Q1. Show that the shortest distance b/w the lines \( x+a = 2y = -12z \) and \( x = y + 2a = 6(z-a) \) is \( 2a \).

Sol. Given lines are

\[
\begin{align*}
\frac{x+a}{2} &= \frac{y}{1} = \frac{z}{-12} \\
\therefore x &= y + 2a, \quad x = 6z + 6a
\end{align*}
\]

or \( \frac{x+a}{12} = \frac{y}{6} = \frac{z}{-1} \)

\( x-y-2a = 0 = x-6z+6a \)

Eq. of a plane through line \( 2 \) is

\[
(x-y-2a) + k(x-6z+6a) = 0.
\]

\( (1+k)x - y - 6kz - 2a + 6ka = 0 \)

Now, ds. of normal to this plane are \( 1+k, -1, -6a \).

If this plane is \( l \) to line \( 1 \)

Then \( 12(1+k) + 6(-1) - 1(-6k) = 0 \)

\( 12 + 12k - 6 + 6k = 0 \)

\( 18k + 6 = 0 \)

\( 3k + 1 = 0 \)
\( K = -\frac{1}{3} \)

Put in eq. of plane

\[(x-y-2a) - \frac{1}{3}(x-6z+6a) = 0\]

\[3x-3y-6a = x+6z-6a = 0\]

\[2x-3y+6z-12a = 0\] is eq. of plane

Through line 2 & 11 to line 1.

Let d be req. shortest distance then

\[d = \text{Distance of pt. } (-a, 0, 0) \text{ from plane } 2x-3y+6z-12a = 0\]

\[= \frac{|2(-a)-0+0-12a|}{\sqrt{4+9+36}}\]

\[= \frac{|2(-a)|}{\sqrt{4+9+36}}\]

\[= \frac{|-14a|}{\sqrt{49}}\]

\[= \frac{14a}{7}\]

\[d = 2a\]

Q2. Find the shortest distance b/w the axis of x & the st. line \( ax+by+c'z+d = 0 = a'x+b'y+c'z+d' \)

We know that eq. of x-axis in symmetric form is

\[\frac{x}{1} = \frac{y}{0} = \frac{z}{0}\] ———(1)
Now given line is
\[ \alpha x + \beta y + \gamma z + d = 0 = a'x + b'y + c'z + d' \quad (2) \]

Now eq. of a plane containing this line is
\[ (\alpha x + \beta y + \gamma z + d) + k(a'x + b'y + c'z + d') = 0 \]

or \[ (a + kd)x + (b + kb)y + (c + kc')z + d + kd' = 0 \]

If this plane is \( \perp \) to \( x \)-axis then
\[ k(a + kd') = 0 \]

\[ \implies \quad k = -\frac{a}{a'} \]

Put in eq. of plane
\[ (\alpha x + \beta y + \gamma z + d) - \frac{a}{a'} (a'x + b'y + c'z + d') = 0 \]

\[ a'x + b'y + c'z + d' - \frac{a}{a'} a'x - \frac{a}{a'} b'y - \frac{a}{a'} c'z - ad' = 0 \]

\[ (a'b - ab')y + (ac' - ac)x + (dd' - ad') = 0 \]

is eq. of plane containing line (2)

Let \( d' \) be req. shortest distance then
\[ d' = \text{Distance of pt.}(0, 0, 0) \text{ from plane} \]
\[ = \frac{(a'b - ab')0 + (ac' - ac)0 + (dd' - ad')}{\sqrt{(a'b - ab')^2 + (ac' - ac)^2}} \]
\[ = \frac{a'd' - ad'}{\sqrt{(a'b - ab')^2 + (ac' - ac)^2}} \]
Q3: Show that the shortest distance b/w the str. 
lines \[ \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad \text{and} \quad \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} \]

1) \( \frac{1}{\sqrt{6}} \) 4 eqs. of the str. line perpendicular to
both are \( 11x + 2y - 7z + 6 = 0 \quad \text{and} \quad 7x + y - 5z + 7 \)

Ssl: Given lines are
\[ \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad \quad \quad (1) \]
\[ \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} \quad \quad \quad (2) \]

A pt. on line (1) is \( A(1, 2, 3) \)

A pt. on line (2) is \( B(2, 3, 5) \)

\[ \overrightarrow{AB} = (2-1)\hat{i} + (3-2)\hat{j} + (5-3)\hat{k} \]
\[ \overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k} \]

New dirs. of (1) are 2, 3, 4

Dirs. of (2) are 3, 4, 5

Let \( \vec{u} \) be a vector \( \perp \) to both lines then

\[ \vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} \]
\[ \vec{u} = (15-12)\hat{i} - (10-12)\hat{j} + (8-9)\hat{k} \]
\[ \vec{u} = -\hat{i} + 2\hat{j} - \hat{k} \]

Let \( d \) be the req. shortest distance b/w lines then
\[ d = \frac{\overrightarrow{AB} \cdot \overrightarrow{u}}{|\overrightarrow{u}|} \]
\[ = \frac{(\hat{1} + 2\hat{j} + 2\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{1 + 4 + 1}} \]
\[ = \frac{(1)(-1) + (2)(2) + (2)(-1)}{\sqrt{6}} \]
\[ = \frac{-1 + 4 - 2}{\sqrt{6}} \]
\[ \left[ d = \frac{1}{\sqrt{6}} \right] \text{ is req. distance} \]

Now eq. of line 1 to both given lines is
\[
\begin{vmatrix}
2 - 1 & y - 2 & z - 3 \\
2 & 3 & 4 \\
-1 & 2 & -1 \\
\end{vmatrix} = 0
\]
\[
\begin{vmatrix}
x - 2 & y - 4 & z - 5 \\
3 & 4 & 5 \\
-1 & 2 & -1 \\
\end{vmatrix} = 0
\]
\[
(x - 1)(-3 - 1) - (y - 2)(-2 + 4) + (z - 3)(4 + 3) = 0 = (x - 2)(-4 - 10) - (y - 4)(-3 + 5) + (z - 5)(6 + 4)
\]
\[
(x - 1)(-11) - (y - 2)(2) + (z - 3)(17) = 0 = (x - 2)(-14) - (y - 4)(2) + (z - 5)(10)
\]
\[
-11x - 2y + 7z + 11 + 4 - 21 = 0 = -14x - 2y + 10z + 28 + 8 - 50
\]
\[
-11x - 2y + 7z - 6 = 0 = -14x - 2y + 10z - 14
\]
\[
+2y - 7z + 6 = 0 = 7x + y - 5z + 7
\]

is req. eq.
\[
\overrightarrow{U} = 4\hat{i} + 6\hat{j} + 8\hat{k}
\]

Let \( \overrightarrow{d} \) be the req. shortest distance b/w lines then

\[
d = \frac{\overrightarrow{AB} \cdot \overrightarrow{U}}{|\overrightarrow{U}|}
\]

\[
= \frac{(-4\hat{i} - 6\hat{j} - 8\hat{k}) \cdot (4\hat{i} + 6\hat{j} + 8\hat{k})}{\sqrt{16 + 36 + 64}}
\]

\[
= \frac{-16 - 36 - 64}{\sqrt{116}}
\]

\[
= -\frac{116}{\sqrt{116}}
\]

\[
= -\sqrt{116}
\]

\[
= -\sqrt{4 \times 29}
\]

\[
d = 2\sqrt{29} \quad \text{(in magnitude)}
\]

Now given lines are

\[
\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} = t \quad \text{---- (1)}
\]

\[
\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} = s \quad \text{---- (2)}
\]

Parametric eq. of given lines are

\[
\begin{align*}
\begin{cases}
x = 3 + t \\
y = 5 - 2t \\
z = 7 + t
\end{cases} \quad \text{---- (3)}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
x = -1 + 7s \\
y = -1 - 6s \\
z = -1 + s
\end{cases}
\end{align*}
\]
Q4 Find the shortest distance b/w the lines
\[
\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad \text{and} \quad \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.
\]
Find eq. of the st. line \(L\) to both the given st. lines \(L_1\) also its pt. of intersection with the given st. lines.

Given lines are
\[
\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad (1)
\]
\[
\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \quad (2)
\]
A pt. on line \(1\) is \(A(3,5,7)\)
A pt. on line \(2\) is \(B(-1,-1,-1)\).
\[
\overrightarrow{AB} = (-1-3)\hat{i} + (-1-5)\hat{j} + (-1-7)\hat{k}.
\]
\[
\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 8\hat{k}.
\]
The dir. of line \(1\) are \((1, -2, 1)\)
The dir. of line \(2\) are \((7, -6, 1)\)
Let \(\overrightarrow{u}\) be a vector perpendicular to both lines then
\[
\overrightarrow{u} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
1 & 1 & 1 \\
7 & -6 & 1
\end{vmatrix}
\]
Expanding from \(R_1\)
Any pt. on \( C \) is \( P(3+t, 5-2t, 7+t) \)

Any pt. on \( C \) is \( Q(-1+3t, -1-6t, -1+8) \)

D.s.s. of line \( P Q \) are \( \begin{align*}
3+t+1-7t, 5-2t+1+6t, 7+t+1-8 \ &= t-7t+4, -2t+6t+6, t-8+8
\end{align*} \)

If \( PQ \) is the line of shortest distance then
then \( PQ \) is perp. to both the lines, so

\[
\begin{align*}
1(t-7t+4) - 7(-2t+6t+6) + 1(t-8+8) &= 0 \\
7(t-7t+4) - 6(-2t+6t+6) + 1(t-8+8) &= 0 \\
6t - 20t &= 0 \\
2t - 8t &= 0
\end{align*}
\]

\[
\Rightarrow t = 0 \quad t = 0
\]

Hence Co-ords. of pts. \( P + Q \) are

\( P(3,5,7) \) \& \( Q(-1,-1,-1) \)

Now eq. of line of shortest distance is

\[
\frac{x-3}{3+1} = \frac{y-5}{5+1} = \frac{z-7}{7+1}
\]

\[
\frac{x-3}{4} = \frac{y-5}{6} = \frac{z-7}{8}
\]

\[
\text{or} \quad \left\{ \begin{array}{l}
\frac{x-3}{2} = \frac{y-5}{3} = \frac{z-7}{4} \\
\text{in ref. line.}
\end{array} \right.
\]
Q5 Find the co-ords. of the pt. on the join of \((-3,7,-13)\) and \((-6,1,-10)\) which is nearest to the intersection of the planes:

\[2x - y - 3z + 32 = 0 \quad \text{and} \quad 3x + 2y - 15z - 8 = 0\]

\[\text{Sol.}\quad \text{Eq. of the line through } (-3,7,-13) \text{ and } (-6,1,-10)\]

\[\frac{x+3}{-3+6} = \frac{y-7}{7-1} = \frac{z+13}{-13+10} \]

\[\frac{x+3}{3} = \frac{y-7}{6} = \frac{z+13}{-3}\]

\[\Rightarrow \frac{x+3}{1} = \frac{y-7}{2} = \frac{z+13}{-1} = t \quad \Rightarrow (1)\]

\[\Rightarrow \begin{cases} x = -3 + t \\ y = 7 + 2t \\ z = -13 - t \end{cases}\]

Any pt. on line (1) is \(P(-3+t, 7+2t, -13-t)\).

Also, given eq. of line is:

\[2x - y - 3z + 32 = 0 \quad \text{and} \quad 3x + 2y - 15z - 8 = 0\]

Let \(l, m, n\) be d.s. of this line. Then since it lies on both planes, so by condition of perpendicularity:

\[2l - m - 3n = 0 \quad \text{and} \quad 3l + 2m - 15n = 0\]

\[\Rightarrow \frac{l}{15+6} = \frac{m}{-3+9} = \frac{n}{4+3}\]
\[ \frac{1}{21} = \frac{m}{21} = \frac{n}{7} \]
\[ \frac{3}{3} = \frac{m}{3} = \frac{n}{1} \]

So d.m.s. of given line are 3, 3, 1.

To find a pt. on line put \( t = 0 \) in above eqs.

\[
\begin{align*}
2x - y + 32 &= 0 \\
3x + 2y - 8 &= 0
\end{align*}
\]

\[
\frac{x}{8-64} = \frac{-y}{-16-96} = \frac{1}{4+3}
\]

\[
\frac{x}{-56} = \frac{-y}{-112} = \frac{1}{7}
\]

\[
\begin{align*}
x &= -8 \\
y &= 16 \\
z &= 0
\end{align*}
\]

So \((-8, 16, 0)\) is a pt. on given line.

Now eq. of given line through \((-8, 16, 0)\) with
d.m. 3, 3, 1 is

\[
\frac{x + 8}{3} = \frac{y - 16}{3} = \frac{z}{3} = t 
\]

\[
\begin{align*}
x &= -8 + 3t \\
y &= 16 + 3t \\
z &= 3t
\end{align*}
\]

Any pt. on this line is \(( -8 + 3t, 16 + 3t, 3t )\)

1st d.m.s. of line \( PQ \) are \(-3 + t + 8 - 3t, 7 + 2t - 16 - 3t, -13 - t\)

\[= t - 3 + 5, 2t - 3 + 9, -13 - t - 3\]

If \( PQ \) is perf. to both lines \( \ell \) & \( \ell' \).
then
\[
\begin{align*}
1(t-3.3+5) + 2(2t-3.3-9) - 1(13-t-3) &= 0 \\
3(t-3.3+5) + 3(2t-3.3-9) + 1(13-t-3) &= 0
\end{align*}
\]

\[
\begin{align*}
6t - 8s &= 0 \\
8t - 19s - 25 &= 0
\end{align*}
\]

Multiply I by 4, II by 2

24t - 32s = 0

\[
\sqrt{24t - 57s} = 75
\]

\[
25s = -75
\]

\[
[3 = -3]
\]

Put in I

6t - 8(-3) = 0

6t + 24 = 0

t + 4 = 0

(t = -4)

Put t = -4 in co-ords of P

P(-3-4, 7-8, -13+4)

or P(-7,-1,-9) is the req. pt.

Q6. Find the length & eq. of the common perpendicular of the lines

L: 6x + 8y + 3z - 11 = 0, x + 2y + z - 3 = 0

M: 3x - 9y + 3z = 0, x + y + z = 0

Solu
Given lines are

\[ L : 6x + 8y + 3z - 13 = 0 = x + 2y + z - 3 \]
\[ M : 3x - 9y + 5z = 0 = x + y - 2 \]

We will write both eqs. in symmetric form.

Let \( l, m, n \) be dir. of line \( L \), since it lies on both planes, so by condition of perpendicularity,

\[
\begin{align*}
6l + 8m + 3n_1 &= 0 \\
l + 2m + n_1 &= 0
\end{align*}
\]

So dir. of \( L \) are \( 2, -3, 4 \).

To find a pt. on \( L \) Put \( z = 0 \)

\[
\begin{align*}
6x + 8y - 13 &= 0 \\
x + 2y - 3 &= 0
\end{align*}
\]

So a pt. on \( L \) is \( \left( \frac{1}{2}, \frac{5}{4}, 0 \right) \).

Now eq. of \( L \) through \( \left( \frac{1}{2}, \frac{5}{4}, 0 \right) \) having dir. \( 2, -3, 4 \) is

\[
\begin{align*}
x - \frac{1}{2} &= \frac{y - \frac{5}{4}}{2} = \frac{z}{4}
\end{align*}
\]

Let \( l, m, n \) be dir. of line \( M \), since it lies on both planes, so by condition of perpendicularity,
\[
\begin{align*}
2.1 - 9m_2 + 5n_2 &= 0 \\
6m_2 + n_2 &= 0
\end{align*}
\]

\[
\begin{align*}
m_2 &= \frac{-m_2}{7+9} \\
\frac{m_2}{3} &= \frac{m_2}{8} = \frac{n_2}{12} \\
\frac{m_2}{1} &= \frac{m_2}{2} = \frac{n_2}{3}
\end{align*}
\]

So dir. of line M are \(1, 2, 3\).
To find a pt. on line M: Put \(x = 0\)
\[
\begin{align*}
3x - 9y &= 0 \\
x + y &= 0
\end{align*}
\]
\[
\begin{align*}
x - 3y &= 0 \\
x + y &= 0
\end{align*}
\]
Subtract, we get
\[
-4y = 0
\]
\[
y = 0
\]
Put in \(x = 0\)
\[
-4x = 0
\]
\[
x = 0
\]

So a pt. on line M is \((0, 0, 0)\).
Hence eq. of M through \((0, 0, 0)\) having dir. \(1, 2, 3\) is
\[
\frac{x}{1} = \frac{y}{2} = \frac{z}{3}
\]
Now we want to find shortest distance b/w \((1, 2, 3)\) and \((0, 0, 0)\).
A pt. on line \((1)\) is \(A\left(\frac{1}{2}, \frac{5}{9}, 0\right)\)
A pt. on line \((2)\) is \(B(0, 0, 0)\)
\[
\overrightarrow{AB} = -\frac{1}{2}i - \frac{5}{9}j + 0k
\]
Let \( \vec{u} \) be a vector perf to both lines, then

\[
\vec{u} = \begin{bmatrix}
\hat{c} & \hat{d} & \hat{k}
\end{bmatrix}
\begin{bmatrix}
1 \\
-2 \\
3
\end{bmatrix}
\begin{bmatrix}
= (-9 - 8) \hat{c} - (6 - 5) \hat{d} + (4 + 3) \hat{k}
\end{bmatrix}
\begin{bmatrix}
= -17 \hat{c} - 2 \hat{d} + 7 \hat{k}
\end{bmatrix}
\]

Let \( d \) be the req. shortest distance b/w lines then

\[
da = \frac{\vec{A}_b \cdot \vec{u}}{|\vec{u}|}
\begin{bmatrix}
= \frac{(-\frac{1}{2} \hat{c} - \frac{5}{2} \hat{d} + 0 \hat{k}) - (-17 \hat{c} - 2 \hat{d} + 7 \hat{k})}{\sqrt{289 + 4 + 49}}
\end{bmatrix}
\begin{bmatrix}
= \frac{-\frac{1}{2}}{\sqrt{342}}
\end{bmatrix}
\begin{bmatrix}
= \frac{11}{\sqrt{342}}
\end{bmatrix}
\]

Now eq. of common perpendicular is

\[
\begin{bmatrix}
x - \frac{1}{2} \\
y - \frac{5}{2} \\
z
\end{bmatrix}
\begin{bmatrix}
2 \\
-3 \\
4
\end{bmatrix}
\begin{bmatrix}
= 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\begin{bmatrix}
= 0
\end{bmatrix}
\begin{bmatrix}
-17 \\
-2 \\
7
\end{bmatrix}
\]

\[
(x - \frac{1}{2})(-21 + 8) - (y - \frac{5}{2})(14 + 6) + z(-4 + 81) = 0 = x(14 + 6) - y(7 + 51) + z(-2 + 34)
\]

\[
(x - \frac{1}{2})(-13) - (y - \frac{5}{2})(82) + z(-55) = 0 = 20x - 58y + 32z
\]

\[
-13x - 82y - 55z + \frac{13}{2} + \frac{25}{2} = 0 = 20x - 58y + 32z
\]

\[
-13x - 82y - 55z + 109 = 0 = 10x - 29y + 16z
\]
Q7. Show that the shortest distance b/w any two opposite edges of the tetrahedron formed by the planes \( y+z = 0, \ x+x = 0, \ x+y = 0 + x+y+z = a \) is \( \frac{2a}{\sqrt{6}} \). Let the three st. lines of the shortest distance intersect at the pt. \((-a, -a, -a)\).

Let the planes \( y+z = 0, \ x+x = 0, \ x+y = 0 + x+y+z = a \) be \( ABC, ACD, ABD + BCD \) respectively.

Then eq. of line \( AC \) is:
\[
\begin{align*}
y + z &= 0 \\
\frac{y}{2} + \frac{x}{2} &= 1
\end{align*}
\]

is symmetric form of line \( AC \).

Now the eq. of opposite edge \( BD \) is:
\[
\begin{align*}
x + y &= 0 \\
\frac{x+y+2}{2} &= a
\end{align*}
\]

Let \( l, m, n \) be the d.s. of this line.

Since it lies on both planes so by condition of perpendicularity:
\[
\begin{align*}
l + m + 2n &= 0 \\
l + m + n &= 0
\end{align*}
\]
\[ \frac{b}{1} = \frac{m}{-1} = \frac{n}{0} \]

So the eq of line is \( x, -1, 0 \)

To find a pt. on this line put \( x = 0 \) in above.

\[ \begin{align*}
0 + y &= 0 \quad \Rightarrow \quad y = 0 \\
0 + z + z &= a \quad \Rightarrow \quad 2z = a
\end{align*} \]

So a pt. on this line \( BB \) is \( (0, 0, a) \)

Hence \( \mathbf{v} \) of this line is:

\[ \mathbf{v} = \frac{x}{1} = \frac{y}{-1} = \frac{z-a}{0} \]

Now we will find shortest distance by \( \mathbf{v} \) \( \mathbf{1} \) & \( \mathbf{2} \)

A pt. on line \( \mathbf{1} \) is \( A(0, 0, 0) \)

A pt. on line \( \mathbf{2} \) is \( B(0, 0, a) \)

Now \( \vec{AB} = 0\mathbf{i} + 0\mathbf{j} + a\mathbf{k} \)

Let \( \mathbf{u} \) be a vector perpendicular to both lines then

\[ \mathbf{u} = \begin{bmatrix} 1 & \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \]

Expanding for \( k \):

\[ = (0-1)\hat{i} - (0+1)\hat{j} + (-1-1)\hat{k} \]

\[ \vec{u} = -\hat{i} - \hat{j} - 2\hat{k} \]

Let \( d \) be the shortest distance b/w lines

Then \( d = \frac{|\vec{AB} \cdot \vec{u}|}{|\vec{u}|} \)
$$d = \frac{(0\hat{i} + 0\hat{j} + a\hat{k}) \cdot (-1\hat{i} - 5\hat{j} - 2\hat{k})}{\sqrt{1 + 1 + 1}}$$

$$= \frac{0\hat{i} + 0\hat{j} - 2a\hat{k}}{\sqrt{1}}$$

$$d = \frac{2a}{\sqrt{1}}$$

in req. distance.

Similarly we can show that the shortest distance b/w opposite edges $AB, CD \& BC, AD$ is also $\frac{2a}{\sqrt{1}}$.

Now eq. of line of shortest distance b/w opposite edges $AC \& BD$ is

$$\begin{vmatrix} x & y & z \\ 1 & 1 & -1 \\ -1 & -1 & -2 \end{vmatrix} = 0 = \begin{vmatrix} x & y & 2-a \\ 1 & 1 & 0 \\ -1 & -1 & -2 \end{vmatrix}$$

$$(-2-1)z - (-2-1)y + (-1+1)x = 0 = (2-a)x - (-2+0)y + (-1-1)(2-a)$$

$$-3x + 3y + 0z = 0 = 2x + 2y - 2z + 2a$$

we see that the pt. $(-a, -a, -a)$ satisfies this eq. So this pt. lies on the line of shortest distance b/w $AC \& BD$. Similarly $(-a, -a, -a)$ also lies on the other two lines of shortest distance. Hence it lies on the intersection of all three lines of shortest distances.

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Q8. Find the shortest distance b/w the str. lines joining the pts. A(3, 2, 1) & B(1, 6, -2) & the str. line joining the pts. C(-1, 1, -2) & D(-3, 3, -2). Also find eq. of the line of shortest distance & Co-ords. of the feet of common perpendicular.

Sol.: Eq. of line AB is:
\[
\frac{x-3}{1} = \frac{y-2}{-4} = \frac{z+4}{-10}
\]

Eq. of line CD is:
\[
\frac{x+1}{-3} = \frac{y-1}{1} = \frac{z+2}{-6}
\]

A pt. on line (1) \(\text{is} (3, 2, 1)

A pt. on line (1) \(\text{is} B(-3, 3, -2)

\[\overrightarrow{AB} = (1, 1, 1) \hat{i} + (1-3) \hat{j} + (3+4) \hat{k}
\]

\[\overrightarrow{AB} = -2 \hat{i} - 2 \hat{j} + 7 \hat{k}
\]

Let \(U\) be a vector, perp. to both lines \(A\) & \(C\)

Then:
\[
\mathbf{U} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k}
\end{vmatrix}
\begin{vmatrix}
1 & 1 & 1
\end{vmatrix}
\begin{vmatrix}
1 & 0 & 2
\end{vmatrix}
\]

\[
\mathbf{U} = \begin{vmatrix}
1 & 1 & 1
\end{vmatrix}
\begin{vmatrix}
1 & 1 & 1
\end{vmatrix}
\begin{vmatrix}
1 & 0 & 2
\end{vmatrix}
\]
\[ \mathbf{U} = (-4, 0) \mathbf{i} - (2 - 1) \mathbf{j} + (0 + 2) \mathbf{k} \]

or \[ \mathbf{U} = -4 \mathbf{i} - \mathbf{j} + 2 \mathbf{k} \]

Let \( d \) be the reg. shortest distance b/w the lines then

\[ d = \frac{\mathbf{R}_2 \cdot \mathbf{U}}{|\mathbf{U}|} \]

\[ = \frac{(-4 \mathbf{i} - \mathbf{j} + 2 \mathbf{k}) \cdot (-4 \mathbf{i} - \mathbf{j} + 2 \mathbf{k}) \sqrt{16 + 1 + 4}}{16 + 1 + 4} \]

\[ = \frac{21}{\sqrt{21}} \]

\[ d = \sqrt{21} \]

As lines are

\[ \frac{x - 3}{1} = \frac{y - 2}{-2} = \frac{z + 4}{4} = t \quad \text{(1)} \]

\[ \frac{x + 1}{1} = \frac{y - 1}{0} = \frac{z + 2}{2} = s \quad \text{(2)} \]

Any pt. on line (1) is \( P(3 + t, 2 - 2t, -4 + 4t) \)

Any pt. on line (2) is \( Q(-1 + s, 1, -2 + 2s) \)

D.s. of \( PQ \) are \( 3 + t + 1 - s, 2 - 2t - 1 + s, -4 + 4t + 2 - 2s \)

or \( t - 5 + 1, -2 + 1, t - 2s - 2 \)

Suppose \( PQ \) is line of shortest distance then \( PQ \) is perf. to both lines (1) & (2)

So, \[ 1(t - 5 + 1) - 2(-2 + 1) + 1(t - 2s - 2) = 0 \]

\[ 1(t - 5 + 1) + 0(-2 + 1) + 2(t - 2s - 2) = 0 \]
\[
\begin{align*}
6t - 5s &= 0 \\
3t - 5s &= 0 \\
\Rightarrow t &= s = 0
\end{align*}
\]
So, coordinates of feet of perpendiculars PA and QA are
\[
\begin{align*}
P(3, 2t - 4) & \quad Q(-1, s - 2)
\end{align*}
\]
Now, eq. of common perp. PA is
\[
\begin{align*}
\frac{x - 3}{-1 - 3} &= \frac{y - 2}{1 - 2} = \frac{2 + 4}{-2 + 4} \\
\frac{x - 3}{-4} &= \frac{y - 2}{-1} = \frac{2 + 4}{2} \\
\text{or} \quad \frac{x - 3}{4} &= \frac{y - 2}{1} = \frac{2 + 4}{-2}
\end{align*}
\]
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