

Lecture 08: Discrete Mathematics

Course Title: Discrete Mathematics

Course Code: MTH211

Class: BSM-II

Objectives

The main aim of the lecture is to discuss

- *Order pair*
- *Product of sets*
- *Relation*
- *Inverse relation*
- *Pictorial representation of relation*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hill, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, McGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

Order Pair:

An *ordered pair* of elements a and b , where a is designated as the first element and b as the second element, is denoted by (a, b) . In particular,

$$(a, b) = (c, d)$$

if and only if $a = c$ and $b = d$. Thus $(a, b) \neq (b, a)$ unless $a = b$. This contrasts with sets where the order of elements is irrelevant; for example, $\{3, 5\} = \{5, 3\}$.

Product of Sets:

Consider two arbitrary sets A and B . The set of all ordered pairs (a, b) where $a \in A$ and $b \in B$ is called the *product*, or *Cartesian product*, of A and B . A short designation of this product is $A \times B$, which is read “A cross B.” By definition,

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

One frequently writes A^2 instead of $A \times A$.

EXAMPLE Let $A = \{1, 2\}$ and $B = \{a, b, c\}$. Then

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

$$B \times A = \{(a, 1), (b, 1), (c, 1), (a, 2), (b, 2), (c, 2)\}$$

Also, $A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

EXAMPLE \mathbf{R} denotes the set of real numbers and so $\mathbf{R}^2 = \mathbf{R} \times \mathbf{R}$ is the set of ordered pairs of real numbers. The reader is familiar with the geometrical representation of \mathbf{R}^2 as points in the plane as in Fig. 2-1. Here each point P represents an ordered pair (a, b) of real numbers and vice versa; the vertical line through P meets the x -axis at a , and the horizontal line through P meets the y -axis at b . \mathbf{R}^2 is frequently called the *Cartesian plane*.

The idea of a product of sets can be extended to any finite number of sets. For any sets A_1, A_2, \dots, A_n , the set of all ordered n -tuples (a_1, a_2, \dots, a_n) where $a_1 \in A_1, a_2 \in A_2, \dots, a_n \in A_n$ is called the *product* of the sets A_1, \dots, A_n and is denoted by

$$A_1 \times A_2 \times \cdots \times A_n \quad \text{or} \quad \prod_{i=1}^n A_i$$

Just as we write A^2 instead of $A \times A$, so we write A^n instead of $A \times A \times \cdots \times A$, where there are n factors all equal to A . For example, $\mathbf{R}^3 = \mathbf{R} \times \mathbf{R} \times \mathbf{R}$ denotes the usual three-dimensional space.

Relation

Definition Let A and B be sets. A *binary relation* or, simply, *relation* from A to B is a subset of $A \times B$.

Suppose R is a relation from A to B . Then R is a set of ordered pairs where each first element comes from A and each second element comes from B . That is, for each pair $a \in A$ and $b \in B$, exactly one of the following is true:

- (i) $(a, b) \in R$; we then say “ a is R -related to b ”, written aRb .
- (ii) $(a, b) \notin R$; we then say “ a is not R -related to b ”, written $a\not Rb$.

If R is a relation from a set A to itself, that is, if R is a subset of $A^2 = A \times A$, then we say that R is a relation *on* A .

The *domain* of a relation R is the set of all first elements of the ordered pairs which belong to R , and the *range* is the set of second elements.

EXAMPLE

- (a) $A = (1, 2, 3)$ and $B = \{x, y, z\}$, and let $R = \{(1, y), (1, z), (3, y)\}$. Then R is a relation from A to B since R is a subset of $A \times B$. With respect to this relation,

$$1Ry, 1Rz, 3Ry, \quad \text{but} \quad 1\cancel{R}x, 2\cancel{R}x, 2\cancel{R}y, 2\cancel{R}z, 3\cancel{R}x, 3\cancel{R}z$$

The domain of R is $\{1, 3\}$ and the range is $\{y, z\}$.

- (b) Set inclusion \subseteq is a relation on any collection of sets. For, given any pair of set A and B , either $A \subseteq B$ or $A \not\subseteq B$.
- (c) A familiar relation on the set \mathbf{Z} of integers is “ m divides n .” A common notation for this relation is to write $m \mid n$ when m divides n . Thus $6 \mid 30$ but $7 \nmid 25$.
- (d) Consider the set L of lines in the plane. Perpendicularity, written “ \perp ,” is a relation on L . That is, given any pair of lines a and b , either $a \perp b$ or $a \not\perp b$. Similarly, “is parallel to,” written “ \parallel ,” is a relation on L since either $a \parallel b$ or $a \nparallel b$.

Inverse Relation

Let R be any relation from a set A to a set B . The *inverse* of R , denoted by R^{-1} , is the relation from B to A which consists of those ordered pairs which, when reversed, belong to R ; that is,

$$R^{-1} = \{(b, a) \mid (a, b) \in R\}$$

For example, let $A = \{1, 2, 3\}$ and $B = \{x, y, z\}$. Then the inverse of

$$R = \{(1, y), (1, z), (3, y)\} \quad \text{is} \quad R^{-1} = \{(y, 1), (z, 1), (y, 3)\}$$

Clearly, if R is any relation, then $(R^{-1})^{-1} = R$. Also, the domain and range of R^{-1} are equal, respectively, to the range and domain of R . Moreover, if R is a relation on A , then R^{-1} is also a relation on A .

Review:

Let A and B be sets. A *binary relation* or, simply, *relation* from A to B is a subset of $A \times B$.

For example:

If $A = \{1, 2, 3\}$ and $B = \{x, y, z\}$, then

$$R = \{(1, y), (1, z), (3, y)\},$$

$$S = \{(2, y), (2, z), (3, y), (3, z)\} \text{ and}$$

$$T = \{(1, x)\}$$

are the examples of relations from A to B as these are the subset of $A \times B$.

Now we discuss different forms of pictorial representations of the relations.

Pictorial Representation of Relation:

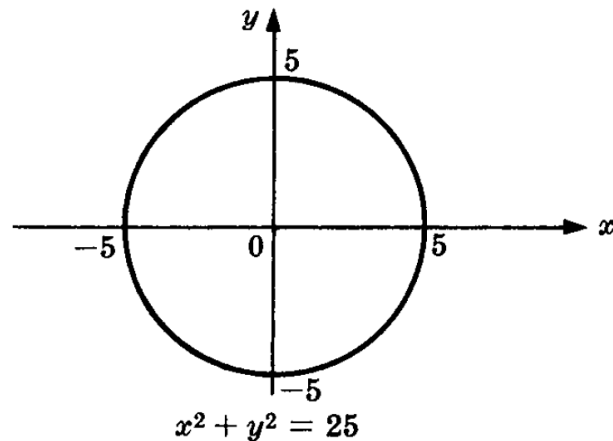
There are various ways of picturing relations.

Relation on \mathbf{R}

Let S be a relation on the set \mathbf{R} of real numbers; that is, S is a subset of $\mathbf{R}^2 = \mathbf{R} \times \mathbf{R}$ such that S consists of all ordered pairs of real numbers which satisfy some given equation

$$E(x, y) = 0 \text{ such as } x^2 + y^2 = 25.$$

Since \mathbf{R}^2 can be represented by the set of points in the plane, we can picture S by emphasizing those points in the plane which belong to S . The pictorial representation of the relation is sometimes called the *graph* of the relation.



For example, the graph of the relation $x^2 + y^2 = 25$ is a circle having its center at the origin and radius 5.

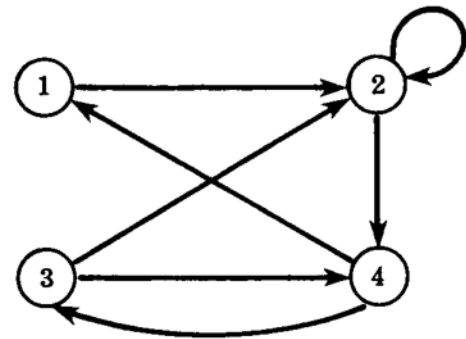
Directed Graphs of Relations on Sets

There is an important way of picturing a relation R on a finite set. First, we write down the elements of the set, and then we draw an arrow from each element x to each element y whenever x is related to y . This diagram is called the *directed graph* of the relation.

For example, consider $A = \{1, 2, 3, 4\}$ and

$$R = \{(1, 2), (2, 2), (2, 4), (3, 2), (3, 4), (4, 1), (4, 3)\}$$

In a figure given on right, observe that there is an arrow from 2 to itself, since 2 is related to 2 under R .



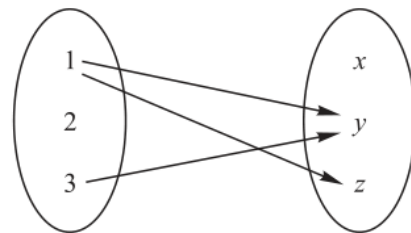
Pictures of Relations on Finite Sets

Suppose A and B are finite sets. There are two ways of picturing a relation R from A to B .

(i) Form a rectangular array (matrix) whose rows are labelled by the elements of A and whose columns are labelled by the elements of B . Put a 1 or 0 in each position of the array according as $a \in A$ is or is not related to $b \in B$. This array is called the *matrix of the relation*.

| | x | y | z |
|---|-----|-----|-----|
| 1 | 0 | 1 | 1 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 1 | 0 |

(ii) Write down the elements of A and the elements of B in two disjoint disks, and then draw an arrow from $a \in A$ to $b \in B$ whenever a is related to b . This picture will be called the *arrow diagram* of the relation.



For example, see for $R = \{(1, y), (1, z), (3, y)\}$.

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THANKS FOR YOUR ATTENTION