \frac{[-1]}{9}$$

$$x_1 = 2 + \frac{1}{9}$$

$$\boxed{x_1 = 2.1111}$$

∴ The value of $x_0 = 2$ given in question.
∴ By putting the value of $x_0 = 2$ in eq (i) we get $= -1$
∴ By putting the value of $x_0 = 2$ in eq (ii) we get 9

At $n = 1$:-

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 2.1111 - \frac{0.0753}{10.3702}$$

$$x_2 = 2.1111 - 0.0072$$

$$\boxed{x_2 = 2.1039}$$

At $n = 2$:-

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 2.1039 - \frac{0.0009}{10.2791}$$

$$\boxed{x_3 = 2.1039}$$

As the root "2.1039" repeated two times so the Required root is $\boxed{2.1039}$ Ans.

$\boxed{x = 2.1039}$ Ans.

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Exercise set 2.4

2. $x^3 - 5x + 3 = 0$ with $x_0 = 0$

Soln.

By using Newton's Raphson method:-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x^3 - 5x + 3 = 0 \longrightarrow$ eq. (i)

$3x^2 - 5 \longrightarrow$ eq (ii)

Put n=0 :-

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$\therefore$ By putting the value of x_0 in eq (i) we get 3

$\therefore$ By putting the value of x_0 in eq (ii) we get -5

$$x_1 = 0 - \frac{3}{-5}$$

$$x_1 = 0 + \frac{3}{5}$$

$$\boxed{x_1 = 0.6000}$$

Put n = 1 in eq (i)

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.6000 - \frac{0.2160}{-3.9200}$$

$$x_2 = 0.6000 + \frac{0.2160}{3.9200}$$

$$x_2 = 0.6000 + 0.0551$$

$$\boxed{x_2 = 0.6551}$$

Put n = 2 in eq (i) :-

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 0.6551 - \frac{0.0056}{-3.7125}$$

$$x_3 = 0.6551 + \frac{0.0056}{3.7125}$$

$$x_3 = 0.6551 + 0.0015$$

$$\boxed{x_3 = 0.6566}$$

Put n = 3 in eq (i)

$$x_{3+1} = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = 0.6566 - \frac{0}{-3.7066}$$

$$x_4 = 0.6566 + 0$$

$$\boxed{x_4 = 0.6566}$$

Since the root 0.6566 is repeated two times so required root is $\boxed{x = 0.6566}$

$\boxed{x = 0.6566}$ Ans.

Exercise 2.4

3. $e^{-x} - \sin x = 0$ with $x_0 = 0.5$

Sol:-

By using Newton's Raphson Method:-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$e^{-x} - \sin x = 0 \longrightarrow$ eq (i)

$-e^{-x} - \cos x \longrightarrow$ eq (ii)

Put n=0 in eq (i) :-

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0.5 - \frac{0.1269}{-1.4840}$$

$$x_1 = 0.5 + 0.0855$$

$$\boxed{x_1 = 0.5855}$$

Put n=1 in eq (i) :-

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.5855 - \frac{0.0041}{-1.3902}$$

$$x_2 = 0.5855 + 0.0029$$

$$\boxed{x_2 = 0.5884}$$

Put n=2 in eq (i) :-

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 0.5884 - \frac{0.0002}{-1.3870}$$

$$x_3 = 0.5884 + 0.0001$$

$$\boxed{x_3 = 0.5885}$$

Put n=3 in eq (i)

$$x_{3+1} = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = 0.5885 - \frac{0}{-1.3868}$$

$$x_4 = 0.5885 + 0$$

$$\boxed{x_4 = 0.5885}$$

Since the root 0.5885 is repeated two times so the required root is $x = 0.5885$

$$\boxed{x = 0.5885}$$ Ans.

Exercise 2.4

4. $e^x - 3x = 0$ with $x_0 = 0$

Sol:-

By using Newton's Raphson method:-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$e^x - 3x = 0 \rightarrow$ eq (i).

$e^x - 3 \rightarrow$ eq (ii).

Put $n=0$ in eq (i).

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 - \frac{1}{-2}$$

$$x_1 = 0 + 0.5000$$

$$\boxed{x_1 = 0.5000}$$

Put $n=1$ in eq (i).

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.5000 - \frac{0.1487}{-1.3513}$$

$$x_2 = 0.5000 + 0.1100$$

$$\boxed{x_2 = 0.6100}$$

Put $n=2$ in eq (i) :-

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 0.6100 - \frac{0.0104}{-1.1596}$$

$$x_3 = 0.6100 + 0.0089$$

$$\boxed{x_3 = 0.6189}$$

Put $n=3$ in eq (i):

$$x_{3+1} = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = 0.6189 - \frac{0.0002}{-1.1437}$$

$$x_4 = 0.6189 + 0.0001$$

$$\boxed{x_4 = 0.6190}$$

Put $n=4$ in eq (i).

$$x_{4+1} = x_4 - \frac{f(x_4)}{f'(x_4)}$$

$$x_5 = 0.6190 - \frac{0}{-1.1433}$$

$$x_5 = 0.6190 + 0$$

$$\boxed{x_5 = 0.6190}$$

Since By the following "0.6190" repeated two times so the the required root is 0.6190.

$$\boxed{x = 0.6190}$$ Ans.

Exercise 2.4

5.) $4\sin x = e^x$ in the intervals $]0, 0.5[$

Sol:-

$f(x) = 4\sin x = e^x \rightarrow$ eq (i)

eq(i) $\Rightarrow f(x) = 4\sin x - e^x \rightarrow$ eq (ii).

Now in the question we have to find the value x_0.
Then Put the value of the intervals $]0, 0.5[$ in eq (ii).

$f(0) = 4\sin(0) - e^0$ $\quad\therefore e^0 = 1$

$f(0) = 0 - 1$ $\quad\therefore \sin 0° = 0$

$\boxed{f(0) = -1}$

Now at interval 0.5 put in eq (2)

$f(0.5) = 4\sin(0.5) - e^{0.5}$

$= 4(0.4794) - 1.6487$

$= 1.9176 - 1.6487$

$\boxed{f(0.5) = 0.2689}$

$\boxed{x_0 = 0.2618}$ $\quad\therefore$ The value of $\boxed{x_0 = 0.2618}$

$f'(x) = 4\sin x - e^x \rightarrow$ eq (ii)

Differentiating eq(ii) w.r.t "x"

$= 4\cos x - e^x$

Now By using Newton's Raphson method:-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Put $n = 0$

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0.2618 - \frac{(-0.264)}{2.5640}$$

$$= 0.2618 + \frac{0.264}{2.5640}$$

$$= 0.2618 + 0.1029$$

$$\boxed{x_1 = 0.3647}$$

Put $n = 1$ in (i):

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.3647 - \frac{(-0.0134)}{2.2968}$$

$$x_2 = 0.3647 + \frac{0.0134}{2.2968}$$

$$\boxed{x_2 = 0.3705}$$

Put $n = 2$ in (i)

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 0.3705 - \frac{(-0.0001)}{2.2802}$$

$$x_3 = 0.3705 + 0.0000$$

$$\boxed{x_3 = 0.3705}$$

Hence the required root is 0.3705 because it repeated two times in the eq. ea.

$$\boxed{x = 0.3705} \text{ Ans.}$$

Exercise 2.4

6). $\sin x = 1 - x$ with $x_0 = 0$.

Sol:-

By using Newton-Raphson method:-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(x) = 1 - x - \sin x = 0 \rightarrow$ eq (i).

$f'(x) = -1 - \cos x \rightarrow$ eq (ii).

Put n = 0 in eq (i) :-

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 - \frac{1}{-2}$$

$$x_1 = 0 + \frac{1}{2} \Rightarrow \boxed{x_1 = 0.5000}$$

Put n = 1 in eq (i).

$$x_{1+1} = x_n - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.5000 - \frac{(-0.0206)}{1.8776}$$

$$= 0.5000 + 0.0109$$

$$\boxed{x_2 = 0.5110}$$

Put n = 2 in eq (i) :-

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 0.5110 - \frac{0}{1.8723}$$

$$x_3 = 0.5110 - 0$$

$$\boxed{x_3 = 0.5110}$$

As "0.5110" is repeated two times so this is a rewuired root.

$\boxed{x = 0.5110}$ Ans. (The End).