Lecture 21: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II (SP20)

Objectives

The main aim of the lecture is to

- *define subgraphs, isomorphic graphs, homeomorphic graphs*
- *define path, connectivity, also give related examples and theorem.*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

Subgraphs

Consider a graph $G = G(V, E)$. A graph $H = H(V', E')$ is called a *subgraph* of G if the vertices and edges of H are contained in the vertices and edges of G, that is, if $V' \subseteq V$ and $E' \subseteq E$. In particular:

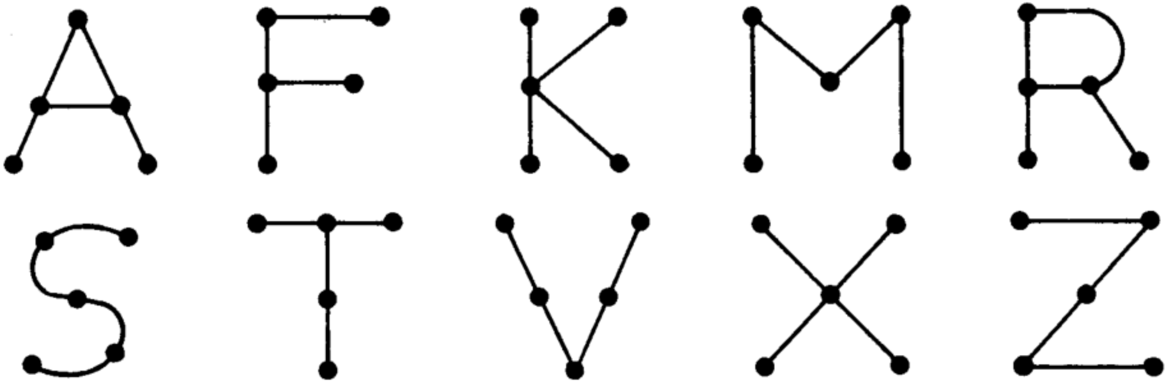
(i) A subgraph $H(V', E')$ of $G(V, E)$ is called the subgraph *induced* by its vertices V' if its edge set E' contains all edges in G whose endpoints belong to vertices in H.

(ii) If v is a vertex in G, then $G - v$ is the subgraph of G obtained by deleting v from G and deleting all edges in G which contain v.

(iii) If e is an edge in G, then $G - e$ is the subgraph of G obtained by simply deleting the edge e from G.

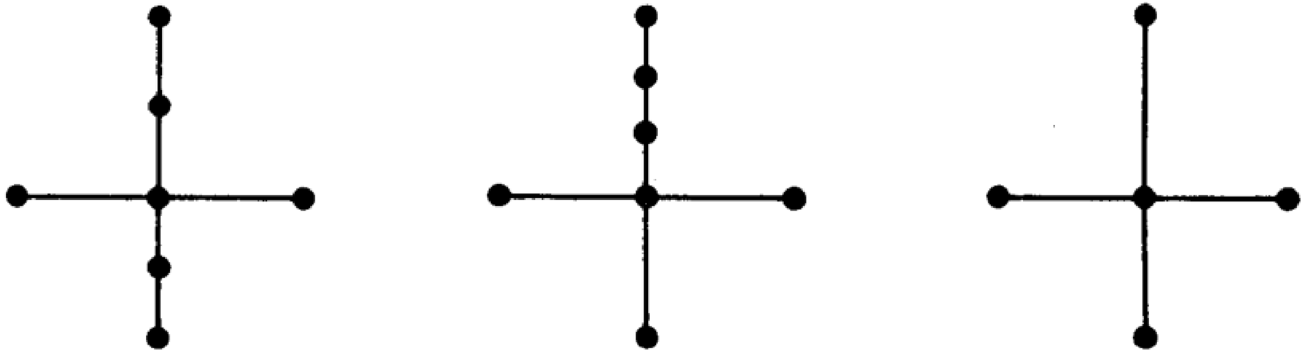
Isomorphic Graphs

Graphs $G(V, E)$ and $G(V^*, E^*)$ are said to be *isomorphic* if there exists a one-to-one correspondence $f: V \to V^*$ such that $\{u, v\}$ is an edge of G if and only if $\{f(u), f(v)\}$ is an edge of G^*. Normally, we do not distinguish between isomorphic graphs (even though their diagrams may "look different"). **Figure below** gives ten graphs pictured as letters. We note that A and R are isomorphic graphs. Also, F and T are isomorphic graphs, K and X are isomorphic graphs and M, S, V, and Z are isomorphic graphs.



Homeomorphic Graphs

Given any graph G, we can obtain a new graph by dividing an edge of G with additional vertices. Two graphs G and G^* are said to *homeomorphic* if they can be obtained from the same graph or isomorphic graphs by this method. The graphs (a) and (b) in the figure are not isomorphic, but they are homeomorphic since they can be obtained from the graph (c) by adding appropriate vertices.



(a) (b) (c)

Path, Connectivity

A path in a multigraph G consists of an alternating sequence of vertices and edges of the form

$$v_0, \quad e_1, \quad v_1, \quad e_2, \quad v_2, \quad \ldots, \quad e_{n-1}, \quad v_{n-1}, \quad e_n, \quad v_n$$

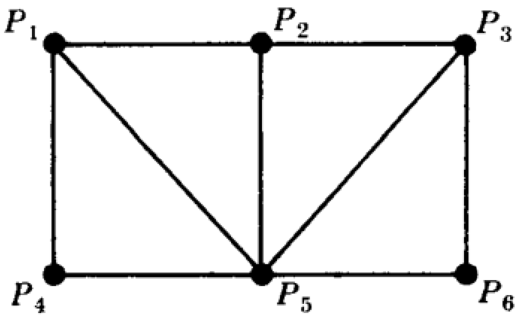
where each edge e_i contains the vertices v_{i-1} and v_i (which appear on the sides of e_i in the sequence). The number n of edges is called the *length* of the path. When there is no ambiguity, we denote a path by its sequence of vertices $(v_0, v_1, \ldots, v_n)$. The path is said to be *closed* if $v_0 = v_n$. Otherwise, we say the path is from v_0, to v_n or *between* v_0 and v_n, or *connects* v_0 to v_n.

A *simple path* is a path in which all vertices are distinct. (A path in which all edges are distinct will be called a *trail*.) A *cycle* is a closed path of length 3 or more in which all vertices are distinct except $v_0 = v_n$. A cycle of length k is called a *k-cycle*.

EXAMPLE 8.1 Consider the graph G in **figure below**. Consider the following sequences:

$$\alpha = (P_4, P_1, P_2, P_5, P_1, P_2, P_3, P_6), \quad \beta = (P_4, P_1, P_5, P_2, P_6),$$
$$\gamma = (P_4, P_1, P_5, P_2, P_3, P_5, P_6), \quad \delta = (P_4, P_1, P_5, P_3, P_6).$$

The sequence α is a path from P_4 to P_6; but it is not a trail since the edge $\{P_1, P_2\}$ is used twice. The sequence β is not a path since there is no edge $\{P_2, P_6\}$. The sequence γ is a trail since no edge is used twice; but it is not a simple path since the vertex P_5 is used twice. The sequence δ is a simple path from P_4 to P_6; but it is not the shortest path (with respect to length) from P_4 to P_6. The shortest path from P_4 to P_6 is the simple path (P_4, P_5, P_6) which has length 2.



Theorem

There is a path from a vertex u to a vertex s if and only if there exists a simple path from u to v.

Thanks for your attention.