COMSATS University Islamabad

Attock Campus

Department of Mathematics

Assignment # 01

Class: BSM-VI
Subject: Real Analysis II
Instructor: Dr. Atiq ur Rehman

Due Date: 3-3-2023 (15:00AM)
Course Code: MTH322
Marks: 10

Note: *Please follow the due date & time strictly.*

Write a brief response to the any two non-consecutive questions from each group in no more than four lines.

Group A

1. Write three partitions of interval $[0,20]$.
2. Write two partitions of interval $[1,5]$ with 100 elements.
3. Give an example to show that if P and Q be two partitions of some interval then $\|P\| \geq \|Q\|$ does not imply $P \subseteq Q$.
4. If $P = \{1,2,3,4,5\}$ is partition of $[1,5]$ and $f:[1,5] \to \mathbb{R}$ is a function defined by $f(x) = x^2$, then find $U(P,f)$.
5. Let $Q = \left\{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi\right\}$ be partition of $[0,\pi]$ and $f:[0,\pi] \to \mathbb{R}$ be function defined by $f(x) = \cos x$. Find $L(Q,f)$.

Group B

1. Draw some suitable diagram to show that if P^* is a refinement of P, then $L(P,f) \leq L(P^*,f)$.
2. Draw some suitable diagram to show that if P^* is a refinement of P, then $U(P,f) \geq U(P^*,f)$.

3. Draw diagram to show that $\int_1^5 f(x)dx = \int_1^3 f(x)dx + \int_3^5 f(x)dx$ for an integrable function f on $[1,5]$.
4. Draw diagram to show that if f and g are integrable on $[a,b]$ such that $f(x) \le g(x)$ for $x \in [a,b]$ then $\int_a^b f(t)dt \le \int_a^b g(t)dt$.
5. State fundamental theorem of calculus.

Group C

1. Write two examples of improper integrals of the second kind.
2. Write two examples of improper integrals of mixed kind.
3. Prove that and integral $\int\limits_1^\infty \frac{1}{x^3}dx$ is convergent.
4. Prove that and integral $\int\limits_1^\infty \frac{1}{\sqrt{x}}dx$ is divergent.
5. Evaluate the convergence or divergence of $\int_{-\infty}^0 \sin x\, dx$.

Academic Honesty Requirements:

You are encouraged to work with others in the completion of assignments, but it doesn't include copying. Academic integrity is an ethical code, whereby the student guarantees that all work submitted is the student's own work. For this purpose, please include the following statement with every submitted assignment on title page:

I worked on this homework myself, and I understand it well.