Lecture 03: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

Objectives

The main aim of the lecture is to

- *define the notion of tautologies and contradiction.*
- *define and discuss logical equivalence.*
- *discuss algebra of propositions.*
- *define conditional and biconditional statements.*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.
- https://www.freepik.com/ (for background image)

Tautologies and Contradictions

Some propositions $P(p, q, \ldots)$ contain only T in the last column of their truth tables or, in other words, they are true for any truth values of their variables. Such propositions are called *tautologies*. Analogously, a proposition $P(p, q, \ldots)$ is called a *contradiction* if it contains only F in the last column of its truth table or, in other words, if it is false for any truth values of its variables.

For example: From the truth table, $p \vee \neg p$ is tautology and $p \wedge \neg p$ is contradiction.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Logical Equivalence

Two propositions $P(p, q, \ldots)$ and $Q(p, q, \ldots)$ are said to be *logically equivalent*, or simply *equivalent* or *equal*, denoted by

$$P(p, q, \ldots) \equiv Q(p, q, \ldots)$$

if they have identical truth tables.

For example, Consider the truth tables of $\neg(p \wedge q)$ and $\neg p \vee \neg q$:

Observe that both truth tables are the same, that is, both propositions are false in the first case and true in the other three cases.

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

(a) $\neg(p \wedge q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

(b) $\neg p \vee \neg q$

Accordingly, we can write: $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

Remark: Let p be "Roses are red" and q be "Violets are blue." Let S be the statement:

"It is not true that roses are red and violets are blue."

Then S can be written in the form $\neg(p \wedge q)$. However, as noted above, $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

Accordingly, S has the same meaning as the statement:

"Roses are not red, or violets are not blue."

Algebra of Propositions

Propositions satisfy various laws which are listed in the table below. (In this table, *T* and *F* are restricted to the truth values "True" and "False," respectively.)

Laws of the algebra of propositions

Idempotent laws:	(1a) $p \vee p \equiv p$	(1b) $p \wedge p \equiv p$
Associative laws:	(2a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$	(2b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	(3a) $p \vee q \equiv q \vee p$	(3b) $p \wedge q \equiv q \wedge p$
Distributive laws:	(4a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	(4b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	(5a) $p \vee F \equiv p$ (6a) $p \vee T \equiv T$	(5b) $p \wedge T \equiv p$ (6b) $p \wedge F \equiv F$
Involution law:	(7) $\neg\neg p \equiv p$	
Complement laws:	(8a) $p \vee \neg p \equiv T$ (9a) $\neg T \equiv F$	(8b) $p \wedge \neg p \equiv T$ (9b) $\neg F \equiv T$
DeMorgan's laws:	(10a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$	(10b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Conditional and Biconditional Statements

A statement of the form “If p then q” is called *conditional* statement and is denoted by $p \to q$. The conditional $p \to q$ is frequently read “p implies q” or “p only if q.”

A statement of the form “p if and only if q” is called *biconditional* statement and is denoted by

$$p \leftrightarrow q.$$

The truth values of $p \to q$ and $p \leftrightarrow q$ are defined by the the following tables:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

(*a*) $p \to q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(*b*) $p \leftrightarrow q$

THANKS FOR YOUR ATTENTION