Lecture 28: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

Objectives

The main aim of the lecture is to define the notion of

- *event.*
- *sum and product rules of the events with examples.*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.
- https://www.freepik.com/ (for background image)

Here our aim is to develop some techniques for determining the number of possible outcomes of a particular event or the number of elements in a set without direct enumeration. Such sophisticated counting is sometimes called ***combinatorial analysis***. It includes the study of ***permutations*** and ***combinations***.

First, recall that an ***event*** is a set of outcomes of an experiment to which a probability is assigned (see the chapter about Permutation, Combination and Probability of your FSc class).

Basic Counting Principles for Events

Here we discuss two counting principles for events. The first one involves addition and the second one multiplication.

Sum Rule Principle:

Suppose some event E can occur in m ways and a second event F can occur in n ways. Suppose both events cannot occur simultaneously. Then E or F can occur in $m + n$ ways.

Example:

Suppose a person enter in a shop to buy soft drink **or** pack of chips. Let E be an event that a person buys one of soft drinks and F be an event that a person buys one of packs of chips.

If there are 9 choices from soft drinks and 8 choices for pack of chips:

Then the sum rule gives us that there are 9+8=17 ways to buy soft drink or pack of chips.

Product Rule Principle:

Suppose there is an event *E* which can occur in *m* ways and, independent of this event, there is a second event *F* which can occur in *n* ways. Then combinations of *E* and *F* can occur in *mn* ways.

Example:

From the previous example, suppose a person enter in a shop to buy a soft drink **and** a pack of chips.

Both the events are independent, therefore the product rule states that there are (9)(8)=72 ways to buy one soft drink and one pack of chips.

The above principles can be extended to three or more events. That is, suppose an event E_1 can occur in n_1 ways, a second event E_2 can occur in n_2 ways, and, following E_2; a third event E_3 can occur in n_3 ways, and so on. Then:

Sum Rule: If no two events can occur at the same time, then one of the events can occur in:

$$n_1 + n_2 + n_3 + \ldots \text{ ways.}$$

Product Rule: If the events occur one after the other, then all the events can occur in the order indicated in:

$$n_1 \cdot n_2 \cdot n_3 \cdot \ \ldots \text{ ways.}$$

Example 1: Suppose a college has 3 different history courses, 4 different literature courses, and 2 different sociology courses.

(a) The number m of ways a student can choose one of each kind of courses is:

$$m = 3(4)(2) = 24$$

(b) The number n of ways a student can choose just one of the courses is:

$$n = 3 + 4 + 2 = 9$$

Example 2: Suppose a shop has three types of colas, two types of chips and six type of biscuits.

(a) The number p representing the ways to buy only one item from all is:

$$p = 3+2+6 = 11.$$

(b) The number q representing the ways a person buys each from the above items is:

$$q = 3(2)(6) = 36.$$

There is a set theoretical interpretation of the above two principles. Specifically, suppose $n(A)$ denotes the number of elements in a set A. Then:

(1) ***Sum Rule Principle:*** Suppose A and B are disjoint sets. Then

$$n(A \cup B) = n(A) + n(B).$$

(2) ***Product Rule Principle:*** Let $A \times B$ be the Cartesian product of sets A and B. Then

$$n(A \times B) = n(A) \cdot n(B).$$

THANKS FOR YOUR ATTENTION