

Lecture 14: Discrete Mathematics

Course Title: Discrete Mathematics

Course Code: MTH211

Class: BSM-II

Objectives

The main aim of the lecture is to

- *define and discuss function,*
- *discuss function as relation,*
- *draw/sketch the graph of function.*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hill, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, McGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

Function

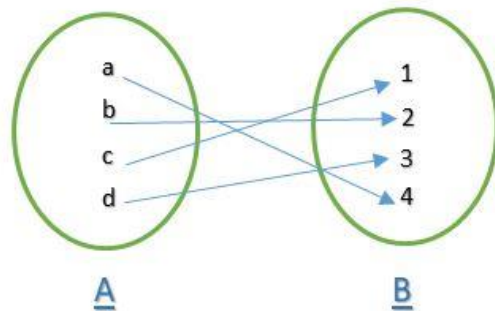
Suppose that to each element of a set A we assign a unique element of a set B ; the collection of such assignments is called a **function from A into B** . The set A is called the **domain** of the function, and the set B is called the **target set** or **codomain**.

Functions are ordinarily denoted by symbols. For example, let f denote a function from A into B . Then we write

$$f : A \rightarrow B$$

which is read: “ f is a function from A into B ,” or “ f takes (or maps) A into B .” If $a \in A$, then $f(a)$ (read: “ f of a ”) denotes the unique element of B which f assigns to a ; it is called the image of a under f , or the value of f at a .

The set of all image values is called the **range** or **image** of f . The image of $f : A \rightarrow B$ is denoted by $\text{Ran}(f)$ or $\text{Im}(f)$ or $f(A)$.



Frequently, a function can be expressed by means of a mathematical formula. For example, consider the function which sends each real number into its square. We may describe this function by writing

$$f(x) = x^2, \quad x \mapsto x^2, \quad y = x^2.$$

In the first notation, x is called a variable and the letter f denotes the function. In the second notation, the barred arrow \mapsto is read “*goes into*”. In the last notation, x is called the independent *variable* and y is called the *dependent variable* since the value of y will depend on the value of x .

Remark

Whenever a function is given by a formula in terms of a variable x , we assume, unless it is otherwise stated, that the domain of the function is \mathbf{R} (or the largest subset of \mathbf{R} for which the formula has meaning) and the codomain is \mathbf{R} .

Examples:

1. Consider the function

$$f(x) = x^3$$

i.e., f assigns to each real number its cube. Then the image of 2 is 8, and so we may write $f(2) = 8$.

2. Let A be any set. The function from A into A , which assigns to each element in A the element itself is called the identity function on A and it is usually denoted by 1_A , or simply 1. In other words, for every $a \in A$,

$$1_A(a) = a.$$

Functions as Relations

There is another point of view from which functions may be considered. First of all, every function

$$f : A \rightarrow B$$

gives rise to a relation from A to B called the **graph** of f and defined by

$$\text{Graph of } f = \{(a,b) \mid a \in A, b = f(a)\}.$$

Two functions $f : A \rightarrow B$ and $g : A \rightarrow B$ are defined to be equal, written $f = g$, if $f(a) = g(a)$ for every $a \in A$; that is, if they have the same graph. Accordingly, we do not distinguish between a function and its graph.

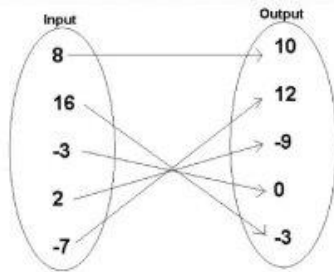
Now, such a graph relation has the property that each a in A belongs to a unique ordered pair (a,b) in the relation such that $f(a) = b$. Consequently, one may equivalently define a function as follows:

Definition: A function $f: A \rightarrow B$ is a relation from A to B (i.e., a subset of $A \times B$) such that each $a \in A$ belongs to a unique ordered pair (a, b) in f .

Functions vs. Relations

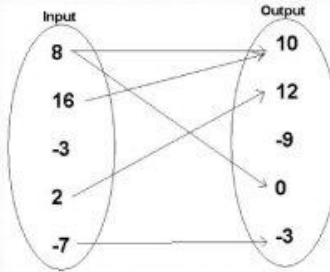
- Input/Output chart

Each input has only one output.



- Input/Output chart

At least one input has more than one output



Graph of function

Although we do not distinguish between a function and its graph, we will still use the terminology “graph of f ” when referring to f as a set of ordered pairs. Moreover, since the graph of f is a relation, we can draw its picture as was done for relations in general, and this pictorial representation is itself sometimes called the graph of f .

How to draw/sketch graph of function?

Use any software to draw the graphs, e.g. Graph 4.2, Maple, Mathematica, Matlab and for Android you may use “Desmos Graphing Calculator” or “Grapher - Equation Plotter & Solver” or search in play store for any other graph.

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Thanks for your attention.